Process Control and Instrumentation
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Lecture - 11
Dynamic Behavior of Chemical Processes (Contd.)

In the last class, we just started the discussion on Dynamic Behavior of first order system that we will continue.

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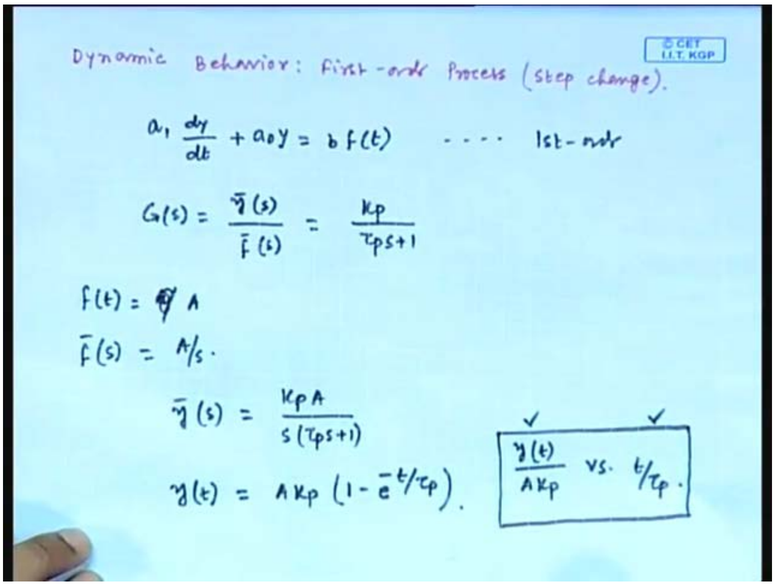

So, the topic is dynamic behavior for first order process introducing step change in input variable. So, you know the first order system can be represented by the first order differential equation and we have considered this equation, previously a naught y equal to b f t this is the model first order process. And transfer function we got, that is G s equal to y bar s divided by f bar s equal to k p divided by tau p s plus 1 and we have considered step change in input variable with magnitude A I mean f t equal to A and in Laplace domain this is A by s.

So, y bar s becomes k p A divided by s tau p s plus 1 inverting this, we get y t equal to A k p 1 minus exponential of minus t divided by tau p up to this we had discussed in the last class. Now, we will try to plot the y versus time. So, to plot this equation I mean we will make the plot in terms of dimensionless quantities. So, dimensionless quantities means, we will consider y t divided by A k p versus t divided by tau p we will make a plot between y t divided by A k p versus t divided by tau p. This is the dimensionless quantity, which we will consider along y axis and this is also the dimensionless quantity which we will consider along x axis.

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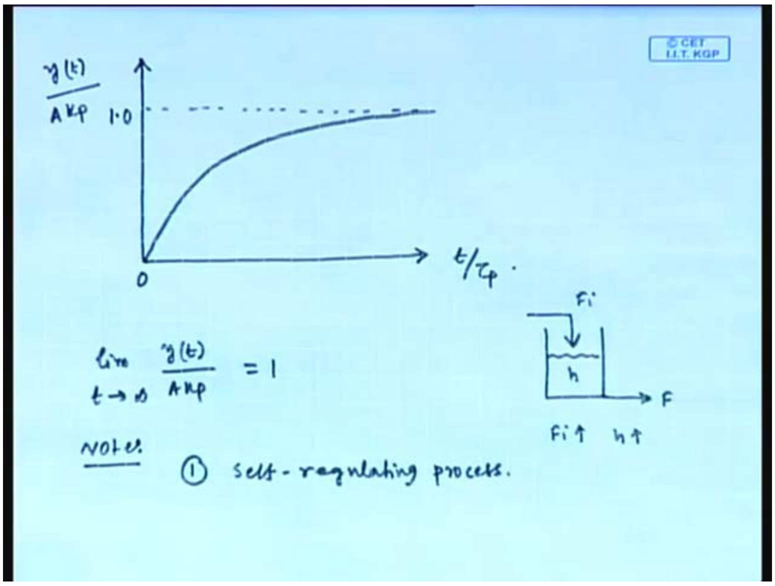

So, what will be the plot y t divided by A k p and t divided by tau p say we are introducing a step change with magnitude A. So, what will be the final steady state, final steady state we can find considering t tends to infinity. So, if we consider y t divide by A k p with t tends to infinity how much it is 1. So, final steady state value is 1 and this is the, starting state I mean t time equals to 0 is a starting point. Now, the dynamics of y t is represented graphically by this curve.

So, this is the dynamic response of the first order system, if we introduce a step change in the input variable with magnitude f magnitude A. Now, you will note down few points, this is basically a self regulating process. Previously we have considered one case that is, pure capacity process and that is non self regulating process. If we take one example, then I think it will be clear, why this is self regulating process, we can consider a liquid tank system, input is f i output is f height is h.

So, if f i increases then what happen, height increases then hydrostatic pressure increases which in term increases the out flow rate. And after a time period and equilibrium is established, upper a time period and equilibrium is established and at that equilibrium state the value of y t by A k p is the new steady state value, that is the reason for which we are considering this is the self regulating process, this is the first remark.

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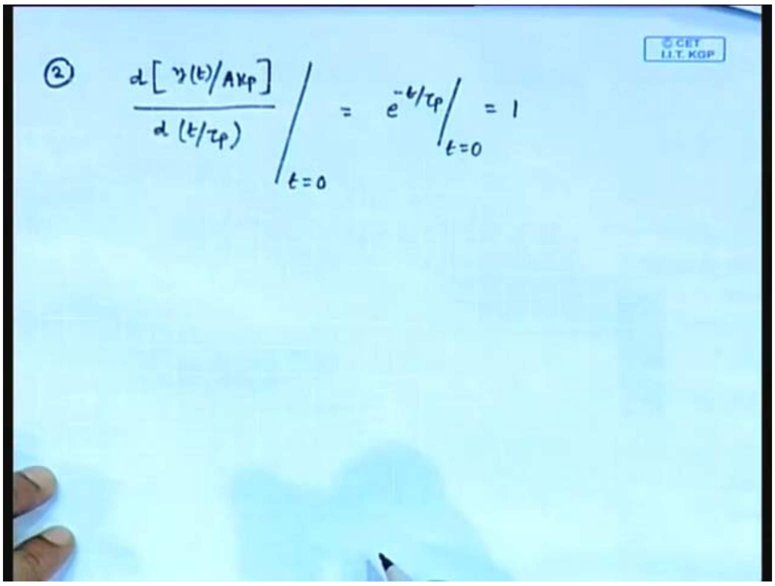

Second remark, what is the slop this response at time t equal to 0 can you calculate the slop at time t equals to 0 I mean d y t divided by A k p d t by tau p at time t equals to 0 how much is this, exponential of minus t by tau p at t equals to 0, so slop is 1.

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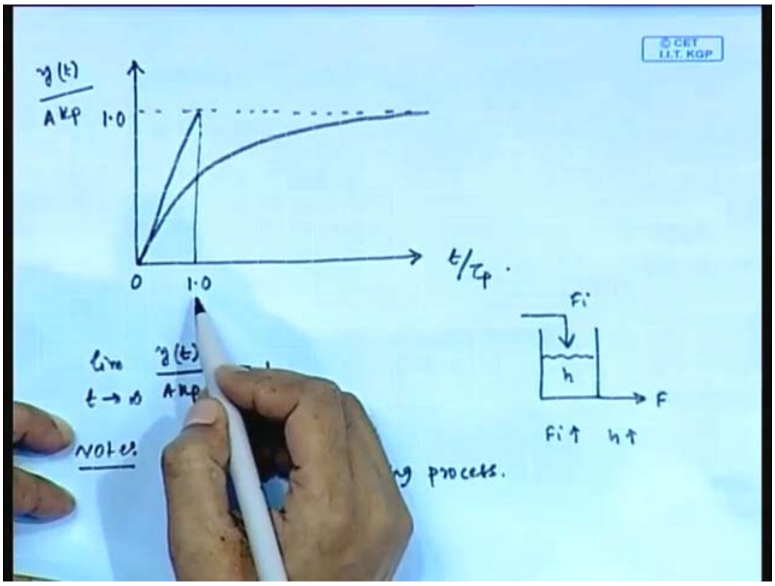

If we see this dynamic response then this state line has the slop of 1, what will be the corresponding t by tau p value, if slop is 1 what is the corresponding t by tau p value 1 because, this value is 1 so, it is 1. So, can we say that, if the initial rate of change of y t wire to be maintained the response would reach it is final steady state value in 1 time constant, can we say if this is 1; that means, t by tau p 1; that means, t equal to tau p.

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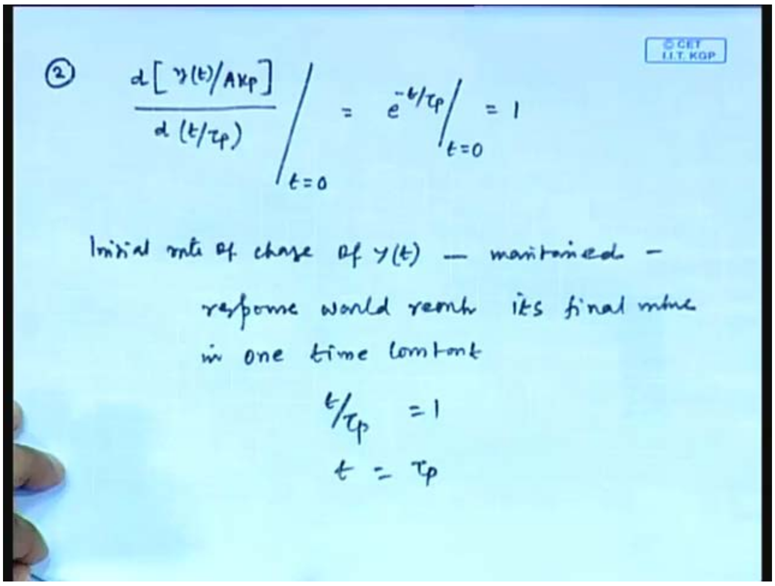

So, we can conclude that, if the initial rate of change of y t wire to be maintained, then the response would reach it is final value in one time constant. Because, t divided by tau p is 1 so; obviously, t equal to tau p this is the second remark. Next we will go for the third remark.

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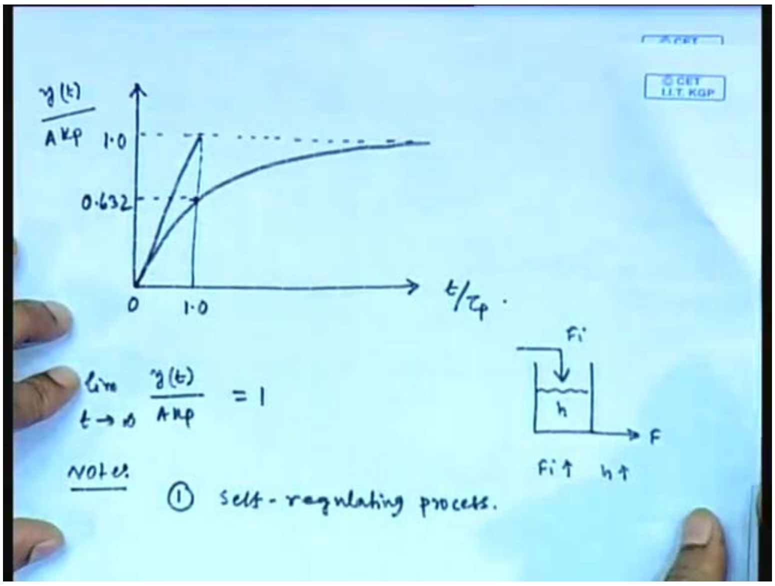

But, what happens originally I mean if we see this plot if this is the response, then only the final steady state value we obtain at t equal to tau p, but that is not the original case, the dynamic response is this 1. So, at t equal to tau p how much is the y t by A k p value, can we calculate that.

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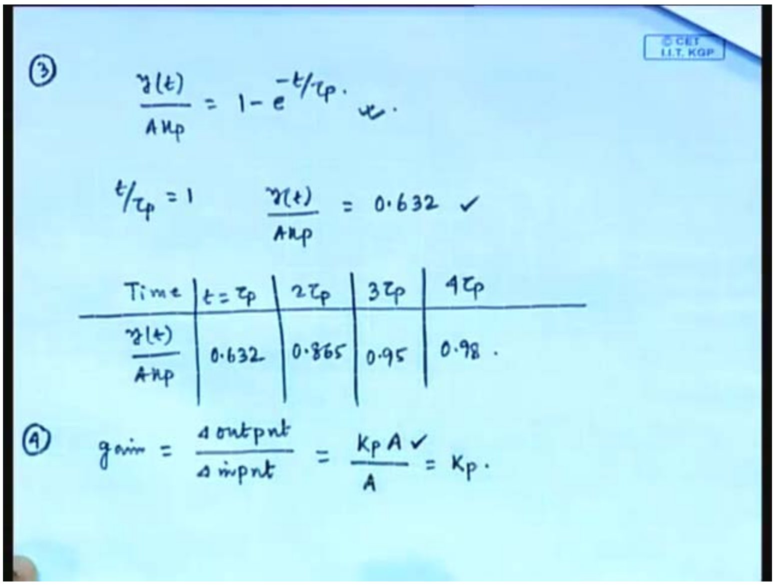

See we have the equation y t divided by A k p equal to 1 minus exponential of minus t divided by tau p. Considering t divided by tau p equals 1, we get y t divided by A k p equals 0.632, considering t by tau p equals 1 we get y t divided by a k p 0.632; that means, originally the process reaches 63.2 percent of it is final value. So, this is 0.632 originally in one time constant, the process reaches 63.2 percent of it is final value.

So, write we can write in this tabular form, that when t equals tau p this value is 0.632. Similarly, considering 2 tau p and using this equation, we get 0.865 I mean in the two time constants, the process reaches 86.5 percent of the final. Similarly, if we consider 3 tau p we will get 0.95 4 tau p this is 0.98 this is our third remark.

Fourth remark, we discussed one thing that, the gain equals change in output divided by change in input. Recall this correlation, which we discussed earlier, gain equals change in output divided by change in input. So, for this system, what is the change in output, what is the change in y that is k p multiplied by A. Because, we have seen earlier that considering t tends to infinity y t divided by A k p equals 1; that means, y equals A k p and how much change we introduced in the input that is A.

So, this gain becomes k p and originally we have considered in the transfer function k p at the gain. But, remember that this is the steady state gain and this is we determined considering steady state only. So, these are four comments, we can make on the dynamic of first order system considering step change in input variable with magnitude A.

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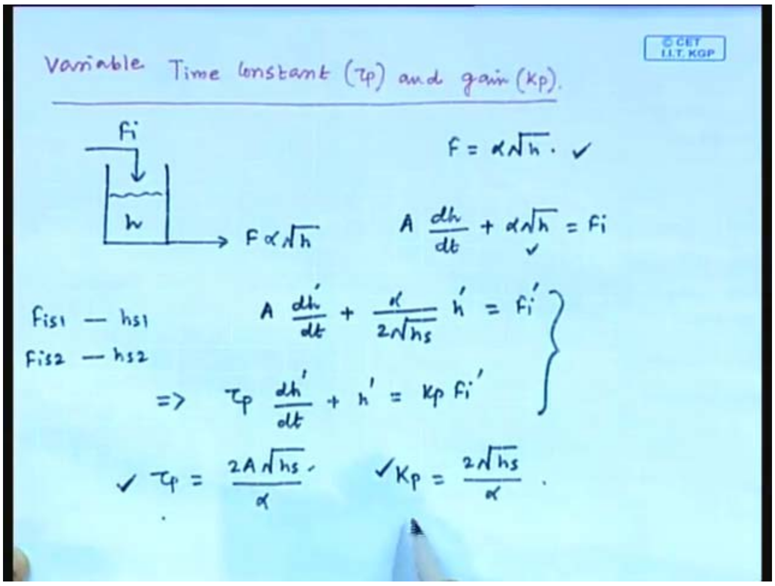

Our next topic is variable time constant tau p and gain k p, so for we have discussed considering time constant and gain as constant quantities. Now, we will consider one case, in which we will observe that tau p and k p are not constants. So, for we have considered tau p and k p both are constants. Now, we consider one example and we will observe that these two are variables, we will consider one that simple liquid tank system input flow rate is f i and output flow rate is f, suppose f is proportional to square of liquid height.

So, f equals to alpha root h, we derived the modeling equation for this system considering f equals alpha root h I am writing that modeling equation which we derived earlier f i. Then in this equation this is the non linear term we linearized it considering Taylor series. And finally, we got A d h d t plus alpha divided by 2 root h s h equals f I we got this equation in linearized form and considering the deviation variables, we can write this equation in this form A d h prime d t plus alpha divided by 2 root h s h prime equals to f i prime.

Now, we can represent this equation introducing tau p and k p by this form tau p d h prime d t plus h prime equals k p f i prime. Now, here tau p equals 2 A root over of h s divided by alpha, if we compare these 2 equations we get the expression for tau p equals 2 A root over h s divided by alpha. Similarly, we get the expression for k p that is k p equals 2 root over h s divided by alpha. So, this is the expression for time constant and this is the expression for gain are they constant or variable. See for a particular f i value, suppose that is f i s 1 we get the corresponding height that is suppose, h s 1.

Now, if we change this f i value to f i s 2 we get different height, that is suppose h s 2; that means, we can conclude that, this h s where is depending on the f i s value the h s is obtained. So, we can say that this tau p and k p both are not constant in this particular example, they are the variables. So, this is all about the dynamic of first order systems, in the next we discuss the dynamics of second order systems.

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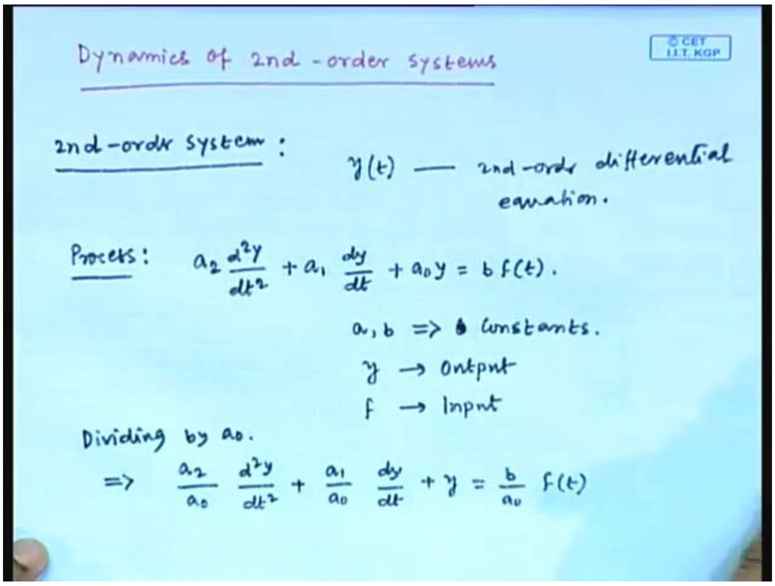

So, next topic is the dynamics of second order systems what is the definition of second order system, second order system is 1 whose output y is modeled by the second order differential equation. So, the output is y t and this should be modeled by second order differential equation. Now, we will consider a second order differential equation. So, for the second order process, the modeling equation we can represent by this form a 2 d 2 y d t square plus a 1 d y d t plus a naught y equals b f t a b both are constant coefficients. y is the output of the process and f is the input to the process.

Now, if we divide both sides of this equation by a naught, we get a 2 divided by a naught d 2 y d t square plus a 1 divided by a naught d y d t plus y equals b by a naught f t. This equation we get by dividing both sides by a naught. Now, we will represent this equation, by this form.

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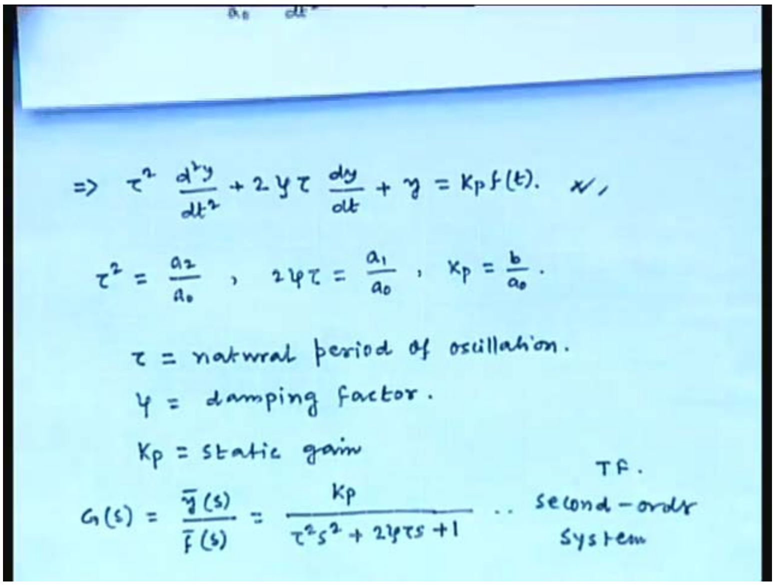

Tau square d 2 y d t square plus 2 zeta tau d y d t plus y equals k p f t. We will represent this equation, the previous equation in this form tau square d 2 y d t square plus 2 zeta tau d y d t plus y equals k p f t here, tau square equals a 2 by a naught, 2 zeta tau equals a 1 by a naught and k p is b a naught.

This tau is called natural period of oscillation of the system, tau is the natural period of oscillation of the system and zeta is here, dumping factor zeta is called dumping factor and k p is the steady state or static gain, considering y and f both are deviation variables. We finally, get the transfer function of this second order system, taking Laplace transform of this equation as G s equals y bar s divided by f bar s equals k p divided by tau square s square plus 2 zeta tau s plus 1, this is the transfer function of the second order system. Now, the second order system transfer function includes one term that is dumping factor.

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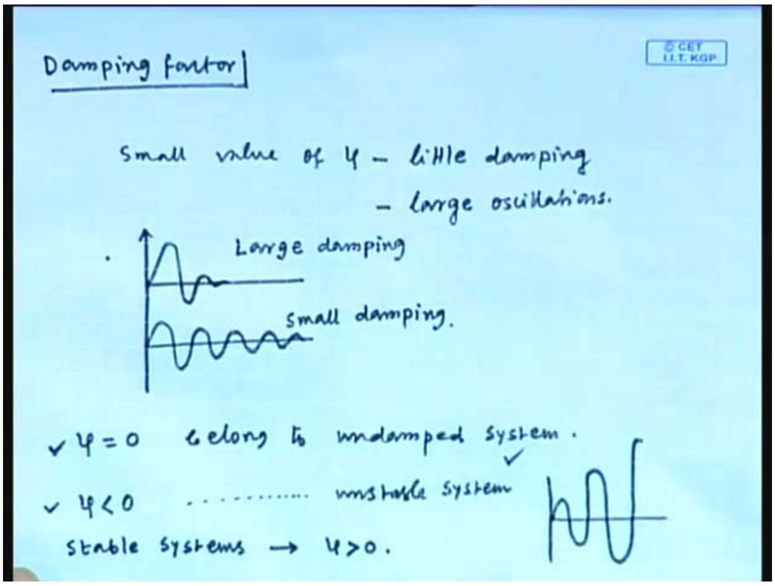

Dumping factor provides a measure of the amount of dumping in the system that is the degree of oscillation in a process response after a perturbation. Dumping factor provides a measure of the amount of dumping in the system, that is the degree of oscillation in a process response after a perturbation. Now, small value of zeta what is indicates, small value of zeta means little dumping small value of zeta implies little dumping, but a large amount of oscillations.

Suppose, this is the output and the response we are getting in this way, another response is like this. So, in this case we can say large dumping, in the second case small dumping, when the dumping factor zeta becomes 0, when zeta equals 0 in that case the oscillation occurs with constant amplitude. So, zeta becomes 0 belongs undumped system, it means oscillation with constant amplitude, for the case of undumped system, we observe oscillations with constant amplitude.

In another case we considered zeta less than 0 what it indicates, oscillations with increasing amplitude. So, it belongs to unstable system, for this case the output is somewhat like this. So, for all stable systems zeta should be greater than 0. Now, earlier we have considered first order systems, presently we are discussing the second order systems now, what processes are called second order systems.

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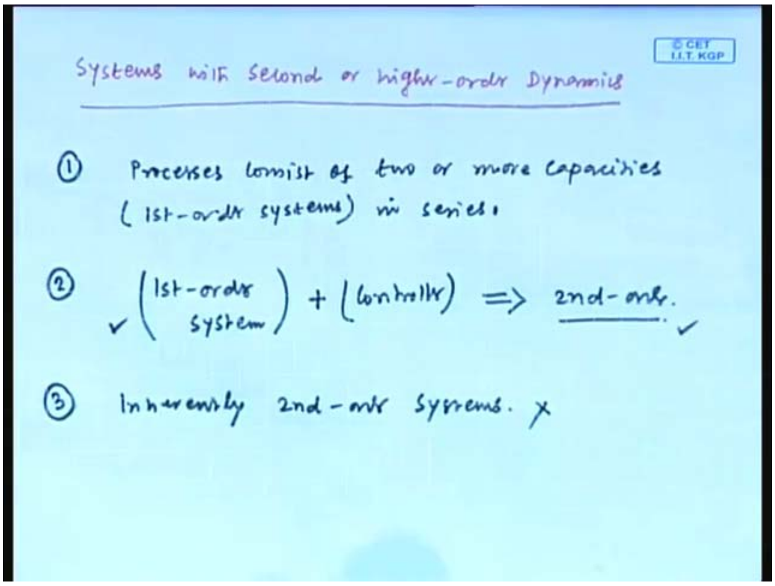

So, we will discuss now the systems with second or higher order dynamics, how we can obtain the second or higher order dynamics, that will discuss now. If 2 or more first order systems are connected in series, then we obtain second or higher order systems, if 2 first order systems are connected in series, the overall dynamics is second order dynamics. If we connect more than 2 first order systems in series, we obtain higher order dynamics. So, this is the first option I mean by this way we can get the higher order dynamics.

So, under this category the processes consist of two or more capacities and they are first order systems in series. If a process incorporates a controller, then also we can obtain higher order dynamics say for example, we have a first order system, if one controller is employed with this first order system, then the overall response may be second order response. If a process includes a controller, then we can obtain second or higher order dynamics and this is the example, if we consider a first order system that includes a controller then the overall response may by second order dynamics.

Thirdly few processes are inherently higher order, few processes are inherently second order systems, but this is very rare in chemical engineering and we will not discuss this third option. We will discuss only first and second options.

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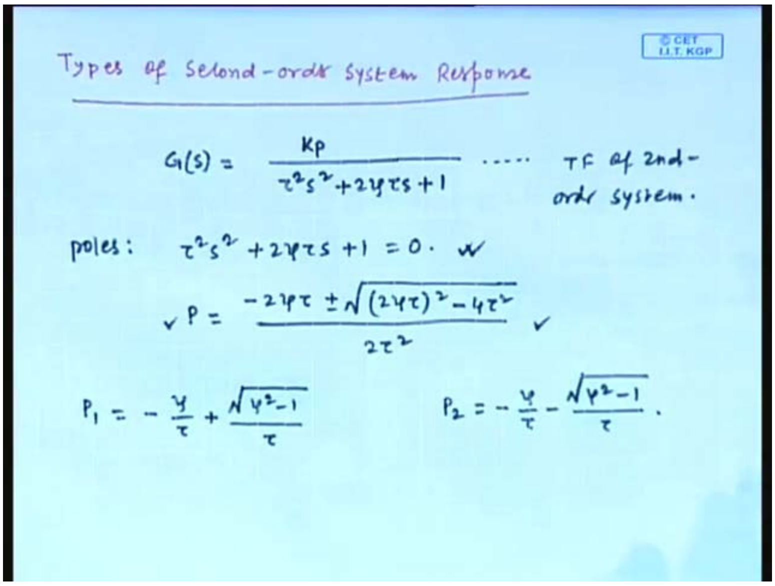

Now, we will discuss the types of second order system response. Previously, we obtained the transfer function for the second order system in general form, that is G s equals k p divided by tau square s square 2 zeta tau s plus 1 this transfer function we derived earlier for the second order system. Now, we want determined the poles of the second order system, for determining the poles of the transfer function, we write tau square s square plus 2 zeta tau s plus 1 equals 0 for determining the poles of the transfer function or poles of the system, we write tau square s square plus 2 zeta tau s plus 1 equals 0.

So, if we represent the pole by p then p equals minus 2 zeta tau plus minus root over 2 zeta tau whole square minus 4 tau square divided by 2 tau square, see this is the quadratic equation. So, we can easily obtain the poles, we have represent the poles by p and the expression for the p is this 1. So, we will get basically 2 poles p 1 equals minus zeta divided by tau plus root over of zeta square minus 1 divided by tau, another pole is p 2 and p 2 equals minus zeta divided by tau minus root over of zeta square minus 1 divided by tau these are the 2 poles. Now, we will categories the system response, based on the value of zeta, we will now categories the system response based on the value of zeta.

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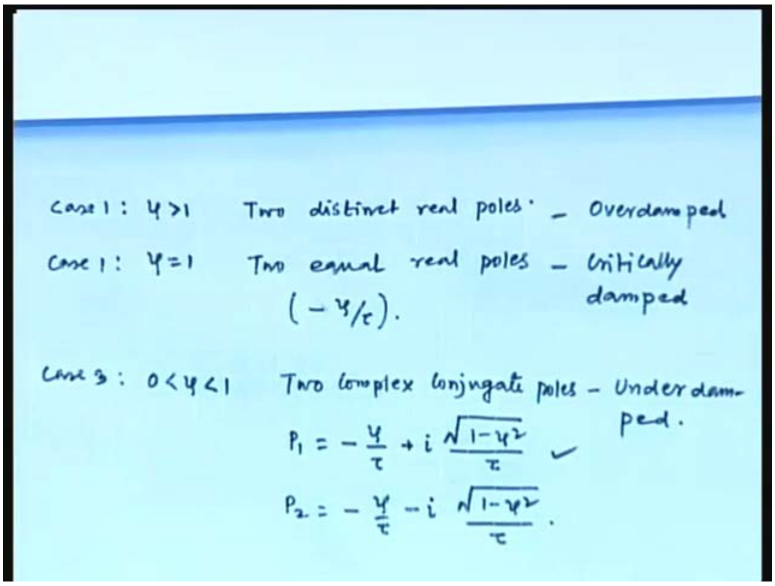

So, first we will consider zeta greater than 1, in case 1 we will consider zeta greater than 1. If zeta greater than 1 then we obtain two distinct real poles, you see from the expression of p 1 and p 2 if we consider zeta greater than 1 we get true distinct real poles. The corresponding system response is called over dumped response, this is case 1 where we have considered dumping factor greater than 1 and dumping factor greater than 1 implies two distinct real poles.

In the next case we will consider zeta equals 1. So, what about the poles, two equal real poles I mean you can say 1 pole. So, we can write 2 equal real poles, it means 1 real pole and that is minus zeta divided by tau, we obtain for the case of zeta equals 1 a single pole and that is minus zeta divided by tau. Then the corresponding system response is called critically dumped response.

So, when zeta equals one the response is critically dumped, in third case we will consider 0 less than zeta less than 1 in the third case we consider zeta is in between 0 and 1 in this case what about the poles, we get two complex conjugate poles and the poles are p 1, 1 is p 1. And another one is p 2 the expression for p 1 is this one p 1 equals minus zeta by tau plus i root over 1 minus zeta square divided by tau, another pole is minus zeta by tau minus i root over 1 minus zeta square divided by tau. So, when zeta is in between 0 and 1 the corresponding response is called under dumped response.

So, this is the under dumped response. So, these are three different cases, depending on the value of zeta. Now, we need to take the invert of inverse of Laplace transform to obtain the expression for y in time domain, that will discuss in the next class, the expression for y in time domain thank you, what you are saying.

Student: ((Refer Time: 49:33))

Yeah, that I will correct.