Process Control and Instrumentation

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Lecture No. # 23

Feedback Control Schemes (Contd.)

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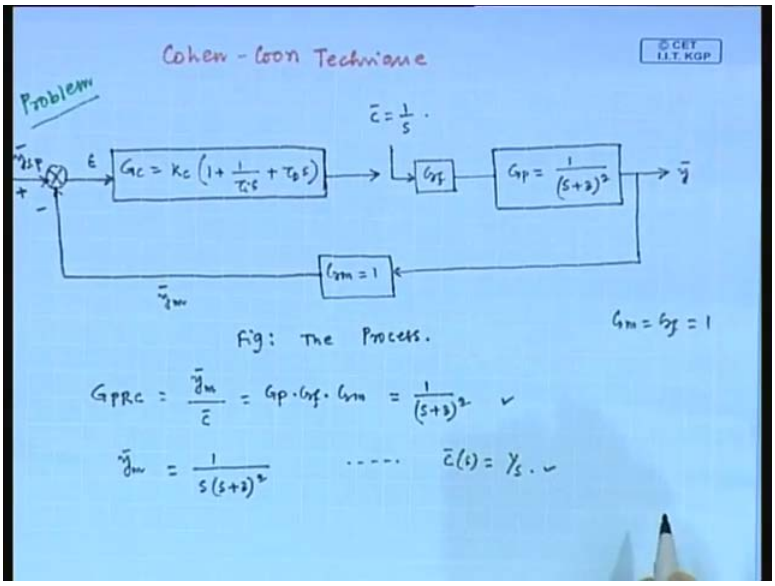

In the last class, we discuss Cohen coon technique. We discussed the algorithm on Cohen coon method. So, today we will solve one problem based Cohen coon technique I mean we will determine the tunic parameter values of the controller using this technique. The close loop black diagram is given with the transfer function of controller as G c equals k c 1 plus 1 divided by tau i s plus tau d s this is the transfer function of PID controller, then as we discussed the final control eliminate and controller blocker disconnected.

So, that is 1 here, then the final control eliminate output goes to the process, having the transfer function G p equals 1 divided by s plus 3 whole square. This is the transfer function of the process. Output of this block is y bar, then this output is measured, the measured signal is then compared with set point of value, so this is basically not the close loop black diagram, we have disconnected here. Suppose, a units step change is introduced in the controller output signal, input a unit step change introduced in input signal to the final control eliminate.

So, this is the process, first we need to form the transfer function of the process reaction curve. So, transfer function Process Reaction Curve of PRC is equal to y m bar divided by c bar, which is equal to G p G f G m for simplicity we have consider G m equals 1 similarly, we will consider G f equals 1. Now, if we substitute individual the transfer functions G p G f g m then we get GPRC equals 1 divided by s plus 3 whole square. So, this is the transfer function of the system example system, as mentioned that unit step change introduced in the input signal to the final control element.

So, the process this transfer function becomes, not transfer function this output becomes 1 divided by s s plus 3 whole square, when c bar s equals 1 by s we are just substitute c bar s equals 1 by s in this process reaction curve. Now, we basically need the y m in the time domain so, for that we need to expand into partial reactions.

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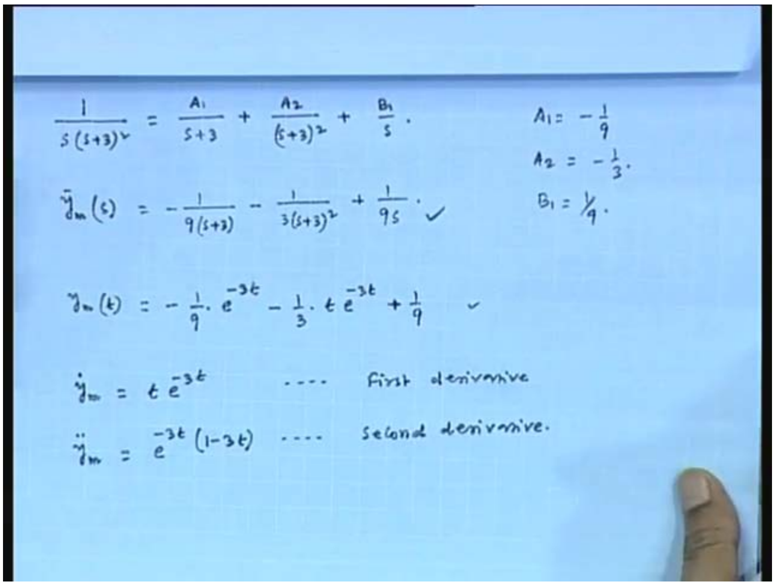

I mean we can write this as A 1 divided by s plus 3 plus A 2 divided by s plus 3 whole square plus B 1 divided by s in the next we need to determine the values of this co efficient say A 1 A 2 and B 1. So, finally, we will get this form y m bar s equals minus 1 divided by 9 s plus 3 minus 1 divided by 3 s plus 3 whole square plus 1 divided by 9 s. That means we got A 1 equals minus 1 by 9 similarly, A 2 equals minus 1 3'rd and B 1 equals 1 by 9. So, the determination of that is co efficiency is left for you.

So, this is the ultimate formed of y m bar. Next we take to inverse of lap lace transform, take the inverse of lap lace of transform. If we take the inverse of lap lace form then we get y m in time domain as y m t equals minus 1 by 9 exponential minus 3 t minus 13’rd t e to exponent especial of minus 3 t plus 1 by 9. So, this is the expression of y m in time in domain, in the next step we need to determine the difference derivative of first derivative and second derivative of y m, find the first and second derivative of y m.

The first derivative of y m represented by y m dot equals t exponential of minus 3 t check this, first derivative of y m we get as t exponential of minus 3 t, this is the first derivative. Now, second derivative we obtain as y m here, approximate use 2 dot exponential of minus 3 t 1 minus 3 t this is the second derivative.

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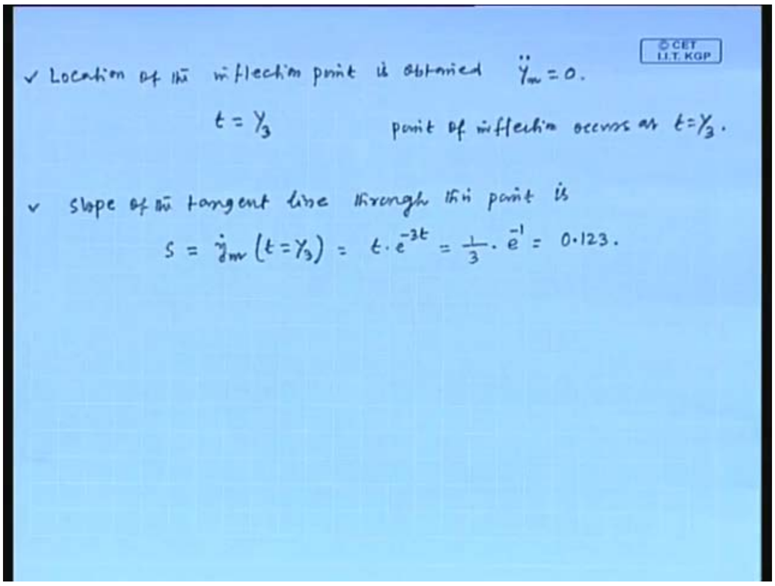

In the location of the inflection point obtain by setting the second derivative equals 0. So, if we do that, then we get t equals 1 3 rd, this is the route of interest in this problem t equals 13’rd. So, the point of inflection of occurs at t equals 1 3’rd. In the next we will determine the slope of the tangent line, the slope of the tangent line through this point is s equals first derivative at t equals 1 3 rd find the value, the expression is t exponential of minus 3 t, if we substitute t equals 1 3 rd then we obtain 0.123. The slope we obtain 0.123 I think at this stage, it is good through drawer response.

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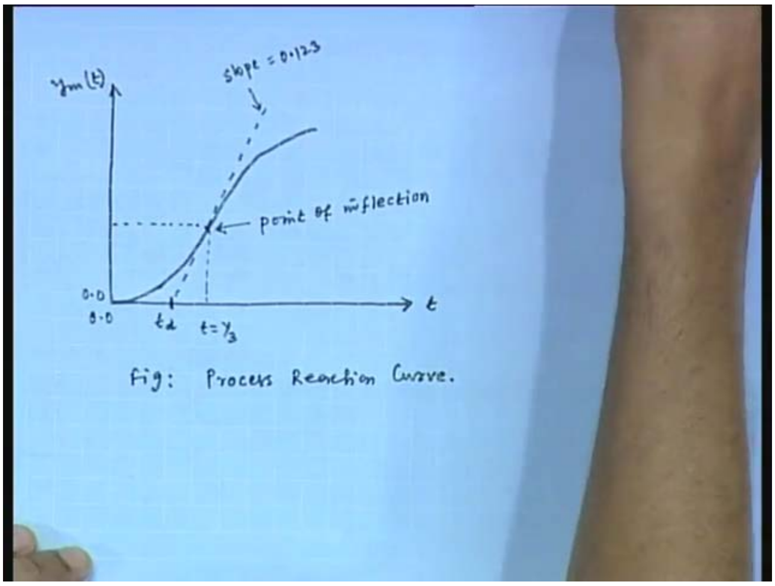

This is time and this is y m t. So, at this point t equals 1 3'rd, this point is correspond to equal 1 3'rd means this is the point of inflection. Now, this point represents the debt time. Now, the slope we obtain 0.123. So, this is the process reaction curve, can you find the corresponding y m value at t equals 1 3'rd, find the corresponding y m value at t equals 1 3'rd.

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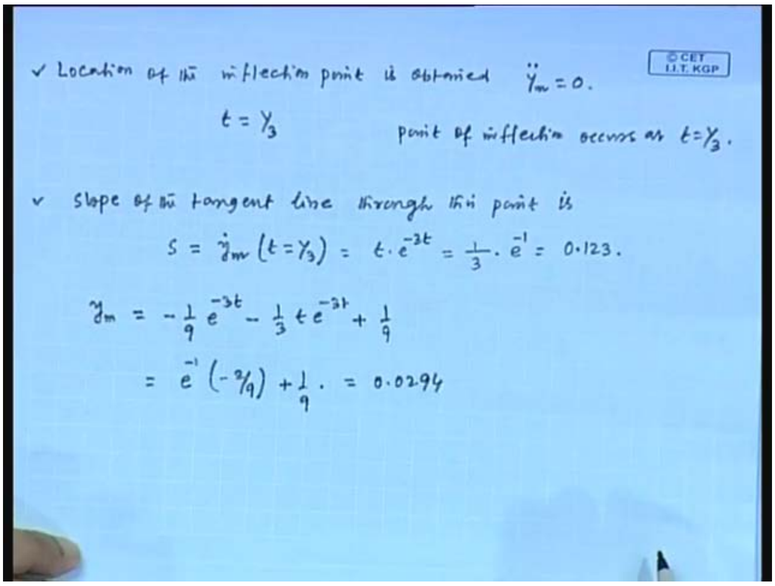

We have the expression for y m that is minus 1 by 9 exponential minus 3 t minus 1 3’rd t exponential minus 3 t plus 1 by 9, if we substitute t equals 1 3’rd than, we obtain this, which is equal to 0.0294. So, the y m at t equals 1 3’rd we obtain 0.0294.

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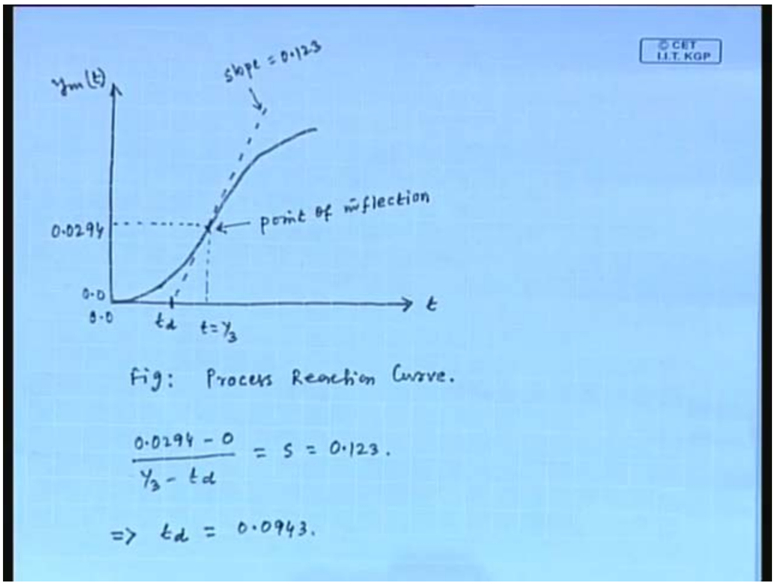

So, this is 0.0294. Now, it is obvious from this flat that 0.0294 minus 0 divided by 1 3’rd minus t d equals what slope, it is obvious from this figure that this equals slope, a slope is how much 0.123 from this co relation we can determine the value of their time, from this co relation we can determine the value of debt time, how much is that 0.0943. So, we obtain the debt time 0.0943 can you determine the ultimate value of y m t.

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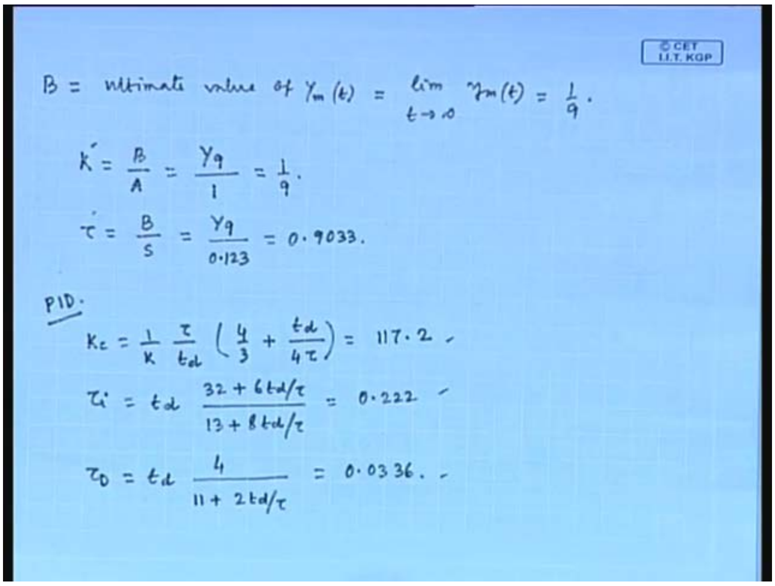

We represent the ultimate value of y m by B. So, B equals how much you calculate ultimate value of y m t that is limit t tense to infinity y m t equals 1 by 9. Now, we determine they gain k, we determine the gain k, k equals to B divided by A, B this is the ultimate value of the response and A is the magnitude of input function.

So, B we obtain 1 by 9 and A is 1 because we have introduced in needs to change; that means, k is 1 by 9. Next we need to determine the time constant tau, which is represented by B by s. Now, B will obtained 1 by 9 and rates is 0.123. So, it gives 0.9033. So, tau is 0.9033 in Cohen coon method basically response of chemical processes is subjected to step input can be approximated by the response of first order plus get time system. So, accordingly we have determine the k tau and t d.

Now, what controller we have used PID controller, they have proposed the expression for k c tau i and tau b, as the function of this k tau and t d like for PID controller the expression for k c is represented as k c equals 1 by k tau by t d multiplied by 4 by 3 plus t d divided by 4 tau. So, we know the values of tau t d and k substituting those values, we obtain k c equals 117.2 substituting the values of k tau and t d obtain k c equals 117.2.

Similarly, the expression for tau i as the form tau i equals t d 32 plus 6 t d by tau divided by 13 plus 8 t d by tau which is equals to 0.222 and the derivative time which is equal to t d 4 divided by 11 plus 2 t d by tau and we can calculate this tau d equals 0.0336 by this way we can tune the controller, using the Cohen-coon technique. Next we will discuss the frequency response analysis of linier systems.

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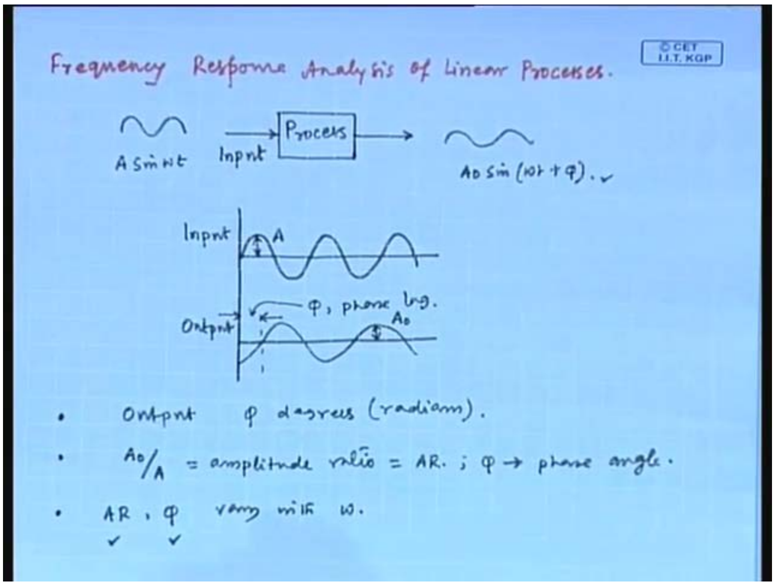

Our next topic is, frequency response analysis of linier processes. So, we need to consider the linier process and the linier process is substituted to a sinusoidal input, this it is ultimate response, after a long time is also a substantial sinusoidal wave, it can be prove that, a linier system a subjected to a sinusoidal input. It is ultimate response after a long time is also a substantial sinusoidal wave A A naught of amplitude, omega is the frequency and phi is the phase leg.

So, this is the input which has the amplitude A, this is the output which has the amplitude A naught and this output is out of phase by phi degrees or radian, this is representing phi, phase leg, these are 2 o f's. Now, it is obvious that both inputs and outputs are sinusoidal and the output is out of phase by phi degrees or radians. The ratio of output and input amplitudes, is known as amplitude ratio enamel a is represented by A R, which is amplitude ratio and phi is call the phase angle.

Now, this amplitude ratio and phi both vary with omega, how the amplitude ratio an phi vary with omega is the main concern of frequency response analysis, amplitude ratio and phi values vary with omega. Now, how the amplitude ratio an phi, vary with omega is the main concern of frequency response analysis. Now, we discuss the fundamental frequency response result.

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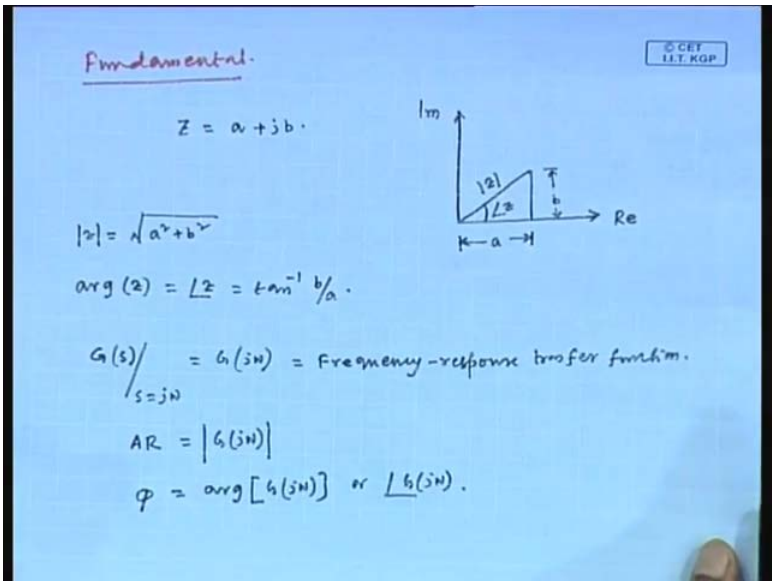

The fundamental frequency response result. A complex number denoted by z whose real part is a and imaginary part is b, can be expressed as z equals a plus j b, j b is the complex number. Now, in the complex plane we will locate the z. So, this is the real part a and this is the imaginary part b. Now, the magnitude of z, we can find as magnitude of z equals root over of a square plus b square and argument of z which we can write by this and it can be determined using this form, I mean argument z equals tan inverse b by a.

This is the argument z. Now, for a particular process we usually have the transfer function G s. Now, if we substitute s equals j omega, then we get frequency response transfer function replacing s by j omega, we obtain frequency response transfer function. Now, amplitude ratio we can determine I mean the amplitude ratio is the magnitude of this G j omega, amplitude ratio is the magnitude of the frequency response transfer function. Similarly, the phase angle phi is the argument of this frequency response transfer function, it can be represented by this notation.

So, this is the fundamental of frequency response result. Next, we discuss the general procedure for frequency response for finding the response frequency.

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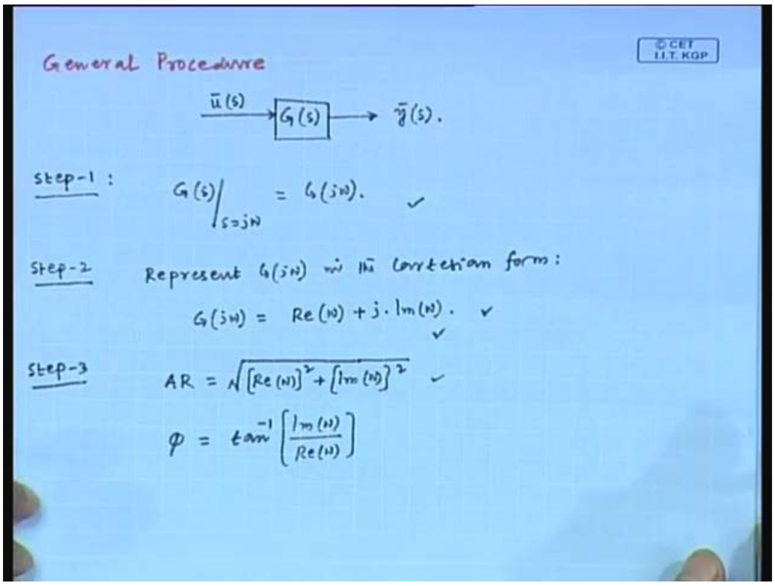

General procedure for finding the frequency response. First we discuss generalizing then we will take few examples to finding the frequency response are to analyze the frequency response, say G s is the transfer function of a process. Now, input to this process we can write as u bar s and output is y bar s in the first step, we need to replace s by g omega in the first step we should replace s by g omega to obtain the frequency response transfer function, this is step one, substitute s equals g omega and find the frequency response transfer function.

In the next step represent this frequency response transfer function in Cartesian form; that means, G j omega equals real part plus imaginary part multiplied by j. Now, this way we can clearly identify the real part and imaginary part, this is step 2. In step 3 we can determine the amplitude ratio and phase angle, we know amplitude ratio equals root over of square of this real part plus square of this imaginary part. We can determine the amplitude ratio using this similarly, the phase and here we can determine from this have phi equals tan inverse imaginary part divided by real part.

So, we just substitute s equals j omega to get the frequency response transfer function, then represent that frequency response transfer function in this form I mean just you try to find out the real and imaginary part, after doing that use these 2 expressions to obtain amplitude ratio and phase angle. Now, the alternative technique in the alternative technique we can represent the frequency response transfer function in polar form.

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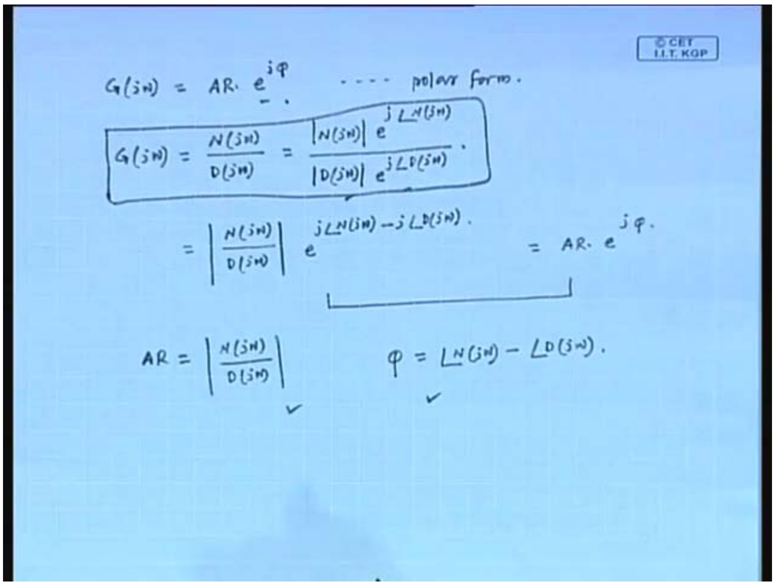

So, we can represent the frequency response transfer function in polar form, amplitude ratio exponential of j phi. Now, we can write this frequency response transfer function as j omega equals the numerator divided by the denominator part. Now, we will apply the polar form I mean we can write the numerator as amplitude and then the exponential term. Similarly, we can write the denominator as first tau a magnitude of the denominator multiplied by the exponential term, this is amplitude, this is magnitude of this numerator multiplied by this exponential term.

Similarly, for the denominator this is the magnitude of the denominator D and this is the exponential term. Now, this we can write again as numerator magnitude divided by denominator magnitude, exponential j in j omega minus j d j omega, the polar form of the frequency response transfer function we have written here amplitude ratio exponential of j 5. Now, we will compare this 2, then we obtained the expression for amplitude ratio as, amplitude ratio equals this, the magnitude of N numerator divided by the magnitude of denominator and we obtained the expression for phi also.

So, we can determine the amplitude ratio phi by this way also, here the steps are substitute s equals j omega to obtain the frequency response transfer function, then you represent the frequency response transfer function by this form, this numerator by denominator, then use this 2 expression to determine the amplitude ratio and phi. So, we will take one example to find the frequency response I mean to discuss the frequency response analysis.

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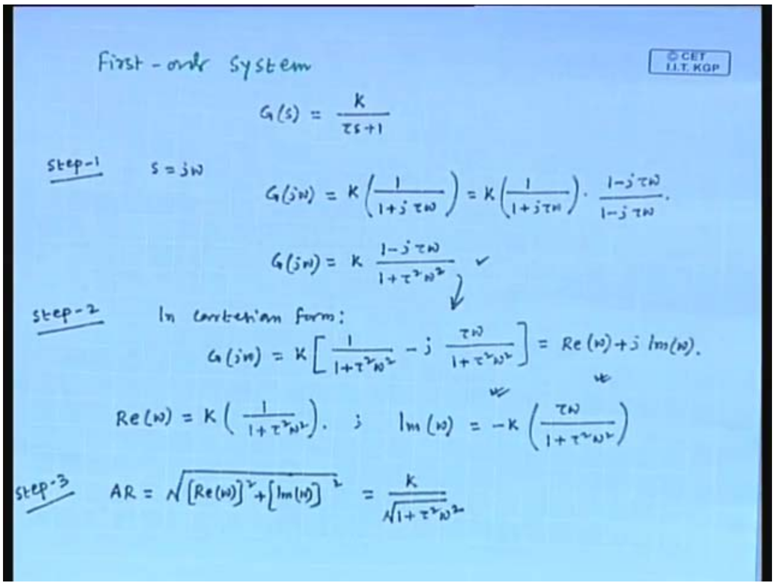

So, we will consider is first order processes, first order system. The transfer function of a first order system suppose, is given as G s equals k divided by tau s plus 1, k is the study state gain of the first order system and tau is the time constant. Now, we proceed step by step, in the first step we need to replace s by j omega. So, you substitute s equals j omega, then we can write G j omega equals k 1 divided by 1 plus j tau omega.

So, we can write this again as k multiplied by 1 divided by 1 plus j tau omega 1 minus j tau omega divided by 1 minus j tau omega, can we write this. So, this is equal to k 1 minus j tau omega divided by 1 plus tau square omega square. After substitute s equals j omega we obtain this form. Now, we need to represent this frequency response transfer function Cartesian form that is step 2. In Cartesian form we can write the frequencies transfer response function has G j omega equals k multiplied by 1 plus tau square omega square minus j tau omega divided by 1 plus tau square omega square, we can write this from this expression.

Now, we need to find the real part and imaginary part. So, if you compare this with this, we obtained the real part s, this real part equal k divided by 1 plus tau square omega square and comparing this 2 again we get the imaginary part also. Imaginary part is equal to minus k tau omega divided by 1 plus tau square omega square. So, you obtain in step

2 the real part and imaginary part. In step 3 we will determine the amplitude ratio and phi using the expressions.

How much is this, amplitude ratio is equal to k divided by root over of 1 plus tau square omega square. Similarly, we can determine the face angle phi.

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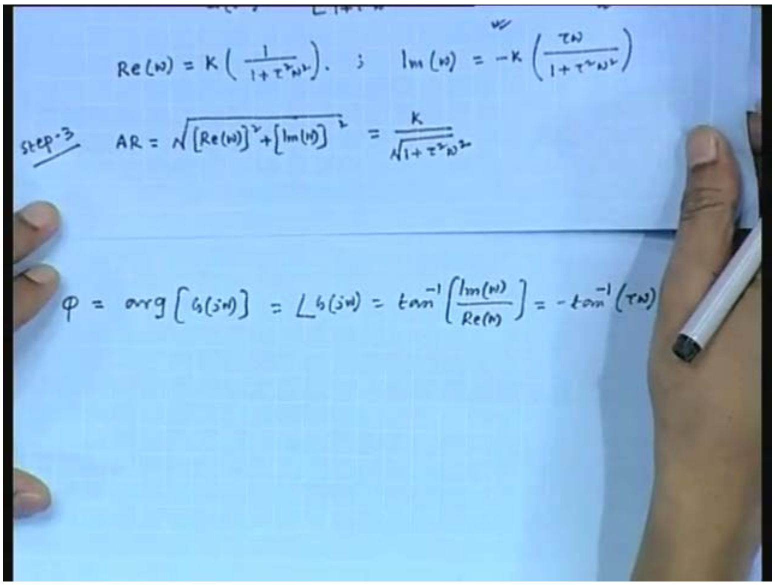

Which is argument and this phi equals tan inverse, imaginary part divided by real part substituting real and imaginary part, we obtain minus tan inverse tau omega. So, for a first order system having the transfer function G s equals k divided by tau s plus 1 we obtained the expression for amplitude ratio and phi, representing the frequency response transfer function in Cartesian form. Similarly, we will represent the transfer function frequency response transfer function in polar form, for finding the amplitude ratio and face angle.

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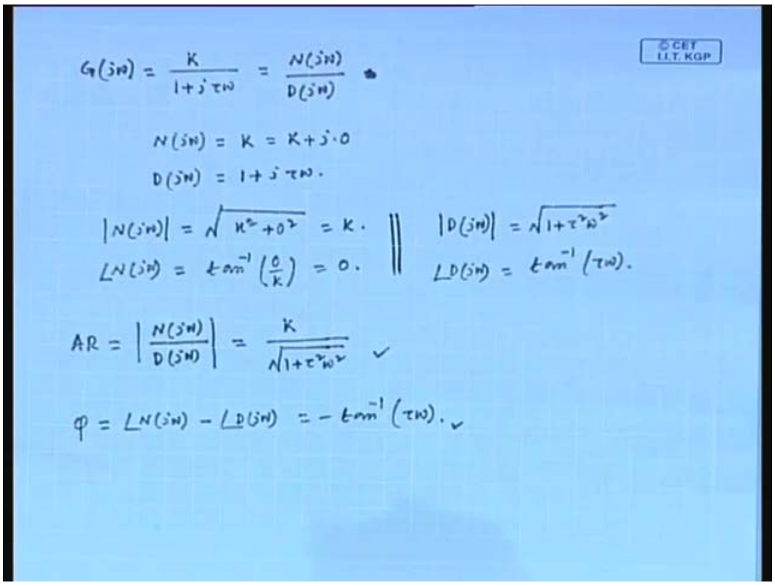

The frequency response transfer function we obtained, as G j omega equals k divided by 1 plus j tau omega, which can be written has N j omega divided by D j omega. Now, what is N j omega, N j omega is equal to k. So, you can write this k plus j multiplied by 0 and we can write the denominator as 1 plus j tau omega, then the magnitude of N j omega is how much k and argument is equal to tan inverse 0 by k which is equal to 0.

So, we obtain the magnitude and amplitude for the numerator. Similarly, for the denominator we obtain the magnitude as 1 divided by tau square omega square agree, the real square plus imaginary square whole to the power half and the argument is equal to tan inverse tau omega. Now, we know the amplitude ratio is the ratio of 2 magnitudes, magnitude of numerator divided by denominator magnitude of denominator, then we obtain k divided by root over of 1 plus tau square omega square and phi is equal to argument of N j omega minus argument of D j omega which is equal to minus tan inverse tau omega.

Now, this 2 expressions we obtained also by representing the frequency response transfer function in Cartesian form.

Thank you.