Process Control and Instrumentation
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Lecture - 18
Feedback Control Schemes (Contd.)

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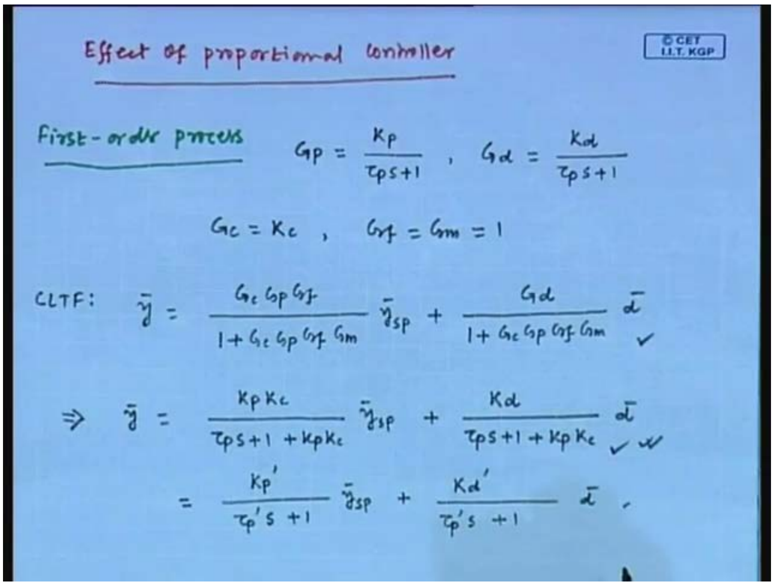

We will continue our topic which we started in the last class. That is the effect of proportional controller, and we will observe first for the first order system, and then for the second order system. So, initially will discuss the effect of proportional controller for the first order process, for the first order process we derived the transfer function G p as K p divided by tau p s plus 1. The transfer function G d equals K d divided by tau p s plus 1 the controller is proportional controller, so the transfer function of the proportional controller is j c equals K c.

The transfer function for P only controller can be written has G c equals K c and for simplicity we are assuming G f and G m both are unity, for simplicity we are assuming the transfer function of measuring device and final control element both are unity. Now, we will try to develop the closed loop transfer function, so for that purpose we will start from the general form of close loop transfer function which can be written as y bar equals G c G p G f divided by 1 plus G c G p G f G m y set point bar plus G d divided by 1 plus G c G p G f G m d bar.

Now, we need to substitute all the individual transfer functions in this generalized form, if we do that than we will get y bar equals K p K c divided by tau p s plus 1 plus K p K c y set point bar plus K d divided by tau p s plus 1 plus K p K c d bar. Substituting all individual transfer functions in this generalized closed loop transfer function, we obtain the close loop transfer function for the first order system in this form.

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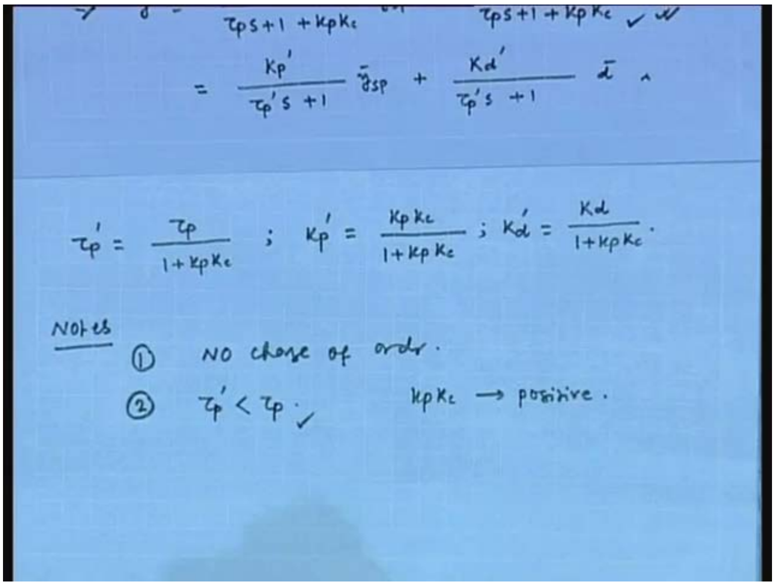

So, this equation we can write again as K p prime divided by tau p prime s plus 1 y set point bar plus K d prime divided by tau p prime s plus 1 d bar, this form we can write again by this form. However, tau p prime equals tau p divided by 1 plus K p K c, K p prime equals K p K c divided by 1 plus K p K c and K d prime equals K d divided by 1 plus K p K c. Now, what conclusion we can make based on the final form of the close loop transfer function, due to the addition of proportional controller is there any change up order in the our own system.

No, there is no change of order due to the inclusion of proportional controller, so no change of order of the overall system, due to the inclusion of proportional action with the first order system there is no change up order. Second conclusion is tau p prime second conclusion can be made based on this, tau p prime less then tau p and K p K c is always positive and therefore, tau p prime is less then tau p.

So, what it indicates, due to the inclusion of proportional action the overall response becomes faster because time constant is decreased, so this is a second conclusion due to the inclusion of proportional action the overall response becomes faster. Now, as I mention that that control performance is observed by conducting two tests one is servo test and other one is regulatory test.

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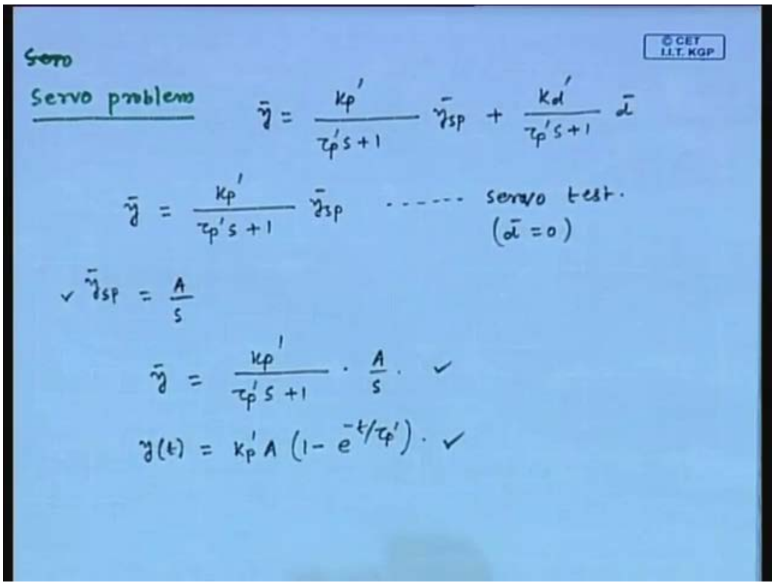

So, in the next you will conduct these two tests for the first order process and the P only control, so first we will discuss the servo problem. Our close loop transfer function is represented as y prime equals y bar equal K p prime divided by tau p prime s plus 1 y set point bar plus K d prime divided by tau p prime s plus 1 d bar. So, for servo problem d bar equals 0; that means, the transfer function becomes y bar equals K p prime divided by tau p prime s plus 1 y set point bar.

This is for the case of servo test, because for servo test there is no change in the disturbance, no change in the disturbance is considered. Now, we will introduce if state change in the set point value and you will observe the transient behavior of first order process under P only control. We will consider a state change in the set point with a magnitude of A, and we will observe the transient behavior of the close loop process.

So, y bar becomes K p prime divided by tau p prime s plus 1 multiplied by A by s agree by considering this set point change in by considering the step change in set point value we get this form. Now, if we take the inverts of Laplace transform, we get y t it is equal to K p prime A 1 minus exponential of minus t divided by tau p prime, taking inverse of Laplace transform we get this form of y in time domain.

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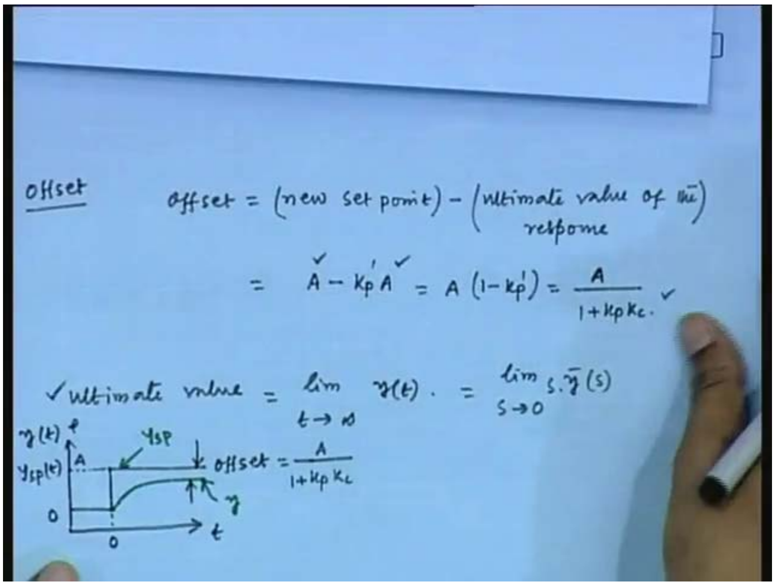

Now, you will try to discuss the offset I mean we will try to determine either the process under P only controller shows any offsets or not. Offset is calculated by representing as new set point minus ultimate value of the response, offset is calculated using this form new set point minus ultimate value of the response. Now, ultimate value of the response means limit t tends to infinity y t ultimate value is calculated by using the final volume theorem, that is limit tends to infinity y t or we can represent in Laplace domain also by limit s tends to 0 s multiplied by y bar s.

So, what is the ultimate value of response for this case, set point value is A, because we have introduced a step change with magnitude A and what is the ultimate value K p prime A. If we apply the final value theorem we can obtain the ultimate value as K p prime A, now offset becomes A multiplied by 1 minus K p prime substituting the expression of K p prime finally, we get a divided by 1 plus K p K c.

Offset is calculated by using this form new set point minus ultimate value, here set point is a ultimate value is K p prime A, substituting the expression of K p prime finally, we get the offset equals A divided by 1 plus K p K c. So, if we produce the plot in terms of transient response and y set point t, now we are introducing step change in y set point, so initially it was 0, now we are giving a change in y set point, this is A at time t equals 0.

Now, the process under P only control response like this, this is the process response under P only control y and this is representing y set point. This plot represents the process behavior under P only control scheme and also by introducing step change in set point value with a magnitude of A. Now, this deviations steady state deviation of the controlled variable from set point value is offset, this difference is offset and mathematically it is A divided by 1 plus K p K c, mathematically, we obtain offset equals a divided by 1 plus K p K c.

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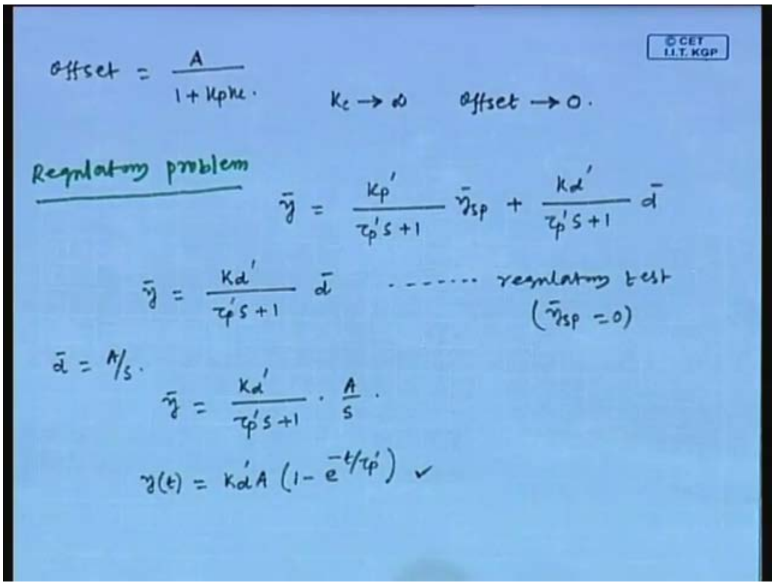

Now, how K c tends to infinity, what about offset we got offset equals 1 divided by 1 plus K p K c offset equals A divided by 1 plus K p K c. Now, how when K c tends to infinity offset approaches 0, but we discussed in the last class that there is a maximum limit of K c beyond which the process goes un stable. So, we cannot arbitrarily select a large value of K c. So, K c tends to infinity this cannot be considered in practice, due to some maximum limit of K c value.

Next we will consider the regulatory test, next we will discuss the regulatory problem, so our close loop transfer function for the first order system under P only control is K p prime divided by tau p prime s plus 1 y set point bar plus K d prime tau p prime s plus 1 d bar. This is a close loop transfer function for the first order system under P only control, now for the regulatory problem there is no change of y set point occurred.

So, for the regulatory problem the close loop transfer function, becomes y set point equals K d prime divided by tau p prime s plus 1 d bar. This is the close loop transfer function for regulatory test, why are no change in y set point is considered, similarly we will consider a step change in the disturbance with a magnitude of A, we get the close loop transfer function as y bar equals K d prime divided by tau p prime s plus 1 multiplied by A by s.

Taking inverse of Laplace transform we get the process output y in time domain has y t equals K d prime a multiplied by 1 minus exponential of minus t divided by tau p prime, taking inverse of Laplace transform we obtain the y in time domain by this, now you will try to determine the offset.

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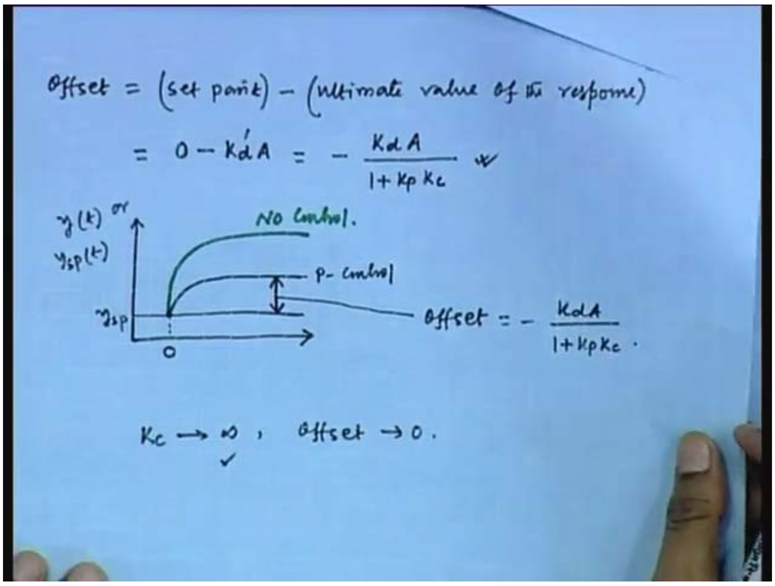

Offset equals new set point minus ultimate value of the response, the ultimate value of the response can be determined by the application of final value theorem. Anyway, how much is the set point, set point is 0 and how much is the ultimate value of the response K d prime A, ultimate value of the response is K d prime A. Substituting the expression of K d prime, we obtain offset equals K d A divided by 1 plus K p K c, substituting the expression of K d prime we finally, get the offset equals minus K d A divided by 1 plus K p K c.

Now, you will produce a plot to observe the transient behavior we did not consider any change in y set point, so y set point remains constant throughout the operation, but we have introduced a step change in the disturbance at time t equals 0. So, the process output changes from t equals 0, this is the process output under p control and if there is no control then the process response behaves like this. We did not introduce any change in the set point therefore, the set point remains constant throughout the operation, but the disturbance is changed at time t equals 0.

Therefore, the process changes to a different steady state value starting from time t equals 0, and if there is no controller then the process response like this. Now, this is the steady state error; that means, this is the offset which is equal to minus K d A divided by 1 plus K p K c, now if you see the expression of offset again we can say that how when K c approaches infinity, the offset approaches 0.

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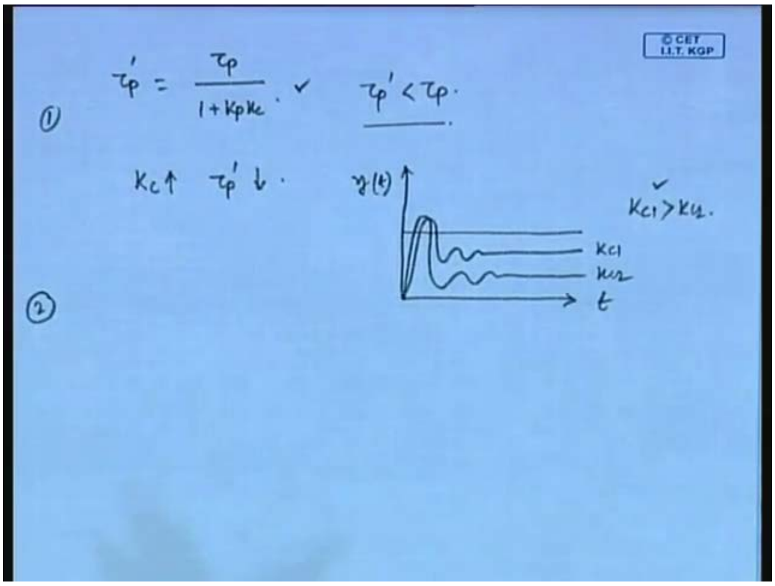

If we see the expression of the offset in the regulatory test we can say that if K c approaches infinity the f offset approaches 0, now you will conclude. So, if we see the expression of tau p prime we can say that the tau p prime is less than tau p, if we see from this expression we can say that tau p prime is less than tau p. So, due to the inclusion of proportional controller the overall response becomes faster, another observation is with the increase of K c the tau p prime decreases.

So, if you recall this plot which includes the process response under different K c value, this is K c 2, so here K c 1 is greater than K c 2. Now, if K c 1 is higher than the process under K c 1 which is the steady state quickly then K c 2 that we need to include also in this figure.

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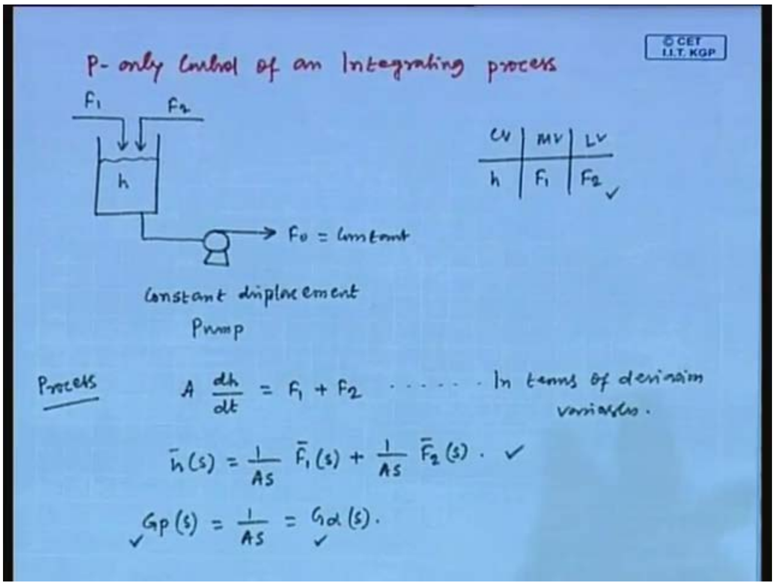

Secondly, we can say that the process under P only controller shows offset, that means it is proved that the P only controller cannot eliminate offset. So, next we will discuss the P only control of an integrating process, so for this purpose we need to consider an integrating process this is a liquid tank, two input streams are involved in this example. One is F 1, another one is F 2, one outlet stream is involved which as the flow rate of F naught.

And one constant displacement pump is installed in the out let section, this is A constant displacement pump, it indicates F naught is a constantan, F naught is constant. Now, in this particular example, we can say that the control variable is liquid height, suppose the manipulated variable is F 1, so load variable will be F 2. If we consider a F 1 as a manipulated variable then F 2 is the load variable, now will consider will develop the transfer functions of the different elements.

So, first is the process performing mass balance, we can write A d h d t equals F 1 plus F 2, all this variables are represented all these variables are here deviation variables. So, this equation is written in terms of deviation variables, and therefore there is no F naught term. So, to represent the deviation variable we are not using super script prime, now if we take Laplace transform and if we rearrange then we get h bar s equals 1 divided by a s F 1 bar s plus 1 divided by A s F 2 bar s, taking Laplace transform we get this form.

So, from this form we get the transfer function of the process, G p with respect to m s 1 by A s and G d s is also equal to 1 by A s, from this equation we get the transfer function G p and G d s and both are equal to 1 by A s.

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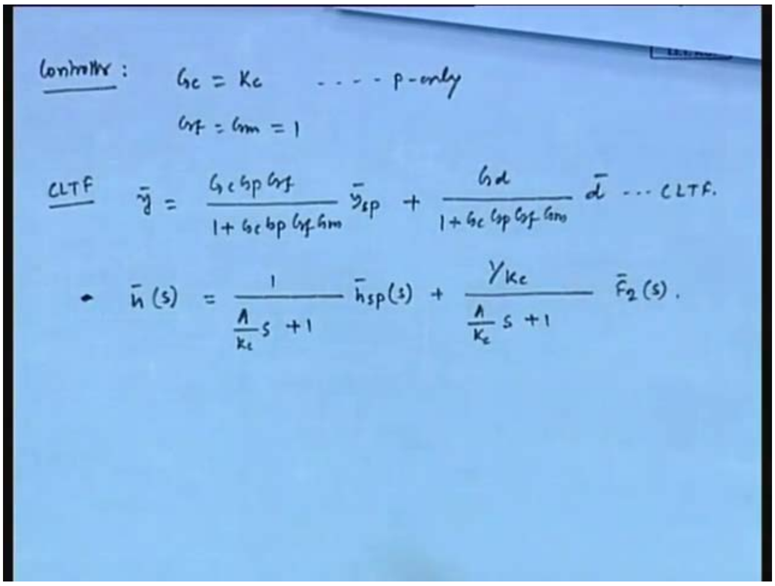

So, next element is the controller, our controller is the P only controller, so the transfer function of P only controller is G c equals K c, this is a transfer function of P only controller. Now, for measuring device and final control element we will consider both the transfer functions as unity, the transfer function of measuring device G m and final control element G f are unity.

Now, we will try to develop the close loop transfer function for the example system, the generalized form is y bar equals G c G p G f divided by 1 plus G c G p G f G m y set point bar plus G d divided by 1 plus G c G p G f G m d bar. This is of generalized form of close loop transfer function, now if we substitute the form of individual transfer function.

Then we get h bar s equals 1 divided by a by K c s plus 1 h set point bar s plus 1 divided by K c A divided by K c s plus 1 F 2 bar s, substituting the expressions of individual transfer functions in the closed loop transfer function, we get this form. Now, we will conduct the servo and regulatory tests to observe the closed loop process behavior, so first we will consider the servo test.

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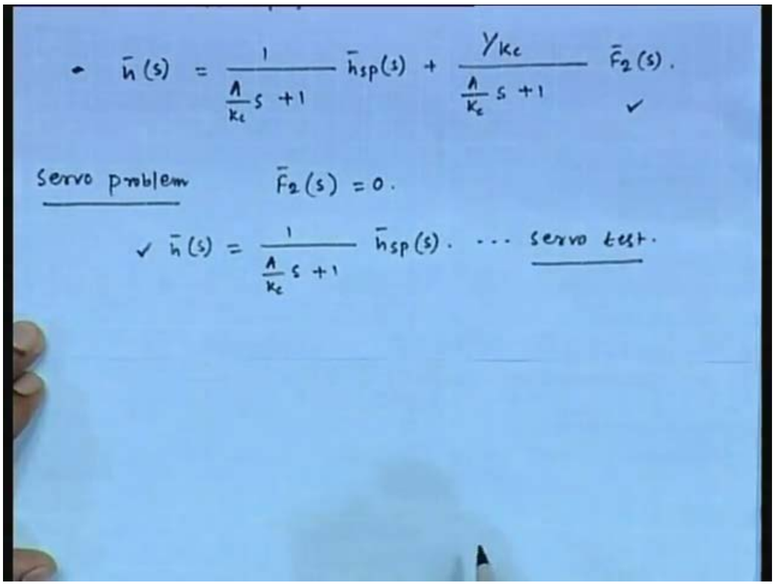

So, this is servo problem, so for servo problem we can write that F 2 bar s equals 0, because there is no change of disturbance, according if the close loop transfer function becomes h bar s equals 1 divided by a by K c s plus 1 h set point bar s. This is for the servo test, considering F 2 bar equals 0, we get this closed loop transfer function for servo problem.

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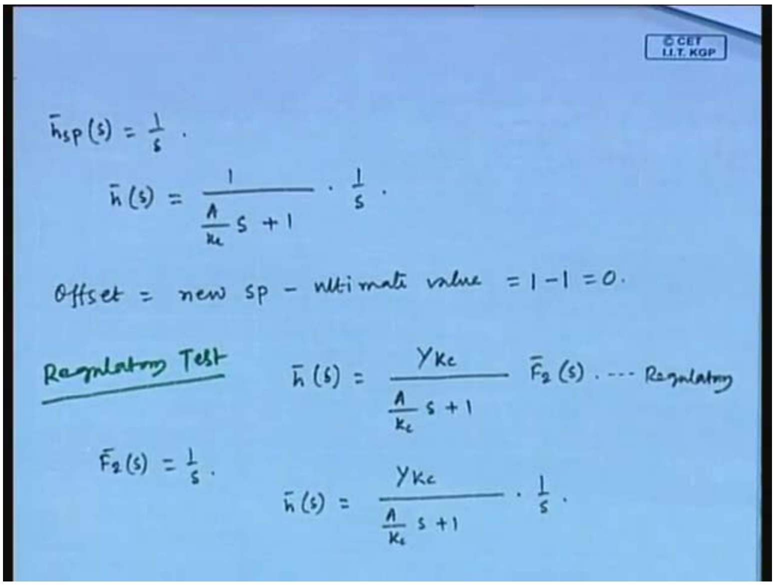

Now, will consider a unit step change in the set point value of in the set point of h; that means, h set point bar s equals by s. So, considering a unit step change in h s p, we get the transfer function s h bar s equals 1 divided by a by K c s plus 1 multiplied by 1 by s. So, what will be the offset, offset we can calculate by finding the new set point minus ultimate value of the response, what is new set point 1, what is ultimate value.

Ultimate value we can find by the application of final value theorem that is also 1, so offset becomes 0, so it is very interesting that for an integrating process the P only controller provides offset free response. It is very interesting that the P only controller provides for an integrating process offset free response, definitely by conducting the servo test we got offset equals 0, but we need to also conduct the regulatory test.

So, in the next we will conduct the regulatory test and we will see what is the offset, our transfer function close loop transfer function is h bar s equals 1 by K c divided by A by K c s plus 1 F 2 bar s. This is the close loop transfer function for the integrating system, how when there is no change is introduced in the set point. So, this is for the regulatory test, introducing a unit step change in F 2 s, we obtain h bar s equals 1 by K c whole divided by a by K c s plus 1 multiplied by 1 by s. Introducing a unit step change in F 2 we obtain the close loop function in this form, can you calculate, now the offset.

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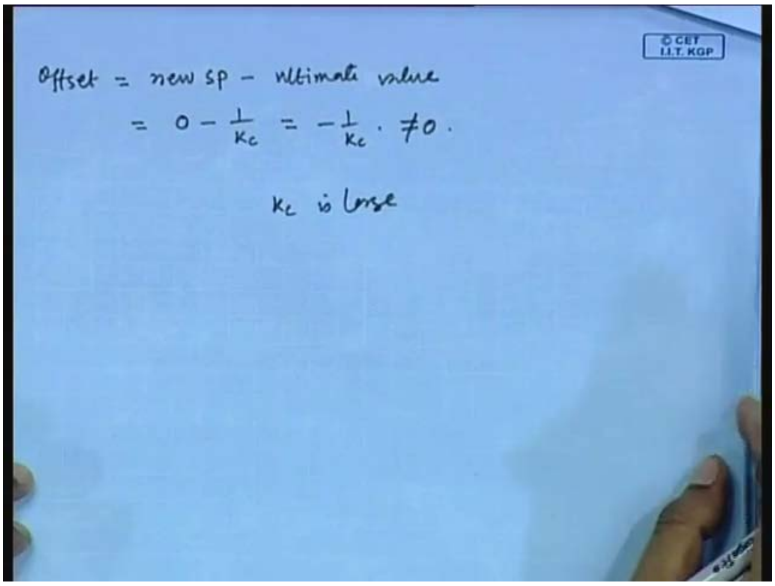

The offset will be calculated by finding the new set point minus ultimate value of the response, so new set point is 0 and ultimate value is minus 1 by K c, so this is equal to minus 1 by K c. So, for the regulatory problem, we obtain none 0 offset, but it is important to note that usually we are not interested in maintaining the liquid level system at the desired value, but within a certain range. We are not interested in maintaining the liquid level exactly at the desired value, but at a certain range in that sense we can say that the P only controller is sufficient to control the liquid level system.

Another important thing is that if K c is quite large, then this offset is acceptable if K c is large then the offset represented by minus 1 by K c is acceptable. So, P only controller can be used to maintain liquid level system, so this is all about the effect of proportional control on the first order system, similarly we will consider we will observe the effect of P only control on the second order system.

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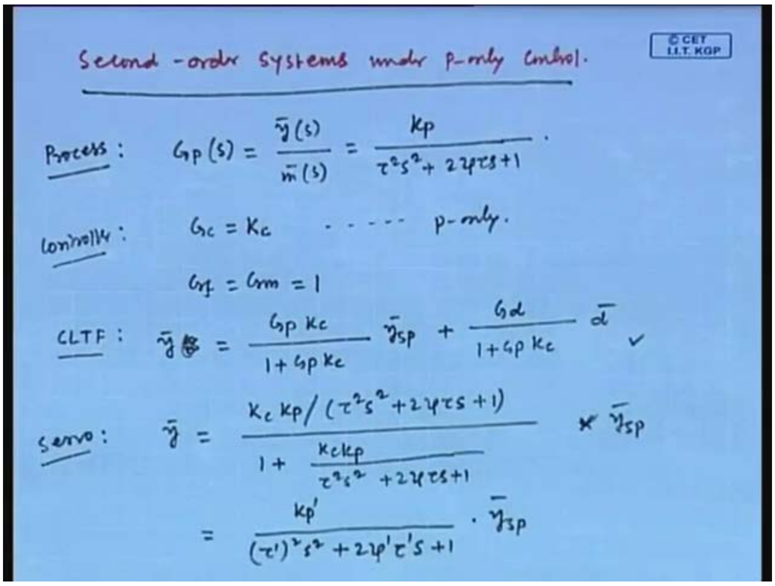

So, next we will discuss second order systems, under P only control next we will discuss the dynamic sub second order system under P only control and here we will only consider the servo problem. The first element of the close loop process is the process I mean the open loop process, the transfer function of a second order process has this form G p s equals y bar s divided by m bar s equals K p divided by tau square s square plus 2 zeta tau s plus 1, this is the transfer function of a second order process.

Next element is the controller and the controller is P only controller, so the transfer function can be written has G c equals K c, this is the transfer function of P only controller. Final control element and measuring device will consider the transfer functions equal unity, to the transfer function of final control element and measuring device are unity. Now, you will write the close loop transfer function for the second order system under P only control, substituting all these individual transfer functions in the generalized form of close loop transfer function.

We obtain y bar equals G p K c divided by 1 plus G p K c y set point bar plus G d divided by 1 plus G p K c d bar, substituting all the transfer functions except the process transfer function, we obtain this form. Now, we will consider only the servo test, we will only conduct the servo test, so for the servo case the transfer function yields y prime equals K c K p divided by tau square s square plus 2 zeta tau s plus 1 divided by 1 plus K c K p divided by tau square s square plus 2 zeta tau s plus 1.

Substituting the transfer function of the open loop process we obtain this expression, now we can represent this transfer function as y prime equals K p prime divided by tau prime square s square plus 2 zeta prime tau prime s plus 1 y set point bar, here we need to multiply y set point.

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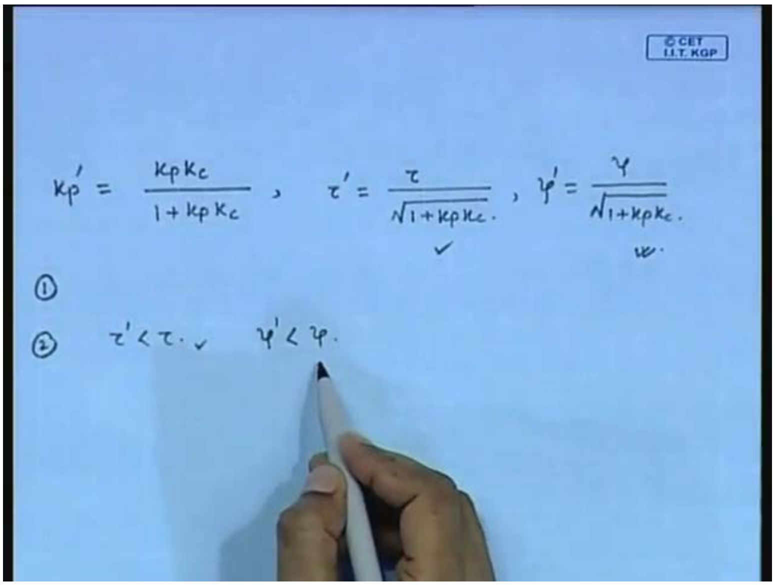

Now, K p prime as the form of K p prime equals K p K c divided by 1 plus K p K c, similarly tau prime as the form of tau prime equals tau divided by root over of 1 plus K p K c and zeta prime equals zeta divided by root over of 1 plus K p K c. So, due to the inclusion of P only controller with the second order process, there is no change of order.

So, first observation is that due to the inclusion of P only controller with the second order process there is no change up order in the overall system.

Another observation we can note that it is tau prime less than tau, if we see the expression of tau prime we can say that tau prime less than tau. Similarly, zeta prime less than zeta, similarly if we see the expression of zeta prime we can say that zeta prime less than zeta; that means, under P only control we may obtain oscillatory response I mean the over dam system may change to under dam system under P only control.

Thank you.