Process Control and Instrumentation

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Lecture No. # 22
Feedback Control Schemes (Contd.)

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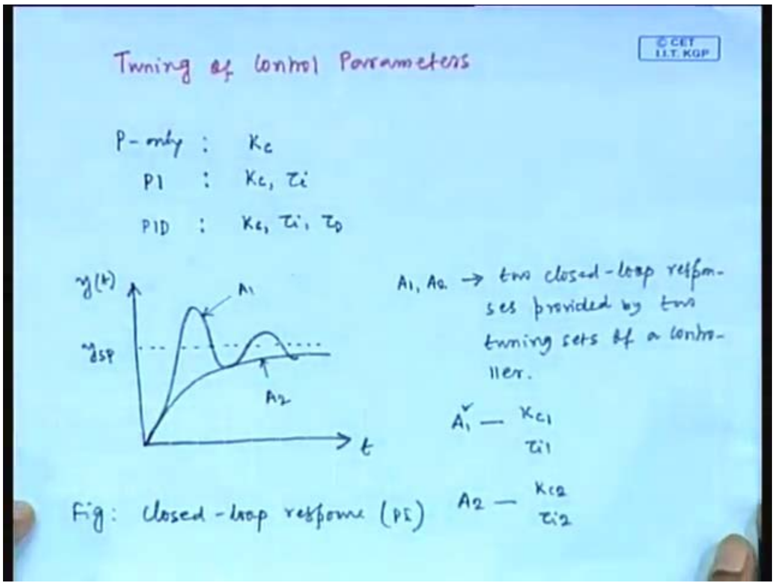

We will start today with the topic tuning of control parameters, tuning of control parameters. We have disused earlier three different control schemes, p only controller, PI controller and PID controller, for p only controller one tuning parameter is there, that is k c propositional gain. In PI controller two tuning parameters are there, one is k c another one is tau i, tau i is the integral time constant. And for PID controller there are three tuning parameters, proportional gain k c, integral time constant tau i and derivative time constant tau D, so these are the tuning parameters.

How will go to apply the control equations, we need to use the values of this tuning parameters, so that we get best possible close loop performance. If we take simple example, here we make a plot between closed loop process output y t and time t. Now, in this process a step change in set point value is introduced, this is the set point value.

We are getting this type of close loop response, for a particular set of tuning parameter values and if we use another tuning set, we get suppose this type of response. Say this is one type of response and this A 1 type of response and this response is suppose A 2, here A 1 and A 2 are two close loop responses provided by two tuning sets of a controller. A 1 and A 2 are two close loop responses provided by two tuning sets of a controller, we get response A 1 and we considers k c 1 and tau i 1 suppose, this is the close loop response of a process under PI controller.

This is the closed loop response and the process includes a PI controller. So, PI controller has two tuning parameters one is k c another one is tau i. Now, we get A 1 response and we consider k c 1 and tau i 1, we get the response A 2 and we consider another tuning set namely k c 2 and tau i 2. So, the close loop response varies with the tuning parameter values. Now, there are different criteria based on which we need to select the tuning parameter values.

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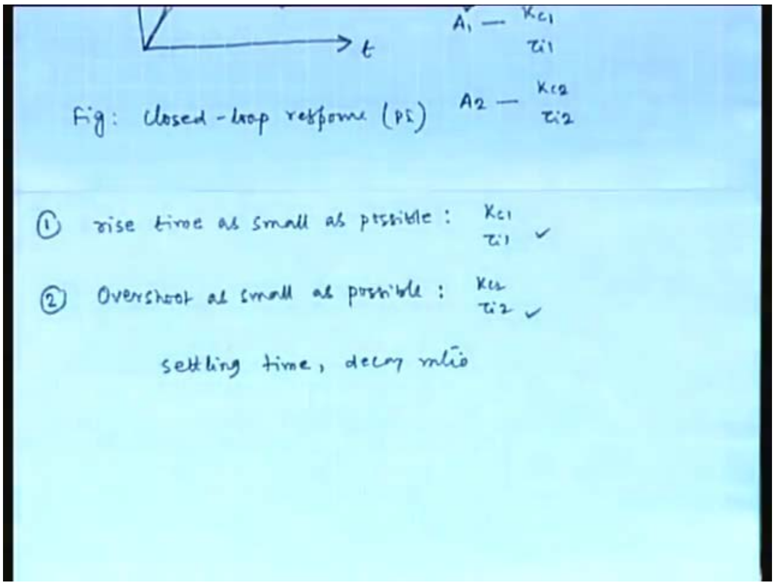

Suppose our first criteria is to keep the rise time as small as possible our target is to keep the rise time as small as possible, this is our target. So, which tuning set we will prefer k c 1 and tau i 1. So, if this is our target I mean, if we need to keep the rise time as small as possible then we will prefer the tuning set corresponding to A 1 I mean we prefer k c 1 and tau i 1. Now, suppose another criteria is keep the over soot as small as possible.

So, if this is our target which tuning set we will use, second tuning set I means k c 2 and tau i 2 agree, like this there are different characteristic features, like rise time over soot there are few more characteristic feature, they are like settling time. Settling time should be as small as possible, another characteristic feature is decay ratio. Now, one thing is clear that we are in dilemma in selecting the tuning set, which one will select k c 1 tau i 1 or k c 2 tau i 2.

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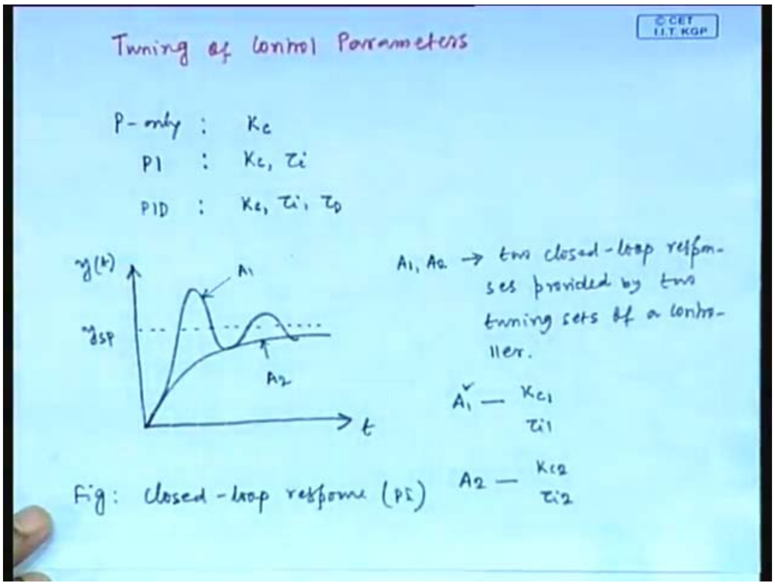

Now, you see one thing, if our target is keep to rise time as small as possible, then we need to consider the close loop response at a particular time instant. And we will proceed to tune the controller; that means, we can tune the controller, considering the response at particular time instant, but it will be nice if we consider the response throughout the operation period, in selecting the tuning parameter values. If we consider the entire time period if we consider the response throughout this time period and if we tune the control parameters, we will defiantly get better performance.

So, in that way the time integral performance criteria is consider for tuning the control parameters.

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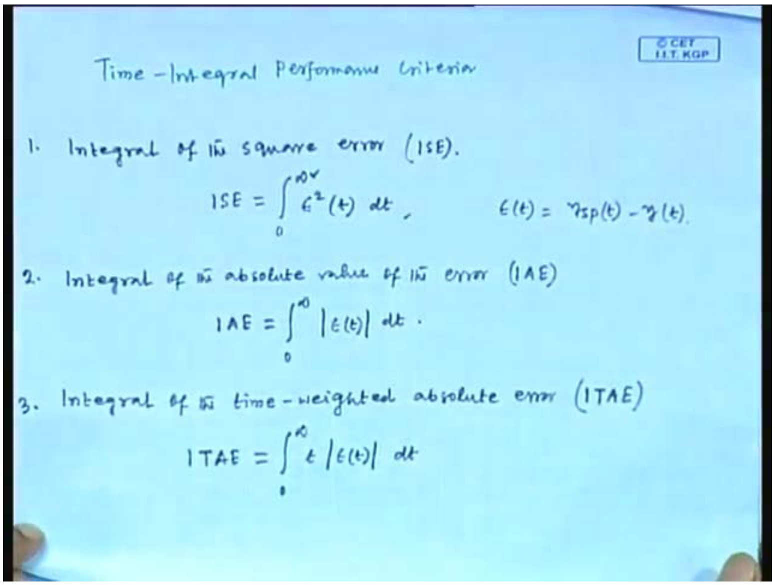

So, we will consider the time integral performance criteria, which take in to account the response throughout the period. Now, there are different time integral performance criteria, one is Integral of the Square Error integral ISE, it is represented as ISE equals 0 to infinity epsilon square t d t, this is basically not infinity, this is the time period for which or based on which we select the tuning parameters. Epsilon t is the difference between set point value of y and process y, this is the integral square error 0 to infinity epsilon square t delta t epsilon t d t.

Another time integral criteria is integral of the absolute value of the error, Integral of the Absolute value of the Error IAE, integral of the absolute value of the error. IAE is equal to 0 to infinity absolute value of epsilon t d t, this is the expression for IAE. Another time integral performance criteria is, Integral of the Time weighted Absolute Error, which is denoted by ITAE in short form ITAE. ITAE is written as ITAE equals 0 to infinity t epsilon t d t this are the three time integral performance criteria which are used in selecting the control parameters values.

Now, we will discuss the suitability of this three time integral criteria

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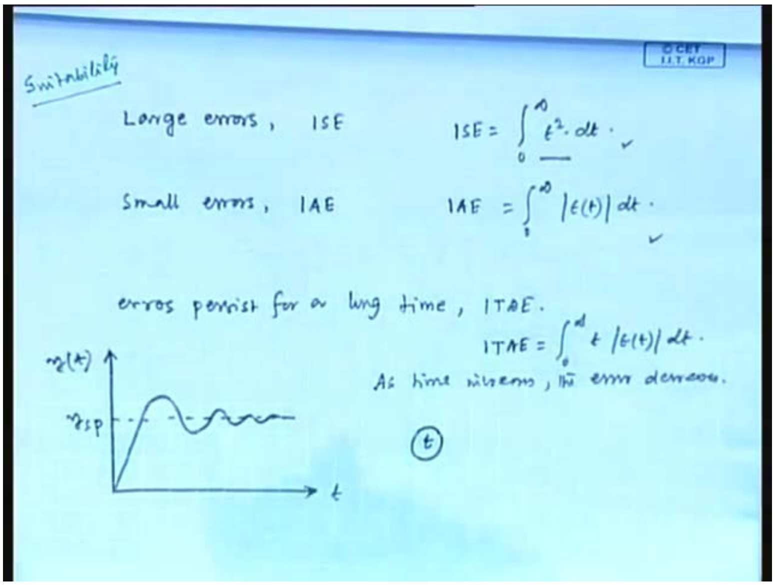

Suitability, if we want to strongly suppress large errors, it is better to use ISE performance criteria. If we want to strongly suppress large errors ISE performance criteria is used. It has this form, now if the errors, if we square the errors the error becomes larger therefore, this ISE performance criteria is used, if we use to suppress small errors, then IAE performance criteria is used.

If small errors are involved ISE performance criteria is not used because if you take squares of small errors, this term because in smaller therefore, this IAE performs criteria gets preference, when small errors are there. And when the errors persist for a long time, errors persist for a long time then ITAE performance criteria is used. Now, we will considers the response of a process suppose this is indicating the set point now, this is the close loop response.

Now, it is obvious from this figure that as time progresses, the error decreases. So, as time increases, the error decreases definitely for it is stable system. Now, in the ITAE performance criteria, the time is multiplied with error it is written by this form t multiplied by e t. Now, in this figure it is clear thus at as time increases the error decreases. Now, if we multiply this time which the error, the enter process response gets weights in selecting the control parameters, if we multiply this time which the error, the enter process response gets weights in selecting the control parameter values.

Therefore, this ITAE performance criteria is used in selecting the control parameter values. Now, how you can tuning the control parameters using this time integral performance criteria, that we will discuss with an example.

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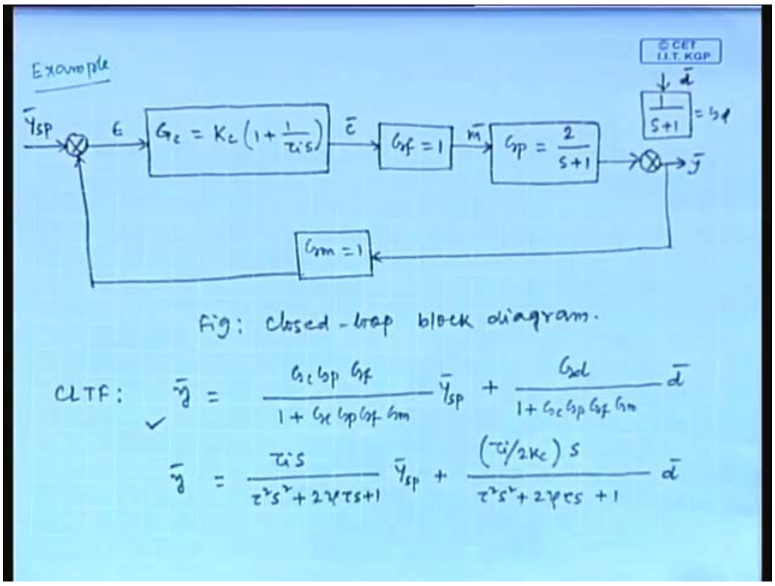

How we can tune the parameters by using this time integral performance criteria that we will discuss in the next. So, in this example first we will consider one close loop block diagram and in this example we consider a PI controller, which has the transfer function G c equals k c 1 plus 1 divided by tau i s this is the controller block, for simplicity we are considering G f equals 1 and the transfer function of the process, has this form G p equals 2 divided by s plus 1 this is the disturbance block and this is equals G d output is y bar, for simplicity again we are considering g m equals 1.

And error signal is going to the control block. This is the closed loop block diagram. Now, we will try to derive the close loop transfer function, the general form of close loop transfer function is written as y bar equals G c G p G f divided by 1 plus G c G p G f G m y set point bar plus G d divided 1 plus by 1 plus G c G p G f G m d bar, if we substitute all the individual transfer functions then finally, we get y bar equals tau i s divided by tau square s square plus 2 zeta tau s plus 1 y set point bar plus tau i divided by 2 k c multiplied by s divided by tau square s square plus 2 zeta tau s plus 1 d bar.

If we substitute the individual transfer function expressions in this general form, we get this close loop transfer function.

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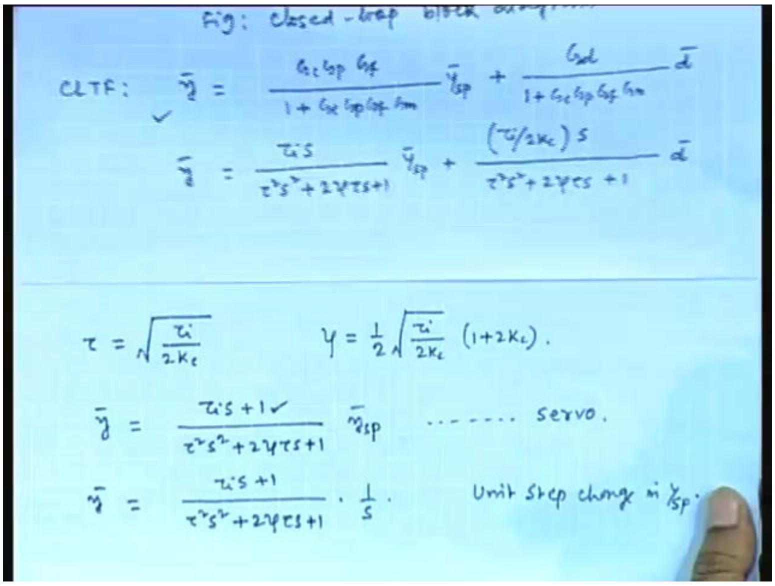

Here, tau equals tau i divided by 2 k c and zeta equals half root over of tau i divided by 2 k c 1 plus 2 k c you see both tau and zeta are the function of k c and tau i. Now, we will consider only the servo problem for tuning, then the close loop transfer function becomes y bar equals tau i s plus 1 divided by tau square s square plus 2 zeta tau s plus 1 y set point bar, this is for the case of this is plus 1 tau i s plus 1 divided by tau square s square plus 2 zeta tau s plus 1 y set point bar, this is the close loop transfer function for the servo case.

In the next we will consider a step change in y set point; in the next step we will consider a unit step change in set point. Accordingly, we will write y bar equals tau i s divided by tau square s square plus 2 zeta tau s plus 1 multiplied by 1 by s and we consider, unit step change in y set point. Next we need to take the inverse of Laplace transform, if we take the inverse of Laplace transform we get the expression for y in time domain.

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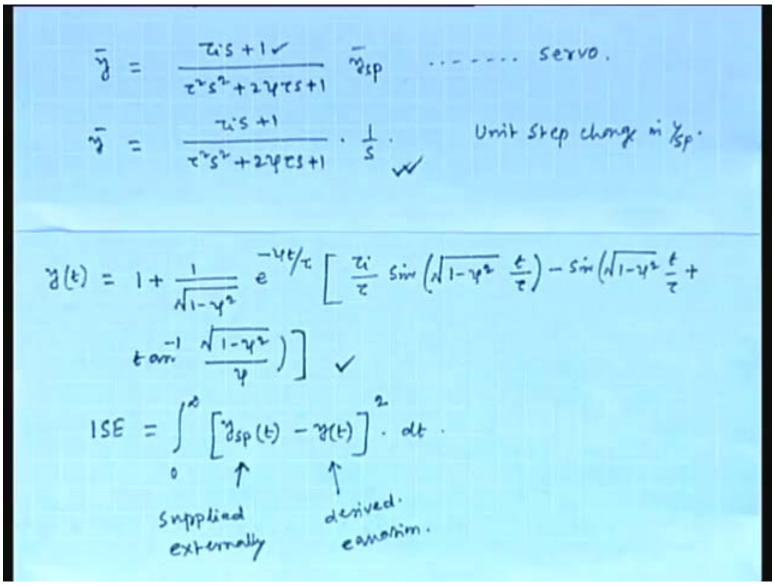

If we take the inverse of lap less transform of this, then we get the expression for y as y t equals 1 plus 1 divided by root over of 1 minus zeta square, exponential minus zeta t divided by tau multiplied by tau i divided by tau sin root over of 1 minus zeta square t by tau minus sin root over 1 minus zeta square t by tau plus tan inverse root over 1 minus zeta square divided by zeta, if we take the inverse of lap less transform of this equation we get y t expression. Now, among the three time integral performance criteria suppose, we is to consider ISE performance criteria.

So, ISE is equal to integration of 0 to infinity y set point t minus y t whole square d t among the three time integral performance criteria, we are interested to tune the control parameters using ISE performance criteria. So, ISE is equal to 0 to infinity y set point t minus y t whole square d t, this y set point t is supplied externally by the person who is in charge of operation, this is supplied externally y set point t and the expression for y t we have derived. So, for y t use the derived equation.

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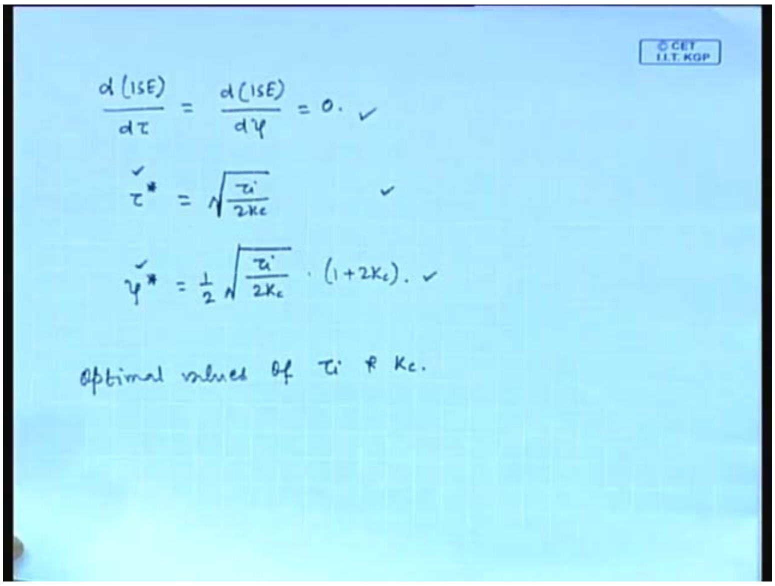

Now, what is our target, what is we required to do, we the ISE should be minimized with respect to tau and with respect to zeta. If we minimize the ISE we will get the optimum values of tau and zeta, if we minimize the ISE with respect to tau and zeta we will get the optimum values of tau and zeta. Suppose, this optimum values are tau star and zeta star. Now, what are the expression of tau and zeta, the expression of tau and zeta we obtained previously as this is the expression for tau and this is the expression for zeta.

Now, we will write tau star equal root over of tau i divided by 2 k c and similarly, zeta star equals half root over of tau i divided by 2 k c multiplied 1 plus 2 k c, this two values tau square and zeta star we obtained by minimizing ISE. Now, we have the expressions for tau and zeta those are written in this form now, you see we have 2 equations and 2 unknowns, we have 2 equations and 2 unknowns tau i and k c solving this two equations we can obtain the optimum values of tau i and k c solving this two equation we get optimal values of tau i and k c.

So, by this way by using the time integral performance criteria, we can obtain the optimal values of tuning parameters. Similarly, we can consider PID controller, we can consider other time integral performance criteria for the approaches like this. Next we will discuss another tuning approach that is Cohen coon technique. Now, so for we discuss routh Hurwitz, we have discus root locus technique and time integral performance criteria.

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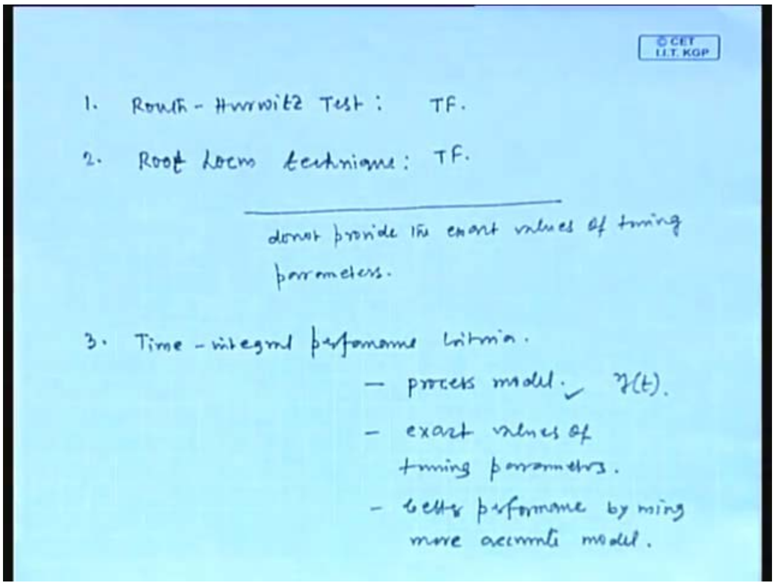

First we discussed routh Hurwitz test. In this test we need the process model, in the form of transfer function. In the routh Hurwitz test, we need the process model in the form of transfer function. Secondly, we started the root locus technique, which is a graphical technique and in the root locus technique also we need the process model because for root locus technique we need to produce the plot by positioning the roots of the characteristic equation.

So, for root locus technique also we need the process model, but these two techniques cannot provide the exact values of tuning parameters, this two techniques only can provide the range of the tuning parameters, this two technique do not provide the exact values of tuning parameters, they only provide the ranges of tuning parameters therefore, there not called as tuning techniques, it is better to call them instability technique.

Thirdly we discuss the time integral performance criteria, I mean controller tuning by time integral performance criteria, this technique requires the process model because in the time integral performance criteria we need the expression for y t in the time integral performance criteria we need the expression for y t; that means, process model is required and this technique provides the exact values of tuning parameters. So, this is a tuning method. Now, best on this first point we can say that, it can provide better performance by using more accurate model, this time integral performance criteria involves the process model.

Now, if we give more accurate model, we will get better performance. So, it provides better performance, when we use more accurate model. So, therefore, we look for an efficient tuning method that uses empirical rules. Since, the time integral performance criteria uses or depend heavily on the process model, we look for an efficient control technique that uses the empirical rules.

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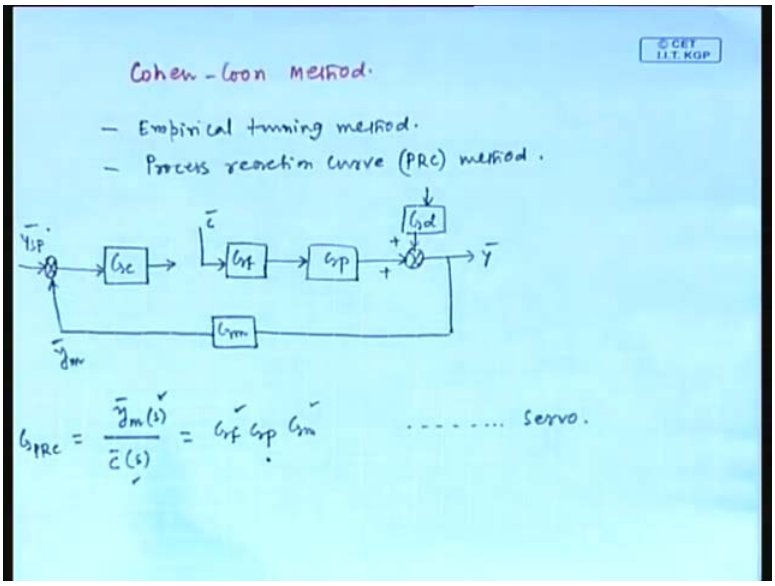

So, next we will discuss one technique which is based on the empirical rules, that is Cohen coon method, this method was proposed by Cohen and coon. This is an empirical tuning method, this method is also called as process reaction curve method. Now, before discussing this method we need to draw the block diagram, close loop block diagram. So, this is y set point, this is controller output, this is the block for final control element then the process, disturbance, process output, this process output is connected with the block of measuring device and the measuring device output is y m bar.

Now, according to this Cohen coon method, the closed loop block diagram is opened by disconnecting the final control element from controller, according to the Cohen coon method this block is this loop is opened by disconnecting the final control element from the controller. Now, if we disconnect the final control element from the controller, then we can write the output y m bar divided by input c bar s equals G f G p G m, definitely this is for the case of servo problem, can we write the output y m bar divided by the input c bar equals G f G p G m and this is the transfer function of the process reaction curve.

Now, it is obvious that the output y m depends not only on the process, it depends also on that final control element and measuring devise, the output y m depends not only on the process, but also on the final control element and measuring device. Now, Cohen and coon they absorbed that for most of the chemical process they got sigmoidal response.

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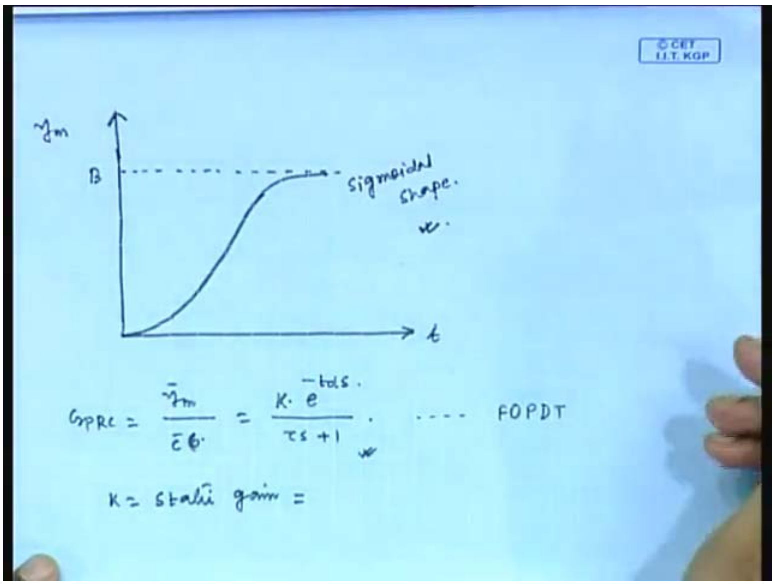

Cohen and coon absorbed that for most of the processes, they obtained sigmoidal response, sigmoidal shape, like this. Suppose, this quantity is B; that means, the set point value, not set point it may be the y value at new study state, B is the value at new study set, for most of the chemical process Cohen and coon absorb that, the process look like sigmoidal shape. Now, this response can be approximated, by the first order plus date time, this is basically the response of a first order plus date time.

So, we can write the first order plus date time as k exponential minus t d s divided by tau s plus 1, so most of the chemical processes the response looks like this. Now, this shape can be approximated by first order plus date time and this is the expression of first order plus date time response, the first order plus date time response can be represented by this equation, here k is static gain. So, what will be the expression for gain, for most of the chemical process Cohen and coon absorb this type of response and a step change is introduce in c.

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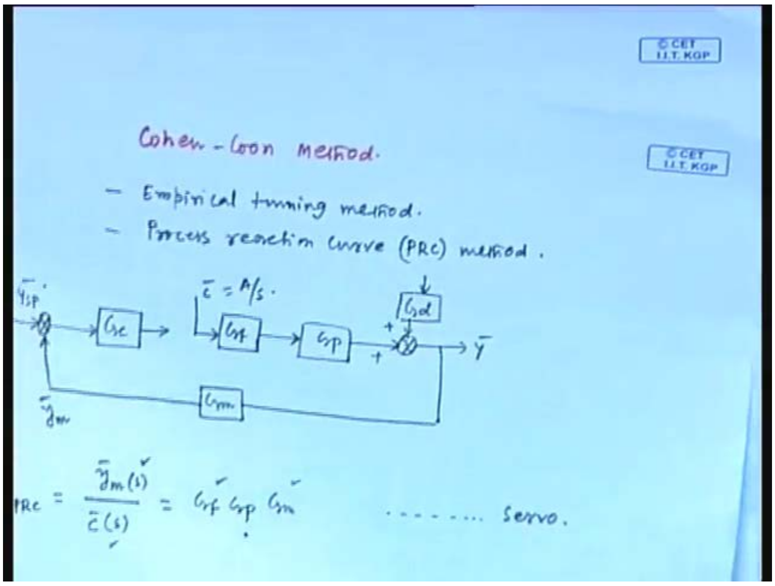

Sigmoidal shape is obtained and a step change with magnitude a is introduced in c.

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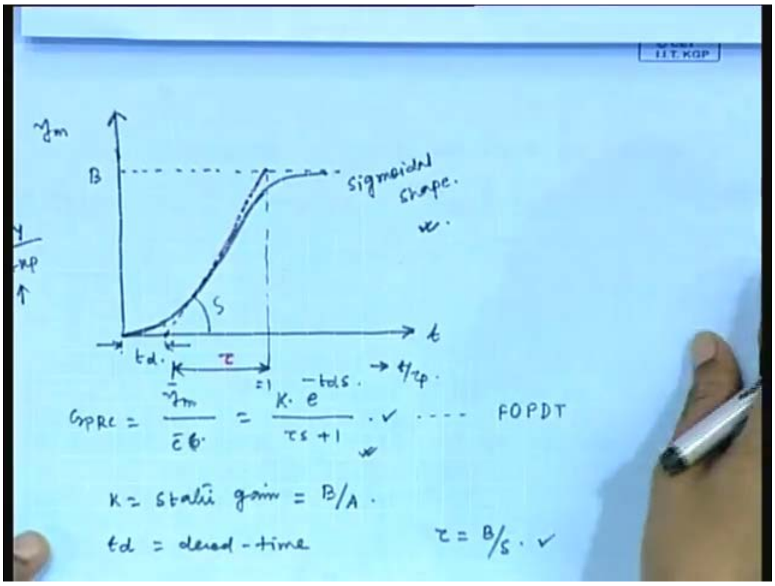

Now, can you find the expression for k change in output that is B divided by change in input that is A agree, gain is the change in output divided by the change in input. Next one is the t d, t d is date time, date time value you can obtain from this plot, this is the date time. Date time value we can obtain from the plot. Now, if we exclude the date time plot, then the response becomes the response of the first order system. If we exclude the date time part from this response, then the response becomes the response of first order system.

If we exclude the date time part from this over all response, then the remaining part is the response of first order system. Now, how much is this quantity for a first order system we have plotted y verses a k p verses t divided by tau p and we got this equals 1 for the first order system we plotted y divided by a k p versus t divided by tau p and we obtained at time t equals 0 this quantity equals 1. So, how much is this quantity, if you plot y verses t this is tau t divided by tau p equals 1; that means, t equals to tau this quantity is tau.

If we consider the slope of this is s this slope of this tangent is s, then we can write tau equals B by s can we write, if s is the slope of this tangent at starting from t equals 0 then we can write tau equals b by s, so by this why you can determine, all the parameters involved in this expression. Now, after finding the parameters of the expression Cohen and coon derived the empirical equations, introducing change in load variable, after finding the parameters involved in the first order plus date time expression Cohen and coon derived empirical equations.

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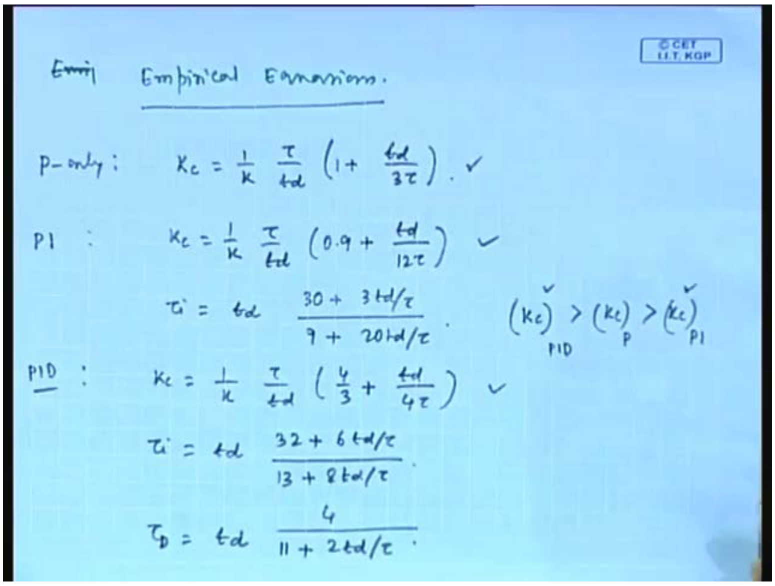

Empirical equations and they have derived the equations introducing change in load variable, for p only controller they got the expression for k c here, k c equals 1 by k tau by t d 1 plus t d by 3 tau for PI controller they got 2 equation, one for k c another for tau

i. This expression they got for k c and for tau i they obtain tau i equals t d 30 plus 3 t d by tau divided by 9 plus 20 t d by tau for PID controller they derive three equations, one for k c one for tau i and one for tau d.

This expression they got from k c for tau i the expression can be written as tau i equals t d 32 plus 6 t d divided by tau divided by 13 plus 8 t d divided by tau and for tau D they got tau D equals t d 4 divided by 11 plus 2 t d divided by tau, this empirical correlations Cohen and coon derived by introducing change in load variable.

Now, if you compare the three equations for p PI and PID 3 k c expressions, which one is the largest I mean which k c is largest k c for p PI or PID k c value of pi controller is the lowest and k c value of PID controller is the highest, reason is very obvious if you introduce integral action there may be some instability problem therefore, k c is taken lowest and PID controller as better stability criteria therefore, highest k c value for PID is consider.

Thank you.