Process Control and Instrumentation
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Lecture - 12
Dynamic Behavior of Chemical Processes (Contd.)

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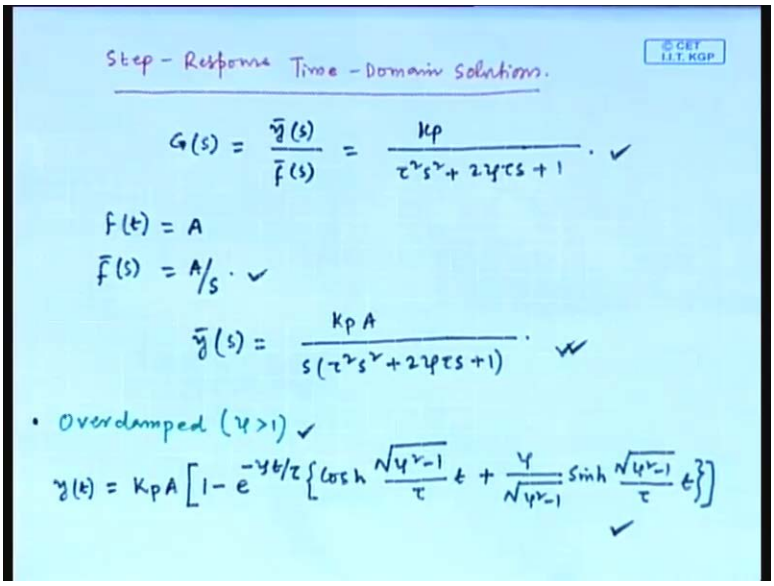

In the last class, we just started to discuss the time domain output. So, we will continue the topic that is step response time domain solutions, fine. Previously we derived the transfer function for the second order system that is represented by G s equals y bar s divided by f bar s equals K p divided by tau square s square 2 zeta tau s plus 1. Now, we will consider a step change in input variable, input variable is f t, if we considers step change with a magnitude of A that means f t equals A, and in Laplace domain, we can write this as A divided by s, fine.

Now, introducing this f bar s expression in this general form of transfer function, we obtain y bar s equals K p A divided by s tau square s square plus 2 zeta tau s plus 1, fine. Now, we will take the inverse of Laplace transform of this expression to obtain output in time domain, I mean y t. Now, we already considered three different cases depending on the value of zeta; one is over damped, second one is critically damped, and third one is under damped. So, we will get three different expressions of y t for these three different cases.

So, first we will write the expression for the case of over damped response in the case of over damped response, we have zeta greater than 1, fine. And the expression is y t equals K p A multiplied by 1minus exponential minus zeta t divided by tau cos hyperbolic root over zeta square minus 1 divided by tau into t plus zeta divided by root over of zeta square minus 1 sin hyperbolic root over zeta square minus 1 divided by tau into t, fine. This is the expression for the case of over damped response the derivation is not shown here, but if we take the inverse of Laplace transform of this equation we get this expression for the case of over damped system, fine. The derivation is left for the students.

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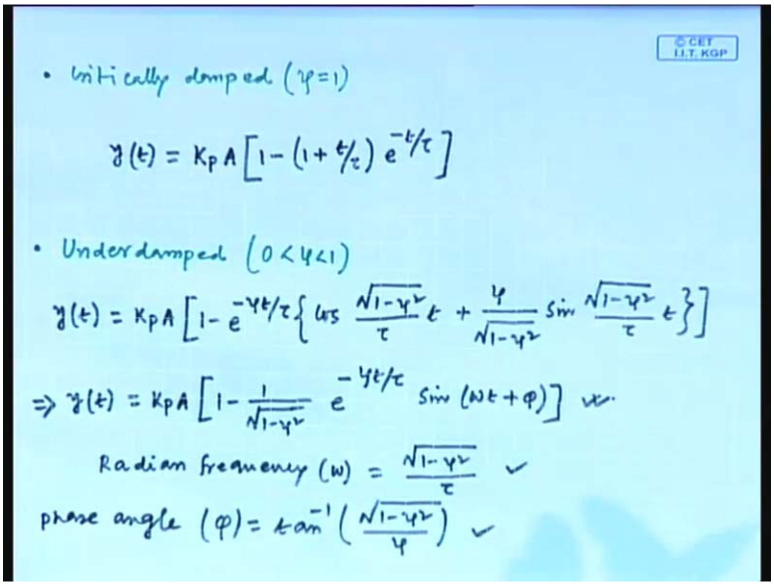

Similarly, for the in the second case, we consider critically damped response where zeta equals 1 and the expression of output in time domain we obtain by inverting as y t equals K p multiplied by A 1 minus 1 plus t by tau exponential minus t divided by tau this is the expression of output y in time domain for the case of critically damped systems, fine. For the case of under damped response, where zero less than zeta less than 1. We obtain the expression for output y in time domain as y t equals k p multiplied by A 1 minus exponential minus zeta t divided by tau cos root over of 1minus zeta square divided by tau into t plus zeta divided by root over of 1 minus zeta square sin root over of 1minus zeta square divided by tau into t.

This is the expression of output y for the case of under damped response, we can write this equation in this form where y t equals K p A1 minus 1divided by root over of 1minus zeta square exponential minus zeta t divided by tau sin omega t plus phi rearranging the expression of y in time domain. We get this last equation where the radian frequency omega radian frequency omega equals root over of 1 minus zeta square divided by tau this is the expression for radian frequency omega. Similarly, if the expression for phase angle phi, we obtain as phi equal tan inverse root over 1minus zeta square divided by zeta this is phase angle this is phase angle, fine. So, these are the three expressions of y in time domain just by inverting the equation of y in s domain.

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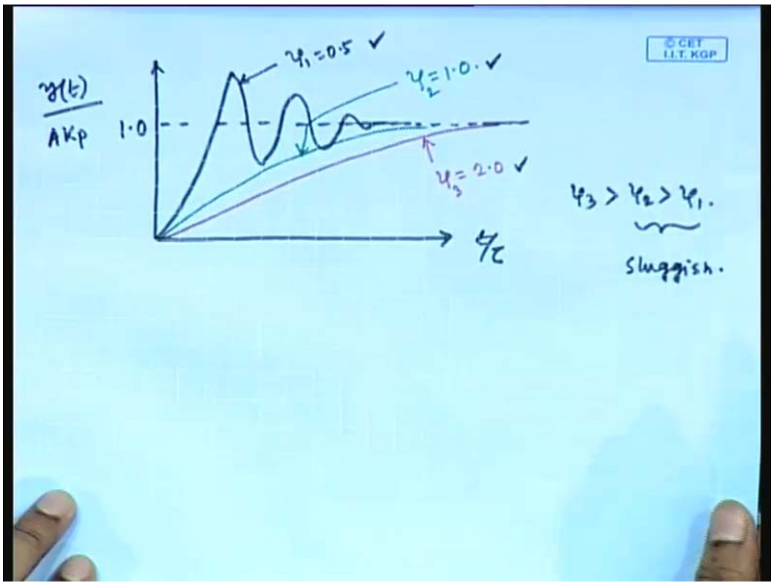

Now, if we graphically represent the outputs of this three y expressions the float looks like this t by tau and in this direction, we are considering y t divided by A K p suppose the final steady state value is 1. So, the output is like this type of response, we obtain for the value of zeta 1 that is suppose 0.5, fine. This type of dynamic response, we obtain for the value of zeta. Say 0.5 in another case, we get the response like this type of response.

We get suppose for the value of zeta equals 1and another type of response we get like this for the value of zeta suppose 2.0. This is a zeta 3 and this is zeta 2. So, obviously here zeta 3 greater than zeta 2 greater than zeta 1, fine. So, this is the case of under damped, where oscillations are there. And zeta 1equals 0.5 that is the case of under damped response and oscillations are there. In the second case, we have considered zeta

2 equals 1. There is no oscillation and that is the case of critically damped response. And in the third case, we have considered zeta 3equals 2 that is over damped response and there is no oscillation. Usually these two cases zeta 2 and zeta 3 are characterized as sluggish. The responses are under critically damped and under damped response are characterized as sluggish, fine. If you reduce the value of zeta 1 then 0.5, you will get more oscillation, fine. So, these are the responses of y at three different cases. Next you will discuss the under damped response in details.

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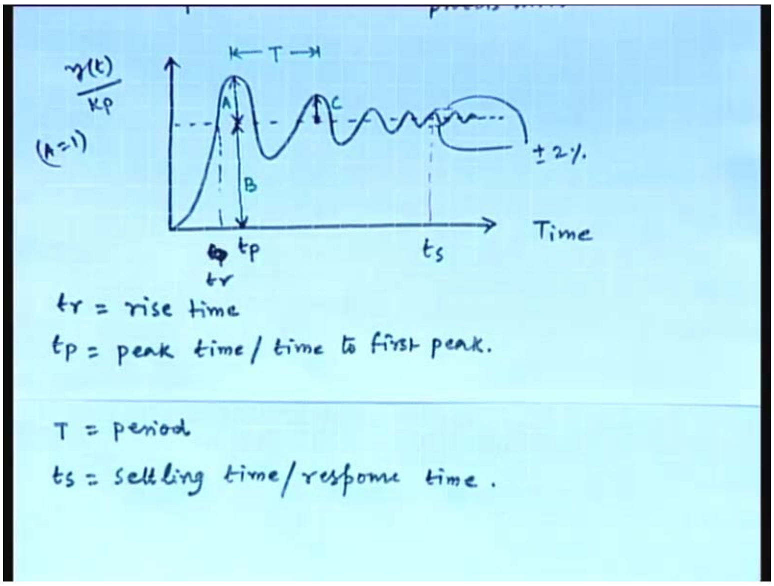

Almost all under damped responses in a chemical plant are caused by the interactions of controllers and process units. Almost all under damped responses, almost all under damped responses are caused in a chemical plant due to the interactions of controllers with the process units. That is why we are more interested to analyze the under damped responses, fine. Now, we will draw the response suppose this is time t this is y t by suppose K p, we are considering here A equals 1, fine. Now this is the new steady state value response is like this the time corresponding to this is say represented by t p.

Sorry, we will use another suffix that is t r, fine; t r is rise time. This is the first maximum peak the corresponding t is or corresponding time is t p. That is peak time or time to first peak t p is peak time or time to first peak, fine. This quantity is suppose B. And this quantity is suppose A. And this 1 is suppose C. This is the period represented by t. So, here t is the period, fine. And these are indicating plus minus 2 percent deviation. And corresponding time, we will represent by t s, where t s is the settling time or response time, fine. Now, we will know of these times t r t p t s along with the quantities a b c 1 by 1. So, first we will know about the rise time, it is very obvious from the figure that it is the time required for the output y to first reach its steady state value.

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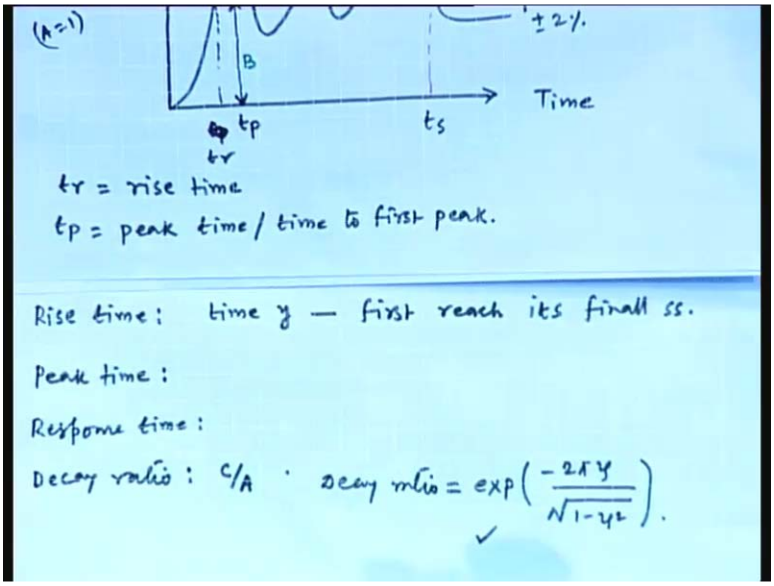

So, the definition of rise time is, it is the time required for the output y to reach, to first reach its final steady state value. Rise time is the time required for the output to first reach its final steady state value and it is very obvious from the figure, fine. So, what about the peak time, it is the time required for the output to reach its first maximum value. It is the time required for the output to reach its first maximum value. What is the response or settling time? Response or settling time, it is the time required for the output to come within some prescribed band of the final steady state value. It is the time required for the output to come within some prescribed band of the final steady state value, fine.

Next term is the decay ratio. Decay ratio decay ratio is define by the ratio of c and a. Decay ratio is defined by the ratio of c and a. We have the expression of y in time domain for the output y, fine. Now, from that equation, we can easily find the expression for this decay ratio that is. So, the expression of decay ratio is equal to exponential of minus 2 pi zeta divided by root over 1 minus zeta square. We have the expression of y in time domain for the under damped response from that expression, we can easily derive the expression for decay ratio and that is this 1 decay ratio equals exponential of minus 2 pi zeta divided by root over of 1 minus zeta square another term is there that is over shoot.

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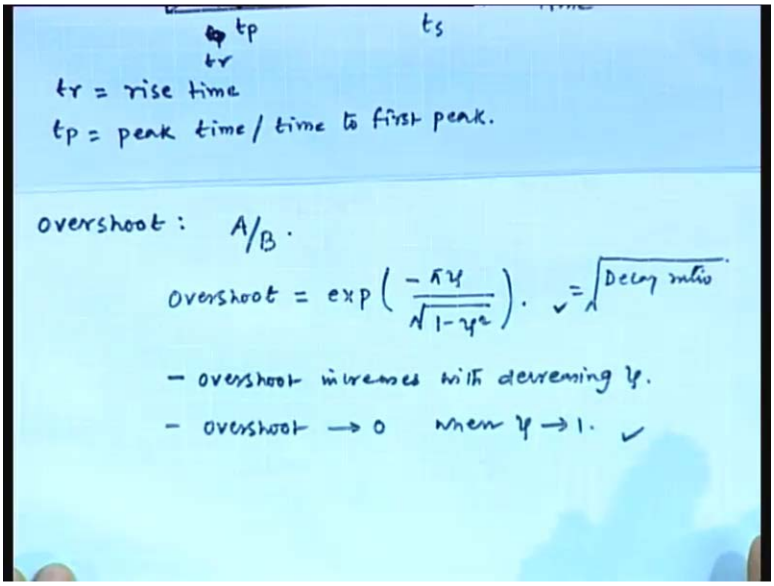

Another term is there that is called over shoot it is defined by the ratio of A and B over shoot is A divided by B, fine. And the expression for the over shoot is mathematical expression for the over shoot is represented by this equation, exponential minus pi zeta divided by root over of 1 minus zeta square. This is the expression for over shoot this expression also, we can derive from the time domain expression of y. If you see the expression of the over shoot and decay ratio is there any correlation between this two decay ratio is the square of over shoot or we can write over shoot is root over of decay ratio square root of decay ratio, fine. Now, over shoot increases with decreasing zeta, if you see this expression it is clear that over shoot increases with decreasing zeta. So, first remark we can make that is over shoot increases with decreasing zeta, fine. What happens? ((Refer Time: 25:27)) zeta becomes 1, there is no over shoot fine.

So, over shoot approaches 0. Over shoot approaches 0 and zeta approaches 1, we made 1 float comparing the responses of different zeta values. In 1 case we considered zeta equals 0.5. In another case, we considered zeta equals 1, and in the third case, we considered zeta equals 2. And you can see that float where zeta becomes 1 there is no over shoot, fine. It is very obvious from this mathematical explanation.

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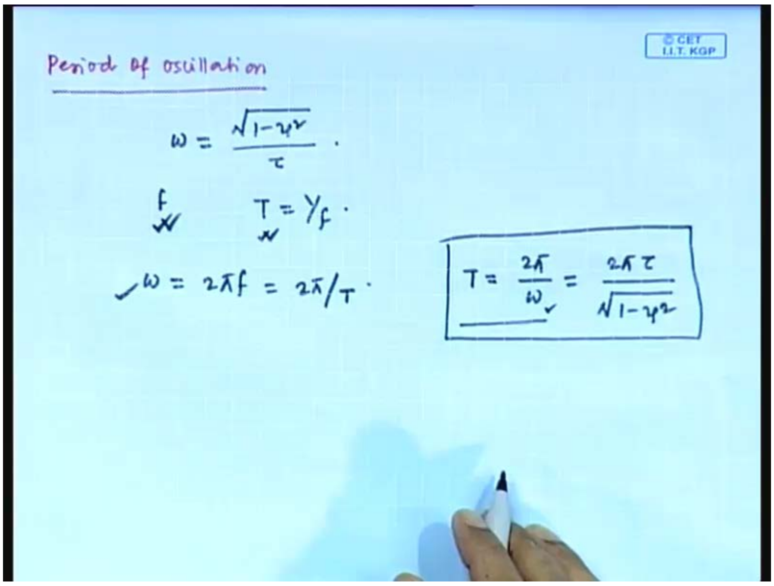

So, next we will discuss the period of oscillation period of oscillation for an under damped response, We got previously, the expression of radian frequency for the under damped response, We got the expression for radian frequency previously, where omega equals root over of 1 minus zeta square divided by tau, fine. This equation we have used in the expression of y in time domain for under damped case. Now, we will consider if cyclical frequency, unit is cycle spark unit time. So, if the cyclical frequency is f. Then the period is 1 by f, if f is the cyclical frequency then; obviously, the period becomes 1 by f fine. So, what is the radian frequency omega. Omega equals 2 pi f again this equals 2 pi divided by t. So, we can write t equals 2 pi divided by omega.

Substituting, the expression of omega, we finally obtain 2 pi tau divided by root over of 2 minus zeta square. f is the cyclical frequency, then we get, we know the period equals 1 by f s. Now, this is the expression for radian frequency omega equals 2 pi f means 2 pi divided by t finally, we can write the period equals 2 pi by omega, and here we substituted the expression of omega. And we finally get the expression for the period, fine. Next we will cover the natural period of oscillation, natural period of oscillation, fine. And in this case zeta equals 0.

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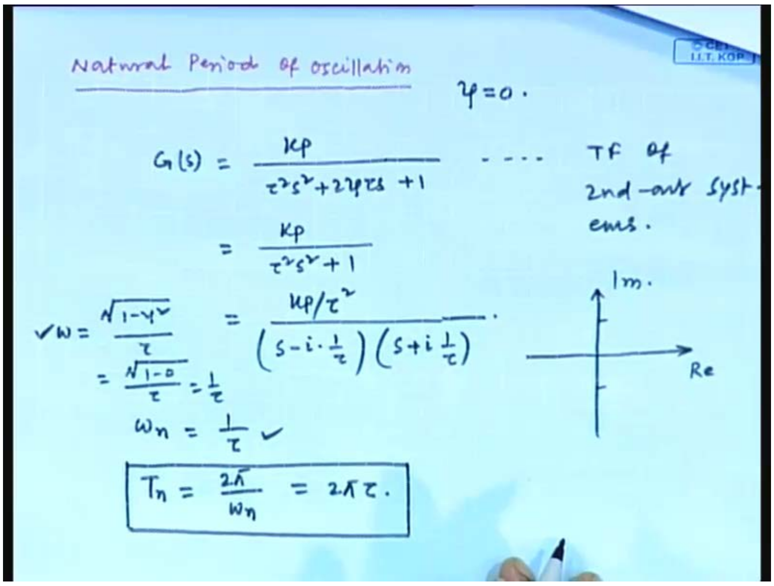

In this case zeta equals 0; that means, we observe the natural period of oscillation for un-damped systems fine. Now we know the transfer function of a second order system, that is K p divided by tau square s square plus 2 zeta tau s plus 1. This is the transfer function of second order systems, if we consider zeta equals 1. Then we get K p divided by tau square s square plus 1, considering zeta equals 1, we get the transfer function K p divided by tau square s square plus 1, we can write this as K p divided by tau square divided by s minus i 1 by tau s plus i 1 by tau, fine.

So, 2 poles are involved and they are complex conjugate poles fine. Can you, can you observe the position of the poles, this is the real axis, this is the imaginary axis there is no real part. So, both the poles lie on the imaginary axis, 1 is here, another 1 is here, the two poles are involved here and for both the poles there is no real part. Therefore, both the poles lie on the imaginary axis. So, what about the natural period? What about the natural frequency? Omega n, we know the expression of omega that is root over of 1 minus zeta square divided by tau.

We know the expression of omega that is omega equals root over of 1 minus zeta square divided by tau, substitute zeta equals 1 then will get omega n, so omega n equal to 1 by tau. Here you just substitute 0 1 minus 0 divided by tau that means, this will 1 by tau which is written here, fine. Then the period we can write by this way t n equals to 2 pi by omega n. that means, 2 pi tau, agree. So, this is the expression for natural period of oscillation t n equals 2 pi tau, fine. Next we will discuss the multi capacity processes as second order systems.

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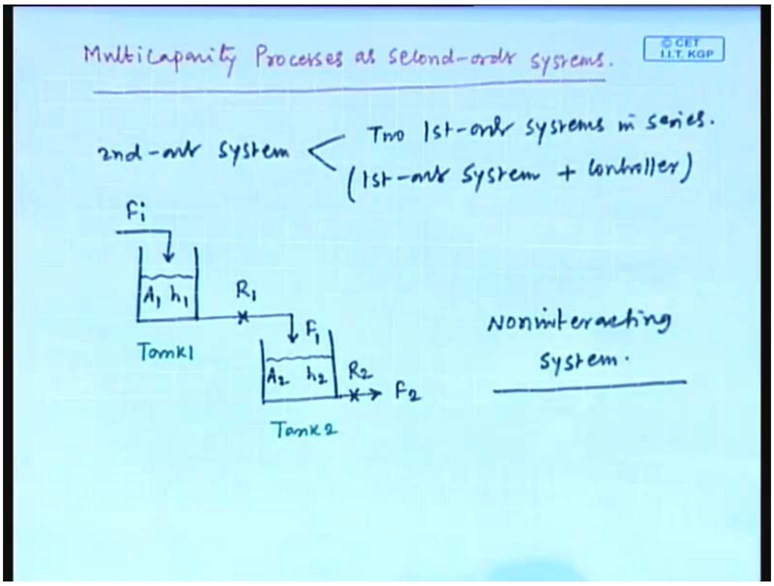

Next we will discuss the multi capacity processes as second order systems. We can obtain overall the second order dynamics if we connect to first order systems in series, fine. So, we can obtain a second order system by connecting to first order systems in series. In another way if we include a controller with a first order process then we obtain the second order dynamics.

So, in the first option if we connect to first order systems in series, we obtain second order dynamics in another case if we include a controller with a first order system. We obtain second order dynamics, fine. So, these 2 options we will discuss in details. So, first we will consider the connection of 2 first order systems in series. We will consider 2 simple liquid tank systems. So, how we can connect the 2 liquid tank systems in series? This is tank 1. Tank 1 input to this tank is F i cross sectional area is suppose A 1 height of liquid in the tank is suppose h 1 the output of tank 1 enters tank 2. This is tank 2 and this flow rate is suppose f 1 here resistance to flow is say R 1. In the tank 2 consider the cross sectional area A 2 and liquid height h 2. the stream which is coming out from tank 2 that has the flow rate of f 2 and resistance is suppose R 2, fine.

So, in this schematic representation to first order systems are connected in series. Now, how the tanks interact to each other? You see tank 1 effects tank 2, but tank 2 does not affect tank 1, because the flow rate to tank 2 inlet stream 2. Tank 2 is the outlet of tank 1. So, what about the interaction? This is 1 way interacting system tank 1 affects tank 2, but tank 2 does not affect tank 1. And we will use the term for this type of systems that is non-interacting system, fine. If the 2 liquid tanks are connected in series in this way, we will use the term non-interacting tanks. In another way also, we can connect the 2 tanks.

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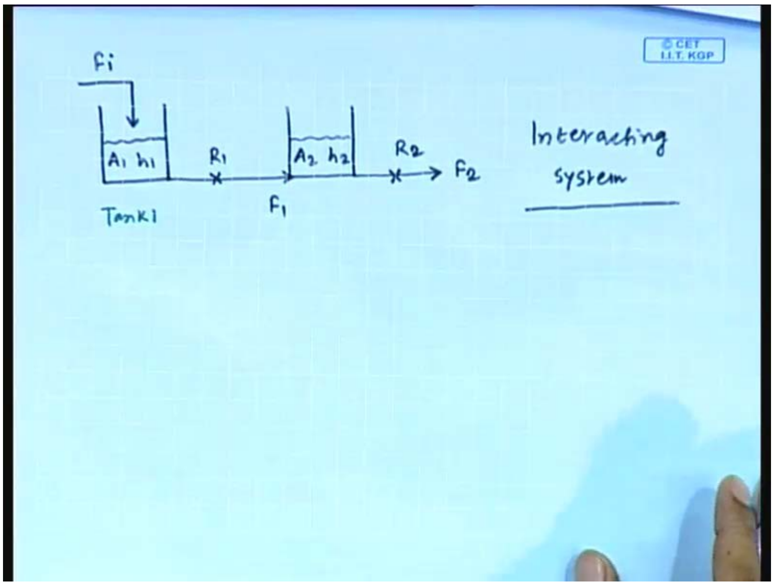

This is tank 1 cross sectional area is A 2 height is h 1 inlet flow rate is f i. outlet is f 1, and resistance is R 1. Second tank is placed here. Cross sectional area A 2 height h 2. Output is f 2, fine and resistance is here R 2. This is the second scheme in which we have connected the two tanks in series. Now, what about the interaction in this system? This is the case of both way interaction I mean tank 1 interacts tank 2 and tank 2 also interacts tank 1. So, for this type of systems we will use the term interacting system. So, for this type of systems we will use the term interacting system or interacting tanks, fine. Now, we will try to derive the overall transfer function for the case of non-interacting tanks and in the next, we will consider for the case of interacting tanks, fine.

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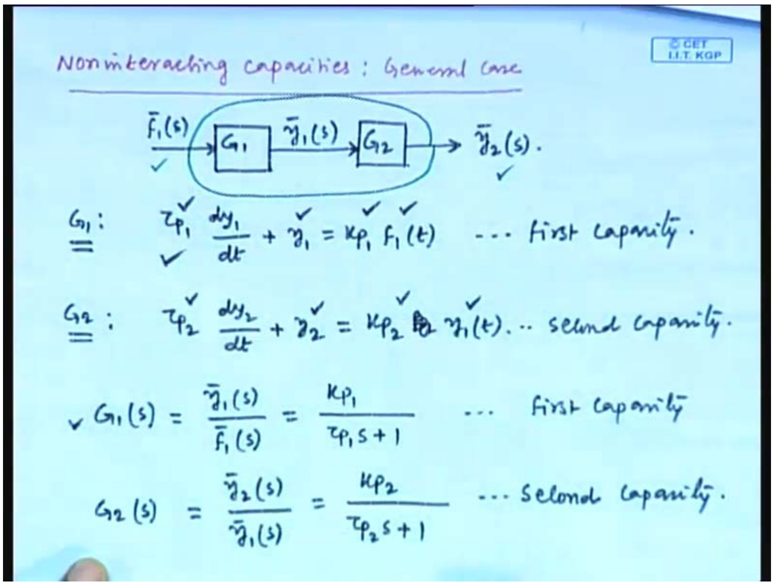

So, now our topic is non-interacting capacities. And we will discuss the general case, fine. So, 2 tanks are or 2 first order systems are connected in series, those 2 first order systems have the transfer functions of suppose G 1 and G 2. So, this is one tank G 1 and this is another tank G 2. input is suppose f 1 par s. Output of the first first order system is y 1 bar s. and output of the second first order system is y 2 bar s, fine. So, if we consider both the systems are first order, then what is the modeling equation for first system I mean for g 1 the modeling equation can be written by this equation, tau p 1 d y 1 d t plus y 1 equals K p 1 f 1 t. So, we can write this is the modeling equation for say first capacity, which has the transfer function of G 1.

Similarly for the second capacity which has the transfer function of G 2, we can write the modeling equation as tau p 2 d y 2 d t plus y 2 equals K p 2 this will be y 1 t, fine. This is for the second capacity. So, for the first capacity the time constant is tau p 1, gain is K p 1, output is y 1, and input is f 1 for the second capacity tau p 2 is the time constant K p 2 is the gain y 2 is the output and y 1 is the input. So, using this two modeling equations it is straight forward to derive the transfer functions. So, we can write the expression for transfer function for the first capacity G 1 is equals y 1 is divided by f 1 s that will be K p 1 divided by tau p 1 s plus 1 fine.

This is for the first capacity the expression of G 1, we can obtain by taking Laplace transform of this mode. Similarly, for the second capacity, we obtain the transfer function s G 2 s equals y 2 bar s divided by y 1 bar s equals K p 2 divided by tau p 2 s plus 1. This is for second capacity, agree. Now, our objective is to find the overall transfer function, if we consider this is a single system. So, what will be the overall transfer function? That will be y 2 bar s divided by f 1 bar s. isn't the overall transfer function will be.

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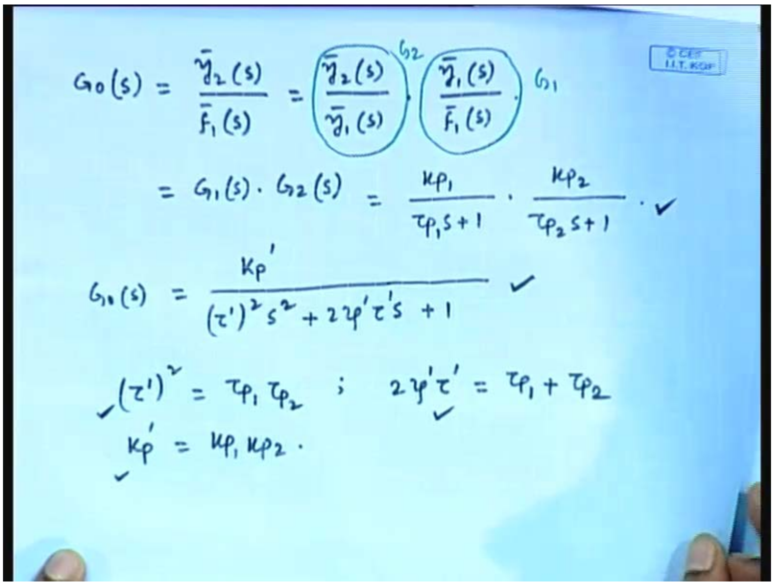

If we represent by G naught s as G naught is equals y 2 bar s divided by f 1 bar s for the overall system y 2 is the output and f 1 is the input. So, we can write this as y 2 bar s divided by y 1 bar s multiplied by y 1 bar s divided by f 1 bar s, agree. So, this is the expression for overall transfer function. This means G naught is equals G 1 s multiplied by g 2 s, this is the transfer function G 1 and this is the transfer function G 2. So, if we substitute the expression for G 1 s and G 2 s, we obtain G naught s as K p 1 divided by tau p 1 s plus 1 multiplied by K p 2 divided by tau p 2 s plus 1, fine.

So, we can write this expression as K p prime divided by tau prime whole square s square plus 2 zeta prime tau prime s plus 1, we can write this equation in this form. G naught is equals K p prime divided by tau prime whole square s square plus 2 zeta prime tau prime s plus 1, where tau prime whole square equals tau p 1 tau p 2, fine. And 2 zeta prime tau prime equals tau p 1 plus p 2 and third term that is K p prime equals K p 1 K p 2, fine. So, this is the prime expression, this is the expression for 2 zeta prime tau prime and this is an expression for K p prime. Now, we will make some conclusions.

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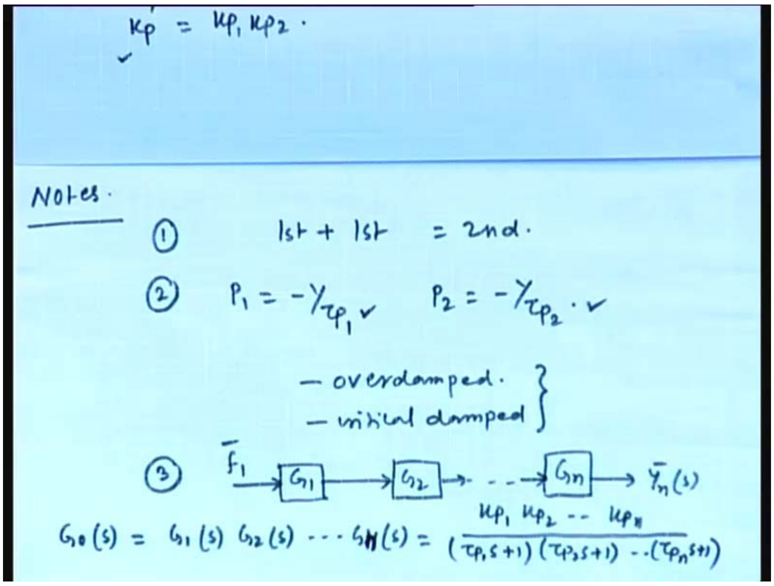

So, what about the overall response is that first order or second order? the overall response is second order. So, it is proved that if two first order systems are connected in series the overall response becomes second order response, fine. So, we have considered 2 first order systems and we got second order response. Secondly, what are the poles in the overall transfer functions? One pole is minus 1 divided by tau p 1. Another pole is minus 1 divided by tau p 2.

If we see the overall transfer function G naught s we have two poles 1 is minus 1 divided by tau p 1, another 1 is minus 1 divided by tau p 2. So, what about the overall response over damped or critically damped or under damped? We have to distinct real poles the response is over damped response, but in some cases tau p 1 may be equal to tau p 2. In that situation the response is critically damped, fine. So, if 2 first order systems are connected in series, we obtain over damped or critically damped response, but never under damped, fine. this is the second naught third one is we have connected two 2 first order systems in series if we connect n number of first order systems what will be the overall transfer function?

Suppose this is G 1 this is G 2 like this way we are connecting n number of systems this is f 1 and finally, we are getting y n s, fine. So, overall we can write G naught s equals G 1 s G 2 s G small n s equals K p 1 K p 2 K p n divided by tau p 1 s plus 1 tau p 2 s plus 1 like this tau p n s plus 1, fine. So, if n first order systems are connected in series we obtain the overall transfer function like this.

Thank you.