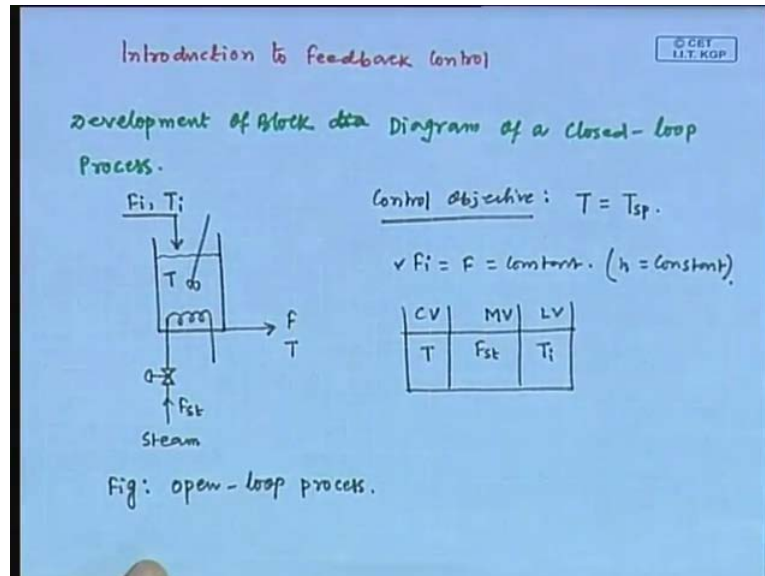


Process Control and Instrumentation
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Lecture - 15
Feedback Control Schemes

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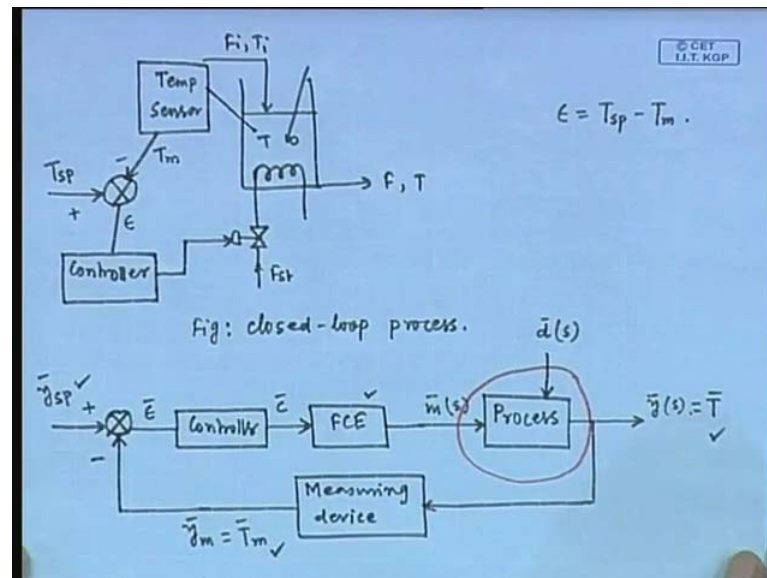


Now, we start the topic introduction to Feedback Control. We will discuss the topic introduction to Feedback Control. Now, before discussing the feedback controlling schemes, we will develop the block diagram for a closed loop system. So, development of block diagram, of a closed loop process we have develop the block diagram for open loop process. So, now we used to develop the block diagram of closed loop process, for this purpose we will consider one example that is hitting tank system, for the development of block diagram, we consider the hitting tank system.

Steam is introduced through this coil with a flow rate of f_{st} in food flow rate is f_i and temperature is T_i outlet stream is going out at the flow rate of f and temperature T . Now, what is the control of objective the control of objective is to maintain the temperature at it is desired value. The control of objective of this example process is to maintain the temperature at it is desired set point value and we assuming that, f_i and if their identical and their constant quantities. We are assuming that a f_i if both are identical and their constant.

So, what is the control variable, what is the manipulated variable and what is the load variable, that we can detect. Now, control variable is temperature, what is the manipulated variable, corresponding manipulated variable is steam flow rate and load variable is T_i . Now, this is the schematic of an open loop system, this is the schematic of the open loop hitting tank system. Now, we have assumed that the inlet and outlet flow rates are identical and they are also constant, it indicates there is no variation of height, it indicates height is also constant. Now, we will configure the control around this process, we have already dictated that control variable, manipulated variable and load variable. Now, we will configure the feedback control scheme for maintaining the liquid temperature.

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So, this the hitting tank system we considered as an example, now first we need to measure the temperature by using a measuring device, we need to measure the temperature using a measuring device. So, this is the block for temperature sensor which is employed to measure, the liquid temperature. Now, this measure temperature usually defers, from the original temperature habit, this measure temperature we can represent by t suffix n , in the next step this measure temperature is compared with it is set point value represented by T_{sp} . So, this is positive and this is negative.

Then the output of this comparator is the output of the comparator is represented by it is epsilon, it is epsilon is equal to set point temperature minus measure temperature, the

comparator output can be represented by ϵ or ϵ is equal to $T_s p$ minus t_m . Then this information is supplied to the controller, this information is supplied to the controller, based on the error signal the controller takes action and that action is physically implemented to these control hall, the controller action is physically implemented to the final control element that is control hall.

This is the configuration of the controller, employed around this hitting tank system. And this is the schematic of the closed loop hitting tank system, this is the schematic of the closed loop hitting tank system or closed loop process, by the introduction of the controller the loop is closed. Now, in the next week we used to develop the block diagram for this closed loop process. So, we will consider a block process, this block is representing the process, input to this process is suppose, m bar is input to the process is represented by m bar s that is nothing, but the final control output.

And another input is considered, that is the disturbance d bar s . So, there are two inputs affecting the process, one is the final control element output, another one is the disturbance. Disturbance is represented by d and final control element is represented by here m , the output of the process is y in Laplace domain we can write y bar s , this is the open loop process, this is the block diagram of the open loop process, there is no controller involve with these open loop process.

Now, what we doing first, we are first measuring the temperature using a measuring device. So, one block we can draw for the measuring device, this measuring device is measuring the temperature, basically this y is T only. The control variable y is here the temperature of the liquid in the tank. So, first we measuring the temperature using this measuring device. What is the output of this measuring device, output is y suffix m that is nothing, but measure temperature sensor output is measure temperature T_m .

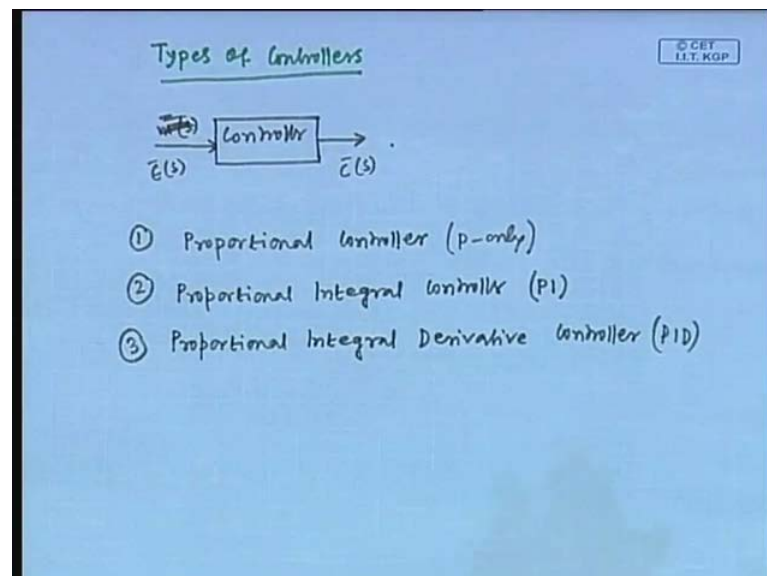
Now, this measure temperature, this measurement signal goes to the comparator and the sign is negative, you just compare the configuration with this block diagram. Now, another input to this comparator is y said point, that is y suffix $s p$ and we use positive sign for this. So, there are two input signals considered for the comparator, what is the comparator output, ϵ in Laplace domain we can write ϵ bar, this comparator output is supplied to the controller.

So, one block we can draw for the controller, the error signal ϵ goes to the controller, then the controller calculates the control action. We can represent the controller output by c and in Laplace domain \bar{c} , this controller output goes to the final control element, that is the control half, the controller output c is supplied to the final control element and final control element output is m .

So, this the block diagram of the closed loop process this the process which as two inputs, one is the final control element output and another one is the disturbance, these two inputs effect the process. If we consider the liquid tank system hitting tank system, the output is the temperature this temperature is first measured by using one measuring sensor.

Sensor output is T suffix m that is measure temperature, then that measure temperature is compared with it is said point value, comparator output is represented by here it is ϵ , that information is goes to the controller. And using that information controller calculates the control action, then control information is physically implemented to this final control element. So, this the closed loop block diagram of a process.

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Next we will discuss the different types of controllers. Now, if we redraw the controller block, then we see that input is \bar{m} s output is sorry input is ϵ bar and output is \bar{c} s. So, the basically the controller relates \bar{c} to it is ϵ . So, various types of

controller differ in the way they relate c to ϵ . So, first we will consider three types of controller, which are classical controllers and which are definitely feedback controllers.

These three controllers are proportional controller, this is also called I mean this is also named as p only controller. Second controller is Proportional Integral controller, which we can call as PI controller, proportional integral. Proportional integral controller additional includes the integral term. Third controller is Proportional Integral Derivative controller, which we can call as PID controller, these are the basic feedback controllers. And we will discuss one by one so, first we will consider proportional controller.

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P-only controller

$c'(t) \propto \epsilon(t)$

$c'(t) = K_c \epsilon(t)$

$\Rightarrow c(t) = c_s + K_c \epsilon(t)$

$\checkmark K_c = \text{proportional gain}$

$c_s = \text{Bias signal}$

$c'(t) = c(t) - c_s$

$K_c = \frac{c'(t)}{\epsilon(t)}$

$\rightarrow c(t) = c_s \text{ when } \epsilon = 0$

PB = proportional band = $100/K_c$

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First we will discuss proportional or p only controller the controller output c which can be written in terms of divisional variables as c prime t is proportional to the error signal. If you see the control block, will observe that its ϵ is the input and c is the output. Now, we have written c in terms of divisional variables. So, c prime t is proportional to the error signal that is the input to the controller and that is why the name I mean that is why it is called proportional controller. So, you can write this against as c prime t equals k_c it is ϵ .

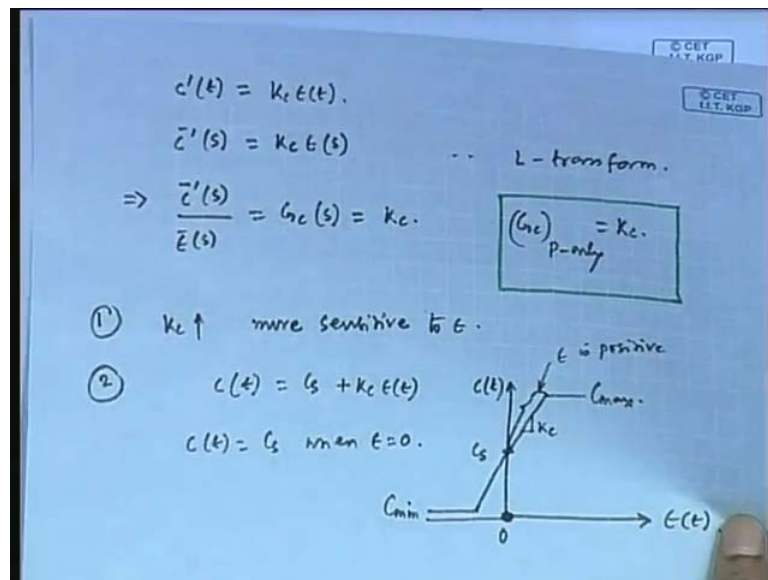
And finally, we write c t equals c s plus k_c it is ϵ or c prime t is equal to c t minus c s . I have mentioned that c prime is the divisional variable. So, c prime t we can write as c t minus c s . Now, k_c is the proportional gain of the controller. It is very

obvious you see k_c we can write as $c'(t)$ divided by $\epsilon(t)$ we can write this, k_c is equal to $c'(t)$ is divided by $\epsilon(t)$ you just see this block $c'(t)$ is the output. If $\epsilon(t)$ is the input I have already mentioned that gain is change in output per units change in input, it is obvious in this correlation.

So, we can say k_c as the proportional gain. Now, what is c_s , c_s is the bias signal, controllers bias signal. We can define it as c_s is the controller output and there is no error, if you see this equation. We can write that $c'(t)$ is equal to c_s when the error is 0. So, bias signal is the controller output and error equals 0. This k_c is one tuning parameter controller tuning parameter, the value of that parameter we need to determine. So, one equivalent term is also used in different soft wares and industrial factories, that is proportional band.

One equivalent term of k_c called proportional band is also used in different process simulator and in industrial practice. Proportional band, which is 100 divided by k_c proportional band is 100 divided by k_c . Now, we used to determine the trans perform of the controller.

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You see for the p only controller we got $c'(t)$ is equal to $k_c \epsilon(t)$. Now, if we take the Laplace transform for this, then we obtain $\bar{c}'(s) = k_c \bar{\epsilon}(s)$, if we take Laplace transform we obtain $\bar{c}'(s)$ is equal to $k_c \bar{\epsilon}(s)$. Now, what is the output of the controller, $\bar{c}'(s)$ what is the input to the controller $\bar{\epsilon}(s)$,

and this can be represented as transfer function of the controller $G_c(s)$ and $G_c(s)$ is equal to k_c . So, for PI controller transfer function is sorry for the p only controller transfer function is equal to k_c .

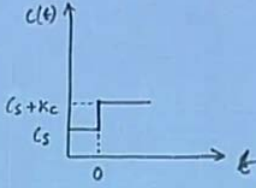
Now, we will conclude the p only controller, with the increase of k_c the controller becomes more sensitive to error, this is the first conclusion which we can draw for the p only controller. With the increase of k_c the controller becomes more sensitive to error epsilon. If we increase the k_c value the controller become more sensitive to it is epsilon. Now, we want to see the behavior of the p only controller, how the controller behaves that we want to represent graphically.

So, remember the controller equation that is $c(t) = c_s + k_c \epsilon(t)$ this is the p only controller equation in time domain. Now, if we plot c verses epsilon, we obtain the c_s when it is epsilon is equal to 0. So, this pointing is representing c_s because, we know that $c(t) = c_s$ when it is epsilon is equal to 0. So, we are considering this or epsilon is equal to 0, so the corresponding c value is c_s and when we increase epsilon, it increase like this, it is a slope of k_c . So, slope is k_c .

And the last value of c is c_{max} maximum value of c we cannot consider c as infinity due to the some physical limitation of final control element, we have to put constants on c . So, the higher limit is c_{max} and lower limit is c_{min} . Now, this variation is obtained when it is epsilon is positive similarly, we obtain this line, when it is epsilon is negative and the minimum value of c is c_{min} . So, this is the second conclusion on p only controller. Next we will discuss how the controller response, which it is take place in error signal.

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$$\bar{c}'(s) = K_c \bar{e}(s)$$
$$\bar{e}(s) = \frac{1}{s}$$
$$\bar{c}'(s) = \frac{K_c}{s}$$
$$c'(t) = K_c \quad \checkmark$$
$$\Rightarrow c(t) = c_s + K_c \quad \checkmark$$


So, we will use the controller equation in Laplace domain. This is the p only controller, equation in Laplace domain. Now, we will consider a unit place change, we will consider a unit step change in error signal, then the controller equation is $\bar{c}'(s)$ is equal to K_c by s . If we take in words of transform, then we get $c'(t)$ equals K_c if we take in terms of transform we obtain $c'(t)$ equals K_c ; that means, $c(t)$ is equal to c_s plus K_c . Finally we get $c(t)$ equals c_s plus K_c .

Now, we will produce the plot to represent this concept, this is time t and this is $c(t)$. Now, initially the process was, initially the controller has the value of c_s . Now, we are introducing a unit step change at time t equals 0 , then the controller output provides the value of c_s plus K_c . This is the controller behavior against a unit step change in error signal. Next we will discuss the second controller that is PI controller Proportional Integral controller.

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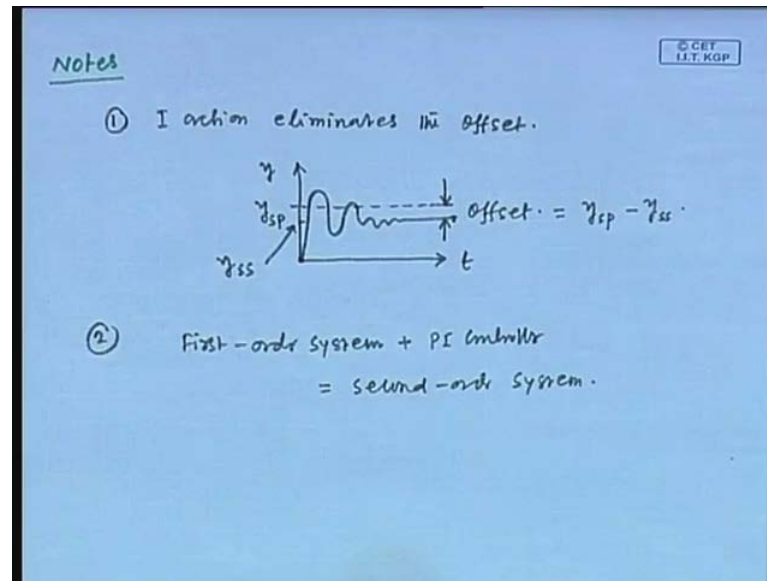
Proportional Integral Controller.

$$c(t) = c_s + \underbrace{k_c e(t)}_{\text{P action}} + \underbrace{\frac{k_c}{\tau_i} \int e(t) \cdot dt}_{\text{I action}}$$
$$\Rightarrow c'(t) = k_c e(t) + \frac{k_c}{\tau_i} \int e(t) \cdot dt$$
$$G_c(s) = \frac{C'(s)}{E(s)} = k_c \left(1 + \frac{1}{\tau_i s} \right) \dots \text{TF of PI Controller.}$$

Next we will discuss, proportional integral controller $c(t)$ equals c_s plus k_c it is $\epsilon(t)$ plus this the representation of p only controller. Now, additionally we include the integral term in p only controller, to obtain PI controller. That integral term is represented by k_c divide by τ_i integration of error $t dt$. So, this is the integral action and this the proportional action, combining this two actions we obtain t_i controller.

Now, we can represent this terms in divisional variables for finding the transform function of PI controller. Accordingly we write, $c'(t)$ equals $k_c \epsilon(t)$ plus k_c divided by τ_i integration of error $t dt$ by taking Laplace transform and rearranging we obtain the transform function, as $G_c(s)$ equals $C'(s) / E(s)$ equals $k_c \left(1 + \frac{1}{\tau_i s} \right)$ by taking Laplace transform and rearranging finally, we obtain the transfer function of PI controller as $G_c(s) = k_c \left(1 + \frac{1}{\tau_i s} \right)$ this is the transfer function of PI controller. Now, we will make some remarks on these PI controller.

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The integral action eliminates the offset, first conclusion is the integral action eliminates the offset. Offset is basically the steady state error. If we plot y versus time t we may get this type of response under a controller, this is the set point value and the value which we are getting finally, that is the steady state value. So, this corresponding value, this value is the steady state value y_{ss} . Now, the difference between the set point value and steady value is called offset.

So, offset is basically $y_{sp} - y_{ss}$. Now, this theory controller can eliminate the offset due to the inclusion of integral action I mean p only controller cannot eliminate the offset due to the inclusion of integral action. I mean the p only controller can not eliminate offset, this is the first conclusion. Now, due to the additional of integral action, the overall response becomes more sluggish, this is the second conclusion due to the addition of integral action the overall response become more sluggish, due to the increase of order by one due to the increase of integral action the overall response becomes more sluggish.

Say for example, if we consider a first order system and if we employ the PI controller, then the overall system becomes second order system. If we consider a first order system and if a controller is employed, then the overall system response becomes second order dynamites, that means order is increased by one. Now, in plot we have compared the dynamites of first order, second order and fourth order systems and we have observed

that increasing the number of capacities in series, increases the sluggishness. So, by that line we can say that by inclusion of integral action the order is increased by one, it means the overall response becomes more sluggish. So, this is the second conclusion. Now, we will consider a unit step change in error signal and we will observe the response of the PI controller.

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$$\textcircled{3} \quad \bar{c}'(s) = \bar{E}(s) \left[k_c \left(1 + \frac{1}{\tau i s} \right) \right]$$

$$\bar{E}(s) = \frac{1}{s}$$

$$\Rightarrow \bar{c}'(s) = \frac{k_c}{s} + \frac{k_c}{\tau} \cdot \frac{1}{s^2}$$

$$c'(t) = k_c + \frac{k_c}{\tau} \cdot t \quad \checkmark \Rightarrow c(t) = (k_s + k_i) + \frac{k_i}{\tau} \cdot t$$

The graph shows the controller output $c(t)$ versus time t . At $t=0$, the output jumps from k_s to $k_s + k_i$. For $t > 0$, the output increases linearly with a slope of $\frac{k_i}{\tau}$.

So, PI controller equation in Laplace domain, we can write as $\bar{c}'(s)$ equals $\bar{E}(s)$ multiplied by $k_c \left(1 + \frac{1}{\tau i s} \right)$. We will consider a unit step change in error signal; that means, $\bar{E}(s) = \frac{1}{s}$, then we obtain $\bar{c}'(s) = \frac{k_c}{s} + \frac{k_c}{\tau} \cdot \frac{1}{s^2}$.

Now, inverting this, we obtain $c'(t) = k_c + \frac{k_c}{\tau} \cdot t$. Taking inverse of Laplace transform we obtain finally, $c'(t) = k_c + \frac{k_c}{\tau} \cdot t$. Now, we will represent this graphically, this is time t this is $c(t)$. Now, at time $t = 0$ the controller as the value of c_s and we are introducing a unit step change at time $t = 0$ accordingly the controller output becomes $c_s + k_c$.

So, $c_s + k_c$ we can obtain from this $c_s + k_c + \frac{k_c}{\tau} \cdot t$. So, it is obvious that the time $t = 0$ $c(t) = c_s + k_c$ and when time is greater than 0 the c increases linearly, with a slope of $\frac{k_c}{\tau}$.

tau i. This is the behavior of PI controller against the unit step change in error signal. And that error signal the change in error signal introduced at time t equals 0.

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$$I = \frac{K_c}{T_i} \int e(t) \cdot dt$$

T_i = Integral time constant / reset time.

$$c'(t) = K_c + \frac{K_c}{T_i} \cdot t$$

--- A unit step change in e

$t = T_i$, $c'(t) = K_c + K_c$

Time required by the controller to repeat the initial proportional action change in its output.

Now, this integral action, we can write as k_c divided by τ_i ϵ t $d t$, k_c is the proportional gain and τ_i is the integral time constant, one additional tuning parameters is there, that is τ_i and τ_i is called as integral time constant, it is also called as reset time, unit of τ_i is time.

Now, for the PI controller we got $c'(t) = k_c + k_c / \tau_i \cdot t$ by introducing a unit step change in error signal. We obtain for PI controller $c'(t) = k_c + k_c / \tau_i \cdot t$. Now, if we consider $t = \tau_i$. The controller output is $c'(t) = k_c + k_c$. So, this term is given this yielding k_c understood or not, for the PI controller one additional term is there that is integral action. The integral action can be represented by this form, integral action equals k_c divided by τ_i integration of error $t d t$.

Now, the tuning parameter k_c is present also for p only controller. Now, the additional term τ_i is present with this PI controller that τ_i is called integral time constant or reset time. The unit of these integral time constant is time. Now, we obtain these form $c'(t) = k_c + k_c / \tau_i \cdot t$ considering it a unit step change in error signal this form we obtain.

Now, we are just considering time t equals τ_i then the controller output is $c'(t)$ equals to k_c can we define now reset time. What is reset time, reset time is a time required for the controller, reset time is the time required by the controller to repeat the initial proportional action change in its output you see the initially we had one proportional gain that is k_c . Now, due to the addition of integral term and considering time equals τ_i we are getting another proportional action, and therefore we can say that, the reset time is the time required by the controller to repeat the initial proportional action change, in its output.

Thank you.