Process Control and Instrumentation
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Lecture - 10
Dynamic Behavior of Chemical Processes (Contd.)

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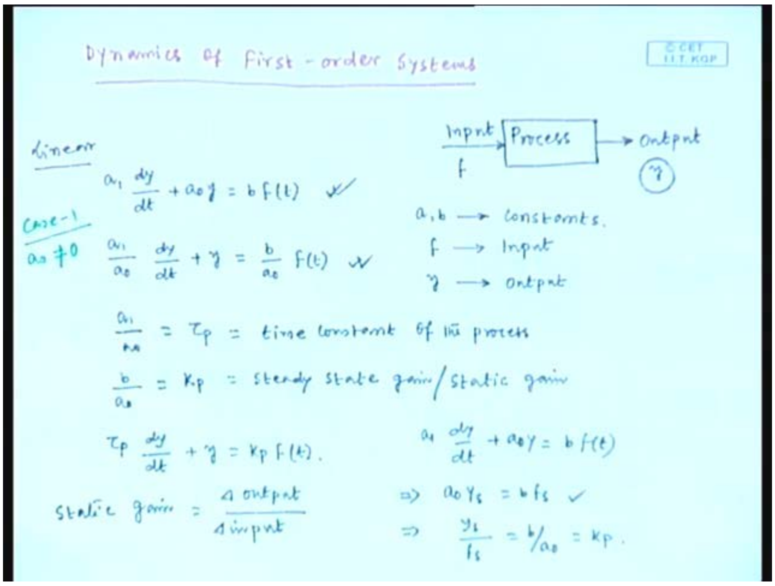

Today, we will study the dynamics of first order systems. So, our topic is dynamics of first order systems. So, before going to discuss the dynamics of first order system, we will know what is first order system, what is first order system? First order system is one whose output y t is modeled by first order differential equations. Suppose, we have a process, this is the input to the process and this is the output to the process. Input is represented by f and output is say by y. Now, first order system is one whose output y, whose output y is modeled by first order differential equation.

Now, we will consider the linear case or if non-linear equation exists, we need to linearize that equation. So, if we consider the linear case, then first order differential equation we can write, by this form a 1 d y d t plus a naught y equal to b f t definitely y is also function of time. Now, in this equation a and b both are constant coefficients, a and b are the constants. F is the forcing function or input f is the input and y is the output.

Now, if we rearrange this equation, then we get a 1 divided by a naught d y d t plus y equal to b divided by a naught function of t. Now, before this I want to mention one thing that we will consider two cases, in one case a 1 equal to 0 and in another case a 1 not is equal to 0.

So, in case 1, we will consider a 1 not is equal to 0, first we will discuss a 1 not is equal to 0 then in the next case we will consider a 1 equal to 0. Now, after rearranging this first order differential equation, linear equation we get this. Now, we will represent a 1 by a naught, by the term tau p and another term that is b divided by a naught will be represented by k p. Then the linearized form of first order differential equation for the case of a 1 not is equal to 0 becomes, tau p d y d t plus y equal to k p f t.

Now, this tau p is called time constant of the process, definitely we are discussing first order process. So, this is a time constant of first order process and k p is called steady state gain or static gain or only gain or I mean gain of the process. Now, what is gain, gain is basically the change in output, per unit change in input, gain is the change in output per unit change in input I mean gain equal to change in output divided by change in input. Since, we have considered this as static gain.

So, this delta output divided by delta input, they should be considered at steady state only. So, we know the modeling equation a 1 by a naught d y d t plus y equals to b by a naught f t. So, what we will be the steady state form of this, steady state form will be if we consider the first equation I mean a one d y d t plus a naught y equal to b f t this is a modeling equation.

So, what will be the steady state form of this a naught y s equal to b f s agree because, this d y d t term does not exist. Now, if we rearrange this y s by f s equal to b by a naught, agree if we rearrange this equation a naught y s equals to b f s y s divided by f s becomes b by a naught. So, what is y s by f s that is the gain, output by input change, that is why we have represented b divided by a naught as steady state gain. And that we have represented by k p.

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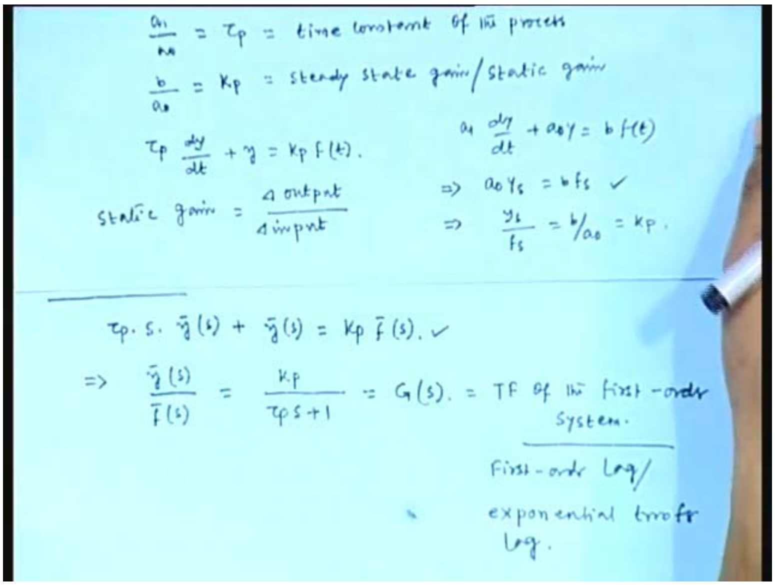

Now, we will go back to the first equation. We got tau p is d y d t plus y equal to k p f t here, y and f both are deviation variables, that is why we have written here this is as delta output and this is as delta input. So, y and f both these variables are deviation variables; that means, y naught equal to f naught equal to 0, can we write this, if y and f both are deviation variables then we can write y naught f naught both are 0. Now, we will take Laplace transform of this Laplace transform.

Then we get tau p s y bar s plus y bar s equal to k p f bar s, if we take Laplace transform of this, then we get this form. Now, rearranging the above equation we get y bar s by f bar s equal to k p divided by tau p s plus 1, y bar s divided by f bar s becomes k p divided by tau p s plus 1 and this is nothing, but the transfer function of the first order system. We will represent this by G s, transfer function of the first order system.

In the next and this is also called as first order lag or exponential transfer lag. We will see later that, this transfer function involves phase lag, that is why this is first order lag transfer function, we will consider that later. So, in the first case we have considered a naught not is equal to 0, in the next case we will consider a naught equal to 0.

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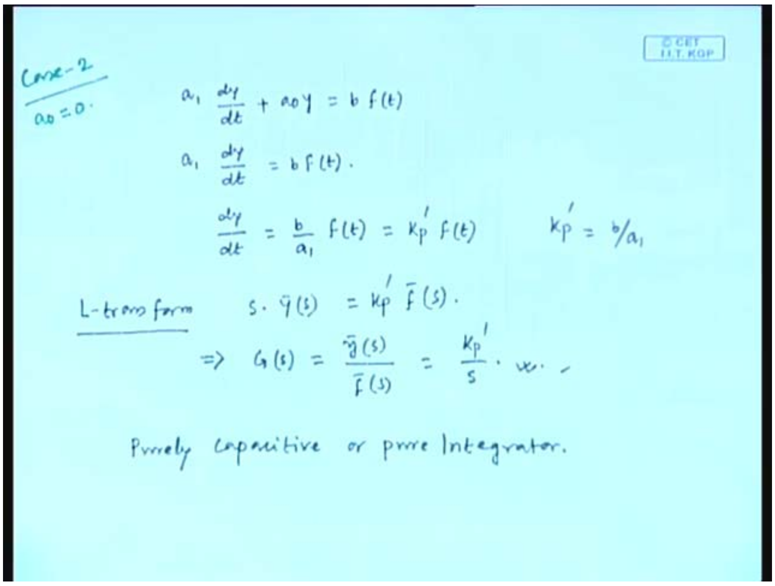

So, in case 2 we will consider a naught equal to 0. Our original equation is a 1 d y d t plus a naught y equal to b f t, this is original equation. Now, if we consider a naught equal to 0 then this becomes a 1 d y d t equal to b f t; that means, d y d t equal to b by a 1 f t this is equal to k p prime f t. Here, gain is represented by k p prime and that is b by a 1 k p prime is here, b by a 1. Now, if we take Laplace transform again, then s y bar s equal to k p prime f bar s, rearranging this form we get the transfer function G s equal to y bar s divided by f bar s equal to k p prime divided by s.

So, this is the transfer function of a first order system, when we consider a naught equals to 0. Now, this type of processes are called purely capacitive or pure integrator. For the example first order system with a naught equal to 0, we got the transfer function this and this type of processes are called pure integrator, we will discuss with physical example what type of processes are called pure integrator processes. Now, these are about the transfer functions in general form, in the next we will discuss with some example.

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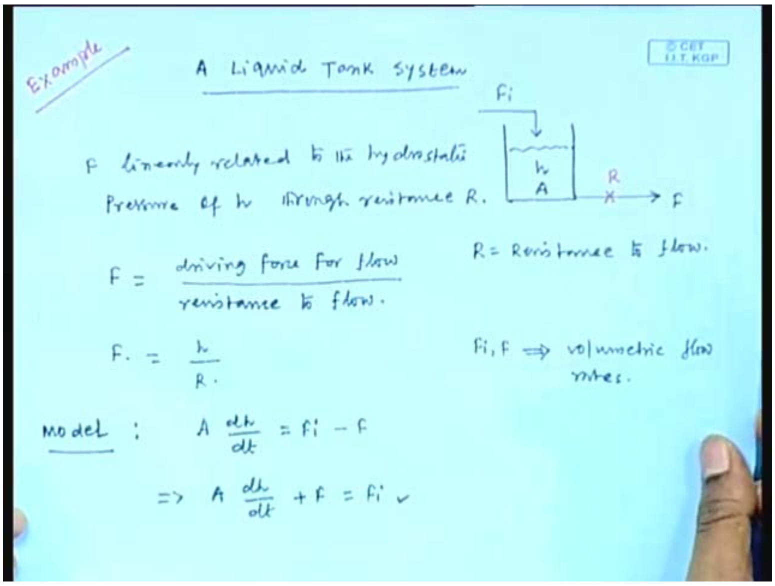

We will first consider one example that is a liquid tank system. Will consider a example that is a liquid tank system. This is a liquid tank height of this liquid is h, cross sectional area of the tank is suppose A, input is entering the tank with a flow rate of f i and output is f and here, we are considering resistance that is R. Now, this effluent flow rate f is related to the hydrostatic pressure, of the liquid height h through resistance R, here R is the resistance to flow.

This f is linearly related, to the hydrostatic pressure of the liquid level h, through resistance R. Effluent stream, effluent outlet we can represent by this form f equal to driving force, for flow divided resistance to flow. It is quite common in heat transfer also, like q equals to delta t divided by r we have started in our heat transfer course, rate of heat transfer equals to driving force that is temperature difference, divided by the resistance.

Similarly, we are writing also the same similar equation, similar a relation that is f equal to driving force is here, h and resistance is R. So, in the modeling equation we will consider f equal to h divided by R. What is the modeling equation of this liquid level system, we have derived the modeling equation for this liquid level system earlier, that is A d h d t equal to f i minus f. F i and f both are volumetric flow rates. Now, if we rearrange this equation, we get A dh d t plus f equal to f I, if we rearrange the modeling equation we get this. Now, we will substitute f equals to h by R.

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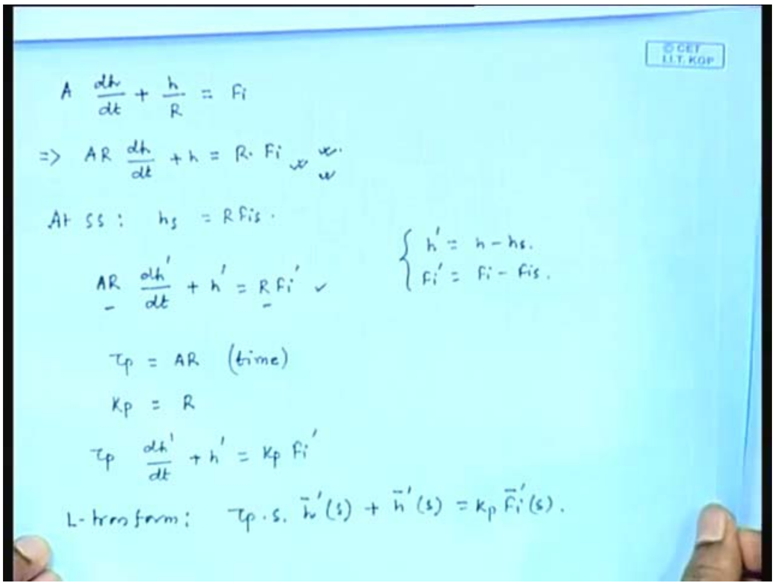

Then we get A d h d t plus h divided by R equal to f i. Multiplying both sides by R we get A r d h d t plus h equal to R f i, multiplying both sides by r we get this equation. Now, we need to represent this equation in terms of deviation variable. So, for that we need to consider the equation as steady state, what will be the form of this model at steady state, it will be h s equal to R f is at steady state condition the above modeling equation I mean this modeling equation becomes h s equal to R f is we are using suffix s to represent the steady state.

Now, subtracting this steady state model from this modeling equation, we get the equation in terms of deviation variable, that is A R dh prime d t plus h prime equal to R f i prime. Subtracting steady state model from this original model, we get this form in terms of deviation variables, where h prime equal to h minus h s and f i prime is f i minus f is this 2 deviation variables are involve.

Now, we can replace A R by the time constant, time constant we have represented by tau p and here, tau p equal to A R. So, what is the unit of this tau p, can you find the unit of this tau p, unit of this tau p is.

Student: Time.

Time. Similarly, we will consider k p, k p is here R. So, that will be the unit of R basically, what is the unit of R, R equals to h by R; that means, time per meter square or time per area we can say. Now, we will just replace this A R term here by tau p and R by this k p tau, then we get tau p d h prime d t plus h prime equal to k p f i prime. Again we will take Laplace transform of this, then we get tau p is h bar prime s plus h bar prime s equal to k p f i bar prime s.

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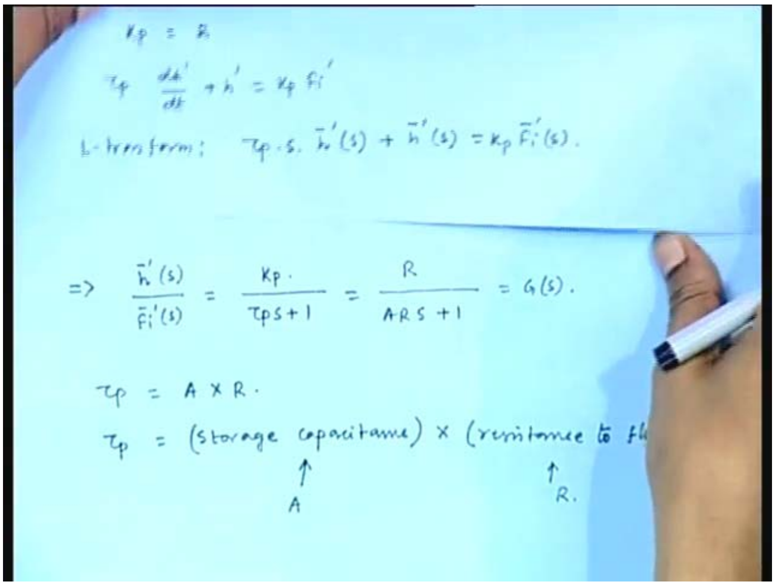

Now, if we rearrange then finally, we get h bar prime s divided by f i bar prime s equal to k p divided by tau p s plus 1. So, here k p is basically the resistance tau p is A R S plus 1, this is the transfer function of this system, liquid level system.

Now, here you see one thing tau p we have considered that is time constant equal to A multiplied by R. Now, we can write this as, storage capacitance, multiplied by resistance to flow. Now, that capacity to store mass, is measured by the help of A the capacity to store, mass is measured by the help of area A and this is; obviously, resistance R.

Similarly, if we consider one example, which includes only the energy balance equation. In that case, this storage capacitance is basically the capacity to store energy. If we consider an example, which is modeled by only energy balance equation, in that case also we can write, this time constant tau p equal to storage capacitance multiplied by resistance to heat, then in that case this is the capacity to store energy not mass. In the next we will take another example, that is the pure capacity system.

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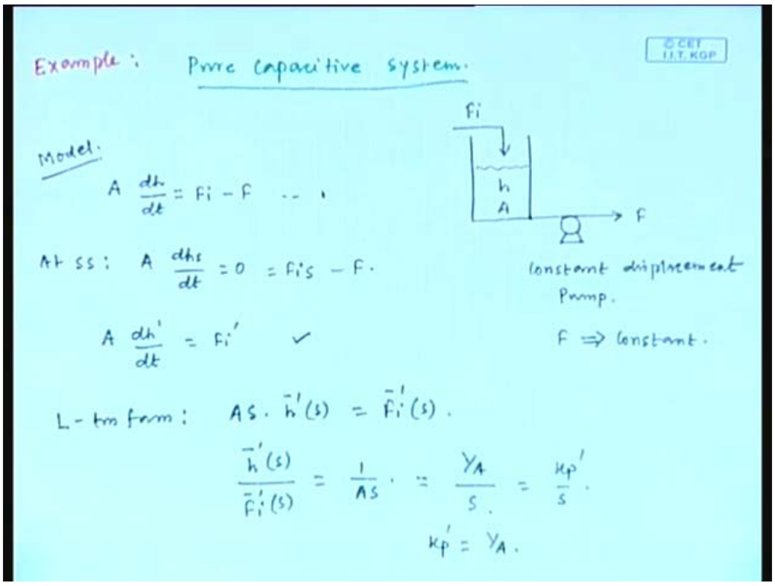

We have consider another example, that is the example of pure capacitive system I mean how we can get the A naught y term equal to 0 physically, we will continue the liquid tank system, which we considered earlier. So, this is a liquid tank system height is h, area is A inlet flow rate is f i in the outlet section one pump is installed, which is delivering outlet with a flow rate of f.

In the outlet section one pump is installed, which delivers the outlet or effluent stream with a flow rate of f and this is a constant displacement pump, it means the flow rate is constant I mean f is constant. So, we have already considered the model of this system in the previous example, that is if A d h d t equal to f i minus f, this is the modeling equation for the liquid tank system and this is a linear equation. So, there is no need of linearization.

Now, this is a modeling equation, what will be the equation at steady state. At steady state we get d h s d t which is nothing, but 0 equal to f is minus f. I told that for the case of constant displacement pump f is a constant or f remains constant. Now, subtracting this steady state model from the actual process model, we get the equation in terms of deviation variables, that is this subtracting the steady state model from the actual process model we get this modeling equation in terms of deviation variables.

Now, what will be the transfer function of this, if we take Laplace transform again A S h bar prime s equal to this. So, h bar prime s divided by f i bar prime s equal to 1 divide by

A S; that means, 1 divided by A whole divided by s. If we write in this form k p prime by s then here k p prime is 1 by A, this is one example of pure capacitive process.

Next we will discuss, the dynamic behavior of first order system. Previously it was told that the transfer function is used to observe the transient behavior of a process. Now, that we will discuss by the use of transfer function, how we can know the transient behavior of a process, that we will discuss in the next. So, first we will consider the dynamics of a first order system, considering input as ramp input.

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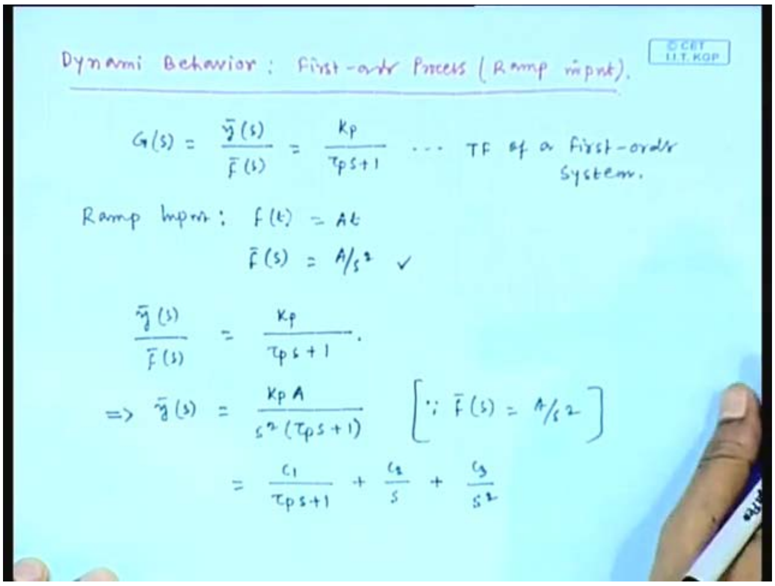

So, next topic is dynamic behavior and we will consider first order process and we will change the input as ramp function, I mean the input is ramp input. This we will discuss in the next. So, we have already derived the transfer function of a first order system in general form, that is G s equal to y bar s divided by input that is k p divided by tau p s plus 1 this is the transfer function of a first order system.

Now, to observe the transient behavior we need to introduce some change in the input, to observe the transient behavior of a process we need to introduce some change in input variable. Now, here we will consider ramp input. So, f is the forcing function, f is a input to this process, we will consider here f t equal to A t and this is a ramp function. Now, in Laplace domain in s domain, we can write this as A by s square, this is the form of ramp function in s domain.

Now, our transfer function is y s divided by f bar s equal to k p divided by tau p s plus 1. If we consider f s equals to A by s square, then the output in s domain becomes k p A divided by s square tau p s plus 1, got it considering f s equal to A divided by s square. Now, we will just write this form in this way c 1 divided by tau p s plus 1 plus c 2 divided by s plus c 3 divided by s square.

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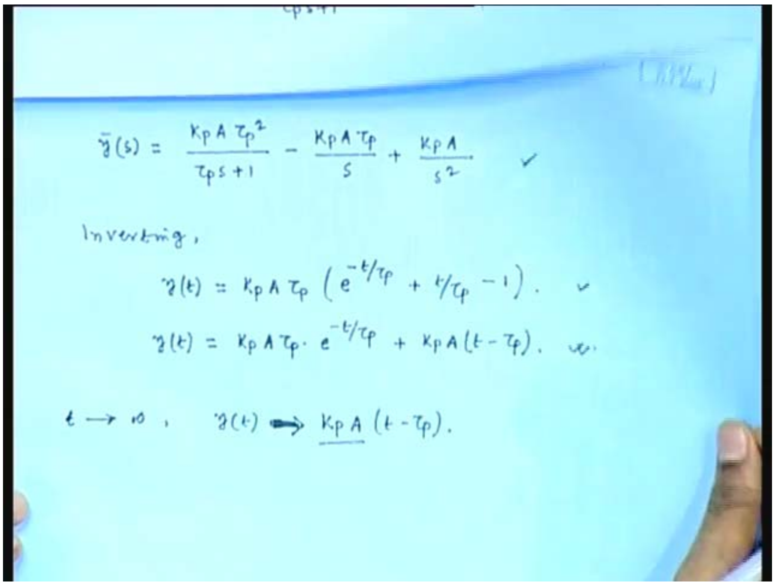

And, finally we will get y bar s equal to k p A tau p square divided by tau p s plus 1 and next term k p A tau p divided by s third term involving c 3 becomes k p A divided by s square, after introducing ramp input to the process, we get this as the output in terms of in Laplace domain. Now, if we inverse, if we consider the inverse of Laplace transform.

We get y t equal to k p A tau p exponential of minus t divided by tau p plus t divided by tau p minus 1. Taking inverse of Laplace transform, we get this form, in time domain. If we rearrange this equation, we get k p A tau p exponential of minus t divided by tau p plus k p A t minus tau p, rearranging this equation we get this form. Now, we will consider one case, that is t tends to infinity. If we consider t tends to infinity, then y t becomes k p A t minus tau p; that means, we can say that the ramp input, yields a ramp output with slope k p A this is the slope.

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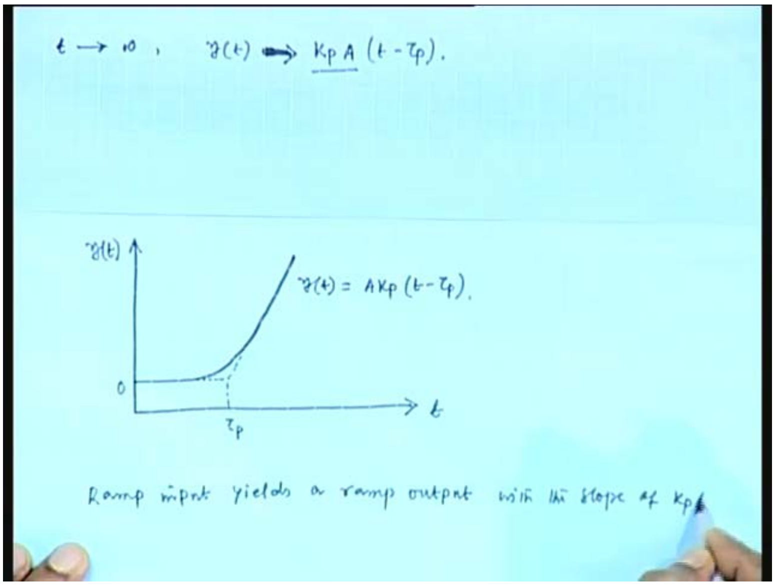

And if we represent this graphically then we get this plot, this is time t, this is y t, this is y t equal to A k p t minus tau p, this is the tau I mean tau p this is 0. So, we can say that the ramp input as I told yields, a ramp output with the slope of k p A. Next we will consider the purely capacitive process, dynamic behavior of a pure capacitive process. This is the dynamic behavior of the first order system considering ramp input, next we will discuss dynamic behavior of a pure capacitive process considering step input.

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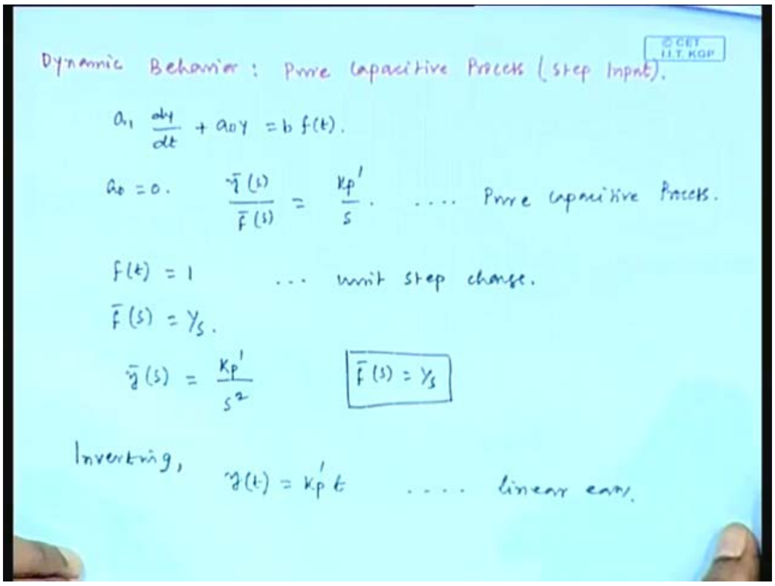

So, dynamic behavior pure capacitive process considering step input. Now, you can recall the first order system model, a 1 d y d t plus a naught y equals to b f t and considering a naught equal to 0, we got y bar s divided by f bar s that is equal to k p prime by s and this is the transfer function of a pure capacitive process, that we have derived earlier, this is the transfer function of a pure capacitive process.

Now, we will introduce a step change in input variable. So, f t, we will consider equal to 1 I mean this is a unit step change, to observe the transient behavior of a process we need to introduce some change in the input variable. So, for that we are considering a unit step change. So, in Laplace domain we can write this as 1 by s then what will be y bar s, y bar s becomes k p prime divided by s square because, f bar s equals to 1 by s. Now, we will take the inverse of Laplace transform, inverting we get, the output in time domain equal to k p prime t, this is a linear equation, with slope k p prime.

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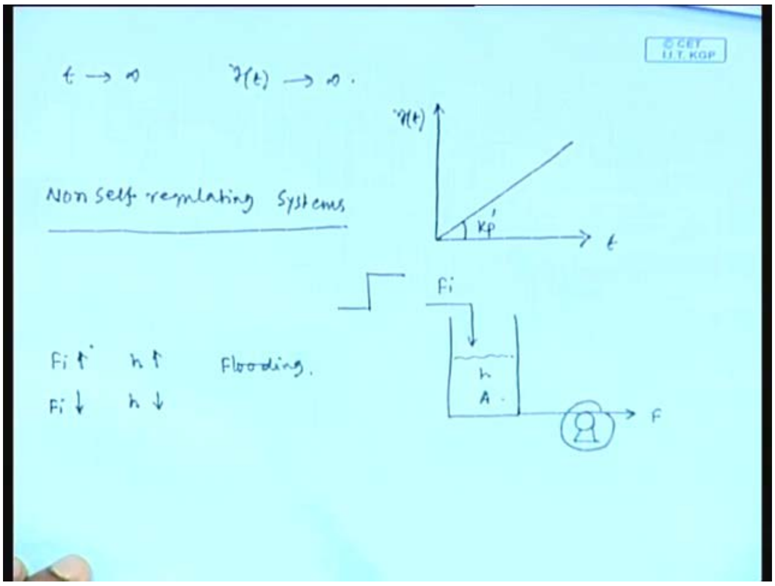

Now, we will represent it graphically, if we consider t tends to infinity what happens y t tends to infinity. So, this is y t and this is time t this is the output and slope is k p prime. This type of processes are called non self regulating processes, this type of processes are called non self regulating systems. Question is why, we will consider one system, that is liquid tank system, to explain this concept we will consider one liquid tank system.

In fact, we have considered this earlier also, one constant displacement pump is installed here, h A. Now, we are introducing a unit step change in f I, then what happens the height increases because, this pump is delivering constant output. So, height gradually increases with time agree. So, if we consider here, positive change I mean if we increase f i then the liquid in the tank I mean the height of liquid in the tank increases and this tank becomes I mean this tank is flooded agree. So, flooding takes place.

Similarly, if we decrease f I, then height decreases because, pump is delivering constant quantity and the tank becomes empty, that is why we called this process as non self regulating process. If there is no pump, you see the fact if there is no pump with the increase of f i the height increases instantly, then hydrostatic pressure increases, which in turn increases the out flow rate and equilibrium is established, that is basically the new steady state, in that case we call the we use the self regulation term, if there is no pump this process is called self regulating process. Anyway next we will consider another dynamic study, that is the dynamics of first order system considering step change, previously we have consider considering ramp change.

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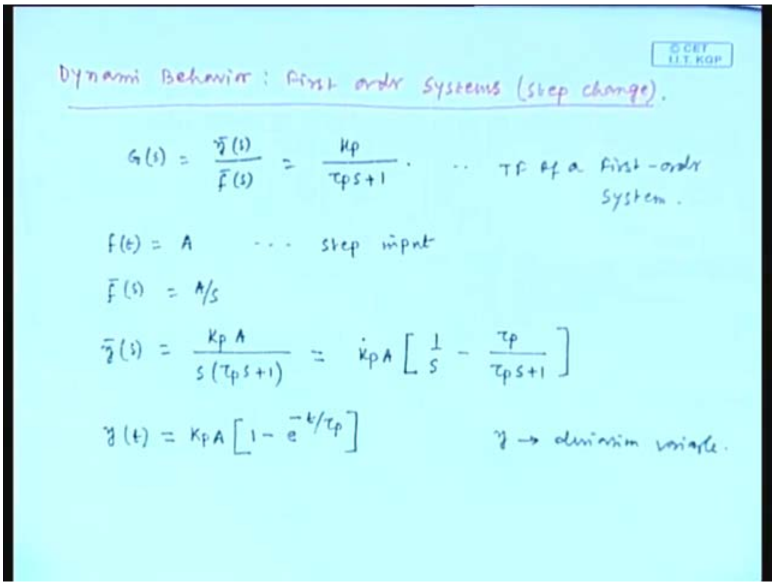

So, now, we will discuss dynamic behavior of first order systems considering step change. So, the transfer function of a first order system is k p divided by tau p s plus 1, this is the transfer function of a first order system. Now, we will consider step change, with suppose magnitude A this is step input; that means, f bar s equal to a by s agree, then y bar s becomes k p A divided by s into tau p s plus 1. Now, we can write this as k p A multiplied by 1 divided by s minus tau p divided by tau p s plus 1.

Taking inverse of Laplace transform, we get the output y in time domain as k p A 1 minus exponential of minus t divided by tau p. This is a process output of a first order system subjected to step input, in time domain. Now, here y is the deviation variable. Now, in the next class we will discuss the, graphical representation of this system I mean first order system subjected to step input.

Thank you, what you told.