Process Control and Instrumentation
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Lecture - 13
Dynamic Behavior of Chemical Processes (Contd.)

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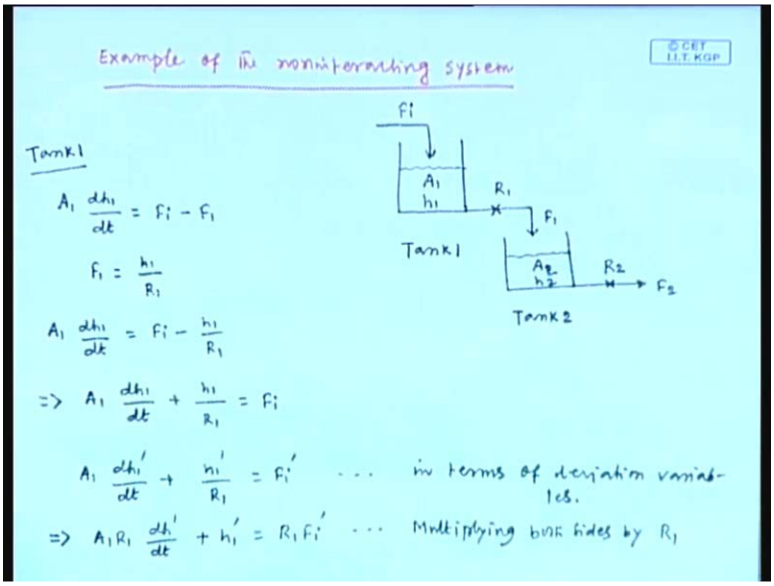

Today, we will continue our discussion on dynamics of second order systems. And we will start today, with a physical example of noninteracting system. Example, of the noninteracting system it was told that if 2 first order systems or 2 first order tanks are connected in series the overall response is second order response. So, today we will take that example I mean, 2 first order systems those are first order tank systems connected in series.

So, this is tank 1, input to the tank is Fi cross sectional area is A1 and height is h1output is F1 and resistant to flow is suppose R1. This outlet from Tank 1 enters Tank 2, this is Tank 2 cross sectional area of this Tank 2 is a 2 and liquid height in the Tank is h2, output stream from Tank 2 is F2 and suppose this resistance is R2 this is a example of a noninteracting system.

Now, what are the modeling equations for these two Tanks? If we consider Tank 1 the modeling equation is A1 dh 1 dt equals input flow rate that is Fi minus output flow rate that is R1. This is a modeling equation for Tank 1. Now, F1 is again we can write by this form h1 divided by R1 equals h1 divided by R1 driving force divided by resistant's to flow. If, we substitute this F1 equals h1 divided by R1 in the modeling equation then, we have A1 dh 1 dt equals Fi minus h1 divided by R1. Substituting the expression of F1 we finally get this, rearranging this equation we have A1 dh 1 dt plus h1 divided by R1 equals Fi fine, rearranging we get this.

Now, if we write this equation in terms of deviation variables, then we obtain A1 dh 1 prime dt plus h1 prime divided by R1 equals Fi prime. This is modeling equation in terms of deviation variables. Fine now, if we multiplied both sides by R1 then we get A1 R1 dh 1 prime dt plus h1 prime equals R1 Fi prime, this equation we get by Multiplying both sides with R1 fine.

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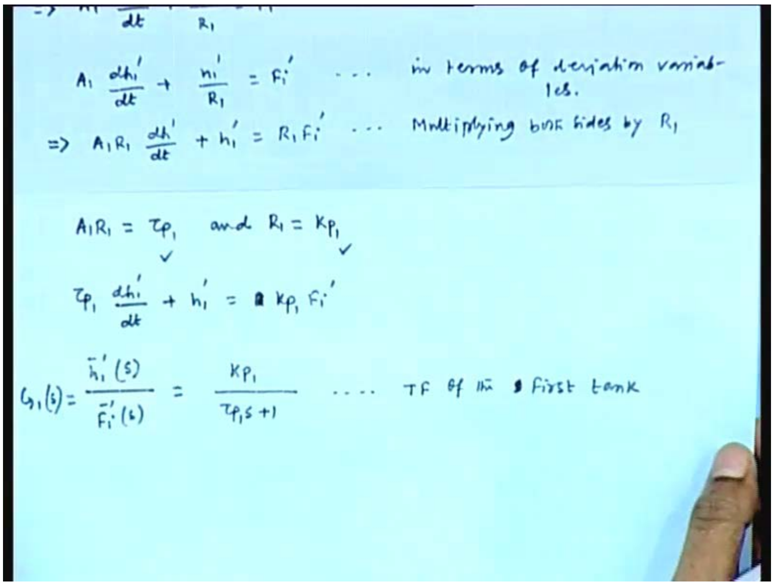

Now, you will assume that A1 R1 equals tau p1 and R1 is equal to Kp 1we assume A1 R1 equals tau p1 which is for the time constant First Tank and R1 equals kp 1 this is the game of the first tank. Then, the equation modeling equation becomes tau p1 dh 1 prime dt plus h1 prime equal to this is Kp 1 Fi prime this is the first order differential equation of the output h1.

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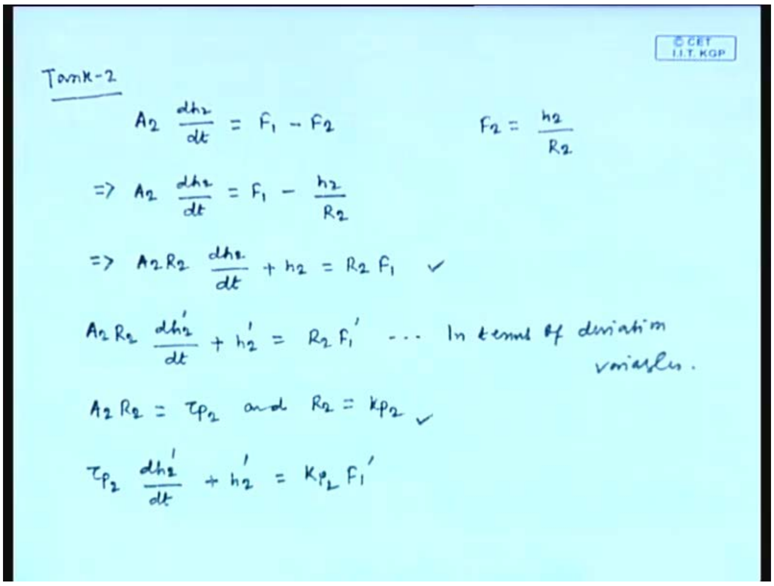

Now, if we take Laplace transform and rearrange then we get the transfer function of the First tank as h a bar prime is divided by Fi bar prime is equals Kp 1 divided by tau p1 s plus 1 this is a transfer function of the First tank that is tank 1. Now, you will represent this as g1 s similarly we will consider tank 2 for tank 2 we can write the modeling equation as A2 dh 2 dt equals input flow rate that is F1 minus output flow rate that is F2.

Now, F2 is again the driving force divided by resistant to flow. So, FT F2 equal to h2 divided by R2 if you substitute that in this modeling equation we get A2 dh 2 dt equal to F1 minus h2 divided by R2. Multiplying both sides R2 and rearranging we get A2 R2 dh 2 dt plus h2 equal to R2 multiplied by F1 fine multiplying both sides by resistant R2 and rearranging we get this equation.

Now, similarly we have to represent this equation in terms of deviation variables then we get A2 R2 dh 2 prime dt plus h2 prime, equal to R2 F1 prime. This is represented in terms of deviation variables fine. Now, we assume A2 R2 equals tau p2 and R2 is equal to Kp 2, we assume A2 R2 is equal to the time constant for tank 2 and R2 as gain of the tank 2. Then we get tau p2 dh 2 prime dt plus h2 prime equal to Kp 2 F1prime.

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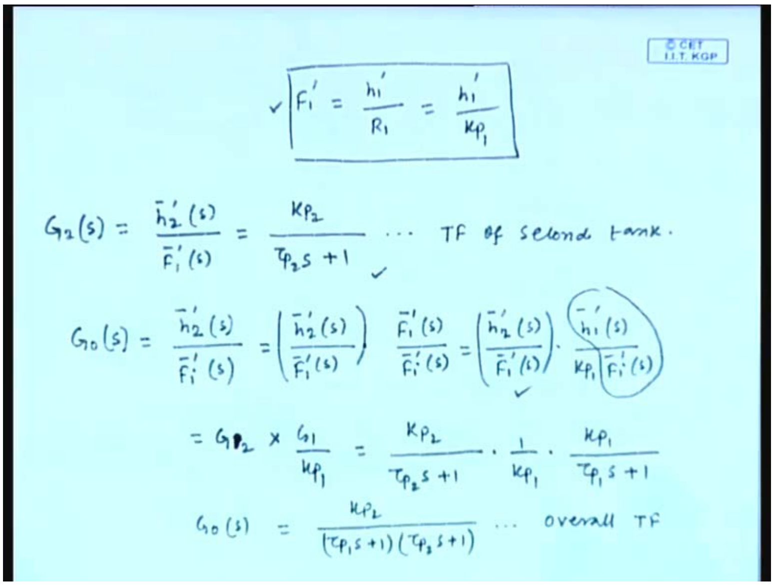

And taking Laplace transform finally, we get the transfer function of tank 2 as G2 s is equal h2 bar prime is divided by F1 bar prime s equals Kp 2 divided by tau p2 s plus 1. This is the transfer function of the second tank that is tank 2. Now, if you see the schematic of the 2 tank system. We can say that the overall transfer function will be h2 bar prime s divided by Fi bar prime s fine. The overall transfer function of this 2 tank systems can be written G naught s equals h2 bar prime s this is a output of tank 2 divided by the input of input 2 tank 1 fine. This is a overall transfer function.

Now, we can write again the overall transfer function by this form, h2 bar prime s divided by F1 bar prime s multiplied by F1 bar prime s divided by Fi bar prime s can we write this, the overall transfer function we can write in this form.

So, this is equal to h2 bar prime s divided by F1 bar prime s this is nothing, but the transfer function of tank 2. And what is F1 bar prime s we know F1 bar equals h1 bar divided by R1, for the tank 1 we have consider this correlation. F1 bar equals h1 bar divided by R1 that means, h1 bar divided by kp 1. Can we write this?

Now, this relationship will substitute here, I mean F1 bar F1 prime this is not bar prime f1 prime equals h1 prime divided by kp 1. So, if we substitute here, h1 bar prime s divided by kp 1 Fi bar prime s. Can we write this? There is no change of this term. Now, in place of F1 prime we have substitute it h1 prime divided by kp 1 and then, Fi prime remains there. So, the first term this 1 represents Gp 2 not Gp 2 G2 and multiplied by this which is G1 divided by kp 1 that means, kp 2 divided by tau p2 s plus 1 multiplied by 1 divided by kp 1 multiplied by kp 1 divided by tau p1 s plus 1 agree. So, the overall transform function G naught s we can write as, kp 2 divided by tau p1 s plus 1 multiplied by tau p2 s plus 1. This is a overall transfer function of the example non-interacting system.

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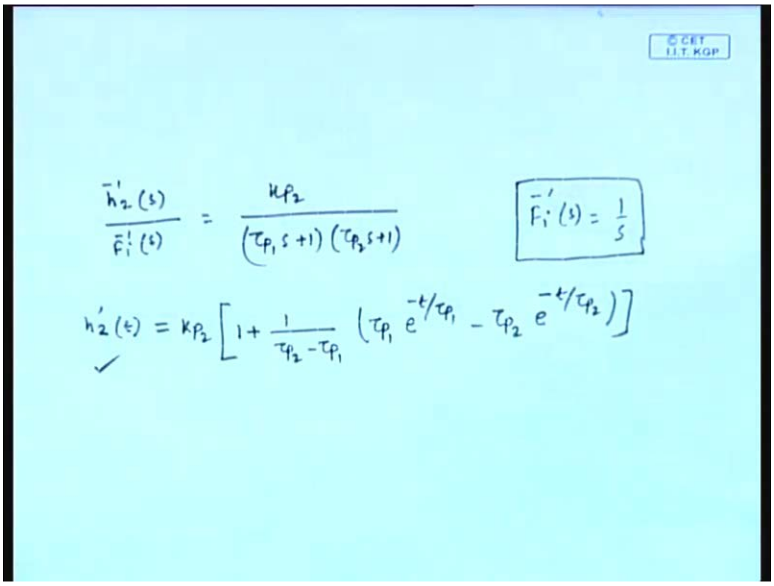

So, G naught s is basically h2 bar prime s divided by Fi bar prime s. We can write in this form now, you will introduce a unit step change in the input variable, considering a unit step change in Fi and taking inverse of Laplace transform we get the output in time domain as h2 prime t equals kp 2, 1 plus 1 divided by tau p2 minus tau p1 tau p1 exponential of minus t divided by tau p1 minus tau p2 exponential of minus t divided by tau p2 considering a unit step change in Fi bar prime s I mean, Fi bar prime s equals 1 by s and taking inverse of Laplace transform we get the expression of output h to in time domain like this fine.

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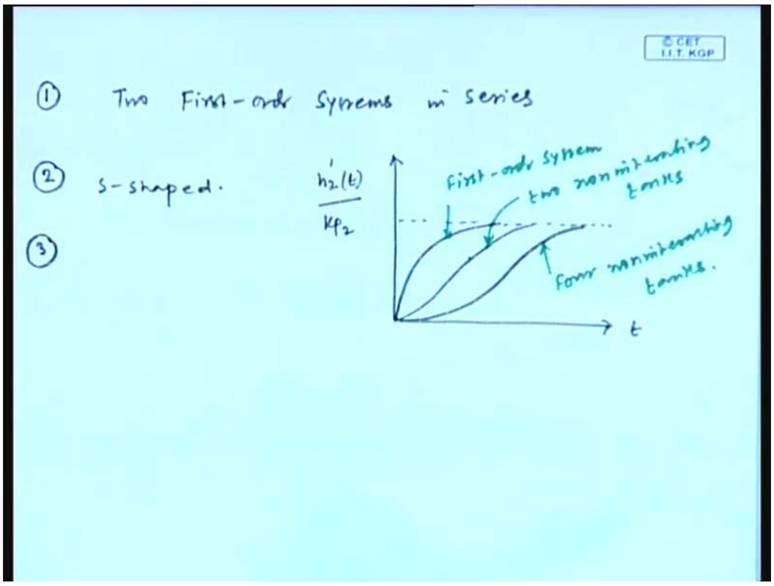

Now, you will conclude based on this discussion. So, First conclusion is that, by connecting to First-order systems in series. We get overall Second-order dynamics fine, by connecting to First-order system in series we obtain Second-order dynamics. So, by connecting to First-order systems in series the overall dynamics becomes Second-order dynamics this is a First conclusion. Second thing is we used to observe the behavior making a plot.

So, this is h2 prime t divided by kp 2 verses time. The steady state is represented by the dotted line and the dynamics is like this, is for First-order system. This is the dynamics of First-order system, this is the dynamics of Second-order or 2 non-interacting 2 non-interacting tanks system and this is the dynamics of 4 non-interacting tanks fine.

For the First-order system this should be h by kp for 2 non-interacting tanks this should be h2 divided by kp 2 or h2 prime by kp 2 for 4 non-interacting tanks this should be h 4 prime t divided by kp 4. So, this is the dynamic behavior of 3 systems. Now, what conclusion we can make initially you see it changes slowly for the 2 non-interacting tank systems initially it changes slowly then it fix up the speed, the behavior for second and higher order systems is s shaped, fine the behavior is s shaped. So, initially it changes slowly then its fix up the speed fine.

So, this is basically the over time response, and 3rd conclusion is as the number of capacities in series increases the delay in the initial response becomes more pronounced.

You see if you compared these two curves, the delay is more for the case of 4 non-interacting tank system. So, that is the 3rd conclusion. As the number of capacities in series increases the delay in the initial response becomes more pronounced this is the 3rd conclusion fine.

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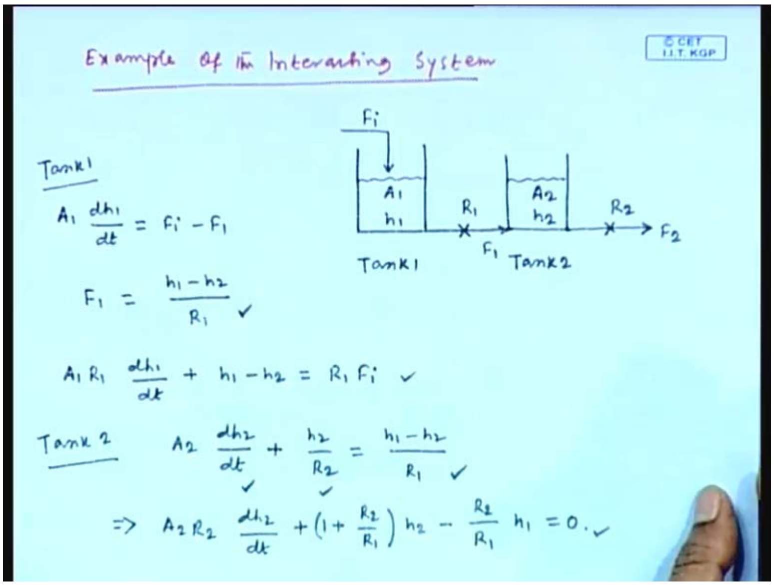

So, this is the example physical example of a non-interacting tank system. In the next we will consider an interacting tank systems; we will connect 2 tanks in series. So, that they are interactive to each other. So, next example is, the Example of the Interacting System fine. For the case of interacting system similarly, you consider 2 tanks which are connected in series. This is tank 1 cross sectional area of A1 and h1 input is Fi resistant to flow is R1, this is Tank 2 cross-sectional area A2 height is h2. This flow rate is F1 and the stream which is coming out from Tank 2 has the flow rate of F2 and resistance is R2.

So, the modeling equation for Tank 1 we can write in this form A1 dh 1 dt equals Fi minus F1. What is F1 here? What is the relationship of F1? F1 equals h1 minus h2 divided by R1. Since h1 is higher than h2 that is why there is a flow. So, if we substitute this, then we obtain A1, R1 dh 1 dt plus h1 minus h2 equals R1, Fi. If we substitute the expression of F1 in the modeling equation. Then we get this equation. Similarly, for Tank 2 the modeling equation we can write in this form A2 dh 2 dt plus h2 divided by R2 equals h1 minus h2 divided by R1. Can we write this, for Tank2 A2 dh 2 dt plus F2 equal to F1 fine. Now, these 2 modeling equation we need to write in terms of deviation variables anyway before that, we can rearrange this equations and we obtained A2 R2 dh 2 dt plus 1 plus R2 divided by R1, h2 minus R2 divided by R1 h1 equal to 0. By rearranging the modeling equation for Tank 2 finally, we get this equation fine.

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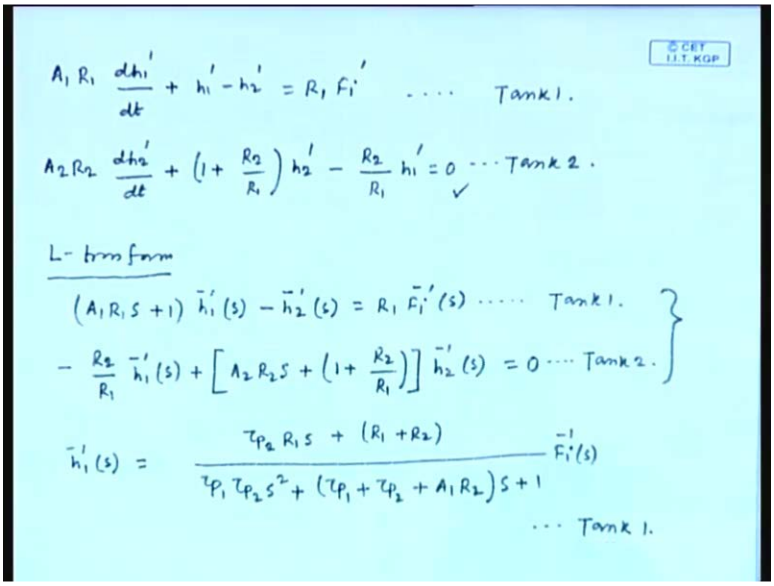

Now, we use to write these 2 equations in terms of deviation variables. So, for Tank 1 we can write the modeling equation in terms of deviation variables, and we obtained A1 R1 dh 1 prime dt plus h1 prime minus h2 prime, equals R1 Fi prime. This is for Tank 1. Similarly for Tank 2, we can write the modeling equation in terms of deviation variables as A2 R2 dh 2 prime dt plus 1 plus R2 divided by R1, h2 prime minus R2 divided by R1 h1 prime equals 0, this is for Tank 2 fine.

Now, we will take Laplace transform of these 2 equations. Taking Laplace transform we get for, Tank 1 A1 R1 S plus 1, h1 bar prime s minus h2 bar prime s equals R1 Fi bar prime s. Taking Laplace transform for the case of Tank 1, I mean the modeling equation of Tank 1 we obtain this form. Similarly if we take the Laplace transform of this, we obtained minus R2 divided by R1 h1 bar prime s, plus A2 R2 s plus 1 plus R2 divided by R1 h2 prime s equals 0. If we take the Laplace transform for the modeling equation of Tank 2 we obtain this expression.

Now, we need to solve these 2 equations, to obtain the transform function with respect to Tank 1. As well as Tank 2, solving these 2 equations we get the transfer function for Tank 1 as h1 bar prime s, equals tau p2 R1 s plus R1 plus R2, divided by tau p1 tau p2 s square, plus tau p1, tau p2, plus A1 R2 multiplied by s plus 1 Fi bar prime s. This is for Tank 1 fine solving these 2 equations we obtain the transfer function for Tank 1 in this form.

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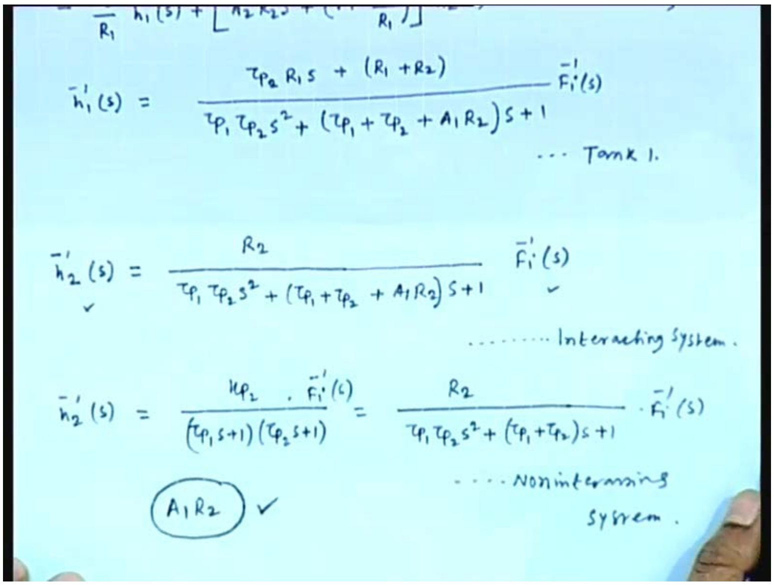

Similarly, for Tank 2 we obtain the transfer function as h2 bar prime s equals R2 divided by tau p1, tau p2 s square plus tau p1 plus tau p2, plus A1 R2 s plus 1, Fi bar prime s. This is a transfer function with respect to h2 fine. So, can say that this is the representation of overall transfer function because, overall transfer function we can represent by h2 bar prime s divided by Fi bar prime s. So, this overall transfer function we obtain for the case of Interacting System. This overall transfer function we obtain for the case of Interacting System. What is the overall transfer function of the non-interacting system? For the non-interacting system we obtained, the overall transfer function in this form kp 2 divided by tau p1 s plus 1 multiplied by tau p2 s plus 1, just we obtain before this discussion.

So, we can write this as R2 divided by tau p1 tau p2 s square plus tau p1 plus tau p2 s plus 1 it will be multiplied with Fi bar prime s. So, Fi bar prime s this is the overall transfer function for non-interacting system. Can we compare these two overall transfer function, if we compare we see that 1 extra term is there for the case of interacting system. That is A1 R2 this term this is the additional term which is present in the overall transfer function of interacting tank system.

So, this term may be thought of as the interaction factor. These term A1 R2 may be thought of as a interaction factor fine. If we compare the 2 transfer functions we see that 1 additional term A1 R2 is present in the overall transfer function of interacting system, and these term may be thought of as the interaction factor.

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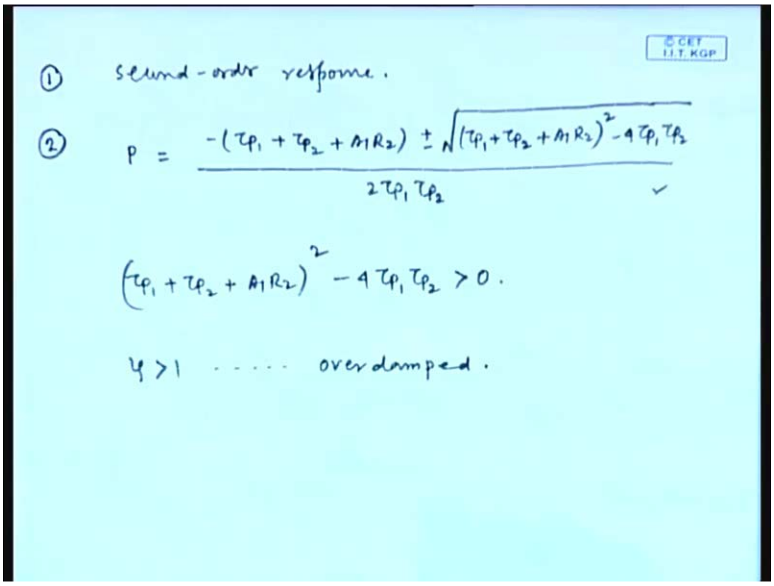

Now, we will conclude based on this discussion for the case of interaction, Interacting System. What is the First conclusion? First conclusion is that the overall response is Second-order response this is the First conclusion. By connecting to First-order liquid tanks in series we obtain the overall response as Second-order response.

So, the overall response is Second-order response. Second conclusion in the second conclusion we use to first find the roots. What are the roots? There are 2 roots because, if you see the denominator that is if Second-order polynomial. That is it quadratic that is given in quadratic form. So, the roots we can write in this form, tau p1 plus tau p2 plus A1 R2 plus minus root over, tau p1 plus tau p2 plus A1 R2 whole square minus 4 tau p 1 tau p2, divided by 2 tau p1 tau p2 fine there are 2 roots and they are represented by this form.

Now, tau p1 plus tau p2 plus A1 R2, whole square minus 4 tau p1 tau p2 is greater than 0 fine. That means, 2 distinct real poles so, what about the overall response? The overall response is over damped response. So, we obtained zeta greater than 1 and the response of interacting capacities is always over damped.. So, these are about, the two tank systems I mean, we observe that if 2 First-order systems are connected in series the overall response is Second-order response.

Another case we mentioned in the previous class that, Second-order system we can obtained by employing one controller around process, we mentioned 3 cases. 1 is by connecting to First-order systems in series, 1 is by employing 1 controller with a process, and in the 3rd case we consider that few systems are inherently higher order systems.

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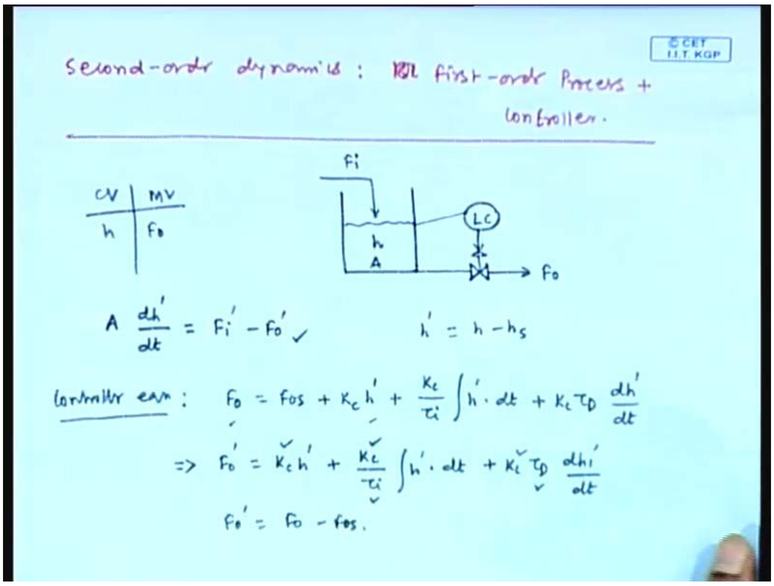

So, now we will consider the process having a controller. We use to observe the higher order dynamics. I mean Second-order dynamics for the case of a first-order process configured with a controller if a controller is employed for a first-order process what will be the overall dynamics that we use to observe.

So, we will first consider 1 example that is the liquid tank system. This is a liquid tank system, inlet flow rate is FI, and outlet flow rate is F naught. This is height cross-sectional area is A, control objective is to maintain the height of the liquid in the tank. So, for that we need to employ 1 level controller. So, our control objective is to maintain the height of liquid in the tank; that means the control variable is h and the corresponding manipulated variable is F naught fine, this is the control pair for this case.

Now, what is the modeling equation? Modeling equation in terms of deviation variables, we can write as A dh prime dt equals Fi prime minus F naught prime fine. Now since, the controller is manipulating F naught. So, we can correlate F naught with the height, by a controller equation suppose a controller equation is given as F naught equals F naught s, plus kc h prime plus kc divided by tau i integration of h prime dt plus kc tau d, dh prime dt. This is the equation of a controller which we will discuss in the subsequent classes fine.

This is a controller equation which is correlating the manipulated variable Fnaught with height; h prime is h minus hs fine. Now, we can write this equation as F naught prime equals kc h prime plus kc divided by tau i integration h prime dt plus kc tau d, dh prime dt ,where F naught prime equals F naught minus F naught s. And in this equation kc tau i and tau d these 3 are controller tuning parameters. In this controller equations kc tau i and tau d are constant parameters fine. Now, we can substitute the expression of f naught prime in the modeling equation, by substituting the expression of F naught prime in the modeling equation.

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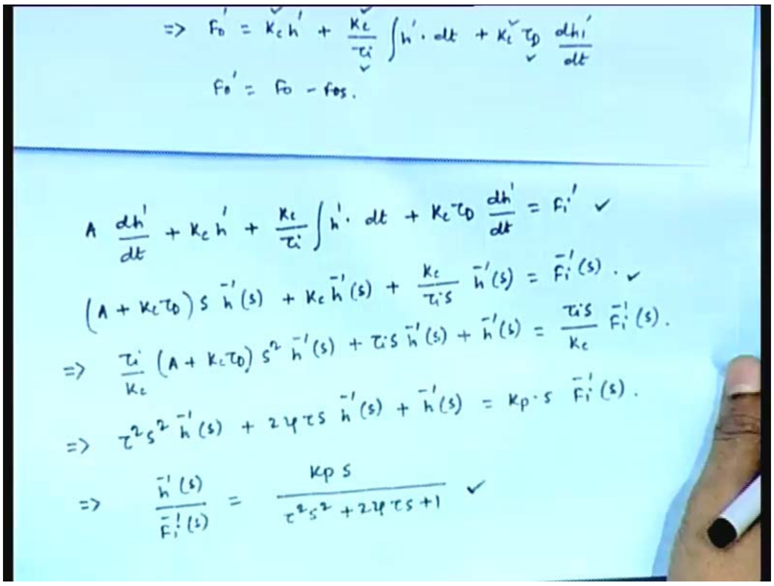

We obtain A dh prime dt plus kc h prime plus kc divided by tau i integration h prime dt plus kc tau d, dh prime dt equals Fi prime. Substituting the expression of F naught prime in the modeling equation we obtain this. Now, taking Laplace transform we get, A plus kc tau d s h bar prime s, plus kc h bar prime s, plus kc divided by tau i s, h bar prime s equals Fi bar prime s fine. Taking Laplace transform we obtain this. So, we can write this as tau i divided by kc A plus kc tau d s square h bar prime s, plus tau i is s h bar prime s plus h bar prime s equals tau i s divided by kc Fi bar prime s.

Now, we can represent this as, tau square s square h bar prime s, plus 2 zeta tau s h bar prime s plus h bar prime s equals kp multiplied by s Fi bar prime s fine. So, the transfer function can be represented by h bar prime s divided by Fi bar prime s equals kp s divided by tau square s square, plus 2 zeta tau s plus 1. The transfer function yields this form, h bar prime s divided by Fi bar prime s equals kp s divided by tau square s square plus 2 zeta tau s plus 1.

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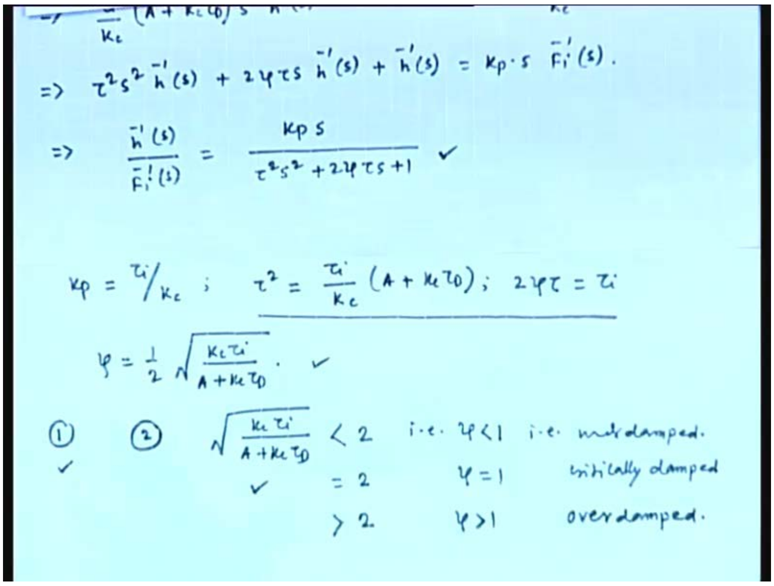

Now, here kp is equals to tau i divided by kc. Similarly tau square is equal to tau i divided by kc A plus kc tau d and 2 zeta tau is equal to tau i fine. We have considered in the derivation of a transfer function these simplified forms. I mean this correlation have been used in the derivation of transfer function.

So, from these equations we obtain, zeta equals half root over of kc tau I, divided by a plus kc tau d fine. These expression for zeta we obtain from these 2 forms. Now we want to conclude. So, our first conclusion is that by employing a controller around a first-order system, we obtain overall second-order response. This is our first conclusion by employing a controller; around a first-order system we obtain overall second-order system. And in the second conclusion we one to know whether, it is over damped or critically damped or under damped response

If, root over of kc tau i divided by a plus kc tau d less than 2 then zeta is less than 1; That means, this is under damped response. If this is equal to 2; that means, zeta is equal to 1. So, it is critically damped fine similarly if this is greater than 2 then zeta is greater than 1 then it is the case of over damped response fine. So, our first conclusion is the overall response become second-order and it may be over damped may be under damped may be critically damped depending on the value of this zeta.