Process Control and Instrumentation

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Lecture No. # 24

Feedback Control Schemes (Contd.)

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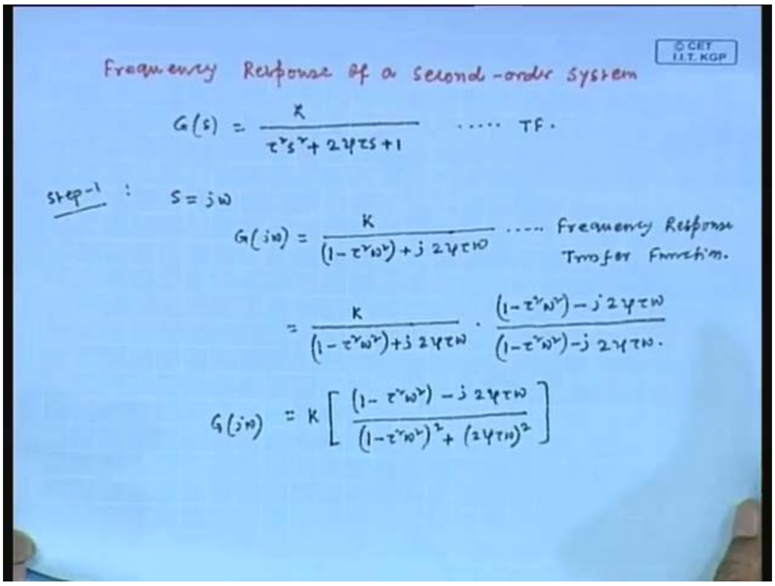

Today, we will continue our discussion on frequency response analysis, we will start with the discussion of frequency response of a second order system. In the last class we discussed the frequency response of a first order system. So, the transfer function of a second order system in general form can be written as K divided by tau square S square plus 2 zeta tau S plus 1 this is a transfer function of a second order system in general form. Now, we will follow the different steps for frequency response analysis. In the first step we replace S by j omega, in the first step we put S equals by j omega in the transfer function.

So, the frequency response transfer function represented by j omega becomes K divided by 1 minus tau square omega square plus j 2 zeta tau omega. If we substitute S equals j omega, we get this form this is frequency response transfer function, in the next step we write the frequency response transfer function as K divided by 1 minus tau square omega square plus j 2 zeta tau omega multiplied by 1 minus tau square omega square minus j 2 zeta tau omega divided by 1 minus tau square omega square minus zeta minus j 2 zeta tau omega.

Can we write rearranging this we obtain K multiplied by 1 minus tau square omega square minus j 2 zeta tau omega divided by 1 minus tau square omega square whole square plus 2zeta tau omega whole square. So, substituting S equals j omega and rearranging finally, we get this form of frequency response transfer function, in the second step we need to represent this frequency response transfer function in Cartesian form.

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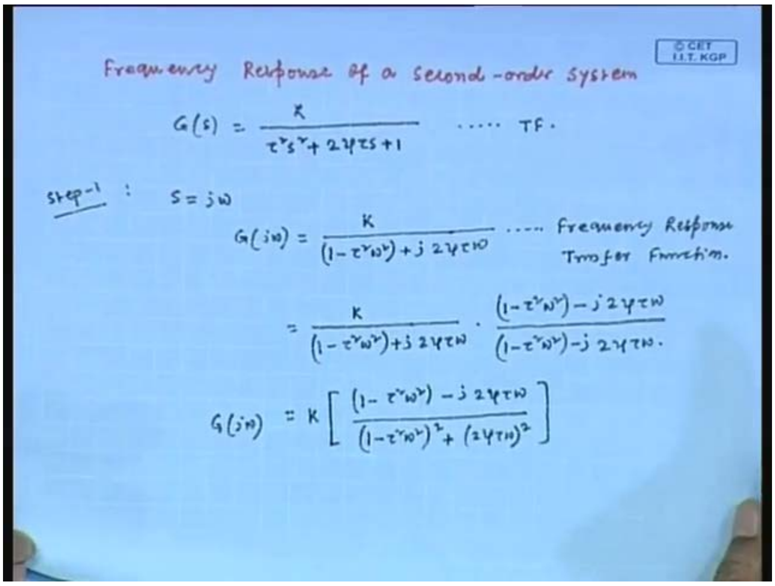

So, we can write the frequency response transfer function as K multiplied by 1 minus tau square omega square divided by 1 minus tau square omega square whole square plus 2 zeta tau omega whole square, this is a real part. And the imaginary part, becomes minus K two zeta tau omega divided by 1 minus tau square omega square whole square plus 2 zeta tau omega whole square. We have written this frequency response transfer function has a combination of real and imaginary parts, so it is obvious that this part is the real part, and this part is the imaginary part real part plus j imaginary part.

Now, in the third step we find the amplitude ratio and the phase angle, amplitude ratio is determined using this form. So, how much is amplitude ratio, if we substitute real and imaginary parts the amplitude ratio becomes K divided by root over of 1 minus tau square omega square whole square plus 2 zeta tau omega whole square, so this is the amplitude ratio for the second order system.

Similarly, we can find the phase angle phi using this form phase angle phi equals tan inverse imaginary part by real part, accordingly we obtain minus tan inverse 2 zeta tau omega divided by 1 minus tau square omega square. So, first we substituted S equals j omega then we represented the frequency response transfer function in Cartesian form, and in the step we determine the expressions for amplitude ratio and phase angle. Now, we will represent the frequency response transfer function in polar form for finding the expressions for amplitude ratio and phi.

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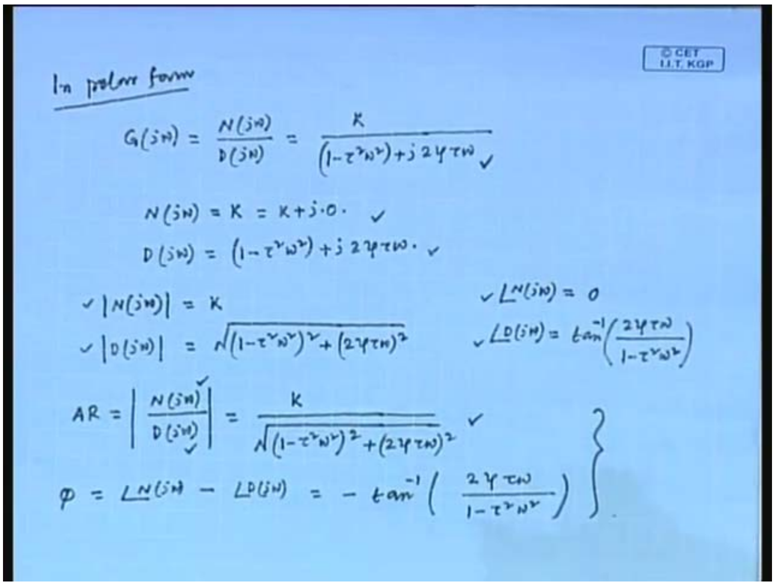

We will represent the frequency response transfer function in polar form and we will find the expressions for amplitude ratio and then phase angle, so first we write the frequency response transfer function as the ratio of numerator and denominator. So, we got this expression K divided by 1 minus tau square omega square plus j 2 zeta tau omega, this expression we got after substituting S equals j omega. Now, if this is the ratio of numerator term divided by denominator term then; obviously, this numerator term is equal to K., so we can write this as K plus j 0.

Similarly, the denominator term we can write as 1 minus tau square omega square plus j 2 zeta tau omega, this is a numerator this is the denominator of the frequency response transfer function. Now, what is the amplitude of this numerator term, amplitude is K what is the amplitude of denominator term that is root over of 1 minus tau square omega square whole square plus 2 zeta tau omega whole square, this is the magnitude of denominator.

Now, what is the argument of this numerator 0 what is the argument of this denominator tan inverse 2 zeta tau omega divided by 1 minus tau square omega square, these are the argument of numerator and denominator and magnitude of numerator and denominator. Now, amplitude ratio is written as magnitude of N j omega divided by D j omega, this is of expression for amplitude ratio, if we substitute the magnitude of a N and D we obtain K divided by root over of 1 minus tau square omega square whole square plus 2 zeta tau omega whole square.

After substituting the magnitude of a N and D we obtain this form for amplitude ratio, similarly the phase angle is obtained as magnitude sorry the argument of N minus the argument of denominator. If we substitute the individual argument then we obtain minus tan inverse 2 zeta tau omega divided by 1 minus tau square omega square, so these are the two expressions for amplitude ratio and phi. The similar expressions we obtained representing the frequency response transfer function in Cartesian form.

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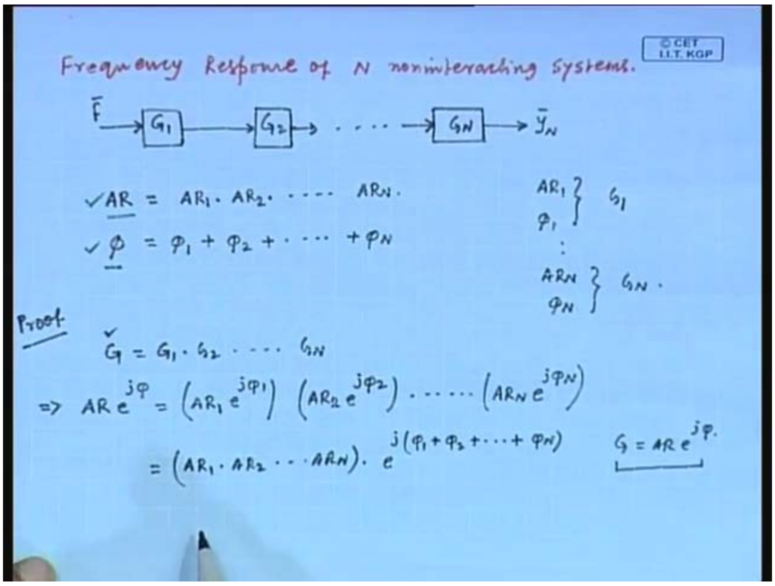

Next we will discuss the frequency response of several systems connected in series, in the frequency response of N none interacting systems, say N none interacting systems are connected in series, and the individual transfer function are G 1 then G 2 like this the last one is G N. So, input to the process G 1 is a prime the output of G 1 is introduced as the input 2 G 2 and finally, we obtain y N bar.

So, this is the, this is indicating the connection of N none interacting systems and now the amplitude ratio of this overall system, can be represented as the multiplication of individual amplitude ratios. The overall amplitude ratio this is the overall amplitude ratio is the multiplication of individual amplitude ratio. Similarly, the overall phase angle phi is the submission of the individual phase angles, this is a overall amplitude ratio, this is the overall phi.

Now, amplitude ratio A R 1 and phase angle phi 1, these 2 correspond to G 1. Similarly, amplitude ratio A R N and phase angle phi N this two correspond to G N, now we will proof this two co-relations, we will try to find the overall amplitude ratio and phi. If N none interacting systems are connected in series, then we can write the overall transfer function represented by G equals G 1 G 2 G N, can we write this, they are multiplied.

Now, we will represent the transfer function in polar form, so we can write amplitude ratio exponential j phi, this we have written for G for the overall system. Similarly, we can write for G 1 as amplitude ratio 1 exponential j phi 1, this is for G 1, for G 2 similarly we can write amplitude ratio 2 exponential j phi 2 by this way the G N becomes amplitude ratio N exponential j phi N. We represented the transfer function in polar form, so generally we write G equals amplitude ratio exponential of j phi in polar form, we can write the transfer function in this way.

Now, the right hand side we can write has amplitude ratio 1 amplitude ratio 2 amplitude ratio N and exponential of j then submission of all the phase angles, can we write? We can write the right hand terms in this way, now if we compare left hand side and right hand side we obtain amplitude ratio is the multiplication of individual amplitude ratios.

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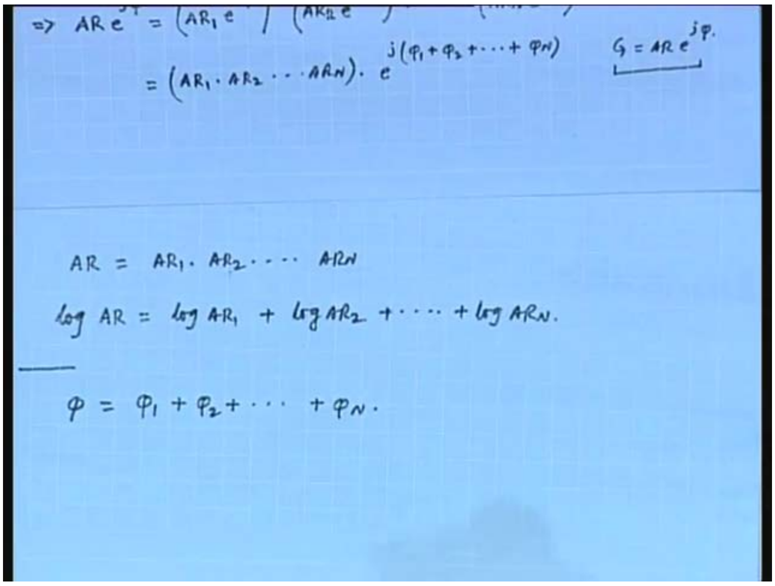

If we compare the left hand side and right hand side we get the overall amplitude ratio as the multiplication of individual amplitude ratios, now if we take logarithm in both sides. Then we obtain this form, similarly for the phase angle we can write phi has the submission of all individual phase angles, comparing left hand side and right hand side. We obtain phi as the submission of all individual phase angles, so these are the two expressions.

Sometimes this amplitude ratio is represented in logarithmic form and this is a expression for phi, so these are basically the fundamentals of frequency response analysis. In the next stage you plot different diagrams and then finally, will go for stability analysis using this frequency response concept.

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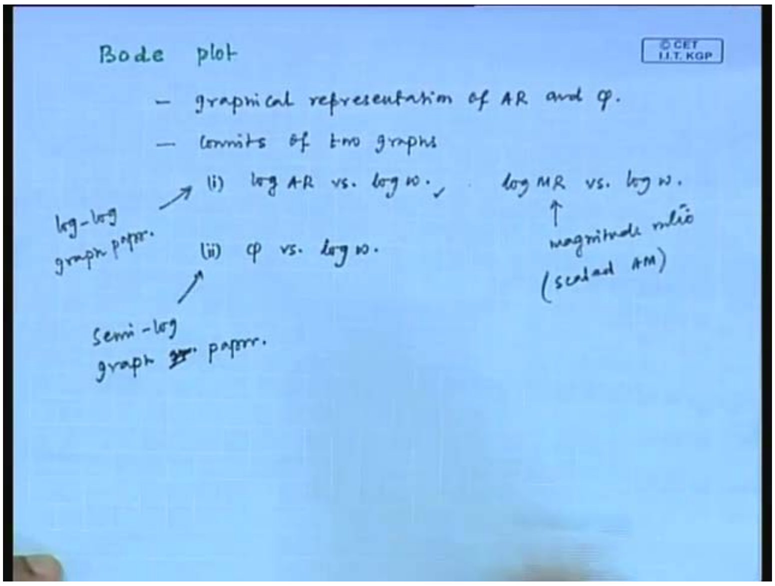

So, in the next we will discuss the bode plot, first we will discuss the bode plot in the next we will discuss nyquist plot, then these two plots will be used for stability analysis. So, this bode plot is a graphical representation of amplitude ratio and phi, bode plot is a common graphical representation of amplitude ratio and phase angle, and this bode plot consists of 2 graphs.

In one graph log amplitude ratio is plotted against log omega, in the bode plot bode plot consists of two graphs in the first graph the amplitude ratio in logarithmic form is plotted with log omega. And in the second plot the phi is plotted with log omega, so these are the two plots which constitute the bode plots. Now, sometimes the amplitude ratio is replaced by magnitude ratio, so we can write sometimes the bode plot includes log magnitude ratio verses log omega.

This is magnitude ratio, it is nothing but the scaled amplitude ratio, and it is interesting to mention also that this omega is also sometimes multiplied by some scaling factor. So, those things we will discuss in different cases different tau A, now since we are interested to plot log amplitude verses log omega therefore, log log graph paper is used for this.

So, for the first plot log log graph paper is used and in the second plot phi is plotted with log omega, so for this semi log graph paper is used. Now, we will take different examples and we will try to make the bode plot, so first we will take a pure gain element, we will make the bode plot taking different examples first we will consider a pure gain element.

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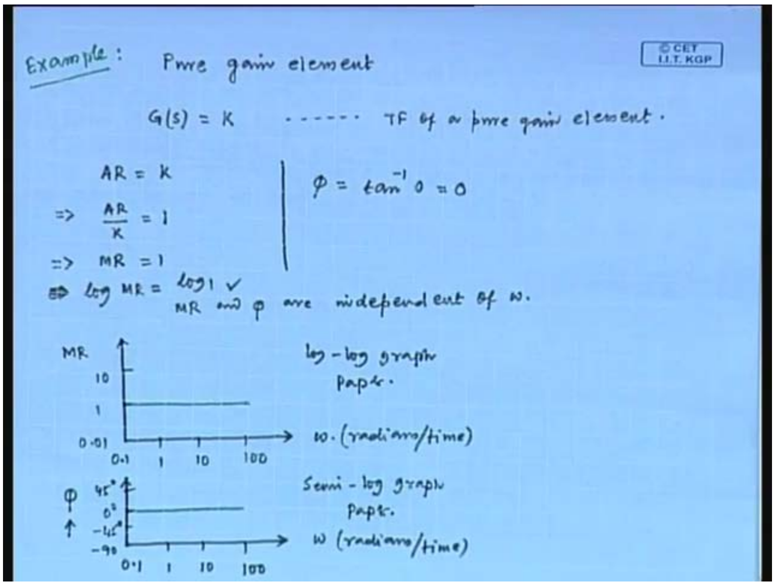

As mentioned the bode plot is produced by plotting the amplitude ratio and phi therefore, first we need to determine the expressions for the amplitude ratio and phi. So, to do those to do that we need to know the transfer function the transfer function of a pure gain element is represented as K. So, this is the transfer function of a pure gain element, now there is no need to express in terms of real and imaginary terms, we can directly find the expressions for amplitude ratio, what is amplitude ratio K.

So, we can write this again as amplitude ratio by K equals 1, so here amplitude ratio by K is the magnitude ratio; that means, magnitude ratio is equal to 1. Now, we will determine the expression for phase angle phi, phase angle will be how much tan inverse 0, because imaginary part is 0 and real part is K, so 0 by K that is 0, so this is 0. Now, bode plot considers the variations of amplitude ratio and phi with omega, but here you see both magnitude ratio and phi they are independent of omega, so here both magnitude ratio and phi are independent of omega.

Next we will produce the bode plot, so first plot is magnitude ratio verses omega, radians per unit time. And this is log log graph paper, now since we obtain magnitude ratio equals 1, so in the next step we can write as logarithmic of magnitude ratio equals log 1, can we write this log of magnitude ratio equals log 1. Now, we will consider this, so what it indicates log magnitude ratio equals log 1, this is a horizontal line.

So, the first plot we obtained like this, this is a horizontal line and it is equals 1, second plot is made between phi verses log omega and remember this is the semi log graph paper. This is the semi log graph paper has commented that phi is independent of omega and that is equal to 0. So, first we will consider the axis this is suppose 0 degree, this is minus 45 degree, this is minus 90, this is 45 degree and it looks like this. It is obvious that both magnitude ratio and phi are independent of omega, so this is the bode plot for the pure gain element.

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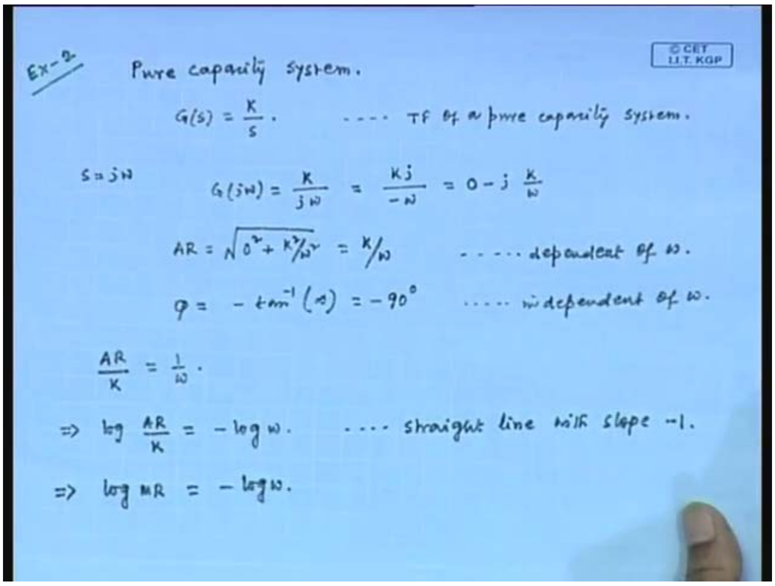

Next we will consider another example that is pure capacity system, this is example two, we will produce bode plot for pure capacity system. What is the transfer function for the pure capacity system, G S is equal to K by S, this is a transfer function of a pure capacity system. So, we will follow the ((Refer Time: 35:23)) steps I mean in the first step we will substitute S equals j omega, then the frequency response transfer function yields G j omega equals K divided by j omega.

Now, this we can write as K j divided by minus of omega a multiplied j in both numerator and denominator, then we can write as 0 minus j K by omega. So, what is amplitude ratio, we can write as the square of 0 square of K by omega and then square root. So, this is equal to K by omega, how much is phi, phi is tan inverse minus infinity minus 90, that is always that this phi is independent of omega, but this amplitude ratio is dependent of omega and phi is independent of omega.

Now, we can write the amplitude ratio as amplitude ratio by K equals 1 by omega, can we rearrange the expression of amplitude ratio in this form. If we take logarithm, then we obtain log amplitude ratio by K equals minus log omega. So, this is basically is straight line with slope minus 1, this is if straight line with slope minus 1, now we will produce the plot bode plot.

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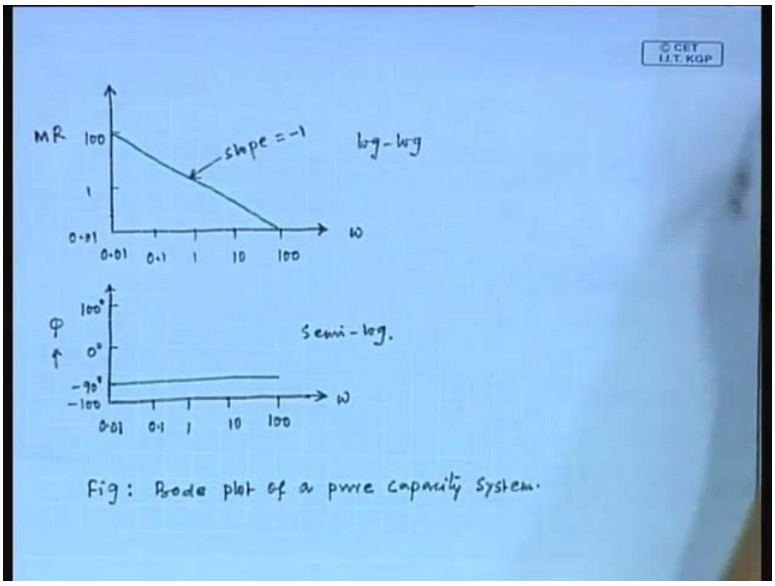

So, this is the log log graph paper this is 0, starting point is 0.01next one is 0.1 1 10 100 0.01 1 and 100, now if we consider the, this equation this type line log magnitude ratio equals minus log omega. Then we obtain this straight line with slope minus 1, if we consider the expression of amplitude ratio in the form of magnitude ratio, we obtain this straight line with slope minus 1, it has the slope of minus

1. In the next plot we consider phi and omega starting value is minus 100 degree another value we are taking that is minus 90 0 degree then suppose this is 100 degree. Since, phi is independent of omega and that is always minus 90 degree, so we get this horizontal line for phi and this plot is produced in semi log graph paper. So, this is the bode plot of a pure capacity system, so for we discussed the construction of bode plot with taking two examples.

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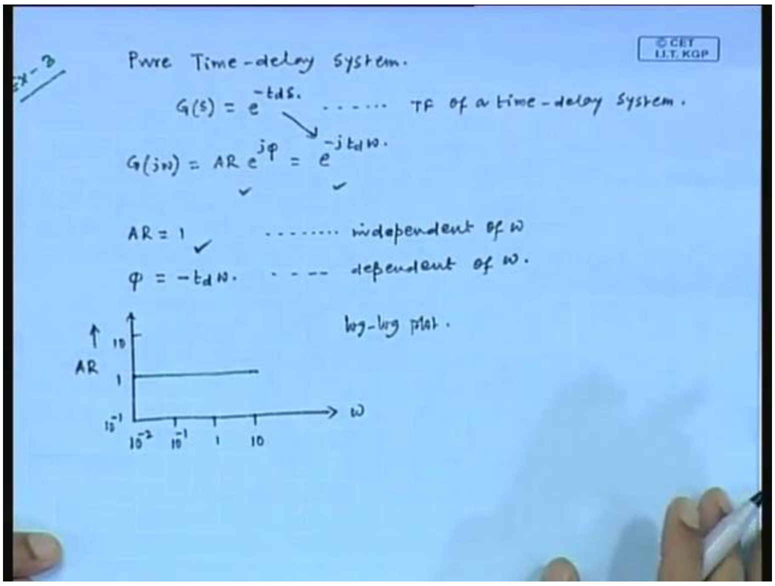

So, in the third example we will consider that time, in example three we will consider pure time delay system, how we can represent the pure time delay system I mean what is the transfer function. G S is equal to exponential of minus t d S t d is the date time, so this is the transfer function of a time delay system. So, what is amplitude ratio, then we can directly represent this transfer function in polar form, so if we express the frequency response transfer function, we obtain G j omega equals amplitude ratio exponential of j phi equals exponential minus j t d omega.

We can write this by this, now if we compare this and this then we obtain amplitude ratio equals 1 and phi equals minus t d omega. So, the expression of amplitude ratio is independent of omega, but phi depends on omega, so this is dependent of omega. Next we will produce the bode plot for this pure time delay system, this is omega and this is amplitude ratio, here we are considering amplitude ratio because no scaling factor is there. This is log log plot, so the x axis starts from 10 to the power minus 2, 2 suppose 10 and this 1 starts from 10 to the power minus 1, 2 10.Now, this is a horizontal line amplitude ratio equals 1, so we get this type of line in the first plot.

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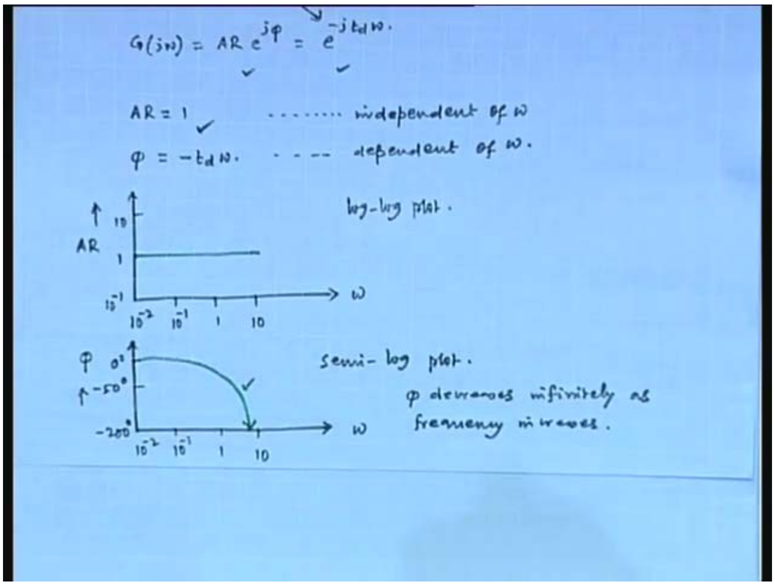

And in the next plot we consider phi verses omega, as usual this is the semi log plot and for the phi we consider from minus 200 degree to 0 degree, this is suppose minus 50 degree. Now, if we produce some intermediate data, then we obtain this type of plot for phi, to know the nature of this plot we need to produce some intermediate data varying the omega value.

Ultimately, we get this type of response for phi, what it indicates, the phase angle decreases infinitely as frequency decreases, as frequency increases, this is an interesting finding for this time delay system. Next you discuss the construction of bode plot for a first order system, which is quite complicated.

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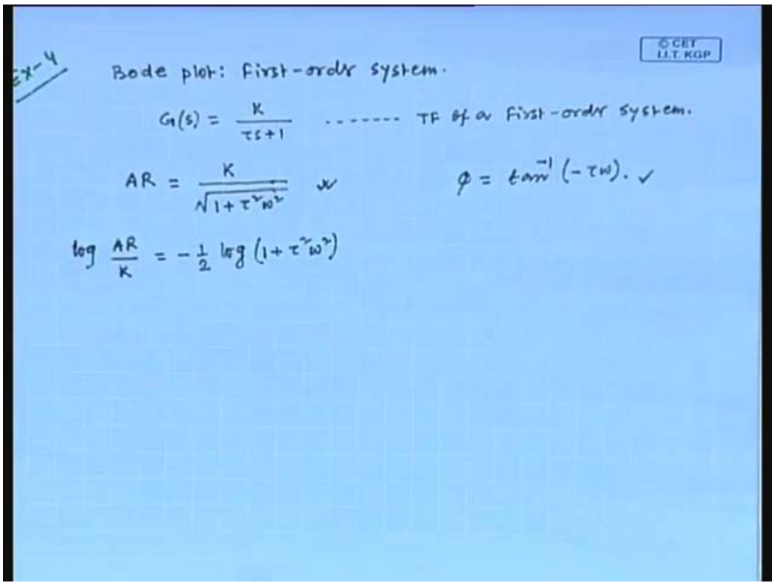

First order system, so to construct the bode plot we need to consider the transfer function of a first order system, the transfer function of first order system is represented as say G S equals K divided by tau S plus 1. We are not considering here the suffix p, which we considered earlier, so this is the transfer function of a first order system. And in the last class we determine the expressions of amplitude ratio and phi, in the last class we obtained the expressions for amplitude ratio and phi for this first order system.

There amplitude ratio equals K divided by root over 1 plus tau square omega square, and phase angle phi equals tan inverse minus tau omega. For a first order system this is the expression for amplitude ratio and this is the expression for phase angle. Now, if we take the logarithm of this expression, then we can write log amplitude ratio by K equals minus half log 1 plus tau square omega square. Can we write this, if we take logarithm of this expression we obtain this form.

So, in the next class we will discuss the asymptotic considerations, for constructing the bode diagram of this first order system. So, today we discussed the frequency response analysis of second order system, for N none interacting systems connected in series, then we took three examples to discuss the construction of bode plot.

Thank you.