Process Control and Instrumentation
Prof. A.K.Jana
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 14
Dynamic Behavior of Chemical Processes (Contd.)

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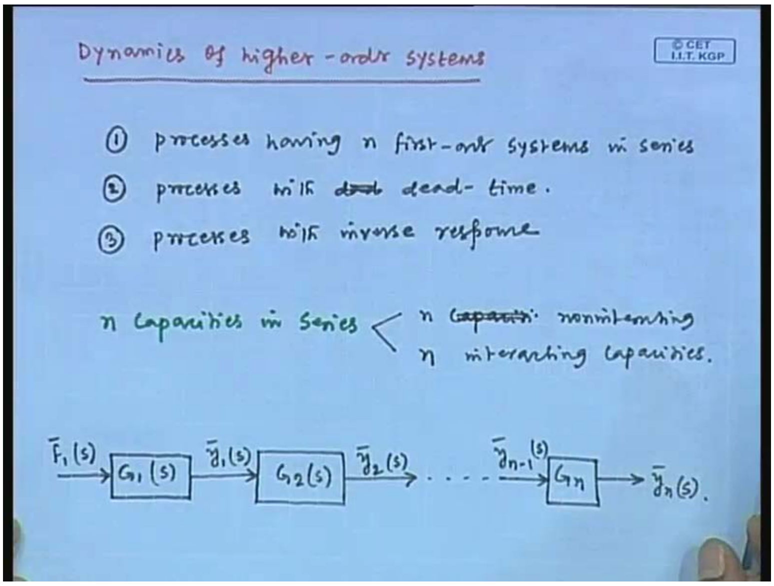

Today, we will start the topic that is dynamics of higher-order systems. Previously, we discuss the dynamics of first order system then the dynamics of second order system. So, today we will discuss dynamics of higher than second order system. Now systems higher than second order dynamics are not uncommon in chemical processes systems with higher than second order dynamics are not uncommon in chemical process example include the processes having n first order systems connected in series.

So, we can say that we are presently interested to discuss n order systems dynamics here n is greater than 2 now, n order system dynamics i mean the examples includes processes having n first order systems connected in series. Second example is processes with dead-time examples includes processes with dead time examples include processes with inverse response. So, these are the examples of n order systems and all these three processes will discuss one by one. So, first we will discuss n first order processes connected in series first we will discuss n capacities in series fine first we used to discuss n capacities connected in series. So, it is it has basically we can classified it into two different systems one is n non interacting capacities in series another one is n interacting capacities in series. So, it has it is two different I mean we can classify these into two classes one is n capacities n non interacting capacities and another one is n interacting capacities.

So, if we consider n non interacting capacities then we need to connect n first order systems in series. So, we will connect this n number of first order systems the transfer function of the first system is G1 S input to this transfer function is say F1 bar S output of these first system is y1 bar S. The second first order system has the transfer function of G2 S. Output of these Second first order system is y2 bar S like this we place the last or n first order system having transfer function of n input to this n first order system is y n minus 1 bar S output is y n bar S. This is n non interacting capacities connecting in series.

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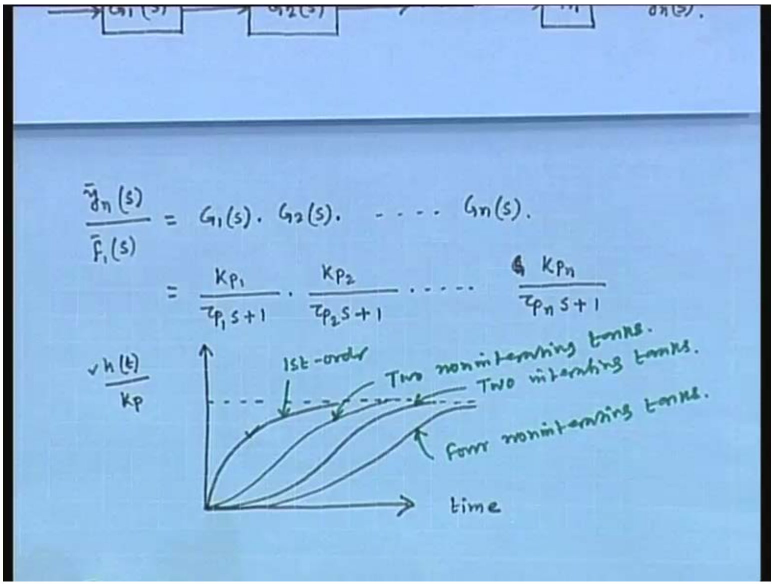

So, what will be the overall transfer function? Overall transfer function of these n non interacting capacities is represented by y n bar S divided by F1 bar S which is equal to G1 S multiplied by G2 S like this Gn S fine multiplication of transfer functions from G1 to Gn.

Now, what is the transfer function of a first order system? I mean G1 equals Kp 1 divided by tow p1 S plus 1 this we form we derived earlier similarly for the second first order system we can write Kp2 divided by tow p 2 S plus 1 and last transfer function is represented by Kpn divided by tow pn S plus 1 fine. So, this is the overall transfer function for n non interacting capacities in series.

Similarly, for the case of n interacting capacities in series we obtain more complex form fine for the n interacting capacities in series we can obtain the overall transfer function in more complex form. Now, we will conclude based on the dynamics presenting in a plot. So, this is the dynamics of different systems here we will consider height divided by Kp say this is the steady state value.

So, for the first order system we get the dynamics like this. This is for the two non-interacting tanks in series this is for two interacting tank in series and this is for four non interacting tank in series. This is for this is the dynamics of first order system see this is the dynamic this is the representation of dynamics of a series of tank system.

So, the first curve represents the dynamics of a single liquid tank system the second curve represents the dynamic behavior of the two non-interacting tanks system this is the dynamic behavior of two interacting tanks system and this is the Dynamics of four non interacting tanks system. So, this representation of Dynamics of liquid tank system.

If we consider a single liquid tank system then we get this type of behavior for two non-interacting two interacting and four non interacting tank system we obtain S staped behavior. And as I mentioned in the last class this h represents the height of first tank for single tank system the height h represents the height of second tank and Kp represents the gain of second tank for two tank system height h represents height Kp represents gain for fourth tank for four tank systems.

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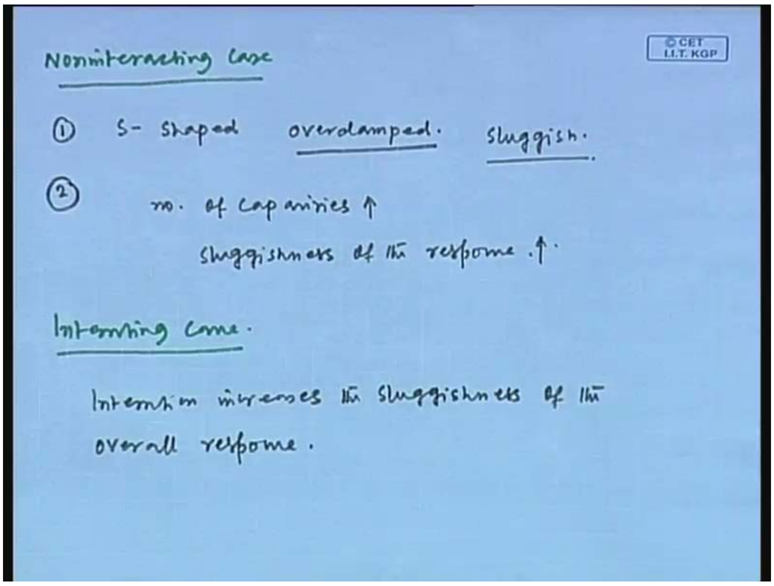

Now, we used to conclude based on these observation. So, first we will consider the non-interacting case. So, as I mentioned that we got S-Shaped response that means, the response as that characteristics of over damped system. So, we can say that the response has the character characteristics of an over damped systems that is the S-Shaped behavior.

And the response is also sluggish over damped means zeta is greater than 1 the overall response is sluggish. This is the first remark fine, second one is increasing the number of capacities increases the sluggishness of the response. If you see if you compare the dynamic behavior of two non-interacting tanks and four non interacting tanks. If, you see the behavior of two non-interacting tanks and four non interacting tanks then we can conclude that increasing the number of capacities increases the sluggishness of the response agree. So, this is the second comment increasing the number of capacities in series increases the sluggishness of the response sluggishness. This is the second conclusion we can make for non-interacting capacities systems.

Similarly, if we can conclude for the interacting case as interacting increases the sluggishness of the response if you again compare the dynamic behavior of two non-interacting tanks and two interacting tanks which is providing slow response two interacting tank. So, we can say we can conclude that interaction increases the sluggishness the response. We can conclude that interaction increases the sluggishness of the overall response this is the conclusion we can draw for the case of interacting tanks system.

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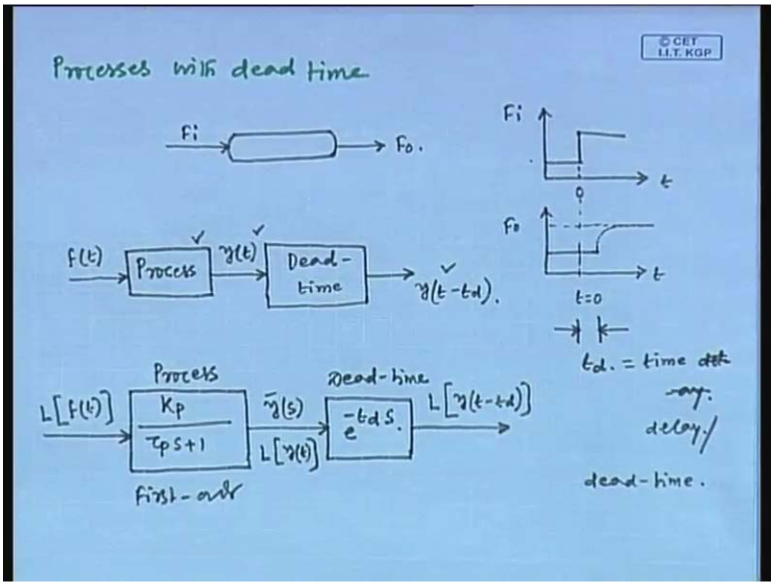

So, next we will discuss the processes with dead time which is also an example of higher order system next we will consider the processes with dead time. To discuss this topic we will first take an example say this is a Fi through which a liquid is flowing this is a Fi through which a liquid is flowing with the inflow rate of Fi and say the outflow rate F naught. Now, we are introducing a stape change at a particular time instant suppose we are introducing a stape change in Fi at time t equals to 0. Initially if it is at steady state I mean there was no change of flow rate with time now we are introducing a stape change in Fi.

So, suddenly we cannot get any change in the output of F naught. If, we give a stape change at time t equal to 0 in Fi at that time instant we cannot get any change in Fi. So, we can also make one plot for F naught verses t this is suppose representing time t equals 0 F naught changes after their time duration of td fine. This td is the time delay.

So, if we introduce a stape change in Fi at time t equals 0 at that time instant we do not get any change in F naught. F naught changes a after time interval represented by t suffix d that is the time delay.

Now, we will represent the time delay writing the number of blocks like we can consider one block for a process input to this process is suppose Ft fine output of the process is say yt now we need to include one block for dead time this time delay we again called dead time. So, one block we need to include for dead time. What will be the output of this dead time block? Output will be yt minus td if td is the dead time.

So, td is time delay or dead time this block is representing a process input to the process is Ft output from the process is yt and another block we have included for the dead time output for the dead time block is yt minus td now, we will represent this block diagram considering the variables in Laplace domain. So, this will be the process.

Say this is a first order process. So, we can write the transfer function Kp divided by tow p S plus 1 this is the transfer function of the process and we can say this is the transfer function of the first order process. Input to this process in is domain we can represent by this form Laplace transform of Ft output of this process we can represent by y bar S and, this is the dead time block output we can write Laplace transform of yt minus td. This is basically Laplace transform of yt fine and this block represent the Dead time in Laplace domain.

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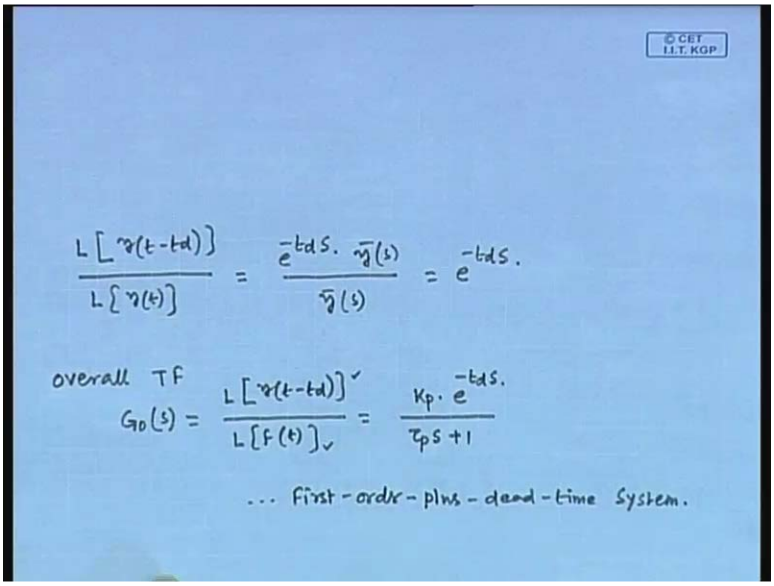

So, what will be the transfer function of dead time. What will be the dead time of Laplace domain? Laplace transform of dead time we can find by this way Laplace transform of yt minus td divided by Laplace transform of yt. So, this is exponentials of minus td S y bar S divided by y bar s that means, exponentials of minus tds. So, this is the transfer function of dead time block. So, we can put here the transfer function as exponentials of minus tds.

Next we need to determine the overall transfer function. What will be the overall transfer function? So, if we represent the overall transfer function by G naught S then G naught S equal Laplace transform of yt minus td divided by Laplace transform of Ft agree. This is the output of the overall system and this is the input of the overall system in Laplace domain if we consider first order process then the overall transfer function becomes Kp exponentials of minus tds divided by tow p S plus 1 if we consider the first order process then the overall transfer function becomes Kp exponentials of minus tds divided by tow p S plus 1 this is called first order plus dead time system.

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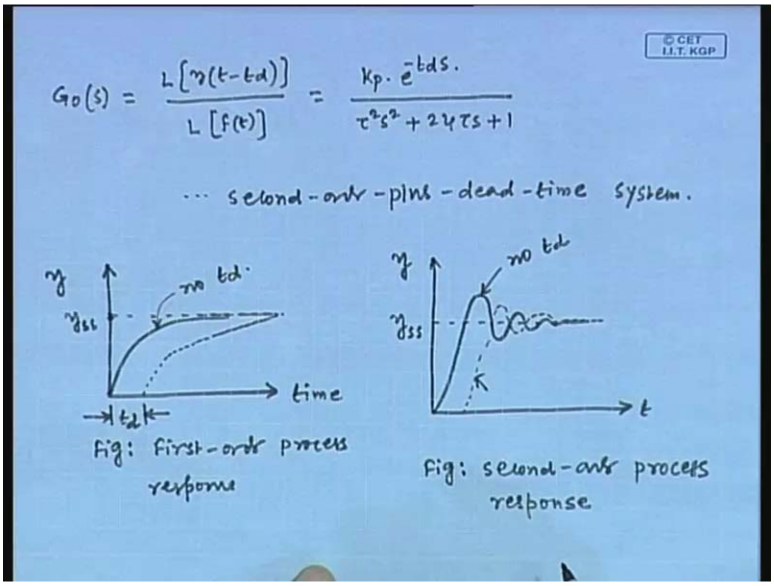

Similarly, if we consider second order process then the overall transfer function we can write as G naught S equals Laplace transform of yt minus td divided by Laplace transform of Ft Kp exponentials of minus tds divided by tow square S square plus 2 zeta tow S plus 1 this is the second order plus dead time system.

Now, we want to represent it graphically the first order and second order plus dead time systems this is time and this is suppose y this is the steady state of y. So, this value is representing yss that means, steady state value of y. Now, if there is no dead time then the response is like this if we (Refer Time: 28:06) the dead time then the response is like this. This is the response of first order system and if we consider the dead time then it becomes like this.

So, this is the first order process response with introducing it a stape change in input variable this is the First order Dynamics with introducing a stape change in input variable and this is the dynamic representation coin we this reader the dead time. So, no dead time I mean no td. So, this difference is the dead time. So, this is the Dynamics of First order system with dead time and without dead time. Similarly if we consider the second order system this is time this is y this value is the steady state value of y then for the second order system we get this type of response definitely this is the second order underdem response fine zeta less than 1 if there is dead time then the response is like this. So, this is representing the second order process response fine this is the Dynamics of second order response and there is no dead time and this is Dynamics of second order response and dead time is there.

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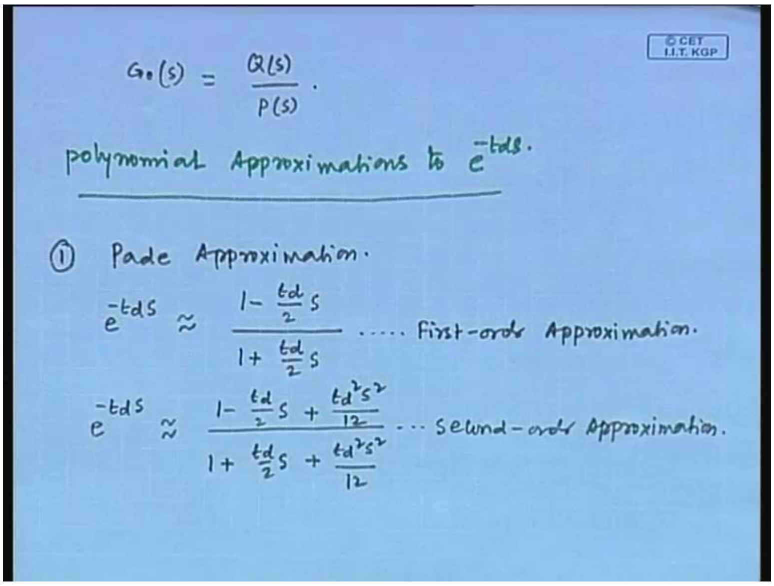

Now, it was discussed earlier that and we commented that it is convenient to represent the transfer function as the ratio of two polynomials it is easy to analyze the transfer function by representing it as the ratio of two polynomials and that is like G naught S equals Q S divided by P S if we represent the transfer function by this form it is easy to analyze.

Now, if there is a dead time then how we can represent the overall transfer function by the ratio of two polynomials that we will discuss next. So are title is polynomial Approximation to the dead time term I mean exponentials of minus tds. So, we can approximate this exponentials term by Pade Approximation. By Pade Approximation we can approximate the exponentials term the forms are like this exponentials of minus tds is approximately equal to 1 minus td divided by 2 S divided by 1 plus td divided by 2 S we can write the exponential term by this form and this is called First order Pade Approximation.

Similarly, if we can write the form Second order Pade Approximation 1 minus td divided by 2 S plus td square S square divided by 12 1 plus td divided by 2 S plus td square S square divided by 12 we can write the exponentials term by this form and this is called Second order Pade approximation fine by another way we can also approximate that is Taylor series expansion by Taylor method also we can approximate this exponential term.

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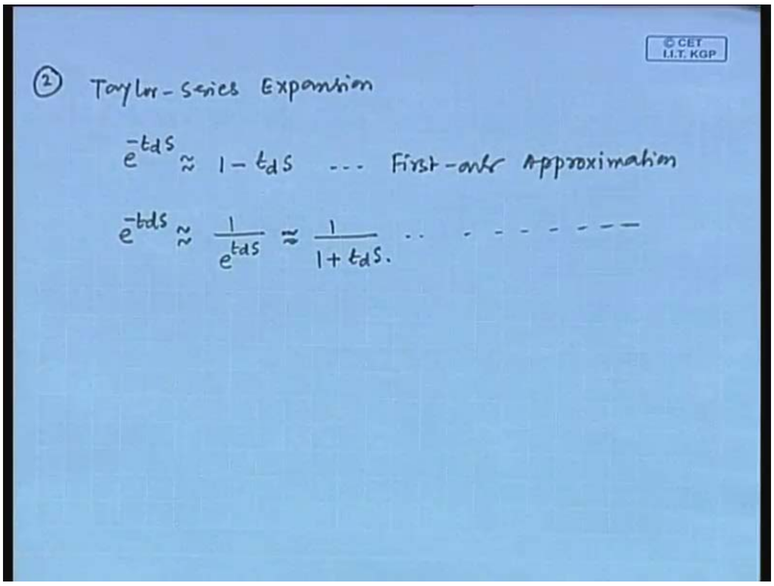

So, that is the second method Tayler series expansion according to this the exponentials of minus tds is approximately equal to 1 minus tds the exponentials of minus tds is approximately equal to 1 minus tds this is also First order Approximation this is first order Tayler Approximation. It as another form the Taylers by using Tayler method we can write this as 1 by exponential of tds which is equal to 1 divided by 1 plus tds exponentials of minus tds equals 1 by exponential of tds and that is equal to 1 divided by 1 plus tds this is also first order approximation first order Taylor approximation. In the next we will discuss in (Refer Time: 37:10) I mean how we can represent the higher order transfer function to lower order transfer function? That we used to discuss.

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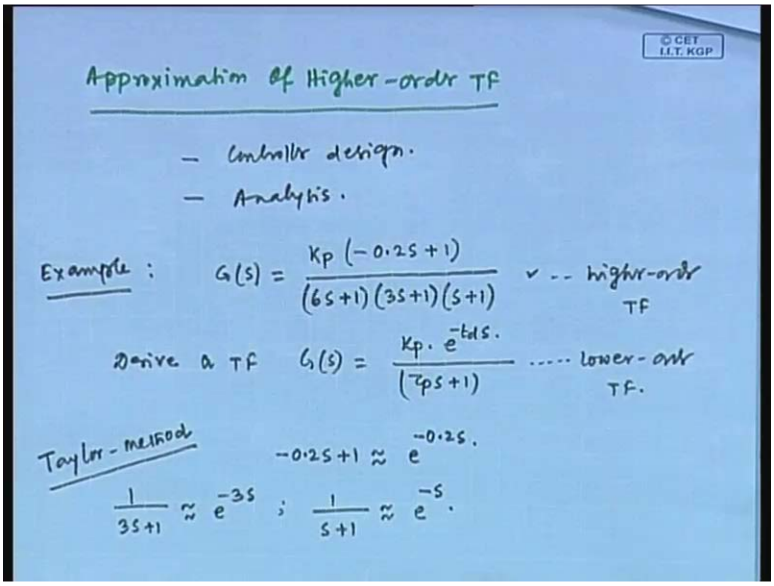

Approximation of Higher order transfer function to lower order transfer function fine next we used to discuss the approximation of Higher order transfer function to lower order transfer function, but why will go for these? By doing this we can easily design the controller. So, the first advantage is by performing the approximation of higher order to lower order we can easily design the controller.

So, it is easy to design the controller by transforming Higher order transfer function to lower order transfer function another advantage also it is easy to analyze it is easy to analyze the lower order transfer function. Now, you just take one example to discuss this approximation we will take one example to discuss this transformation suppose the overall transfer function is even for a particular system by this forms equals the process gain Kp minus 0.2 S plus 1 divided by 6 S plus 1 multiplied by 3 S plus 1 multiplied by S plus 1.

To discuss the transformation of higher order to lower order transfer function we have taken this example having the overall transfer function of these now we used to transform this transfer function to a first order plus dead time system. So, we we used to transform this transfer function in the form of Gs equals Kp exponentials of minus t ds divided by tow pS plus 1. So, this is higher order transfer function and our purpose is to transform these to lower order transfer function.

Now, this we can do by the use of Taylor method. So, first we will use the Taylor method. Now according to the Taylor method minus 0.2 S plus 1 is approximately equal to exponentials of minus 0.2 S agree according to the Taylor method we can write minus 0.2 S plus 1 is approximately equal to exponentials of minus 0.2 S. Similarly, we can write 1 divided by 3 S plus 1 approximately equal to exponentials of minus 3S again we can write 1 divided by S plus 1 equals exponentials of minus S.

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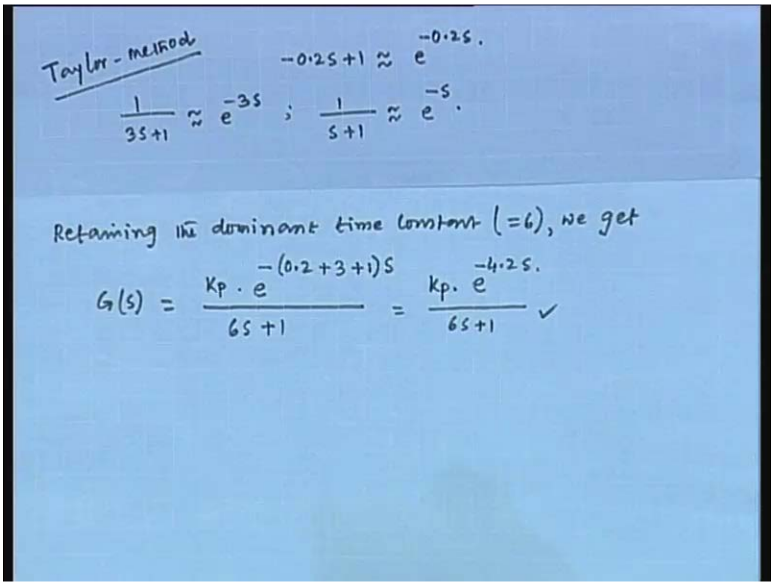

Now, retaining the dominant time constant what is that? That is equal to 6 we get Gs equal to Kp exponentials of minus 0.2 another one is 3 and 3rd one is 1 multiplied by S and the largest time constant remains in the denominator. So, can we write this from the higher order transfer function. So, it becomes Kp exponentials of minus 4.2 S divided by 6 S plus 1. So, this is the representation of higher order transfer function by the first order plus dead time system transfer function.

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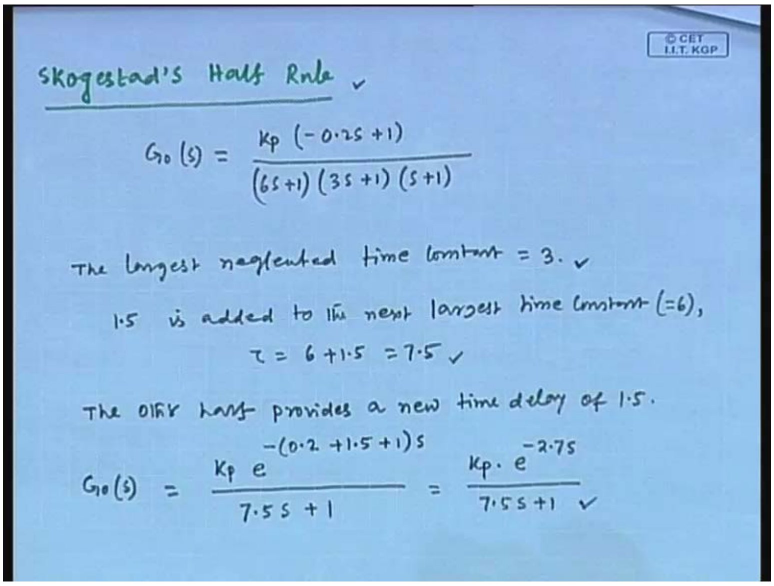

This is one method another method we want to discuss that is Skogestad's half rule method using which we can also transfer the higher order transfer function to lower order transfer function. So, next we will discuss another technique that is Skogestad's half rule.

The overall transfer function which we have taken is represented again I am writing that divided by 6S plus 1 3S plus 1 S plus 1. Now we will transform this transfer function to first order plus dead time system by the use of Skogestad's half rule. So, what is the neglected highest neglected time constant.

The largest neglected time constant considered in the Taylor method is equal to 3 fine. In the earlier discussion we have neglected this time constant which is largest one now, according to this Skogestad's half rule half of these I mean 1.5 is added to the next largest time constant try to remember the stape in the Taylor method the largest neglected time constant considered is 3.

According to this Skogestad's half rule 50 percent of this value that is 1.5 is added to the next largest time constant to the next largest time constant what is the next largest time constant? 6 next largest time constant is 6 that means, the time constant becomes 6 plus 1.5 that is 7.5.

The time constant becomes 6 plus 1.5 that is 7.5 and the others half provides a new time delay provides a new time delay of 1.5 that means, another half is considered as time delay that will be added within the exponentials terms. Then, what will be the reduced transfer function according to this half rule transfer function finally, becomes Kp exponentials of minus 0.2 is there half of 3 that is 1.5 is added and 1 remains there the time constant is already obtained 7.5.

So, the denominator becomes 7.5 S plus 1 that means, Kp exponentials of minus 2.7 S divided by 7.5 S plus 1 it has been observed that this transfer function obtained by the use of Skogestad's Half Rule provides better performance then the Taylor than the transfer function obtained by the use of Taylor method.

It has been observed that the performance provided by the reduced transfer function obtained by the use of Skogestads Half Rule is better than the response provided by the reduced transfer function obtained by the use of Taylor method.

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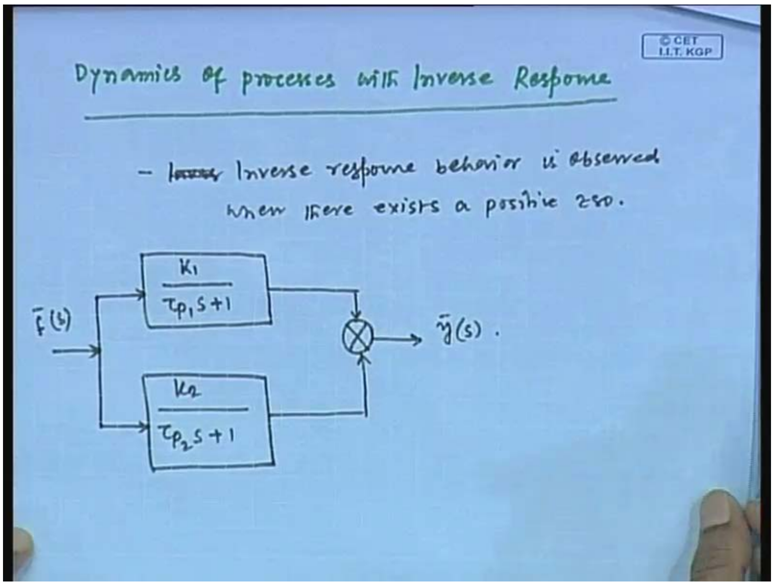

So, next we will discuss the Dynamics of processes having Inverse Response. This type of processes also provide nth order Dynamics. Now, this type of behavior is observed when there is a positive 0 this is the main point associated with this topic. This type of behavior I mean the Inverse Response behavior is observed when there exists a positive 0.

So, we will consider a process we will consider a two processes acting in parallel one has the transfer function of K1 divided by tow p1 S plus 1 another one has the transfer function of K2 divided by tow p two S plus 1. Now, we want to find the condition at which this process shows Inverse Response behavior we used to find the condition at which this process provides Inverse Response behavior. So, input to this process is f bar S and this process is output is y bar S.

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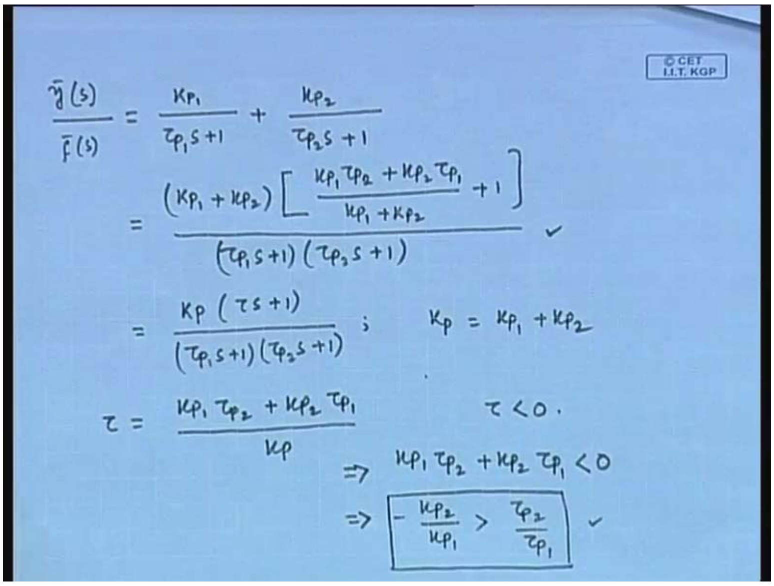

The two processes are acting in parallel. Now, first we need to find the overall transfer function the overall transfer function y bar S divided by f bar S which is equal to K1 divided by tow p 1 S plus 1 this is Kp 1 and another transfer function is Kp 2 divided by tow p two S plus 1. Now, we get from this Kp 1 plus Kp 2 multiplied by Kp 1 tow p 2 plus Kp 2 tow p 1 divided by Kp 1 plus Kp 2 plus 1 whole divided by tow p 1 S plus 1 tow p 2 S plus 1.

We get this transfer function this overall transfer function now, we can represent this again as Kp tow S plus 1 divided by tow p 1 S plus 1 multiplied with tow p 2 S plus 1. Here Kp is Kp 1 plus Kp 2 where Kp is Kp 1 plus Kp 2 and tow is Kp 1 tow p 2 plus Kp 2 tow p 1 divided by K. This is the tow. So, for positive 0 tow should be less than 0 agree that means, we can write Kp 1 tow p 2 plus Kp 2 tow p 1 less than 0 agree.

So, rearranging this again we obtain Kp 2 divided by Kp 1 greater than tow p 2 divided by tow p 1 rearranging finally, we get this condition. So, at this condition the process shows Inverse Response behavior. So, Inverse Response is the net result of the two opposing effects if you see the sign tow p 1 tow p 2 both are positive. So, if one Kp is positive another Kp should be negative. So, we can conclude that Inverse Response is the net result of the two opposing effects.