**Process Control and Instrumentation**
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**Lecture - 17**
**Feedback Control Schemes (Contd.)**

In the last class we discussed three different control schemes, proportional controller proportional integral controller and proportional integral derivative controllers.

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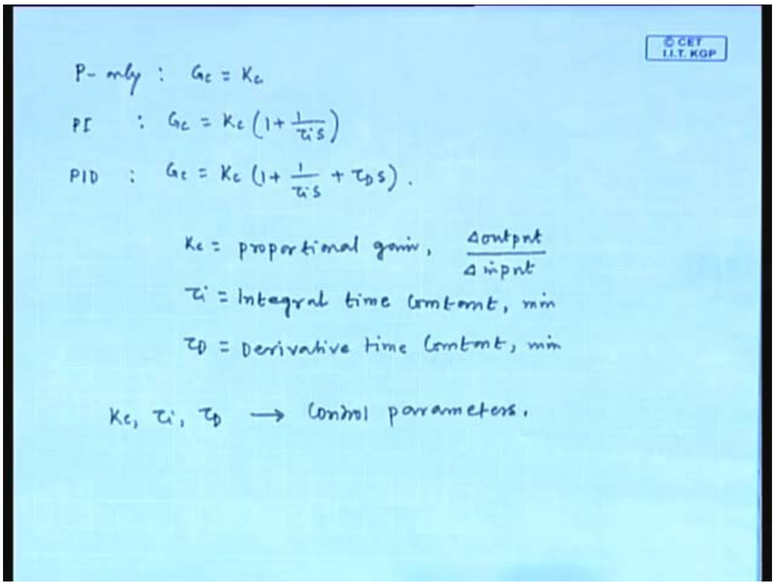


We discussed P only controller having the transfer function of G c equals K c, we discussed P I controller having the transfer function of G c equals K c 1 plus 1 divided by tau i s. We discussed the P I D controller having the transfer function of G c equals K c 1 plus 1 divided by tau s plus tau d s. Now, here K c is basically the proportional gain and this gain equals del output I mean change in output bar unit change in input, so the unit of proportional gain K c depends on the unit of output as well as the unit of input.

And tau i is the integral time constraint, the unit of tau i is time, I mean it may be minute may be hour, so the unit of integral time constant is say minute. Another term which is involved in derivative term that is tau d, tau d is derivative time constant, unit of derivative time constant is minute or it may be hour fine. So, these three parameter are basically the control parameters, K c tau i and tau d are the control parameters, tuning of this parameter will be discuss later. How you can select the values of these three parameters that will be discuss later fine.

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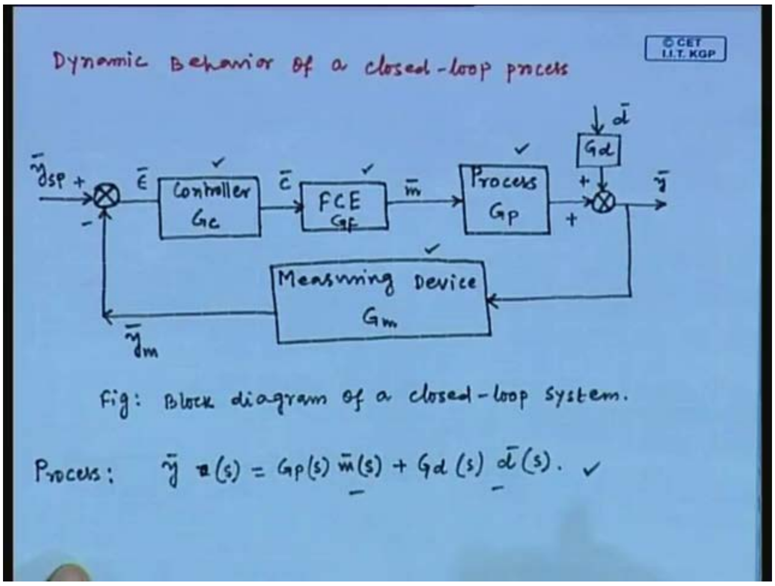


So, today we will start with the dynamic behavior of closed loop process, first we will discussed in generalized form, then we will considered few specific cases, we will considered few examples. Now, to discuss this we need to develop the block diagram of the close loop process, the block diagram of the close loop process we already discuss in the previous class, so I am just redrawing the block diagram of a typical closed loop process.

So, this block is representing the process having the transfer function of G suffix P, this block is representing the process having the transfer function of G suffix P, in is the input to the process m is the input to the process which is basically the output of final control element. Now, the output of this is added with this output fine, this two outputs are added d is the disturbance and G d is the transfer function of the process with respect to disturbance, output is y bar process output is represented by y, and in Laplace doming that is y bar.

Now, what is run first the process output is measured using tape measuring devise, the process output is measured using a measuring devise, say the measuring devise as the transferring function of G m. Now, the measure output, usually defers from the actual output, so we will represent the measured output by y m, then this measured output is compared with the step one value of process output. The measured output is compared in the comparator with its step one value, the output of this comparator is epsilon, and this output is supply to the controller, the controller transfer function we are representing by G c, controller output is say c.

And this c is implemented through the final control element FCE, and this final control element has the transfer function of suppose G suffix f. So, this is the block diagram of a closed loop system, now we will discuss different elements of this close loops transfer function, different elements or process, measuring device, controller final control element. These are the different elements of this close loop process one is the process, another one is measuring device, then controller and final control element and we will try to represent this elements by mathematical forms.

So, if we considered the process we can write for the process y bar s equals G p s multiplied by m bar s plus G d s d bar s, can we write this, the process can be for the process; we can write this equation this two signals are added, this and this signal is added now this equation is representing the process.

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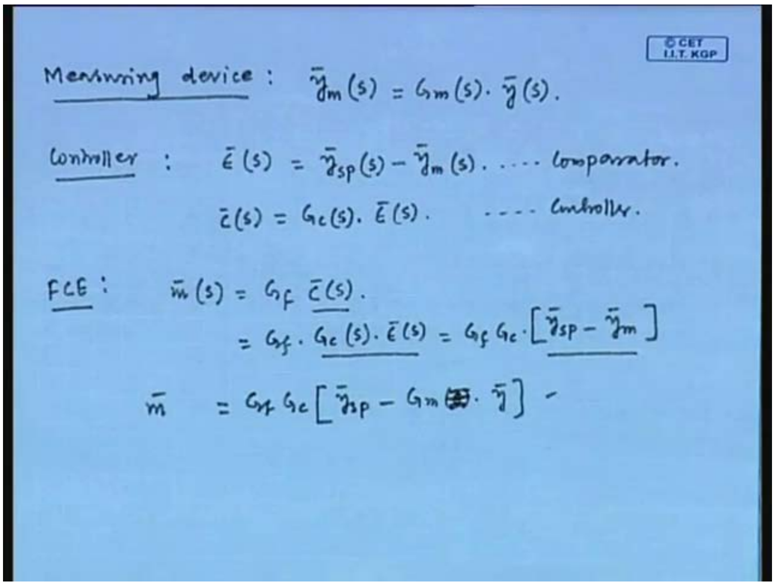


Next one is the measuring device output is y m, so in Laplace domine y m bar is equals G m s y bar s, for the measuring device we can write this equation output equals transfer function multiplied by input. Next element is the controller, the input signal to the controller epsilon bar equals y set point bar s minus y m bar s, input signal to the controller epsilon equals y set point minus y ,m this is representing the comparator. Similarly, the controller output is c bar s controller transfer function is G c s and input to the controller is epsilon, so c bar s equals G c s multiplied by epsilon bar s this is representing the control block.

Now, final control element FCE, for final control element the output is m bar transfer function is G F and input is c bar s, so for final control element we can write output m equals transfer function G c multiplied by c fine. Now, this m bar is we can write again as G F multiplied by c s, c s means what c s means G c s multiplied by epsilon bar s, can we write this, c s equals G c s multiplied by epsilon bar s.

Now, we will substitute the expression for epsilon bar s now, so G F G c epsilon bar s is y set point bar minus y m bar fine, epsilon bar s is y set point bar s minus y m bar s. In the next step we will substitute the expression of y m, so G F G c y set point bar minus y m bar, what is the expression of y m bar G m s multiplied by y bar s, we are not writing s anymore, so this is the expression of m bar agree.

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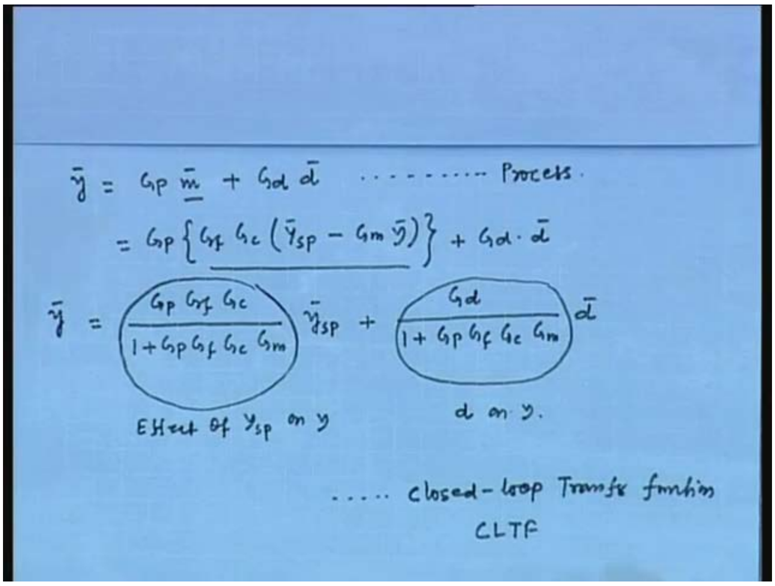


Now, we have written the expression for the process as y bar equals G p m bar plus G d d bar, we are written equation previously for the process fine, now here will substitute the expression of m bar, so G p multiplied by m bar; that means, G F G c y set point minus G m y bar, this is m bar. Now, the second term is G d d bar fine, we are just substituted the expression of m bar in the equation of process. Now, if we rearrange this equation we finally, get y bar equals G p G F G c divided by 1 plus G p G F G c G m y set point bar plus second term is G d divided by 1 plus G p G F G c G m d bar, by rearranging we get this equation.

Now, the first right hand term this provides the effect of y set point on y, similarly the second term second right hand term provides the effect of disturbance on y, now this equation is the closed loop transfer function, this equation is called closed loop transfer function CLTF fine, this is the expression of closed loop transfer function.

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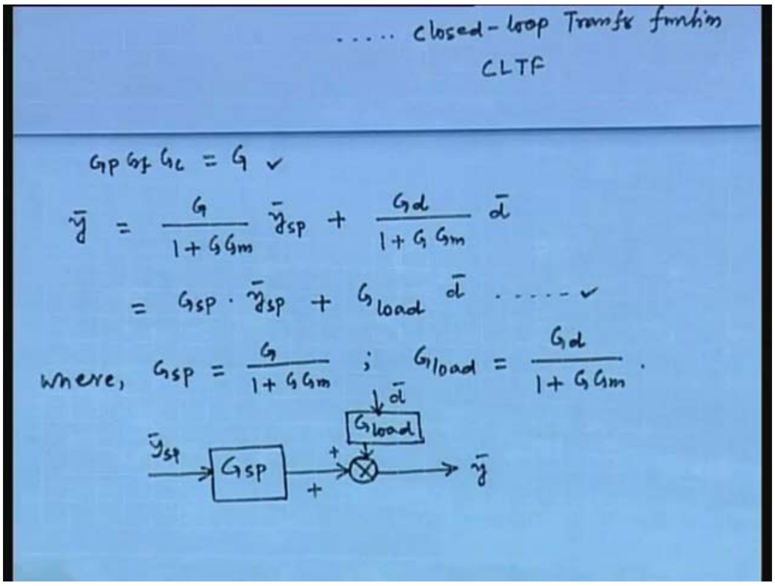


Now, if we considered G p G F G c equals G considering G p G F G c equals G, we get the closed loop transfer function s y bar equals G divided by 1 plus G G m y set point bar plus G d divided by 1 plus G multiplied by G m d bar fine. Considering G p G F G c equals G we get this form of close loop transfer function, now again will consider this as G set point multiplied by y s p plus G load multiplied by d bar.

Here, G set point equals G divided by 1 plus G G m and G load equals G d divided by 1 plus G G m, considering this two expression for G set point and G load finally, we get this closed loop transfer function. Can we corresponding block diagram what will be the block diagram of this final form of closed loop transfer function? It is very simple we can consider one block for G set point, what is the input? Input is y set point, now this output is added, which the output of the this load block I mean this two outputs are added and finally, we get y fine.

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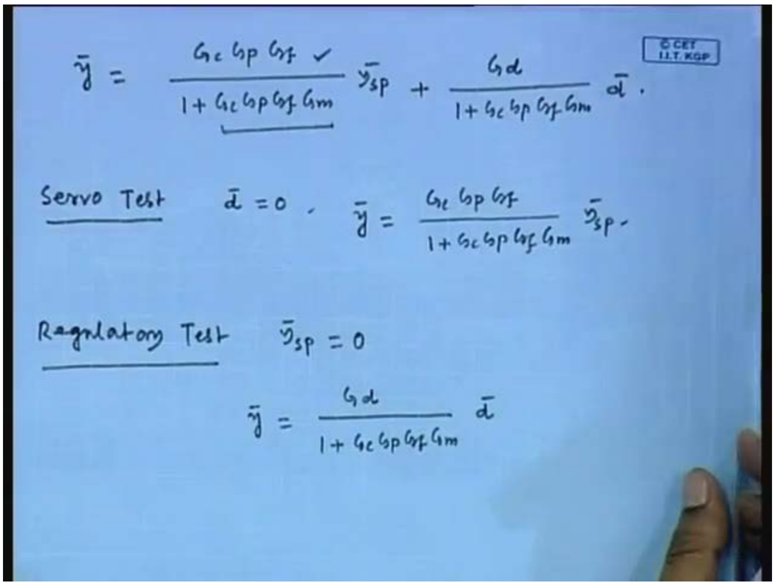


This is the block diagram of the final closed loop transfer function, so ours original transfer function is G c G p G F divided by 1 plus G c G p G F divided by 1 plus G c G p G F G m y set point bar plus G d divided by 1 plus G c G p G c G m d bar. We will use this transfer function mostly in the analysis of different feature fine, you see here the transfer function of measuring devise does not exist in the numerator. And in the denominator in this term particularly all the transfer functions are included, all the individual transfer function are included in the denominator fine.

Now, the control performance is usually investigated by performing two testes, one is servo test another one is regulatory test, so what is servo test, in the case of servo test there is no change introduce in the disturbance, in the servo test there is no change introduce in the disturbance. That means, d bar equals 0 only the set point change is considered, now if d bar becomes 0, what is the closed loop transfer function for the servo problem.

So, close loop transfer function for the servo problem, become G c G p G s divided by 1 plus G c G p G F G m y set point, this is the close loop transfer function for the servo problem. Now, introducing step change may be sinusoidal change in y set point and by taking inverse of Laplace transform we can know the tangent response y t by that wave we can test the controller. This is called servo problem and sometimes another term is use that is set point tracking, so set point tracking performance is absorbed by performing servo test.

Another test is regulatory test, regulatory test is conducting by considering no change in set point; that means, y set point bar equals 0. Regulatory test is conducting by considering no change in set point value accordingly, the closed loop transfer function become y bar equals G d divided by 1 plus G c G p G F G m d bar. So, this is the closed loop transfer function for the case of regulatory problem, another term is again used for this that is disturbance rejection performance, so to absorb the disturbance reaction performance of a controller regulatory test is conducted, set point tracking performance is absorbed by conducting servo test and disturbance rejection performance is absorb by conducting regulatory test.

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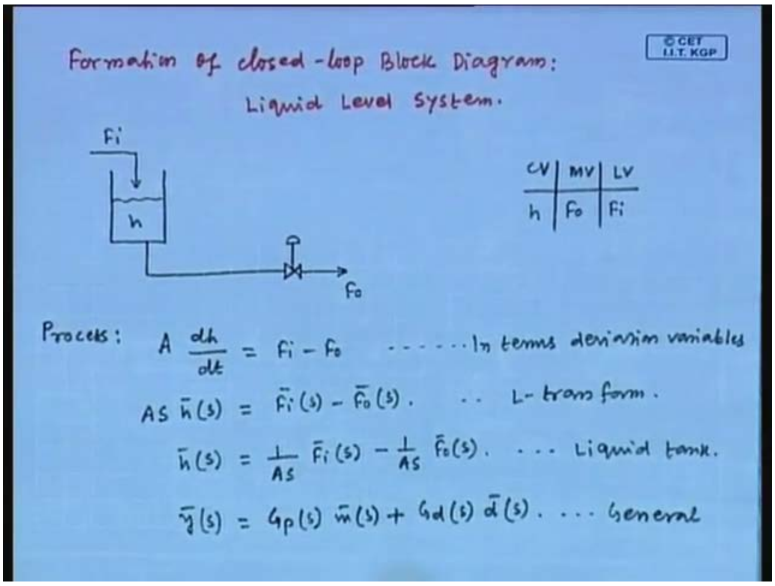


So, next we will considered one example, to discuss the formation of closed loop block diagram, so formation of formation of closed loop block diagram, for a liquid level system. So, first we will draw the schismatic of liquid level system, this is the liquid tank, F i is the inlet floret and out let floret is say F naught, height of liquid in the tank is h. Now, the objective is to maintain the liquid height in the tank by manipulating the outlet floret the objective is to maintain the liquid height in the tank at its desired value, accordingly height is the controlled variable.

So, there are two options of manipulated variable I mean we can select either F i or F naught, suppose F naught is the manipulated variable, then what will be the load variable F i. If we considered F naught as the manipulated variable then F i will be the load variable, now we will consider the different elements. So, first we will considered the process I mean, we need to develop the transfer function of the process in terms of G p and G d.

So, for developing the transfer function of a process we need the model, so model we can represent by this ordinary differential equation A d h d t equals F i minus F naught, this is the modeling equation for the liquid level system. Now, here we are not using the prime to represent the deviation variable, we are not representing the prime superscript to indicate the deviation variable, I mean a h F i F naught, these are deviation variables.

So, this equation is written in terms of deviation variable, we are not representing prime superscripts, if we take Laplace transform then we get A S h bar s equals F i s minus F naught s. Taking Laplace transform we get this form, now rearranging h bar s becomes one by a is multiplied by F bar s minus 1 bar A s F naught bar s, dividing both sides by A S we get this equation.

Now, what is the general form, see general form is y bar s equals G p s m bar s plus G d s d bar s, this is the general form and this is for the liquid tank system. Comparing this last two equation we can easily get the transfer function G p n G d, so what is G p, see all G p is 1 by s. Yes, G p is minus 1 by A s, because our disturbance is F i and aim is F naught.

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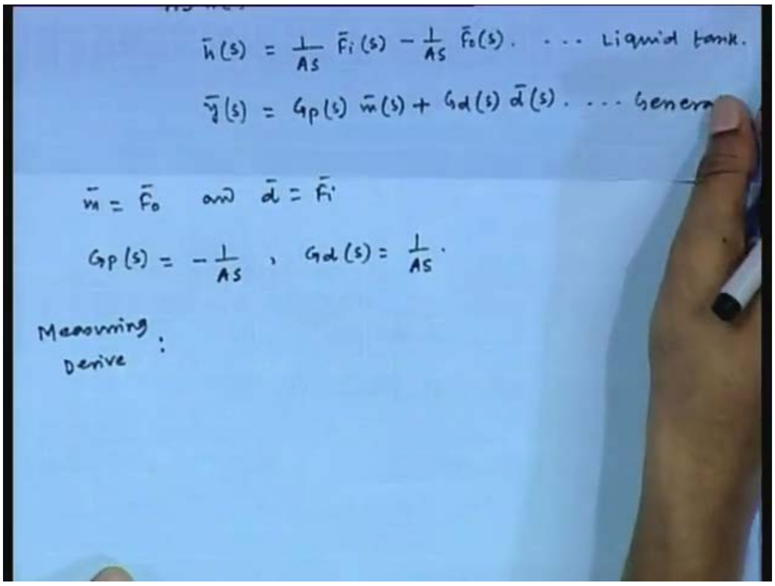


Here aim is F naught and disturbance d is F I, so we get the transfer function G p s equals minus 1 by A s, and we get G d s equals 1 by A s. So, these two transfer function we obtain, now what about the next element, I mean what is the next element next element is the measuring device. So, we will considering the measuring device for finding the transfer function.

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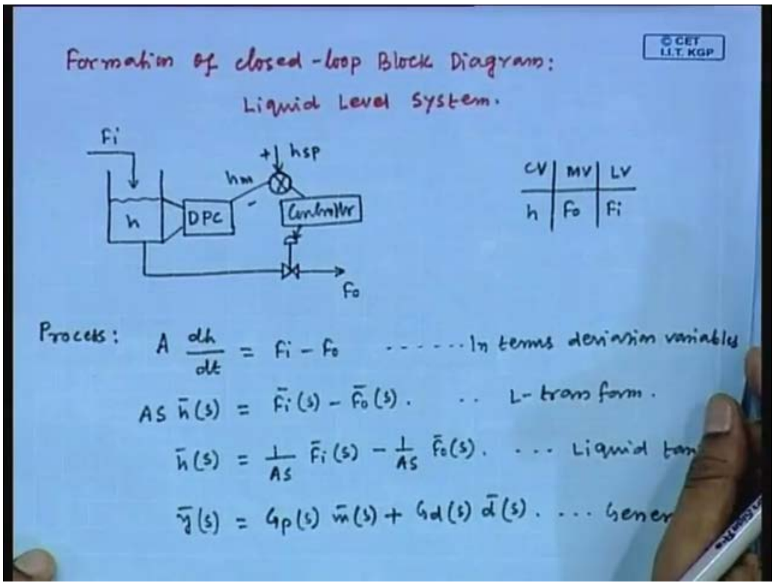


I mean in this liquid level system, first we can configure the control scheme, if we configure the control scheme then we need the final control element, sorry first we need the measuring device. So, to measure this liquid height the differential pressure cell is extensively used to measure the liquid height differential pressure cell d p c can be use as a sensor, so what is the output of this differential pressure cell h m, the output of this differential pressure cell is h m.

Original height is h and measured height is h m, this output is compared with the set point value h s p, measured value is compared with set point value. Then this error signal is supplied to the controller and controllers action is implemented through the final control element, this is the control configuration of the example liquid level system. So, here measuring device is the differential pressure cell, we need to find the transfer function of that differential pressure cell.

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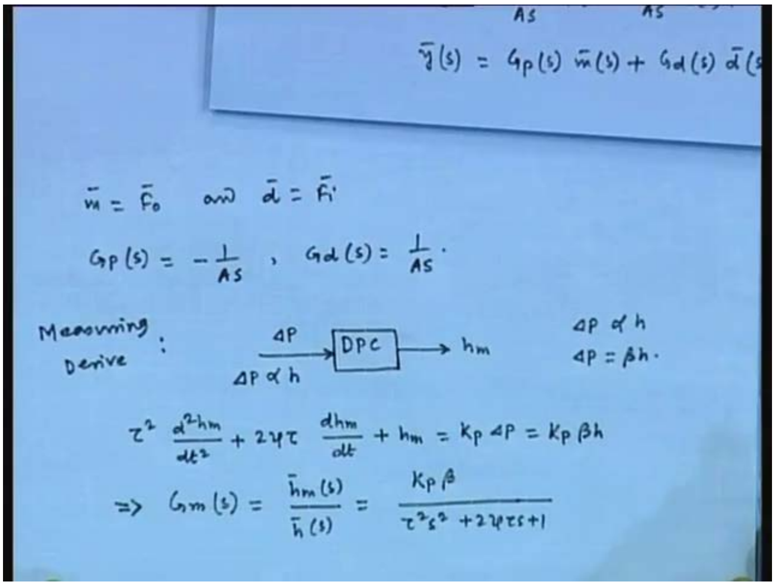


What is the input to this differential pressure cell, input is the pressure difference del p and we can considered del p proportional to liquid height. This del p proportional to liquid height, what is the output of this differential pressure cell h suffix m input to this differential pressure cell is differential pressure and output is h m. Now, considering second order dynamics of this differential pressure cell, we can write the modeling equation as tau square d 2 h m by d t square plus 2 zeta tau d h m d t plus h m equals k p del p.

Considering second order dynamics of the differential pressure cell, we can write this equation now the right hand side is k p del p, so we can write again that is equal to k p beta h, del p is proportional to height, so del p equals beta height beta multiplied by h. Taking Laplace transform and rearranging we get the transfer function as G m s equals output h m bar s divided by input h bar s which is equal to K p beta divided by tau square s square plus 2 zeta tau s plus 1. Taking Laplace transform and rearranging we get finally, this transfer function for the measuring device, next element is the controller, we know the transform function for P PI PID controllers.

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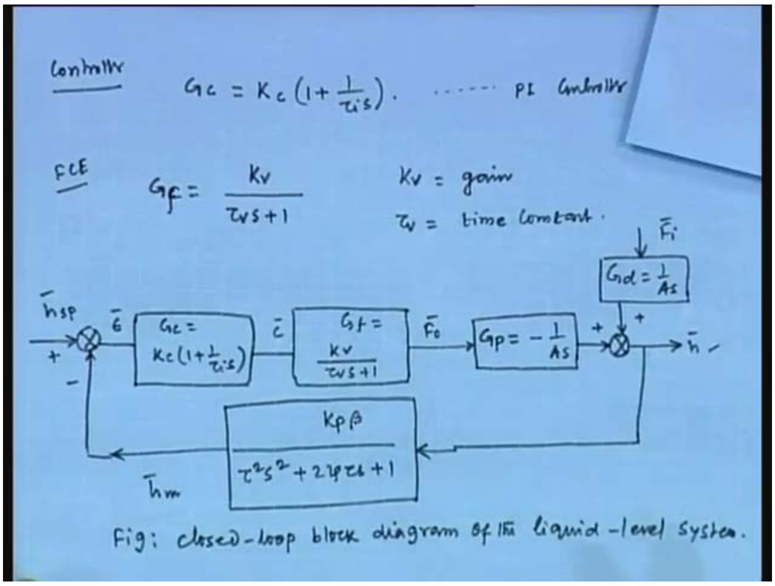


Now, if we considers the P I controller then the transfer function we can write as G c equals K c 1 plus 1 divided by tau i s, if we consider p i controller for the example liquid level system then we can write the transfer function in this form. Another element is the final control element assuming first order dynamics of the control bulb, we can write the transfer function as G c equals K v divided by tau v s plus 1. Assuming first order dynamics of the control bulb we can write the transfer function as G this is f this is G f, so G F equals K v divided by tau v s plus 1, K v is the gain of the control and tau v is the time constraint.

Can we make the close loop block diagram, first we need to develop the process having the transfer function of G p and G d, how much is G p. G p is minus 1 by A s, what is G d is 1 by A s and what is the disturbance, disturbance is F i what is the output, output is height and what is the input to the process, I mean what is the final control element output that is F naught. So, this is the process, the transfer function of the process with respect to F naught is minus 1 by A s, transfer function of the process with respect to F i is 1 by A s output of that process is h.

For the measuring device we have derived the transfer function that is K p beta, K p is the gain of the measuring device, hole divided by tau square s square plus 2 zeta tau s plus 1. So, this is the transfer function of measuring device I mean this is equal to G m, output is measured height, this is the comparator h set point is compared with measured height. Then the error signal goes through the controller which has the transform function of K c 1 plus 1 divided by tau y s, this is the transfer function of the controller P I controller output is c bar.

The transfer function for the final control element, we have derived that is K f equals K v divided by tau v s plus 1, this is the transfer function of the final control element. So, this is the block diagram of the example liquid level system, this is the closed loop block diagram of the liquid level system. In the next we will discuss the effect of proportional action, we have we discussed the three controllers P PI and P I D, basically there are three actions one is proportional action, another one is integral action and derivative action. So, we use to discus effects of all these actions individually, so first we will discuss the effect of proportional action.

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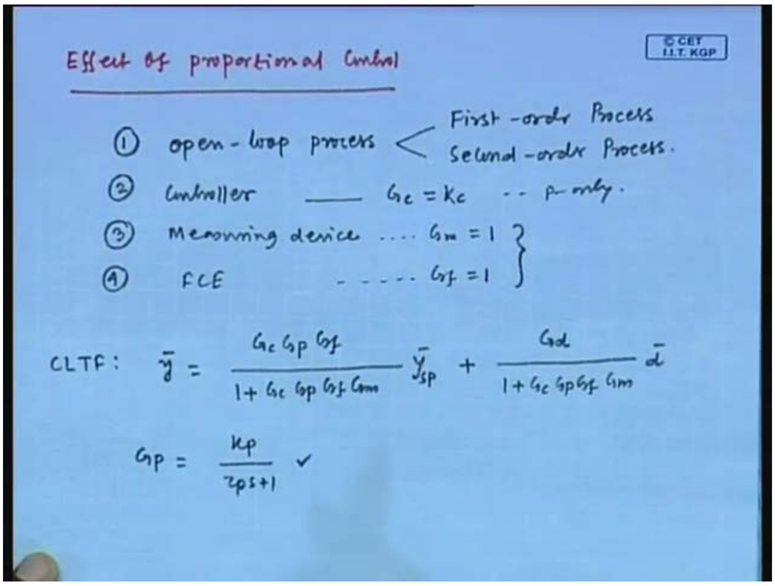


So, first we will discuss the effect of proportional control we can write proportional action or proportional control, because it only includes the proportional action. For this purpose we will consider one process one open loop process we need to considered. So, first we will considered an open loop process, now to absorb the effect of proportional controller we need to close the loop. So, for closing what do we need one controller, that is p only controller because we are going to discuss the effect of p only controller, then we need one measuring device and also we need one final control element to implement the control action.

So, for absorbing the effect of proportional controller, we need to consider a process that is open loop process, now to close the loop we need to include the controller, we need to include the measuring device and final control element. So, fort open loop process we will consider, we will consider two processes one is first order process and second one is second order process and we will discuss one by one.

So, first we will consider first order process in the next we will consider second order process example, next element is the controller. So, here we will include the p only controller, so for p only controller what will be the transfer function for p only controller the transfer function G c equals K c. So, for measuring devise we will consider G m equals 1 and for final control element we will consider G F equals 1. So, for simplicity we are assuming the transfer function of measuring device and final control element both are one fine.

So, we will start from the closed loop transfer function we will start from the closed loop transfer function, the general form of close loop transfer function is y bar equals G c G p G F divided by 1 plus G c G p G F G m y set point bar plus G d divided by 1 plus G c G p G F G m d barThis is the general form of close loop transfer function. Now, in this close loop transfer function we need to substitute the individual transfer functions, like for the process it is mention that first we will consider first order process; that means, G p equals K p divided by tau p s plus 1.

For a first order process this is the transfer function, now if we substitute the transfer function G p G c equals K c G f G m equal to 1, then what we will get if we substitute all the transfer functions then will get the close loop transfer function for this example system. Now, for the process these is the transfer function with respect to manipulated variable, another transfer function is involved that is G d with respect to G d, so what that will be.

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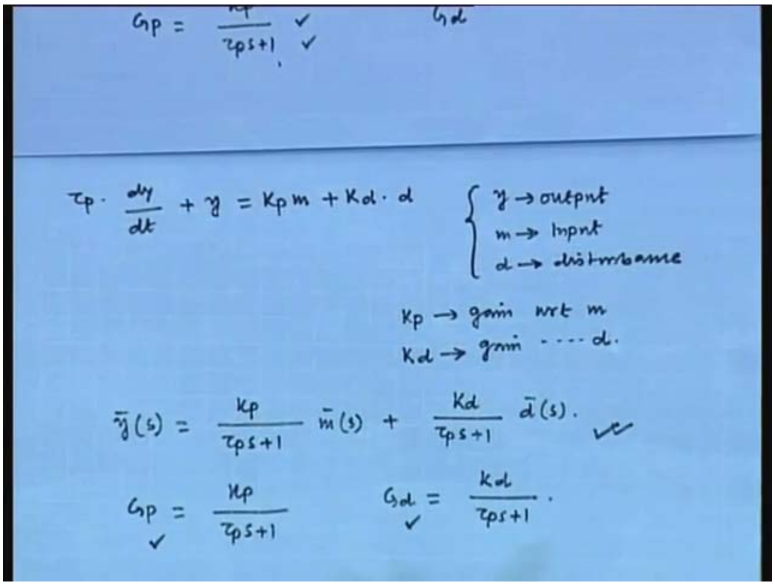

So, for that purpose we will first considered the process, we need to ride the modeling equation for the process, the modeling equation for the process we can write as tau p divide d t plus y equals K p m plus K d d. The first order process we can represent in time domain by this form, where y is the output, m is the input to the process m is basically the output of the final control element and d is the disturbance.

And y m and d all are in terms of deviation variables, now K p is the gain of the process with respect to m, another gain is involved in this equation that is K d, K d is the gain with respect to disturbance. Now, if we take Laplace transform and if you rearrange then finally, we get y bar s equals K p divided by tau p s plus 1 m bar s plus K d divided by tau p s plus 1 d bar s, taking Laplace transform and rearranging we get y bar s equals K p divided by tau p s m bar s plus k d divided by tau p s d bar s.

So, G p is equal to k p divided by tau p s plus 1 and G d is equal to K d divided by tau p s plus 1, from this equation we can get the transfer function of the process with respect to m that is G p and the transfer function of the process with respect to disturbance that is G d. Now, all individual transfer functions are known to us G p G d G c G F and G m, so in the next step we need to substitute all this transfer functions in the close loop transfer function, so that we will discuss in the next class.

Thank you.