Process Control and Instrumentation

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Lecture No. # 21

Feedback Control Schemes (Contd.)

In the last class we discussed Routh-Hurwitz test, today we will solve one problem based on that test.

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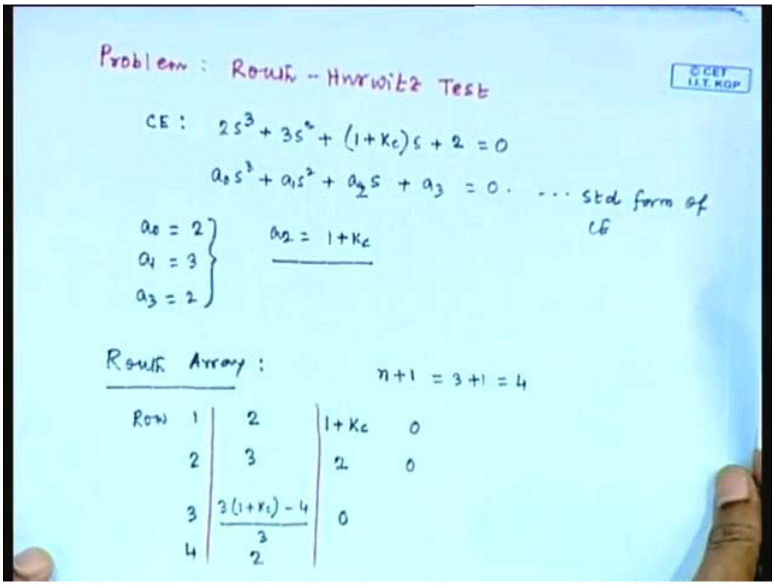

So, today will discuss a problem on Routh-Hurwitz test, now in this problem the characteristics equation is given, it is 2 s cube plus 3 s square plus 1 plus K c s plus 2, which is equal to 0, and K c is the proportional gain of the controller. Now, we need to compare this characteristics equation with the standard polynomial form of characteristics equation. What is the standard form, that is a naught s cube plus a 1 s square plus a 2 s plus a 3 which is equal to 0, this the standard form of characteristics equation.

Now, it is obvious that, a naught equals 2, a 1 equal to 3, a 2 equal to 1 plus K c and a 3 equal to 2. a naught, a 1 and a 3 these are positives coefficients but we cannot say about a

2. But, if a 2 is negative, there is no need of further analysis, we can directly say that, the close loop process is definitely unstable. Therefore, for the time being, we will assume that 1 plus K c is positive quantity and we will try to find the stability condition based on the value of K c.

So, we are assuming all coefficients starting from a 1 to a 3 are positive coefficients then we cannot say whether system is stable or unstable. If all the coefficients are positive, we cannot say, whether system is stable or unstable. So, to make any conclusion, we need to go for further analysis and in the next step, what is require to do, we need constitute routh array. Routh array consists of n plus 1 number of rows, it includes n plus 1 number of rows.

Here, n is 3 so, routh array should constitute with 4 rows so, this is a row 1, row 2, row 3, row 4. Now, you will just write all the coefficients of characteristics equation in the first and second row only. So, first one is 2 which is a naught, second one is a 1 which is 3 then next one is 1 plus K c, next one is 2 and there is no more coefficient. So, we can put 0, 0. So, what will be the first element of third row, can you determine the first element of third row, 3 multiplied by 1 plus K c minus 4 divided by 3.

What will be second element, second element will be 0 then what is the first element in fourth row, 2 so, we got all the elements in the first column based on which we proceed for stability analysis so, we got all the elements of the first column.

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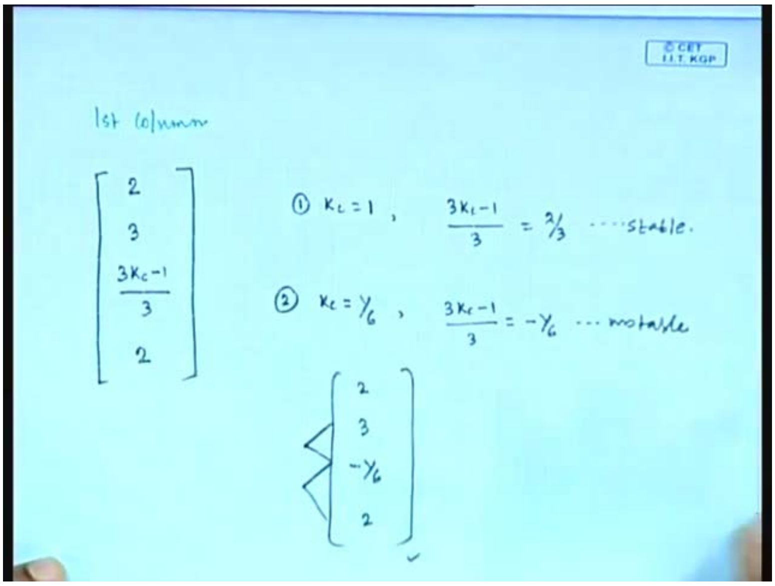

So, the first Colum we can write as 2, 3 then third element is 3 K c minus 1 divided by 3, can I write the element of third row and last element is 2. So, this is the first column in the routh array now, you will try to consider some values of K c arbitrarily. Suppose, if K c is equal to 1, what about the third element in the first column, the third element in the first column becomes two third. So, the system is stable or unstable, system is stable because all the elements in the first column are positive.

So, the close loop system is stable and K c is equal to 1 because all the elements in the first column are positive. We will consider another value suppose, K c is equal to 1 by 6, if K c is equal to 1 by 6, the third element in the first column becomes how much, minus 1 by 6. So, the process is, the close loop system is stable or unstable, unstable if K c is equal to 1 by 6, the close loop system is unstable. Now, if this is unstable, how many poles lie in the right half plane, 2 because if we consider K c is equal to 1 by 6 then the first column becomes this.

If we consider K c equals 1 by 6 then first column becomes this now here, two sign changes are involved one sign changes is from second row to third row, second sign changes is from second row to fourth row. Since two sign changes are involved so, if K c is equal to 1 by 6, 2 poles lie in the right half plane. In the next, we will try to find the critical stability condition no, before critical stability condition, we will find stability condition.

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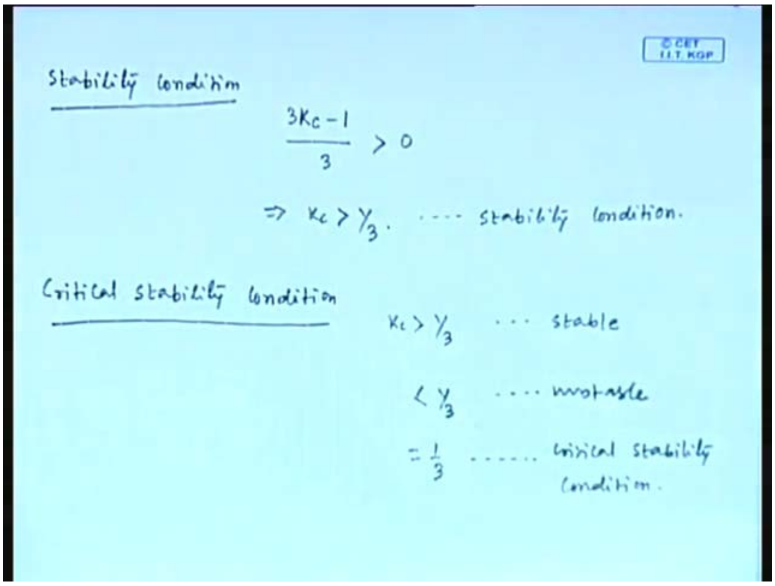

I mean, for what value of K c, the close loop system suite always be stable, can you find that. So, the system will always be positive, always be stable when 3 K c minus 1 divided 3 is greater than 0 then we can say that, the close loop system remains always stable. So, from these, we can find that K c should be greater than one third so, this is the stability condition. If K c is greater than one third then the close loop system remains always stable in the next, we will discuss the critical stability condition.

See, one thing is clear that, if K c is greater than one third then the close loop system is stable, if K c is less than one third then system is unstable. If K c is equal to one third then we can say that, this the critical stability condition so, this the critical stability condition. And K c is equal to one third, this is the stability condition, can you determine the third element of first column when K c is equal to one third, 0. When K c is equal to one third, the third element in the first column becomes 0 then what will be the all elements in first column.

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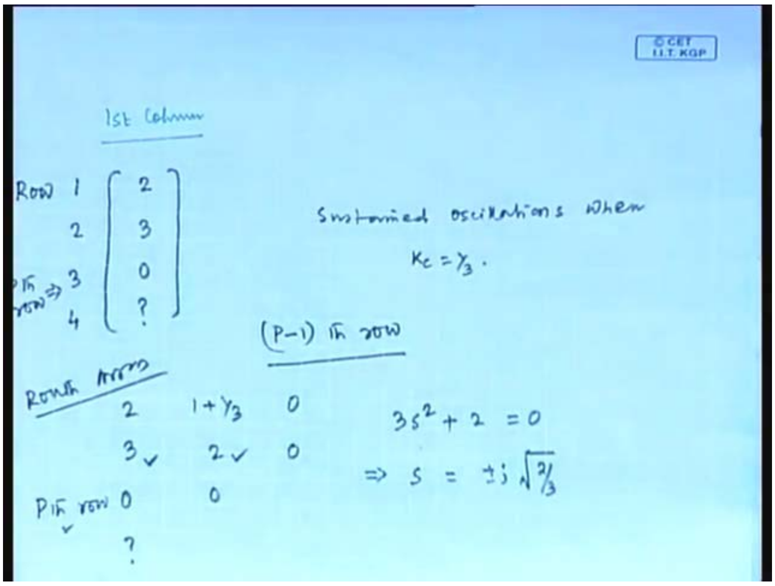

First element is 2, second element is 3, third element is 0 so, upto third row, we can determine the elements in the first column. If the elements in the third row vanish then what about the close loop response, we have discussed in last class, sustain oscillation. So, if all the elements from the third row vanish then we can conclude that, the close loop response provides sustain oscillation I mean, oscillatory response with constant amplitude.

In this situation, we can say that, we get sustain oscillations when K c is equal to one third. When K c is equal to one third, the close loop response provides oscillatory response with constant amplitude that means, the poles lies on the imaginary axis, agree it indicates that the poles lie on the imaginary axis. Can you determine those poles which are basically complex conjugate poles with real part is 0, we can determine say, this is first row, this is second row, this is third row, this is fourth row suppose, the elements vanish from p th row suppose, all the elements banish from p th row.

Now, to determine the complex conjugate poles with real part is equals 0, we need to take the elements from p minus 1 th row, to determine the complex conjugate poles with real parts equals 0 we have to take the elements from p minus 1 th row. That means, if we write other elements then routh array becomes like this agree, this is the routh array we are saying that, this is p th row. Now, all the elements vanish from this row now, we will take the elements just from p minus 1 th row so, one element is 3, another element is 2 which is equal to 0.

And you remember, we only take the elements from p minus 1 th row and from first and second column. All the elements vanish from p th row, we will take elements from p minus 1 th row and from first and second column. So, we can write 3 s square plus 2 equals 0 by solving this, we can write, s equals plus minus j root over of 2 by 3. So, these are the two complex conjugate poles, which lie on the imaginary axis.

So, when K c is equal to one third, we get sustain oscillation and the corresponding complex conjugate roots are this, which lie on the imaginary axis. In the next, we will discuss another technique which is root locus technique.

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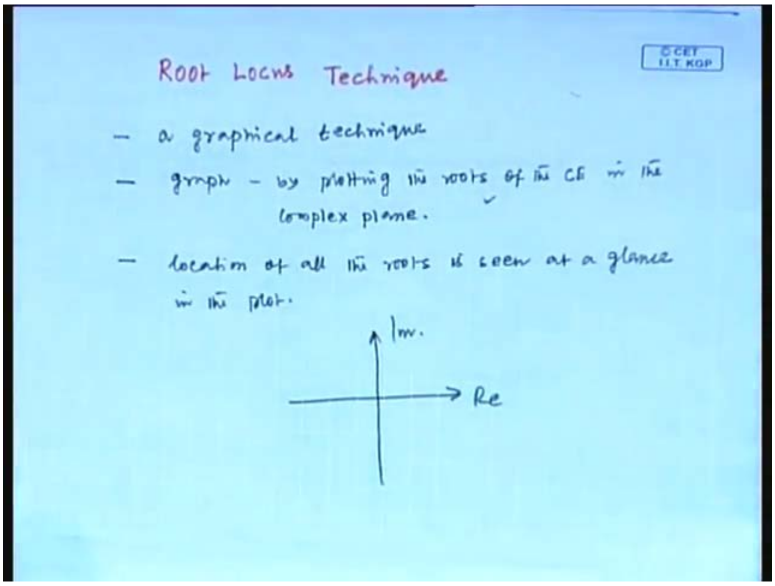

The Routh-Hurwitz taste was purely algebraic but this root locus technique is a graphical technique, root locus technique is a graphical technique. Now, question arise is, how we can produce the graph, we can make the graph by plotting all the poles, all the roots of characteristics equation in the complex plane. So, we can produce the graph by plotting the roots of the characteristics equation in the complex plane. This is a graphical technique and we can produce the graph by plotting all the roots of characteristics of equation in the complex plane.

The location of all the roots is seen at a glance in the plot and we can conclude on stability, by seeing the overall picture. We have the characteristics equation, we can determine the roots now, we will consider one complex plane and we locate all the roots in the plane. That means, at a glance, we can see the position of all the roots in this graph and we can conclude on stability based on this overall picture, this is the root locus technique.

Now, we will start to discuss this root locus technique taking one example, we will discuss this root locus technique taking one problem.

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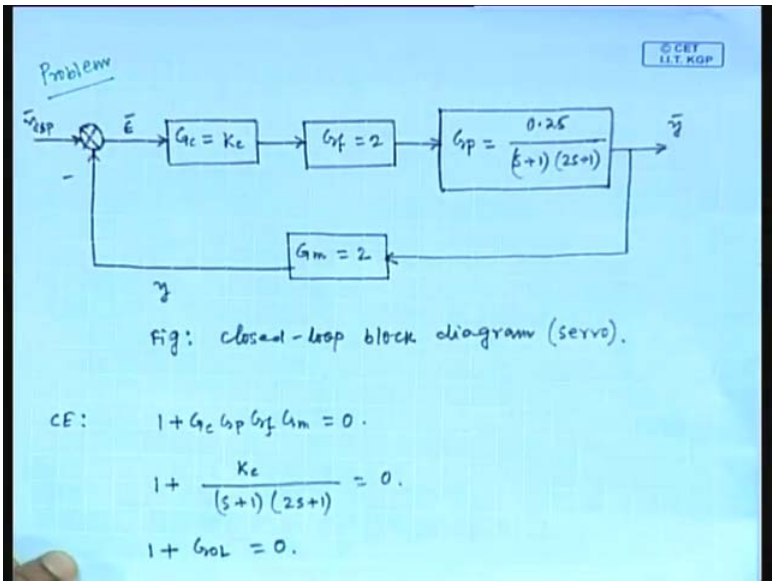

The close loop block diagram is require to develop first suppose, we are considering the process is under P only controller, the process is under P only controller. That means, transfer function of the controller G c equals K c, the transfer function of the final control element G f equals 2. The transfer function of the process we are considering as G p equals 0.25 divided by s plus 1 multiplied by 2 s plus 1. So, this is a second order process, this is the transfer function of the process, output is y bar.

Transfer function of measuring device G m equals 2, the output of these measuring device is suppose, y and the output of the comparator is epsilon. So, this is a close loop diagram, this is the closed loop block diagram for servo case or regulatory case or for both, this for servo case, because we considering no disturbance. Now, you will analyze the closed loop system based on root locus technique, can we find characteristics equation, find the characteristics equation.

The general form of characteristics equation is 1 plus G c G p G f G m equals 0, this is the general form. Now, we substitute all the individual transfer function in this general form, if we substitute all the individual transfer functions, we get 1 plus K c divided by s plus 1 multiplied by 2 s plus 1 equals 0. This is a transfer function, close loop transfer function now, another form also we write that is, 1 plus G suffix O L equals 0 where, G suffix O L is the transfer function of the open loop system.

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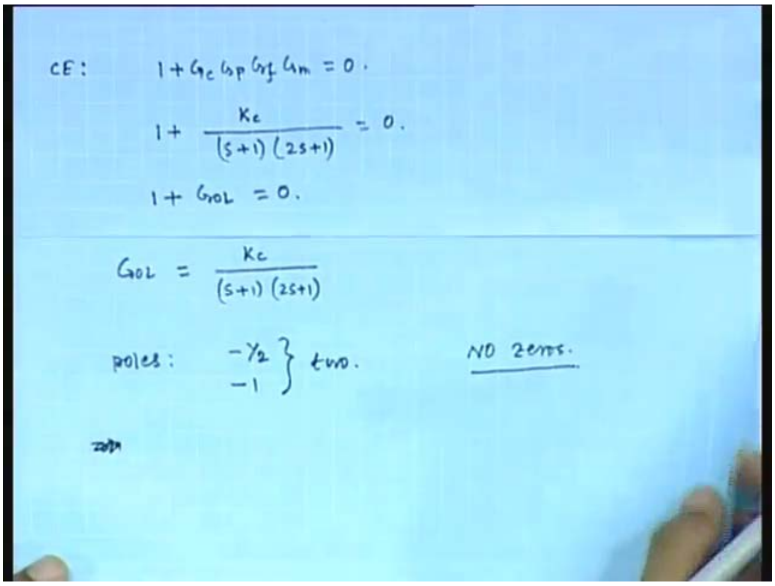

Now, comparing the last two equations we can write, G OL equals K c divided by s plus 1, 2 s plus 1, this the form of the open loop system. Can you find the poles of open loop system transfer, what are the poles, the open loop transfer function has two poles, one is at s equals half, another one is at s equals minus 1 so, two poles are there. What about zeros, no zeros so, there are two poles and no zeros of the open loop transfer function remember, open loop transfer function G OL. Now, we will try to find the roots of characteristics equation.

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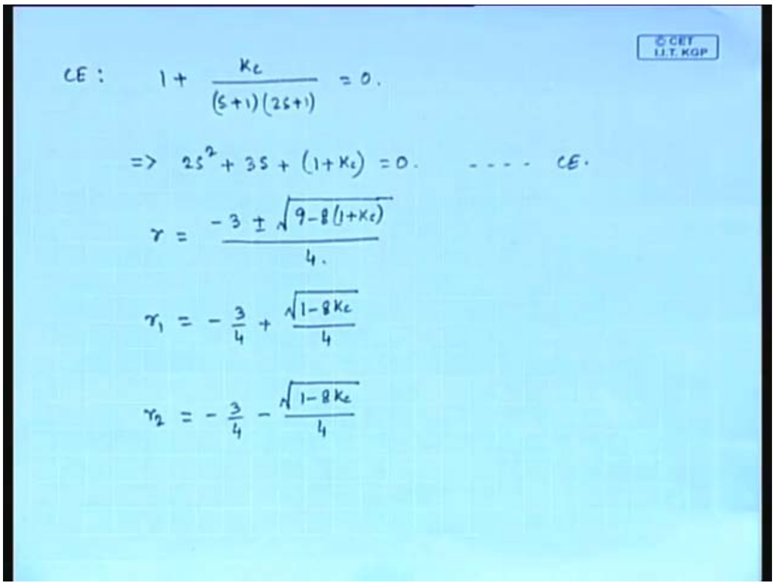

So, our characteristics equation is 1 plus K c divided by s plus 1 multiplied by 2 s plus 1 equals 0. We can write this equation by simplifying s, 2 s square plus 3 s plus 1 plus K c equals 0, can we write the characteristics equation in this form so, this is the characteristics equation which we got after simplifying. Then we can write the root as r equals minus 3 plus minus root over 9 minus 8, 1 plus K c divided by 4.

The characteristics equation in quadratic form obviously, there are two roots so, we will write as r 1 and r 2, r 1 is minus 3 by 4 plus root over of 1 minus 8 K c divided by 4 this is the first root. Second root r 2 is equal to minus 3 by 4 minus root over of 1 minus 8 K c divided by 4 this is second root. Now, you will generate the roots varying K c value from 0 to infinity, the root locus diagram is produced varying K c from 0 to infinity.

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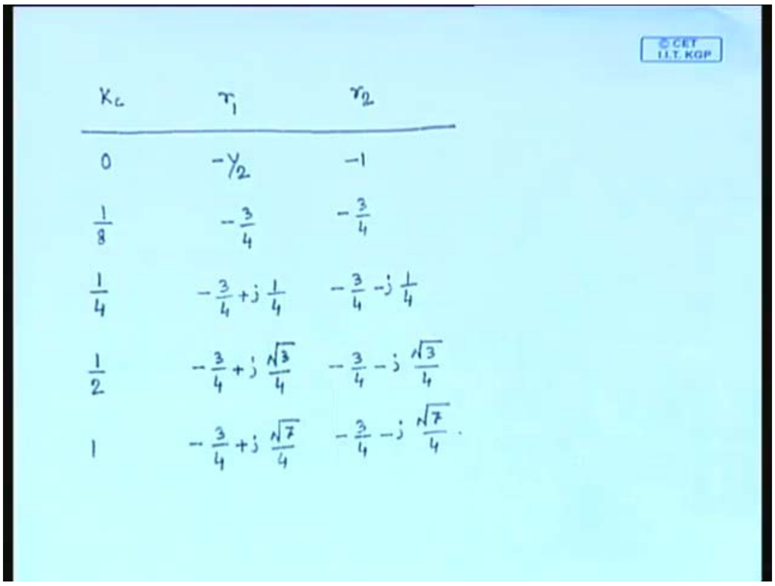

So, we will just produce one plot varying K c and we will determine the corresponding r 1 and r 2 values. So, first we will first consider K c equals 0 then what is r 1, how much is r 1 you keep on determining the values. If K c is equal to 0 how much is r 1, minus half, how much is r 2, minus 1. If K c is 1 by 8, how much is r 1, minus 3 by 4 and r 2 so, both roots are equal. If K c is equal to 1 by 4, how much is r 1, minus 3 by 4 plus j 1 by 4, another one will be minus 3 by 4 minus j 1 by 4 basically, they are complex conjugate roots.

If we consider K c equals half, r 1 becomes minus 3 by 4 plus j root 3 by 4 and second root will be minus 3 by 4 minus j root 3 by 4. Next, we will consider K c is equal to 1 then r 1 becomes minus 3 by 4 plus j root 7 by 4 and second root is minus 3 by 4 minus j root 7 by 4 like this, we can go up to infinity. Now, based on these data I mean, we will produce in next, the root locus diagram putting these two roots.

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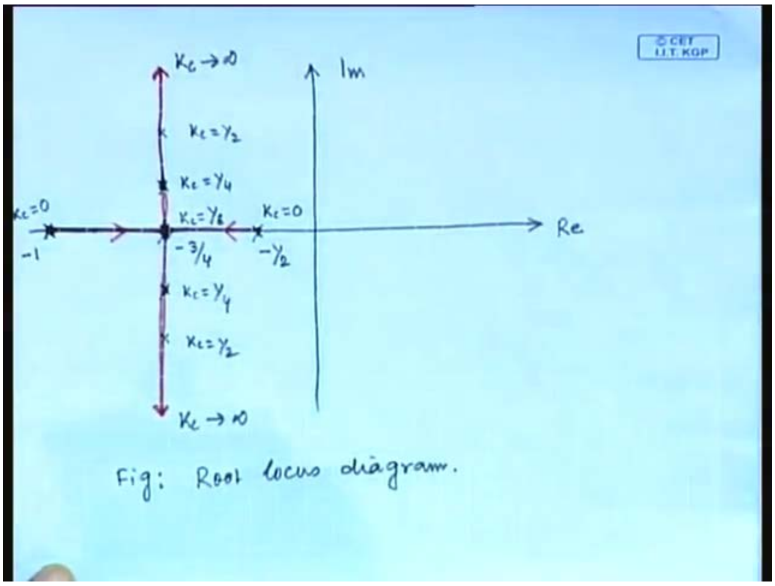

So, our complex plane is this one, this is real axis, this is indicating imaginary axis when K c is equal to 0, r 1 is minus half say, if r 1 is minus half indicates this point, r 1 is minus half and corresponding K c value is 0. When K c is equal 0, another root r 2 is minus 1, corresponding K c value 0 so, when K c equal 0, these two roots we have determine and they are located now in the complex plane. In the next, we will consider suppose, K c equals 1 by fourth, before that we will consider K c equal to 1 by 8.

If K c equal to 1 by 8 then we have only one root that is, minus 3 by 4 and minus 3 by 4 is in between minus 1 and minus half agree, corresponding K c values I am writing here that is, 1 by 8. Next, we will locate the roots when K c equals one fourth see, when K c equals one fourth, we have two complex conjugate roots. Real part is minus 3 by 4 that means, this point and complex part is 1 by 4 and another one is minus 1 by 4. So, I am just writing here, this root corresponds to K c equals one fourth, this is also we got when K c equals one fourth.

So, these two points we obtained when K c equals 0, these two points we obtained when K c equals one fourth, the single root we obtain when K c equals 1 by 8. Next, we will consider K c equals half then these two roots we can locate in complex plane, K c equals half, K c equals half. You see, the real part is same minus 3 by 4, only imaginary part gradually increasing with the increase of K c value. So, by this way, it will increase upto infinity as K c approach is infinity now, you will just connect all these points.

So, we get branches like this, the deduction is this, like this so, these are the branches, by connecting all the points we got this picture. So, this is when K c approaches infinity, this is when K c approaches infinity. Now, this is the root locus diagram for the example system, which we produced locating the roots of characteristics equation. Now, we will make some remarks based on this plot.

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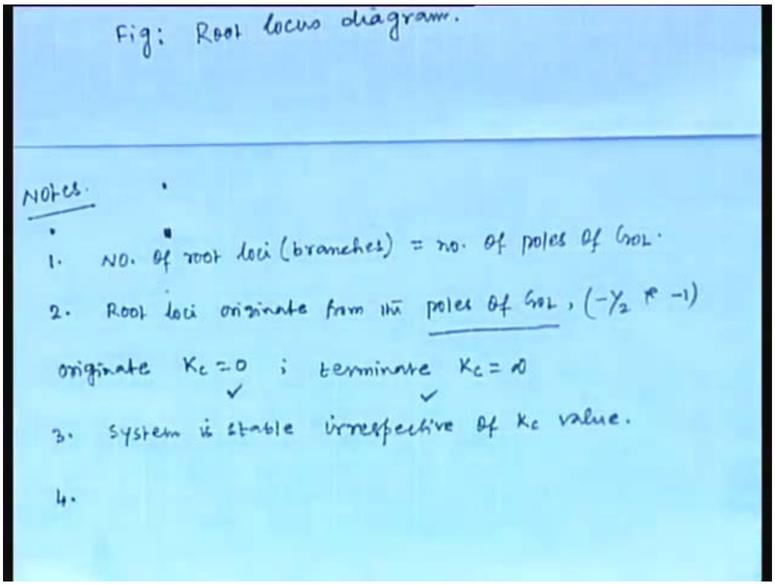

We will note down few important points based on this plot so, how many root loci are there, how many branches are there. Number of root loci or number of branches is equal to 2 so, that means, number of poles of G OL. So, we can conclude based on these root locus diagram that, the number of root loci or branches equals number of poles of G OL that is, 2. Two branches are there and we obtain also two poles of the open loop transfer function here second point is, the root loci originate from which point, root loci originate from the poles of G OL that is I mean, there minus half and minus 1, agree.

The root loci originates from the poles of G OL, the poles are minus half and minus 1 can I say, the root loci originate when K c equals 0, yes. The root loci originate when K c equals 0 and the root loci terminate when K c is equals infinity recall that, the root locus diagram is produced varying K c from 0 to infinity. So, when K c equals 0 I mean, the root loci originate when K c equals 0 and root loci terminate when K c equals infinity.

What about the stability, is there any crossing of root loci from left side to right side no, if you see the root locus diagram, it is clear that, there is no crossing of root loci from left side to right side. That means, the close loop system is stable irrespective of K c value I mean, you can take any large value of K c, there will not any in stability problem. So, third point is the close look system is stable irrespective of K c value, there is no in stability problem.

You can observe another thing that, as the value of K c increases, the poles of the closed loop system move away from the real axis.

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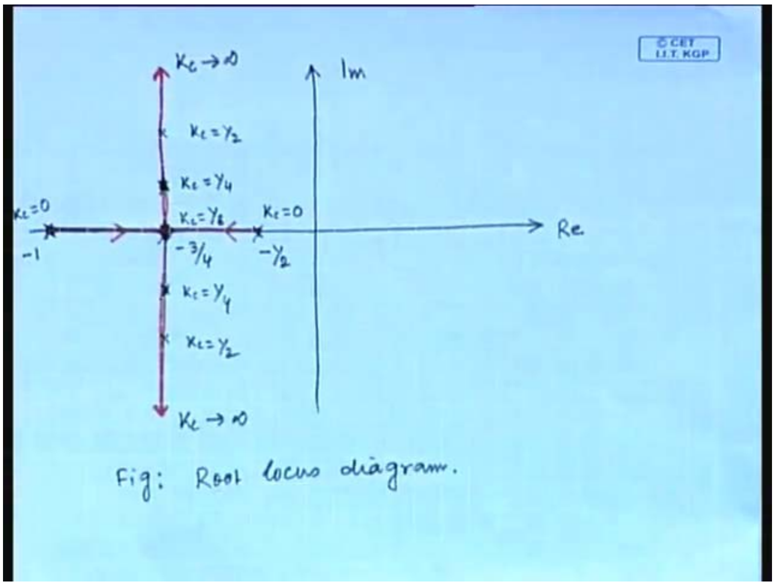

With the increase of K c value, the poles of the closed loop system they move away from the real axis. See here, this root we got when K c equals one fourth, if we increased to K c half, the root is moving away from the real axis in both sides what it indicates, oscillatory response

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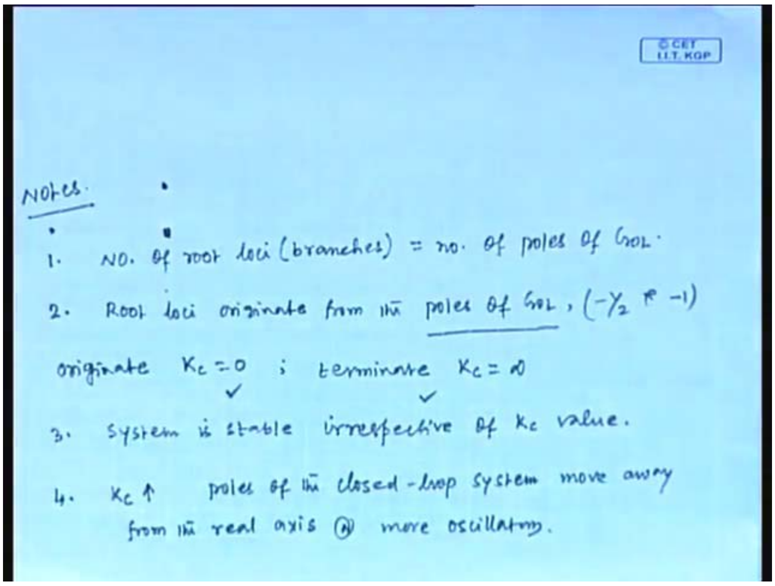

So, you write as the value of K c increases, the poles of the closed loop system move away from the real axis that means, the response becomes more oscillatory so, these are the conclusions, we can make best on the results we obtained. In the next, we will try to find some general features, best on the result we obtained in solving this problem.

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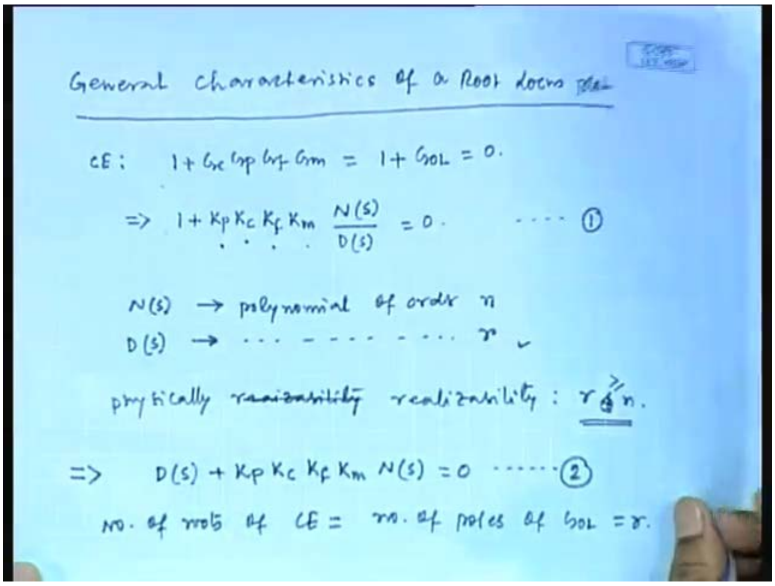

We will try to obtain some general characteristics based on the results we obtain in solving the pervious problem, general characteristics of a root locus plot. Now, root locus plot is produced by plotting, by locating the roots of characteristics equation. The general form characteristics equation is written as 1 plus G c G p G f G m equals 1 plus G OL equals 0, this the general form of the characteristics equation. Now, we will write this equation as 1 plus K p K c K f K m, N s by D s equals 0.

We are writing the characteristics equation in this form where, K p K c K f K m are the gains of process, gain of the controller, gain of the final control element, gain of the measuring device and N s is a polynomial of order n. Similarly, if in the denominator, another polynomial exist that is D s, D s is the polynomial of order r, these are the two polynomials. Now, for any physically realizable system, r should be physical realizability, r should be less or equals to n. For any physically realizable system it will be just opposite, r should be greater or equals to n.

Now, we will just simplifying characteristics equation, if we simplifying characteristics equation we get, D s plus K p K c K f K m N s which is equals to 0. Suppose, this the equation 1 and this is the equation 2 now, what is the order of this equation, r. If r is greater than n then definitely order of the equation is r. So, the number of roots of this characteristics equation is number of poles of G OL.

We can write that, number of roots of characteristics equation equal to number of poles to G OL. And that is equal to r so, this is the first characteristics next, I mean, the second characteristics that we will discuss.

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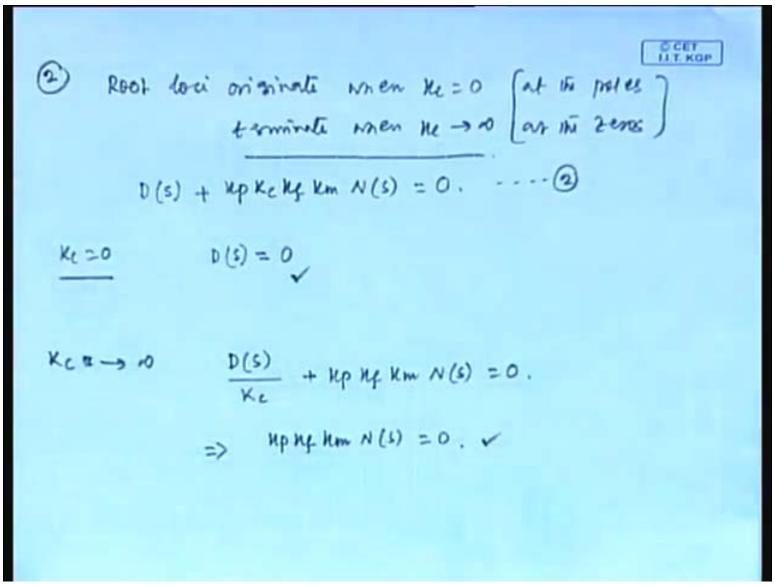

So, in the next, we will discuss that, the root loci originate when K c equals 0 and it terminate when K c approach is infinity that means, it originates at the poles, terminate at the zeros. Now, we got the characteristics equation D s plus K p K c K f K m N s which is equal to 0, which we got in the last slide as equation 2. Now, we will consider K c equal to 0 so, what the characteristics equation gets I mean, the characteristics equation becomes D s equals 0.

When K c equal to 0, D s becomes 0 so, we should start the root loci diagram, plotting the roots of D s equals 0 that means, the root loci originate at the poles of G OL. Next, we will consider K c tends to infinity, if K c approaches infinity what form of the characteristics equation we get, we have to write the characteristics equation by this way, D s by K c plus K p K f K m N s equals 0.

Now, when K c approaches 0, we get K p K f K m N s equals 0 so, we should aim the roots locus diagram plotting the roots of the N s equals 0. So, it proves that, the root locus diagram terminate when K c tends to infinity.

Thank you.