Process Control and Instrumentation

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Lecture No. # 20
Feedback Control Schemes (Contd.)

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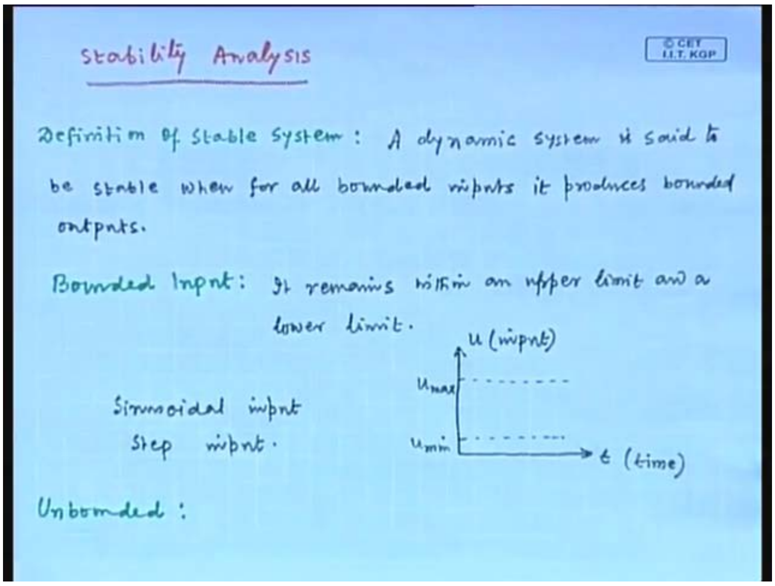

Today, we will covered the topic stability analysis, today will discuss stability analysis. So, first we will try to know the definition of stable system, what is the definition of stable system? A dynamic system is said to be stable when for all bounded inputs, the dynamic system produces bounded outputs, and this is the definition of stable system. So, here to new terms or inward in this definition; one is bounded input and another one is bounded output, these are quite new. So, next we will know what is bounded input, bounded input is an input that remains within an upper limit and lower limit.

Similarly, we can defined the bound output, bounded output is an output, that remains within an upper limit and a lower limit. If we produce a plot between time t versus input u, then it looks like is this, see u is the input. So, it has upper limit, that we can denote by suppose u max and it has the lower limit I mean the lower value that is a u min. Now, the input variable always remains within this upper limit and lower limit, it should not exceed the upper as well as lower limit that is why it is called bonded input, can we give one example of bounded input.

Student: Sinusoidal

Sinusoidal input is one bounded input, step input is another bounded input, what about ramp input, ramp input is not a bounded input. Now, just opposite of this bounded, that is un bounded, there is no existence of the term un bounded in reality, because of the physical limitation of all instruments and equipments. There is no existence of the term un bounded in reality, it only exist in theory, because of the physical limitation of all instruments and equipments.

So, for we have discussed a little about the stability of open loop system, we have determined the roots of the denominators of a transfer function. And we discussed about the position of those roots in the complex plane, best on who which we can comment on stability of open loop system, that we have discussed, so for. Now, we will discussed the stability of closed loop system, so before discussing we if net to know few important points.

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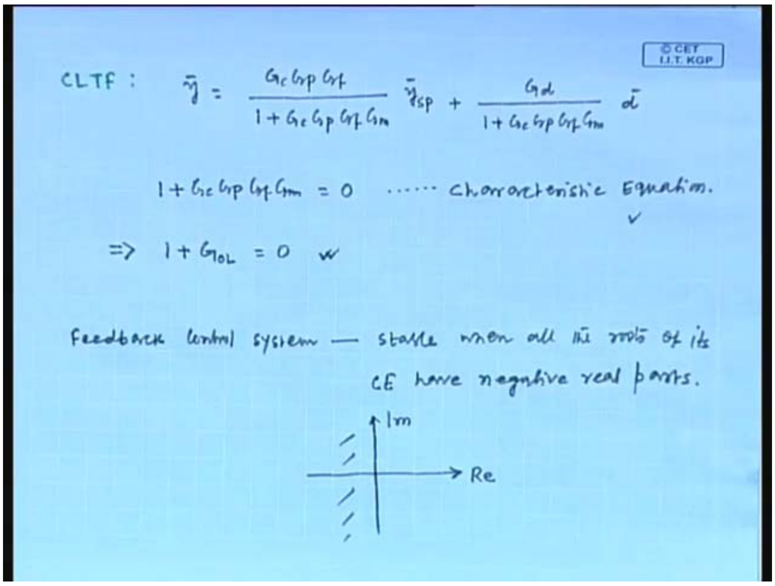

We derive the closed loop transfer function in generalize form, the closed loop transfer function we can represent by this expirations, y bar is equal to G c G p G f divided by 1 plus G c G p G f G m y set point bar plus G d divided by 1 plus G c G p G f G m d bar, this the general form of the closed loop transfer function. Now, the denominator of these transfer function, we can write as 1 plus G c G p G f G m, if we write this equals 0, then that is called the characteristics equation. This is the characteristics equation of the closed loop transfer function for stability analysis we will use this equation.

This is the characteristics equations of the closed loop transfer function, these equation can also be write in this form 1 plus open loop transfer function G suffix O L open loop, which is equal to 0. The characteristic equation we can also write by this form 1 plus the transfer of open loop equals 0. Now, we will write the definition of stable closed loop system, a feed back control system is stable, when all the roots of it is characteristic equation, lie in the left side of complex plane, a feed back control system is stable, when all the roots of it is characteristic equation, have negative real part.

A dynamic a feed back control system is set to be stable, if all the roots of the characteristic equation lie in the left side of the complex plane. So, if this is real axis and this is imaginary axis all the roots would lie in the left side, this is the left side, then we can say the feedback control system stable.

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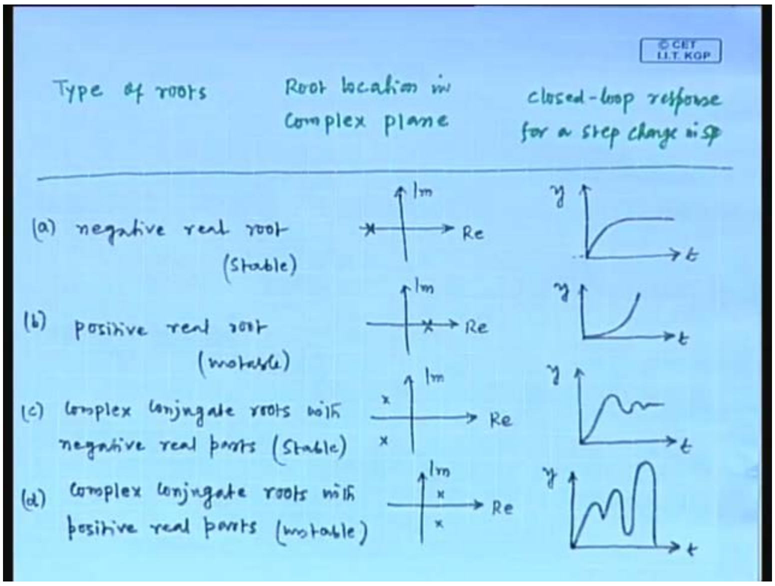

Now, we will just discus different situations, so first we will consider the type of roots, then what will be the corresponding root location in complex plane. And we will know the corresponding closed loop response, for a step change in set point SP, SP means Set Point. Suppose, it characteristic is equation has a number of roots, among them say a characteristic equation has a number of roots and the roots are lie in the left side of the imaginary axis; that means, all the roots have negative real parts negative real root not parts negative real root.

So, this is the complex plane this is real axis and this is imaginary axis, so suppose we are considering a single negative real root is there. So, the position of that would be here what it indicates the system is the feedback system is stable are un stable, this is stable. So, if the characteristic equation has negative real root then the system is stable, what will be the transient response time t verses y, it will be like this definitely against step change in set point value the transient response will be like this.

The it was the earlier state I mean the process was add this positions now we are going to change, we are going to introduce is step change in set point that is why the process is going towards achieving a new steady state. In the second situation we will considered among the roots one has positive real part, so among the root one is positive real root, now we need to locate that root in the complex plane, this will be the location positive real root. So, this is stable or un stable this is un stable since the root has positive real part, so the feedback control system is un stable. What will be the corresponding transient response?

This is time verses y plot, so it will response like this, it is going to un stable zone, in the third situation we will consider complex conjugate roots, with negative real part. Now, if will locate the roots in the complex plane, these are the two roots who which are complex conjugate with negative real parts. Now, what about the stability stable, because the real parts of negative, so the feedback control system is stable, what will be the transient response t verses y, it will reach a new steady state with decreasing oscillation, oscillation with decreasing amplitude.

And in the fourth situation will consider complex conjugate roots with positive real parts, so if we locate the roots in the complex plane, it looks like this these are two roots complex conjugate roots, which have positive real parts. What about the stability un stable, the closed loop system is un stable when it has two complex conjugate roots with positive real parts. And what will be the corresponding transient behavior, oscillation with growing amplitude like this, so these are the four deferent situations created in a closed loop system.

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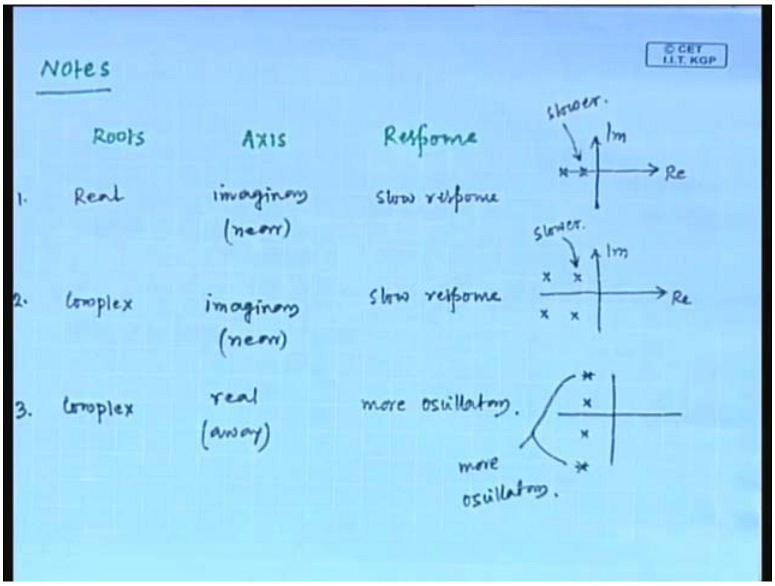

We will note down 5 important points on stability roots, axis, response real roots near the imaginary axis, result in slow response. This is the first important point real roots near the imaginary axis result in slow response, if we compare in the complex plane it looks like this, one root is this one another root is this one. So, the root which is near to the imaginary axis that will provide slower response if we consider two roots, so this is near to the imaginary axis, so it will provide slower response compared to another root, complex roots near the imaginary axis result in slow response.

Suppose, this is one set this is another set, so this is near to the imaginary axis, so this set will provide slower response, among these two complex conjugate roots this one is near to the imaginary axis. So, this have the complex the closed loop system having these roots will slow with, so slower response, another important point is when the complex roots are away from the real axis.

The more oscillatory the transient response will be say this is one set and another one is this one, so this set is away from the real axis, so it this set will provide more oscillatory response. So, these are three important points on the transient response, speed of the transient response you can say. Now, in the next we will discuss the one very popular stability analysis technique, that is Routh Hurwitz test.

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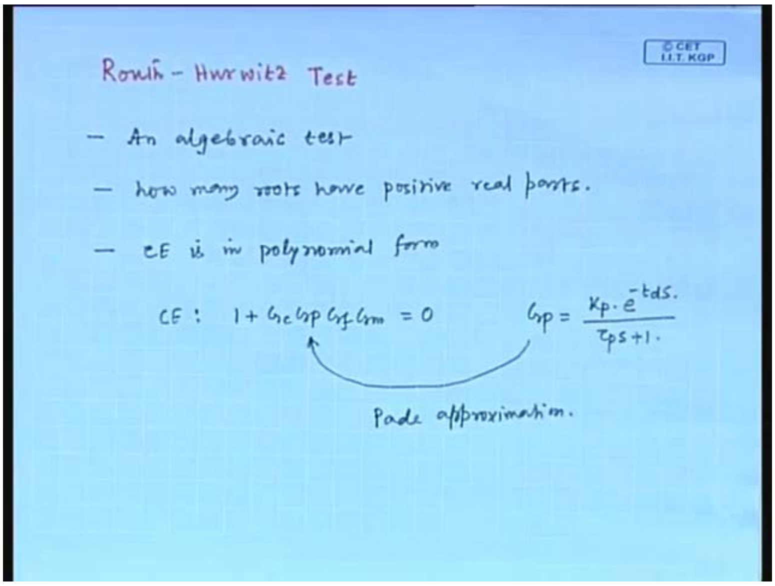

This is purely an algebraic test Routh Hurwitz test is purely an algebraic test, by perfuming this test we can know how many roots have positive real roots, how many roots have positive real parts, it is interesting that without determining the values of the roots. We can know, we can say how many roots have a positive real parts by perfuming this test, there is known it to determine the roots, just by perfuming this test we can say how many roots have positive real parts.

And another point is there is known in of any production of plot, there is known it to production any plot for this particular case and this method this test is applicable when the characteristic equation is expressed in polynomial form. This roots test is applicable when the characteristic equation is in polynomial form, you can say this is the limitation of these roots test, we discussed the characteristics equation that it has this form, I mean we can express these as 1 plus G c G p G f G m equals 0.

Now, suppose the process is a first order get time process suppose the transfer function of the process has this form, G p equals K p exponential of minus tau d s divided by tau p s plus 1 the process is the first order plus date time system. Now, if we substituted G p in this characteristic equations, we do not get polynomial form, so what we can do in this case, in this case we can go for approximation may be Pade approximation may be trailer series expansion. Remember in that case we can do all the approximate stability analysis, if we approximate this exponential term say by Pade approximation, in that case exact stability analysis is not possible ally approximate stability analysis is possible.

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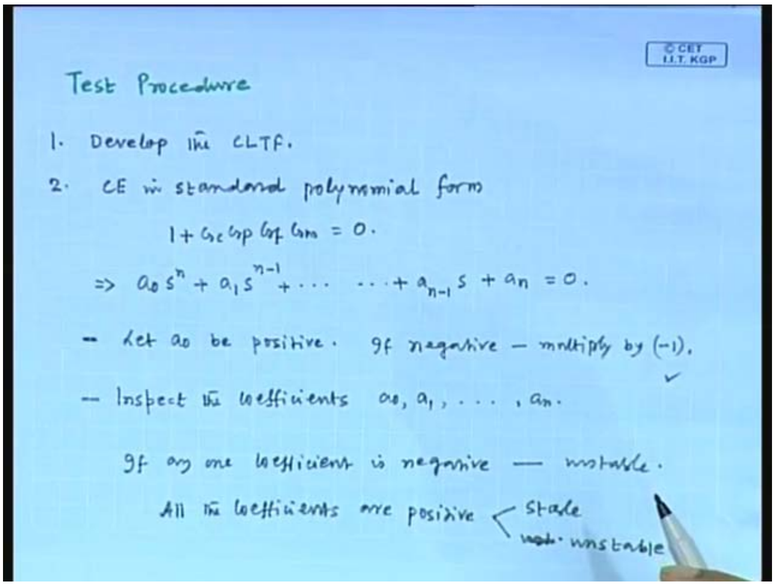

Now, we will known the algorithm I mean the test procedure, we will know the test procedure, what are the stets we need to follow to perform this Routh stability analysis. First point is we need to develop the closed loop transfer function, so in the first we need to develop the closed loop transfer function this is the first point. Secondly write the characteristics equation in standard polynomial form, that is 1 plus G c G p G f G m equals 0, now we will write this in standard polynomial form, as a naught s to the power n plus a 1 s to the power n minus 1 a n minus 1 is plus a n equals 0.

We need to the represent the characteristics equations in this standard polynomial form, which have which is in thought are polynomial. Let a naught be positive, under the second point few important ((Refer Time: 31:31)) we need to include, first point is let a naught be positive, if a naught is negative multiplied both sides of this polynomial by minus 1. Now, inspect the coefficients, in the next inspect the coefficients a 1 a naught a 1 up to a n, inspect the coefficients if any 1 coefficient is negative then the closed loop system is unstable, there is no need of any further analysis needed.

If all the coefficient are positive, the system may be stable or unstable, if all the coefficients are positive, it is difficult to say whether the system is stable or unstable. So, we can say that if all the coefficients are positive, it may be stable may be un stable, so in this situation who in need to go for further analysis. So, this is the second point a naught should be positive we need to assuming a naught be positive, if that is not positive we need to multiply both sides of the characteristics equation by minus 1.

In the next step we need to inspect the coefficients in any one coefficients is negative the system is unstable, so there is no need of further analysis, if all the coefficients are positive the system may be stable or may not be stable. So, to conclude weathered the system is stable or unstable we need to go for further analysis.

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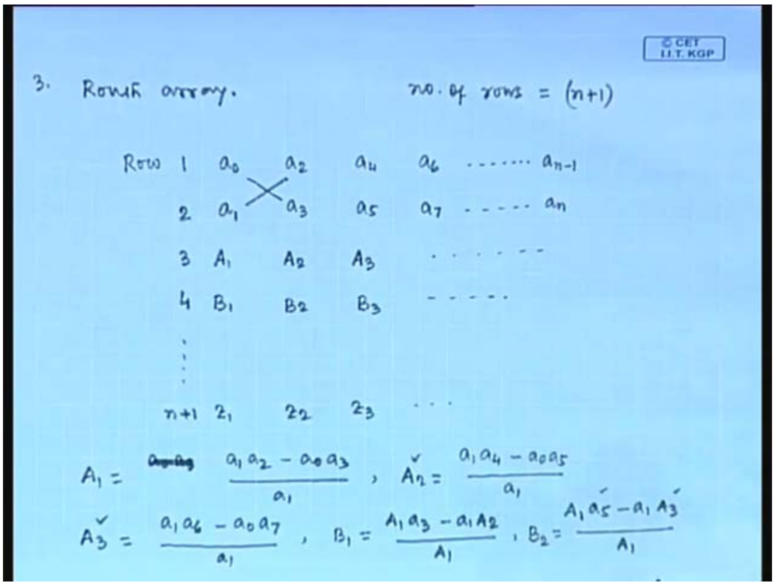

So, that is basically the third point I mean third step of this test, in the third step we need to form Routh array, the Routh array consist of a number of rows. So, we will write the row one by one this is first row, second row, third fourth like this in plus 1, so you remember if the polynomial is in order the number of rows should be n plus 1. So, number of rows is equal to n plus 1, where in is the order of the polynomial, now we will write in this array all the coefficients of the standard polynomial. You we just re visit the characteristic equation the characteristic equation, we have return in this standard polynomial form which in clouds the coefficients starting from a 1 to a n.

Now, we will write the coefficients like this, in the first row we will write a naught, in the second row we will write a 1, then again a 2 a 3 a 4 a 5 a 6 a 7 like this, last one will be a minus 1, a n minus 1 and this one will be a n. So, the elements of the first row include a 1, a 2 a 4 a 6 a n minus 1, the second row includes a 1 a 3 a 5 a 7 through a n.

So, this is the ((Refer Time: 38:14)) we need to write all the coefficients in first row and second row, the elements of the other rows we need to calculate.

The elements of the other rows starting from 3 to n plus 1, we need to calculate, that calculation procedure has been propose by Routh suppose elements are A 1 capital A 1 capital A 2 capital A 3 like this. The elements of the fourth row are assumed B 1 B 2 B 3 and the elements of the last row n plus 1 row or Z 1 Z 2 Z 3, this is the root array. Now, if how we can I mean what are the expirations for calculating the elements starting from third to n plus 1 row.

Capital A 1 which is the first element of third row is calculated as a naught they are calculated as a 1 a 2 minus a naught a 3 divided by a 1, a 1 a 2 divided by minus a 1 minus a naught a 3 divided by a 1. This the expiration for a 1 the expiration of capital A 1 is a 1 a 2 minus a naught a 3 divided by a 1, the expiration for capital A 2 is a 1 a 4 minus a naught a 5 divided by a 1, I think now we can write the expiration for all other elements.

Expiration for A 2 you we just left that column within which a 2 is present, so a 1 a 4 minus a naught a 5 divided by a 1, similarly the expiration for a 3 is a 1 a 6 minus a naught a 7 divided by a 1. a 3 is present in the third column, so you will left the elements of third column and we will stared from first column always that is why A 3 has the expiration and that is expect by a 1 a 6 minus a naught a 7 divided by a 1.

What will B the expiration for B 1 then B 1 will be capital A 1 small a 3 minus small a 1 capital A 2 divided by capital A 1, the expiration for B 1 we can write as capital A 1 a 3 minus small a 1 capital A 2 divided by capital A 1. Similarly, we can write B 2 equals a 1 a 5 minus a 1 capital A 3 divided by capital A 1 capital B 2, we can write as capital A 1 small a 5 minus small a 1 capital A 3 divided by capital A 1.

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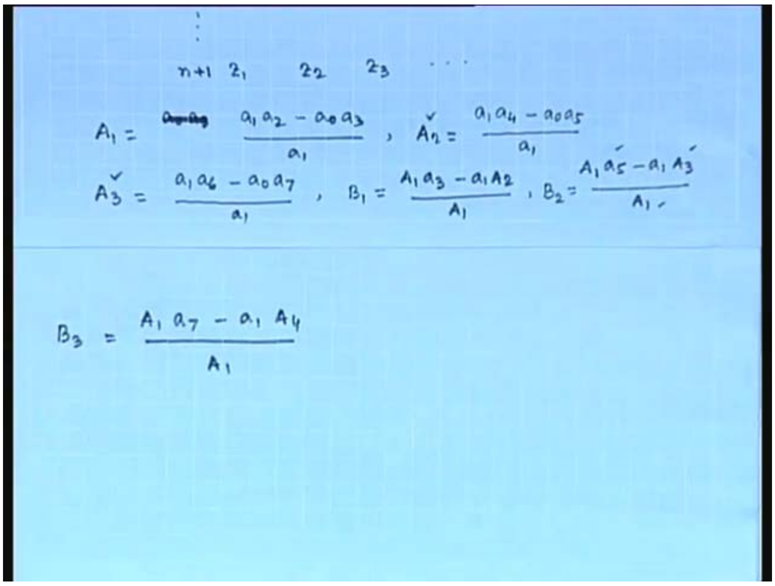

Another element is B 3, so B 3 we can write as Capital A 1 capita small a 7 minus small a 1 capital A 4 divided by capital A 1 B 3, so we need to write here the element a 4, so capital A 1 small a 7 minus small a 1 capital A 4 divided by capital A 1, that is that expiration for B 3. So, this is the third point in which we have discuss the formation of root array, next we will analyze the stability in the forth point.

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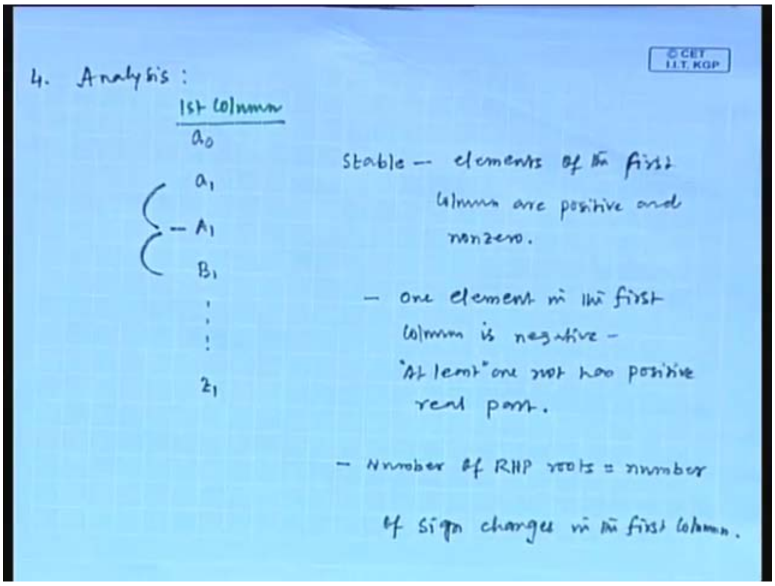

In the forth point we will discuss the analysis of this root array, so what are the elements are a naught a 1 capital A 1 capital B 1 last element is z 1, these are the elements in the fast column. Now, the close loop system is stable if all the elements are positive and non 0, so this is another point is if any one element in the first column is negative then at least one root has the positive real part.

If any one element in the first column is negative, then at least one root has positive real part, another point is if one element in the first column is negative. Then at least one root at least one root has positive real part, I mean that root lie in the right half plane, see we are mentioning here at least; that means, there may be more than one roots lie in the write half plane.

Now, question is how we can say more than one root are off ward situation more than one roots lie in the right half plane, so best on that there is one point number of roots lie in the write half plane equal to the number of sign changes in the first column. So, in the next point we will write the number of right half plane roots is equal to number of sign changes in the first column, suppose this is the first column which includes the elements a naught a 1 capital A 1 capital B 1 2 z 1, among these element one root among these elements one is negative.

That mean say this capital A 1 is negative, so how many sign changes are involved to one sign change from second column to third column, another sign change from third column to third row to fourth row. So, two sign changes involved; that means, two roots have positive real parts, suppose among this elements in the first column one element is 0, then what about the stability of that closed loop system. See, if all the roots are positive then the system is stable, if an one root is negative system is un stable, so if one element is 0 that is the critical stability condition.

So, at critical stability condition we get sub stain oscillation, so if one root one element in the first column is 0 then we get sub stain oscillation, I mean oscillation with constant amplitude. This is all about the Routh Hurwitz stability test and one point is very clear that according to this Routh Hurwitz test we can only now whether the closed loop system is stable or un stable. Using this test we cannot determine the values of tuning parameters, see we need to conduct the root stability test, to know the stability of a close loop system.

That means, the process including the controller, we want to know the stability of a closed loop system, closed loop system means the process which includes the controller.

Now, according to this test we can only say that the power all system in stable or un stable, using or conducting this test we never determined of exact values of the tuning parameters. So, this is not basically the tuning test, this is basically a stability test.

Thank you.