Process Control and Instrumentation

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Lecture No. # 19

Feedback Control Schemes

In the last class we are discussing in the P control of a second order process, in the last class we are discussing of proportional control of a second order process we will continue the topic today.

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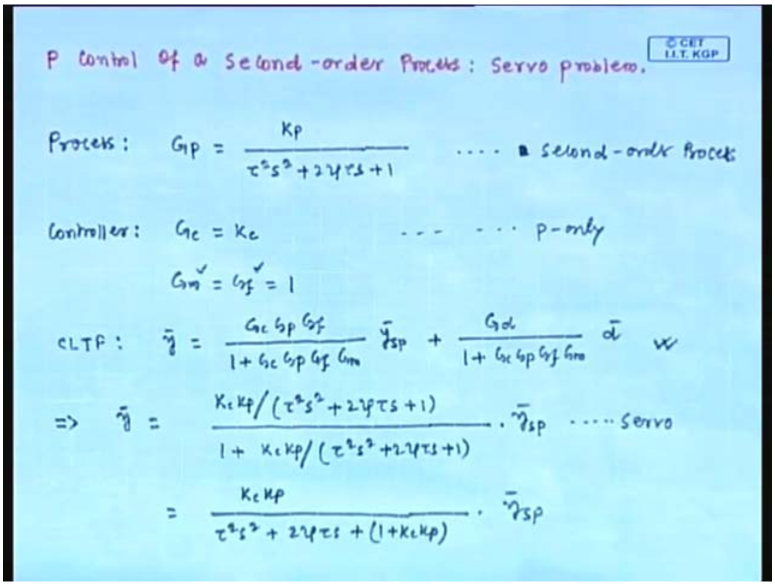

So, p control of a second order process and we will consider servo test, basically you want to see the closed loop response of a second order process under P only control. So, what are the different element of closed loop process, first element is the process that is basically open loop process. The open loop process with second order system has the transfer function G p and that is equal to K p divided by tau square s square plus 2 zeta tau is plus 1.

This is the transfer function of base a second order process, next element is the controller we are interested to observe the close loop performance under the P only control.

Accordingly, the controller equation has the form of G c equals K c, this is the transfer function of P only controller, for simplicity will consider G m and G f both are equal to unity.

The transfer function of the measuring device that is the G m and the transfer function of the final control element that is G f or equal to 1, now what is the expression general expression for closed loop transform function. General expression of close loop transfer function is represented as y bar equals G c G p G f divided by 1 plus G c G p G f G m y set point bar plus G d divided by 1 plus G c G p G f G m d bar, this is the general form of closed loop transfer function.

Now, considering or substituting all the individual transfer function in this generalized form, we obtain y bar equals K c K p divided by tau square s square plus 2 zeta tau s plus 1 whole divided by 1 plus K c K p by tau square s square plus 2 zeta tau s plus 1 multiplied by y set point bar. And this is the close loop transfer function for servo problem. Substituting all the individual transfer function G c G p G f G m in the generalized form of this close loop transfer function, we obtain this form for the servo vocals. Now, if we simplify it than we get K c K p divided by t square s square plus 2 zeta tau is plus 1 plus K c K p multiplied by y set point bar.

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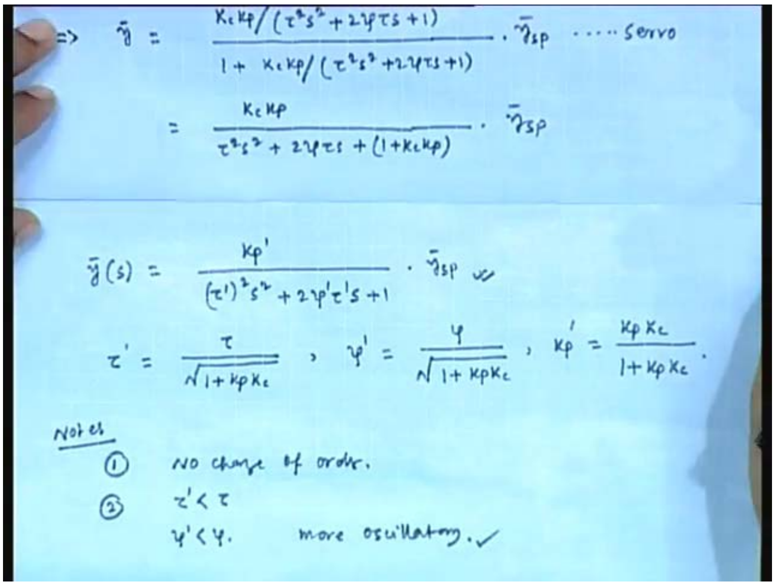

If we divide numerator and denominator by 1 plus K c K p, dividing numerator and the denominator by 1 plus K c K p, than we obtain y bar s equals K p prime divided by tau prime square s square plus 2 zeta prime tau prime s plus 1 multiplied by y set point. But, dividing numerator and the denominator 1 plus K c K p, we obtain this form higher tau prime equals tau Divided by root over of 1 plus K p K c. Zeta prime equals zeta divided by root over of 1 plus K p K c and prime K p prime equals K p K c divided by 1 plus K p K c.

These are the expression with which happen used in this closed loop form, now you will conclude, so due to the inclusion of proportion controller with the second order process, the order does not change the order remains same. So, we will note down few points due to the inclusion of proportion controller there is no change of the order overall system, this is the first point.

Secondly, if we see the expression of tau prime we can say that tau prime less than tau, secondly zeta prime less than zeta it means overall response under P only control becomes more oscillatory, because damping factor is decreased. As a result the overall response under the p control becomes more oscillatory in the last class we discuss up to this.

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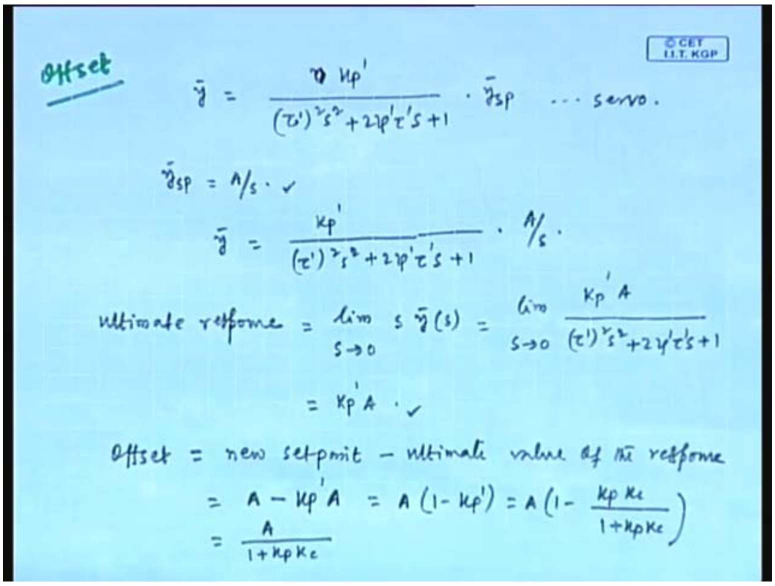

So, next will determine the offset, now out tools loop transfer function is y bar equals K p prime divided by tau prime s square plus 2 zeta prime tau prime s plus 1 y set point bar, this is the close loop transfer function we which we derived for servo case. Now, we will consider a step change in set point with magnitude A, we use to determine the offset introducing a step change for set point with magnitude A. So, considering this y set point equal A by s that close loop transfer function, becomes K p prime divided by tau prime square s square plus 2 zeta prime tau prime s plus 1 multiply by A by s.

Now, according to final value theorem the ultimate response we determine as limit s tends to 0 s y bar s, so limit s tends to 0 K p prime A divided by tau prime s square plus 2 zeta prime tau prime s plus 1, which is equal to K p prime A. The ultimate response for the case of second order system under P only control with the introduction of set p change point with the magnitude A.

We obtain K p prime A than what about of offset we know the offset equals new set point minus ultimate value of the response to new set point is A ultimate value is K p prime A. , so A multiplied by 1 minus K p prime. Now, we will substitute the expression of K p prime, so A 1 minus K p prime is K p K c divided by 1 plus K c, what is the expression of K p prime, K p prime is K p K c divided by 1 plus K p K c. So, the offset becomes A divided by 1 plus K p K c, this is the offset, now when the offset becomes 0 or when offset approach is to 0 ,when K c is to infinity.

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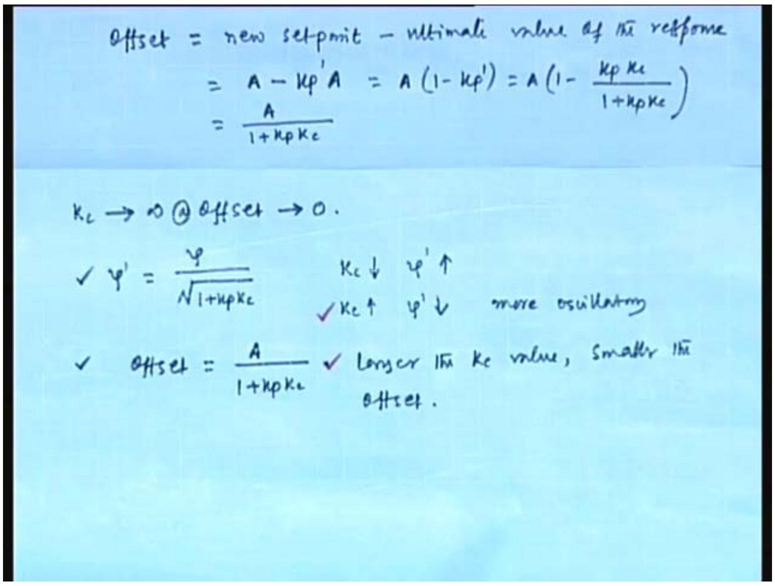

When K c tends to infinity, the offset approach is 0, but this is not through in practice due to some limitation of K c value, now you will try to produce one plot to observe the dynamic behavior of the closed loop system. Now, we obtain the expression of zeta prime as zeta prime equals zeta divided by root over of 1 plus K p K c, this expression we obtain. So, with the decrease of K c, zeta prime increases; that means, with the increase of K c zeta prime decreases.

So, if the increase of K c value the overall response may be more oscillatory, so with the increase of K c the overall response becomes more oscillatory. This is one point, another point is we know the offset equals A divided by 1 plus K p K c, the larger the K c value the smaller the offset. From this expression we can say that the larger the K c value the smaller the offset, but the considering this two points with the increase of K c, the overall this becomes more oscillatory and another point is the larger the K c the smaller the offset.

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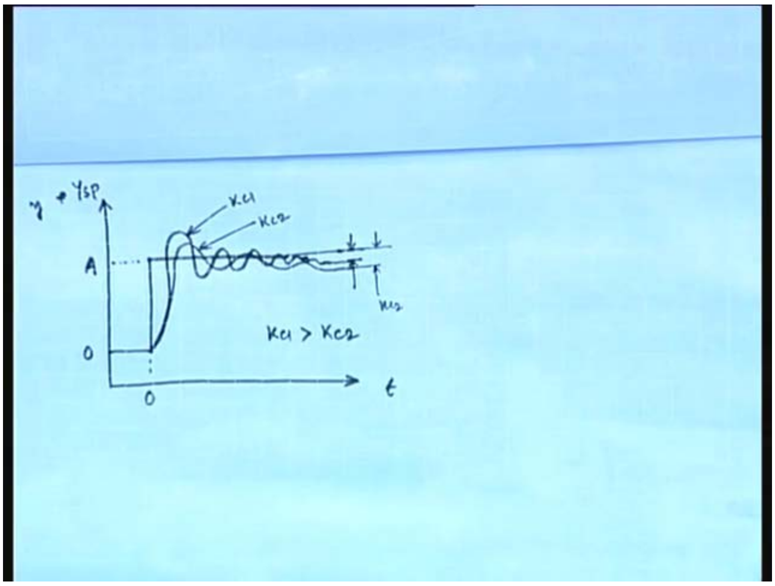

We will try to produce the plot and in the close loop response keeping in mind this two points. So, we will produce y verses time y and y set point will consider and here time, now we consider a step change in set point with magnitude A, so this is magnitude A, initially it was say 0 and we are introducing step change a time equals 0.

Now, the close loop look response is like this, for a particular K c that is suppose K c 1 this is for another K c suppose K c 2, so here K c 1 is greater than K c 2 fine. So, for the last K c that is K c 1 we obtain more oscillatory response, one point an second point is for larger K c the offset is relatively small and this is for the case of K c 2 offset is large for small K c.

So, above two points are taken into account in the development of this close loop response, in the next we will discuss the effect of integral control action. So, for we discussed the effect of proportion action on the overall response of a process, in the next we will discuss the effect of integral action on the overall response of a process.

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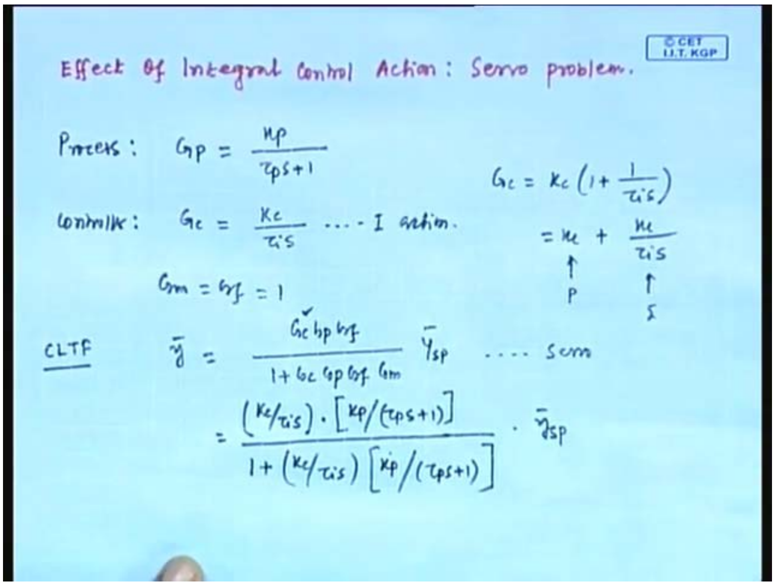

So, next topic is effect of integral control action and we will consider some servo problem only, the regulatory problem is left for the students. So, we will consider different elements and there expressions first, first element is the process and discuss to element integral control action we consider first order process. So, for the first order process is transfer function can be written as G p equals K p divided by tau p s plus 1, this is the transfer function of a first order process.

Next element is the controller, the controller is the integral controller what is the transfer function of integral controller, the transfer function of the integral controller is written as G c equal K c divided by tau i s. For G i control is the transfer function is K c multiplied by 1 plus 1 divided by tau I s; that means, K c plus K c divided by tau i s. So, this is the transfer function of p and this is the transfer function of i, so if we consider only the integral controller than the transfer function is G c equal K c divided by tau i s.

And simplicity we will consider the transfer function of measuring device and the transfer function of final control element, both are unity; that means, G m and G f equal to 1. Now, we will try to develop the close loop transfer function substituting all this individual transfer functions, in the next we will develop the closed loop transfer function.

The general form is G c y bar equals G c G p G f divided by 1 plus G c G p G f G m y set point bar, now we will substitute the individual transfer function in this close loop transfer function for the case of servo problem G c is K c divided by tau i s. This is G c G p is K p divided by tau p s plus 1 and G p is equal to 1 than 1 plus K c divided by tau i s multiplied by K p divided by tau p s plus 1 and y set point.

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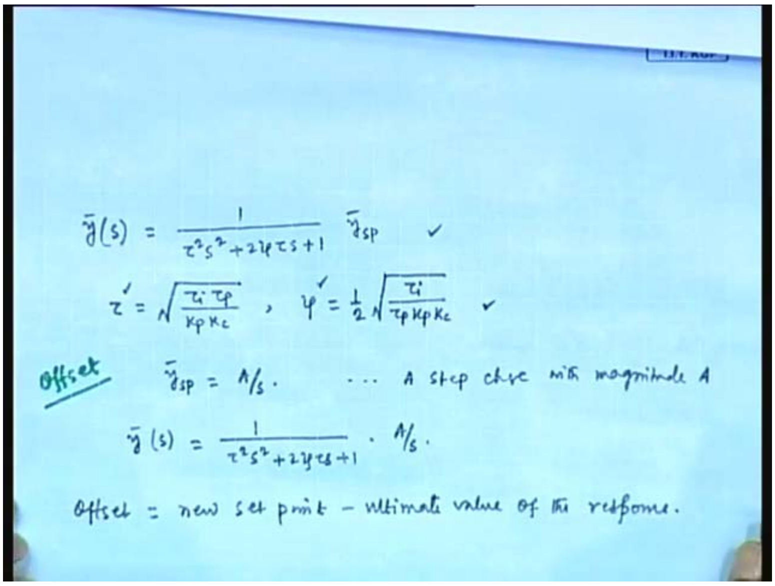

If we simplify this expression than we obtain y bar s equals 1 divided by tau square s square plus 2 zeta tau s plus 1, simplifying we obtain this expression multiplied by y set point bar. First we substituted all the individual transfer function in generalized from of those transfer function, than after simplification we are obtain this expression, where are tau is root over of tau i tau p divided by K p K c. Here, tau has the this expression I mean tau has this relationship, similarly zeta is equal to half root over of tau i divided by tau p K p K c.

So, these are the two expressions which have in consider in the development of close loop transfer function, you see found the co-relation is clear that the tau and zeta both depend on the value of K c and tau y. From these two co-relation, it is obvious that the tau and zeta both depend on the value of K c and tau y. Where, K c and tau y are the tuning parameters, controller tuning parameters, in the next we will try to determine the offset.

Now, for the determining offset we will consider the step change in set point with magnitude A; that means, y set point bar is equal to A bar s. We are consider in step change with magnitude A, than the close loop transfer function becomes 1 divide y bar is equals 1 divide by tau square s square plus 2 zeta s plus 1 multiplied by A by s. Then offset is equal to new set point minus ultimate value of the response, we know this co-relation offset equals minus ultimate value of the response.

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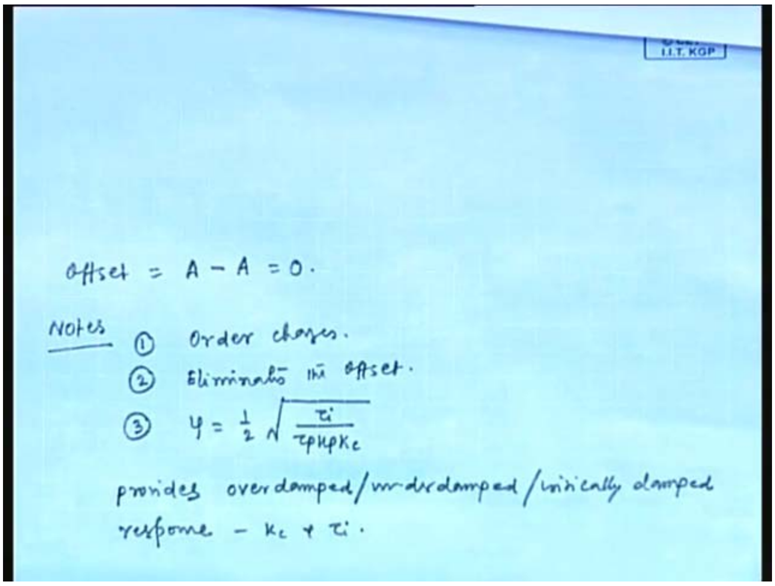

So, new set point is A, and ultimate value of the response is how much A, so offset becomes 0. So, we can say that the integral control action can eliminate the offset, it is very obvious from this that the integral control action can eliminate the offset. Now, we will note down some few important points, is any change of order due to the inclusion of integral action. Yes, so due to the integral action the order changes, the order of the overall system changes, due to the inclusion of integral action this is the first point.

Second point is we have seen that due to the inclusion of integral action the offset become 0, so second important point is the integral action can eliminate the offset, integral control action eliminates the offset. Third important point is as I mentioned zeta want depends on the value K c and tau i, if we see the expression of zeta we can say that zeta depends on the value of K c and tau i.

So, close loop system provides over dammed or under damped, critically damped response depending on the value of K c and tau i, closed loop system provides over damped or under damped or critically damped response depending on the value of K c and tau i. So, the selection of K c and tau i is very important.

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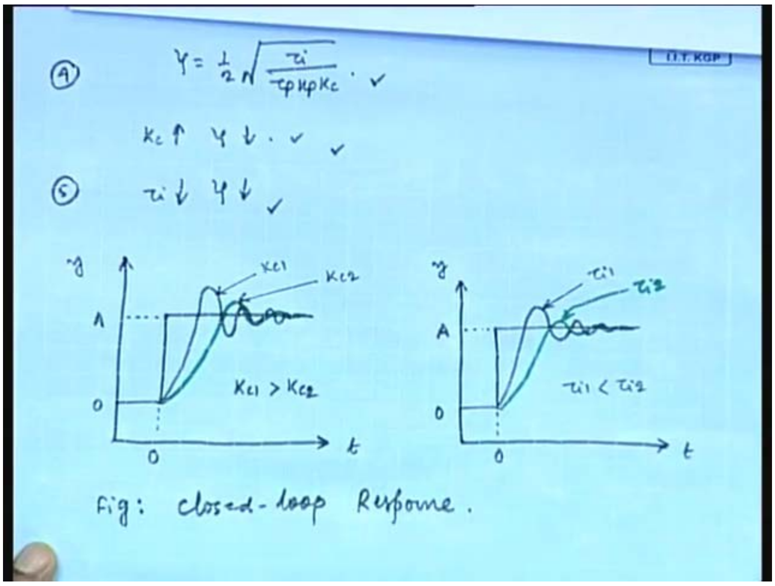

Fourth point is if we again see the expression of zeta that is zeta is half root over of tau i divided by tau p K p K c, we can say that with the increase of K c, zeta decreases. So, the over damped response may move to under damped response with large over suite and big ratio. If K c is sufficiently large the over damped response may move to under damped response with large over suite and big ratio, in the same line we can conclude again that with the decrease of tau i zeta decreases. You see the expression of zeta it is clear that with the decrease of tau i zeta decreases.

So, if the tau i is sufficiently small the over damped response may move to under damped response with large over large suite and big ratio. Now, we is to produce plots best on the common 4 and 5, in the fourth point we have mention that with the increase of K c zeta decreases; that means, more oscillatory response.

So, that we will reflect through this plot y verses time, a step change is introduced with magnitude A, this is suppose 0 and the step change is introduce that time equals 0. Now, this is the close loop response for a particular K c value, suppose that is K c 1 another close loop response we obtain for the value of K c 2. So, which one is greater K c 1 or K c 2, K c 1, because our fourth point is with the increase of K c we get more oscillatory response. So, the K c 1 is larger than K c 2.

So, that is why we obtain more oscillatory response, but definitely here tau y is kept constant we only varied the K c value. Similarly, we will produce the close loop response based on 0.5 that is with the decrease of tau i zeta decreases; that means, with decrease of tau i the overall response become more oscillatory. So, we will try to produce y verses t, similarly a step change is introduce in set point with magnitude, suppose A at that time t equals 0. This is the close loop response for a particular tau value, suppose that a is tau i 1, another sponge will we obtain one tau i 2.

So, which one is larger among this two tau i 2; that means, tau i 1 is less than tau i 2, with the decrease of tau i zeta decreases; that means, more oscillatory response. So, first one is the close loop response with the valuation of K c and second one is the close loop response with the valuation of tau i. So, these two points 4 and 5 clearly say that we need judicially tune the values of K c and tau i. So, we discuss effect of proportion action and integral action and the close loop response, the third action is the derivative action.

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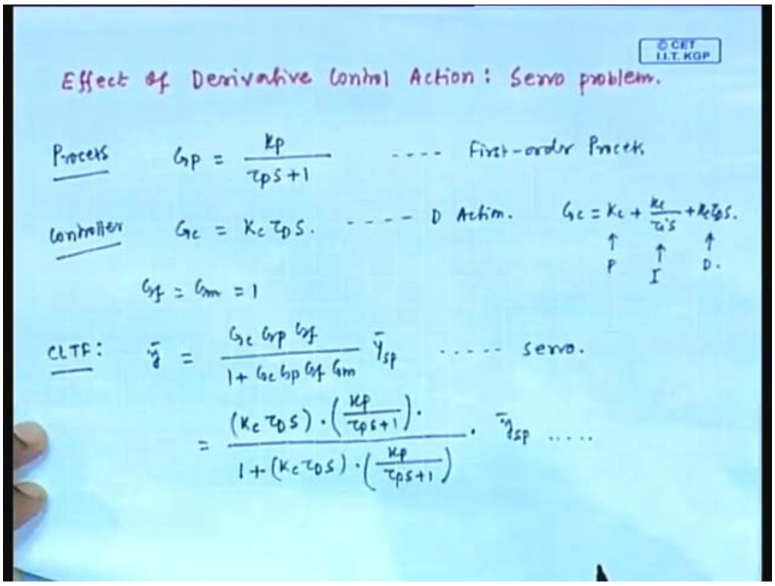

We will discuss that I mean discuss the next derivative action on close loop response, effect of derivative action, derivative control action and we will consider the servo problem. In the discussion of derivative action we will consider the first order process and servo problem, in a close loop black diagram the important element is process. The transfer function of the process of a first order process can be written as G p equals K p divided by tau p s plus 1, this is the transfer function of a first order process.

Next element is the controller recently we discuss derivative action, so the controller is only the derivative controller, what is the function of derivative control, K c tau p s. This is derivative action, because the transfer function of P I D controller, we have written K c K c plus divided by tau i s plus K c tau p i s. This is the transfer function of proportion action, this is the transfer of integral action and this is the transfer function derivative action.

And simplicity for will come consider the transfer function G f and G m both are equal to 1, now we will try to develop the close loop transfer function substituting of all these individual expressions. In the next will develop close loop transfer function which is express in generalized form as y bar equals G c G p G f divided by 1 plus G c G p G f G m y set point bar, this is generalized close loop transfer function for servo base.

Now, if we substitute the individual transfer functions, than we obtain G c as K c tau D s, next transfer function is G p; that means, K p divided by tau p s plus 1 and G f is 1, whole divided by 1 plus K c tau D s multiplied by K p divided by tau p s plus 1 multiplied by y set point bar. Now, if we this is the close loop transfer function we obtain, substituting all individual transfer functions.

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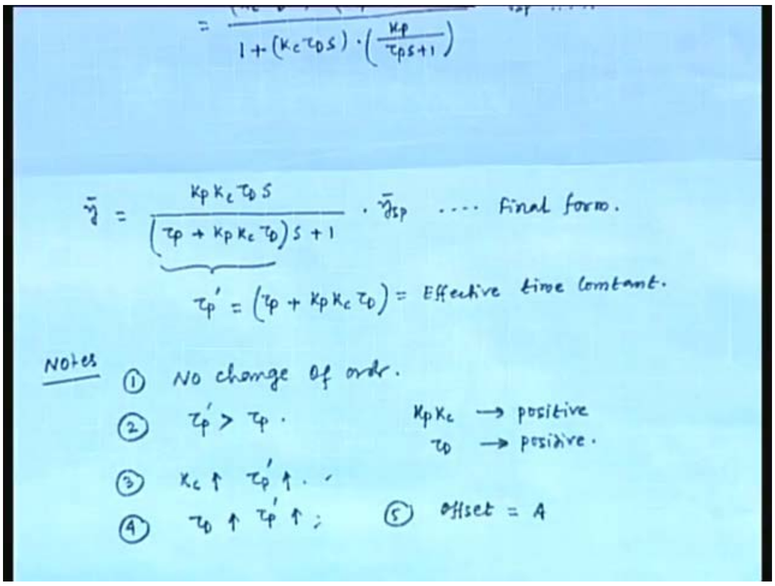

Now, if we rearrange further than we obtain the final close loop transfer function as y bar equals K p K c tau D s divided by tau p plus K p K c tau D multiplied by s plus 1 y set point bar, this is the final form of close loop transfer function. Now, this is the time constant of the close loop transfer function, so this time we can say we can call as effective time constant denoted by tau p prime. So, tau p prime equals tau p plus K p K c tau D, this is called as effective time constant we can call this time constant as effective time constant.

So, which one is larger tau p or tau p prime, tau p prime now we will note down few important points, due to the inclusion of derivative action is there any change of order. No, the order of overall system does not change, we have taken first order process and overall processing is first order. So, there is no change of order of the overall system due to the inclusion of derivative function there is no change order of the overall system, this is the first point.

Second point is regarding the time constant, the time constant tau p prime is greater than tau p, because K p K c is positive and tau D is also positive. K p K c is positive multiplication of K p K c I mean multiplication of K p and K c that is positive and tau D is time constant time constant is always positive. So, this is also positive therefore, tau p prime is greater than tau p, so what it indicates the overall response becomes ((Refer Time: 47:34)).

Third point is as K c increases, then what happens for tau p prime, if K c increases then tau p prime also increases with the increase of K c tau p prime increases; that means, the overall response becomes slower due to the increase of K c. Similarly, what happens for tau D, if tau D increases the effective time constant increases. So, in the derivative action there are two tuning parameters involved, one is K c and another one is tau D. So, that is why we observe the effects of K c as well as tau D with the increase of K c tau p prime increases and with the increase of tau D also tau D increases; that means the overall response becomes slower due to the increase of K c and tau D.

Fifth point is related to the offset, can you calculate the offset with introducing step change point with set point magnitude A. What about offset? Offset is A, so fifth point is the derivative controller cannot eliminate offset, these are the five important points related to the close loop response and under derivative controller action. So, we have discussed about three particular controller proportions one is P only, second one is P I and third one is P I D.

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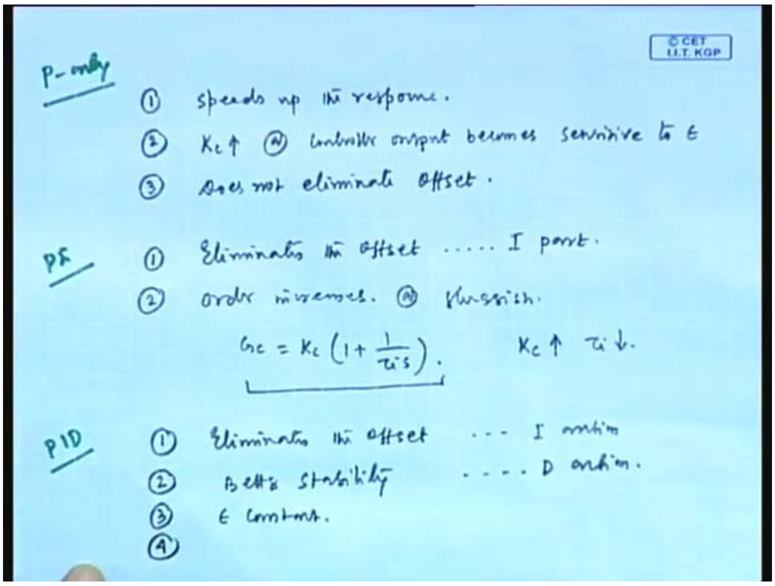

Now we conclude these control actions one by one, so first one is P only controller, what you conclude the P only controller. It speeds up the response, if you see the response of over loop system and if you see the response of a process, same process under P only controller, we obtain first response. In case of process under P only controller, if we include a proportional controller with a process the overall response become faster than the open look response. Therefore, we conclude that the P only controller speeds of the response.

Secondly, we can say that with the increase of K c value, the proportional only controller output becomes more sensitive to error, with the increase of K c value the P only controller output becomes more sensitive to the actuating error signal. This is the second conclusion on P only controller, third one is it cannot eliminate offset, it does not eliminate offset, these are the three important points related to P only controller.

Next is P I controller, most important point related to P I controller is that eliminate the offset, so which part eliminate the offset integral part integral action eliminates the offset. Second point is the order for P only controller there is no change of order, but P I controller order is increased. So, second point is order increases and as the order increases the overall response will sluggish.

Now, how we can improve this situation, due to the inclusion of a integral action with p action, the overall response becomes sluggish. Now, we want to speed of the response, how we can do that either by increasing K c and decreasing tau i is in did. We can speed of the response either by increasing K c or by decreasing tau i, if you see the controlled equation, P I it is controller equation cleared that with the increase of K c the overall system response become faster. But, if K c is larger than the sorter value or if tau i is sufficiently small, the process those constant I mean there some instability problem.

So, if the K c value is sufficiently large or tau i is sufficiently small the instability problem may raise, for the case of P I controller. Next is the P I D controller, so first we can say that it eliminates the offset P I D controller have eliminates the offset and this is the definitely effect to the integral action.

Now, for the case of P I controller, we observe that with the increase of K c value I mean when K c is larger the certain limit or tau i is sufficiently small there is some instability problem, that can be improve by the addition of derivative action. So, P I D control has batter stability criteria and this is due to derivative action only. Third point is if error is constant, then there is no derivative action because deep silent d t is there, if error is constant then there is no derivative action. And last point is for a noisy is almost 0 error, the derivative term is needs to large control action although that is not require. So, these are the concluding remarks on three proportion controller actions.

Thank you.