Novel Separation Processes
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Lecture No. # 09
Membrane Separation Processes
(Contd.)

Well Good morning everyone. Now in last days class we have seen that how to use, how to use the similarity transformation in order to reduce the partial differential equations, that will be a governing equation of solute balance over the mass transfer boundary layer into an ordinary differential equation. We have also looked into the solution of the ordinary differential equations, and then we looked into the algorithm, of how to implement that in the solution, of governing equation in order to calculate the permeate flux and permeate concentration. In fact, what we have done, we wrote an algorithm by which, you can obtain a by using an iterative method to you can obtain the profile of permeate concentration and permeate flux along the membrane length.

Once you get the profile as the function of x the flux and the permeate concentration then you can do an in-numerical integration by using you know Simpson's one- third rule or something like that, or trapezoidal rule. And one can get the length averaged permeate flux and permeate concentration. Then, what we argued that still this method is a bit more complicated. We kept on arguing that you should seek something a simpler method, but which will be accurate, simpler but it will be accurate, which will give the full feature of whatever we are trying to achieve. So, what we did we have that there will be, we can recast the whole solution into two coupled non-linear algebraic equations.

If you know solve these two algebraic equations here by an iterative technique, then you can get the length averaged permeate flux and length averaged permeate concentration directly. So, we have try we are basically going to do that. In order to do that the first thing that, you have to do you have to get an expression of length averaged Sherwood number or length averaged mass transfer coefficient.

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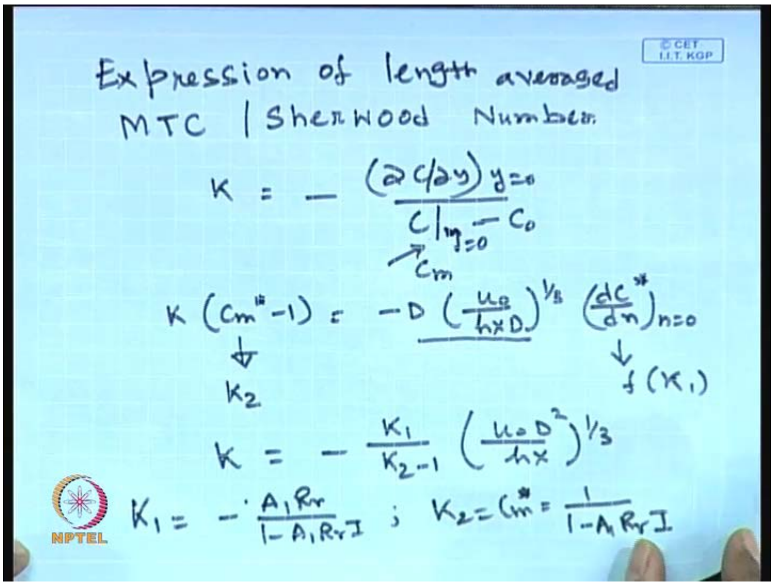

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Expression of length averaged MTC | Sherwood Number

$$K = -\frac{(\partial C/\partial y)_{y=0}}{\underset{\nearrow C_m}{C|_{\eta=0}} - C_0}$$

$$\underset{\underset{K_2}{\downarrow}}{K\,(C_m^* - 1)} = -D\,\underline{\left(\frac{u_0}{h x D}\right)^{1/3}}\;\underset{\underset{f(K_1)}{\downarrow}}{\left(\frac{dc^*}{d\eta}\right)_{\eta=0}}$$

$$K = -\frac{K_1}{K_2 - 1}\left(\frac{u_0 D^2}{h x}\right)^{1/3}$$

$$K_1 = -\frac{A_1 R_r}{1 - A_1 R_r I} \;;\; K_2 = C_m^* = \frac{1}{1 - A_1 R_r I}$$

NPTEL

So, we are above we are doing the derivation of expression of length averaged Sherwood number or mass transfer coefficient or the non dimensional version of mass transfer co coefficient is Sherwood number. So, first thing we wrote yesterday to we wrote down the definition of mass transfer coefficient it will be minus del C del y evaluated at y equal to 0 divided by C at y equal to 0 that is nothing but membrane surface concentration, C at y equal to 0 minus C at the bulk. The C at y equal to 0 is nothing but C m. So, therefore you can write down K C m star minus 1 is equal to minus D u naught h x D rest to the power 1 upon 3 D C star D eta at eta equal to 0.

How did you get this expression? What we did now? We express Del C del y the derivative in terms of dc D eta. So, we expressed this derivative with respect to y in terms of the similarity parameter eta. Once you do that all these things will appeared, so i told you to take it as a home assignment and get this derivation and see at y is equal to 0 is nothing but C m and what we did we just divide both expression by C naught. So, that we can express the concentration as a non dimensional quantity. So, C m star is nothing but C m by C naught and C star is nothing but C by C naught. So, once you do that and if you substitute the expression of C m star, what you C m star, C m star is nothing but, the constant K 2 if you remember. And del C dc star d eta you know it is related to the constant function of the integration constant K 1.

So, therefore one can write down K is nothing but K 1 divided by K 2 minus 1 u 0 D square h x rest to the power 1 upon 3. I just omitted a couple of number of steps, couple of steps here so just ex get the expression of d C star D eta D eta equal to 0 that we have derived in the last class. In yesterdays class we have derived the expression of d C star D eta D eta equal to 0 and that will be the function of the numeric the constant of integration K 1 and C m star is nothing but, the constant of integration K 2. So, once you substitute that you will be getting this expression.

Now if you look into the expression of K 1 and K 2 that we have derived in the last class this is nothing but minus A 1 R r divided by 1 minus A 1 R r I and K 2. K 2 is nothing but your C m star and this will be 1 minus 1 divided by 1 minus A 1 R r the integration I. The integration I is basically a definite integral that will be in the form of exponential minus eta cube by 3 minus A eta D eta. So, we will be in a position now if you simplify this expression if you substitute K the value expression of K 1 and K 2 into the definition of mass transfer coefficient, K 1 minus K 1 divided by K 2 minus 1 will turn over to be 1 over I. Just substitute that you will find it, substitute the expression of K 1 these expression of K 1 here, substitute the expression of K 2 here and simplify it.

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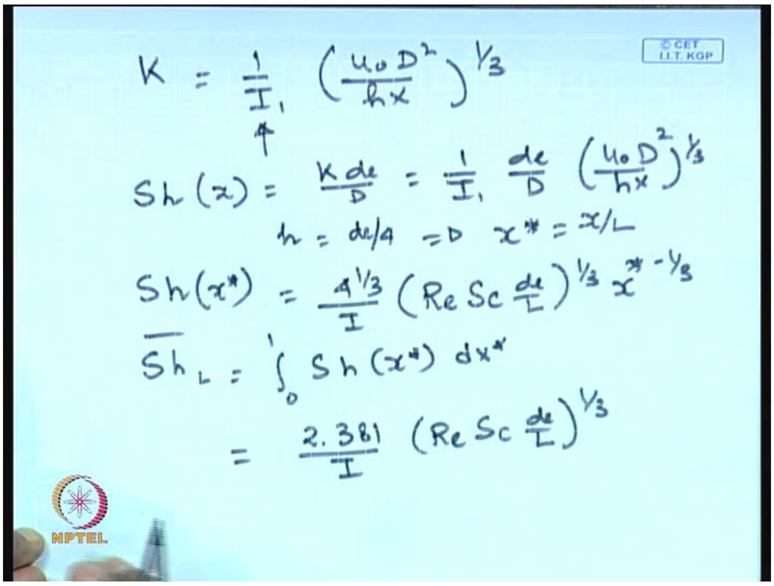

$$K = \frac{1}{I_1}\left(\frac{u_0 D^2}{hx}\right)^{1/3}$$

$$Sh(x) = \frac{K d_e}{D} = \frac{1}{I_1}\frac{d_e}{D}\left(\frac{u_0 D^2}{hx}\right)^{1/3}$$

$$h = d_e/4 \Rightarrow x^* = x/L$$

$$Sh(x^*) = \frac{4^{1/3}}{I}\left(Re\, Sc \frac{d_e}{L}\right)^{1/3} x^{*\,-1/3}$$

$$\overline{Sh}_L = \int_0^1 Sh(x^*)\, dx^*$$

$$= \frac{2.381}{I}\left(Re\, Sc \frac{d_e}{L}\right)^{1/3}$$

So, what you get is that you will be getting K as minus as 1 over I 1 u 0 D square divided by h x rest to the power 1 upon 3. So, once you get the expression of K and these expression will give this I 1 is basically a definite integral so it will be returning a value, u 0 is constant D is constant h is constant. So, K is inversely proportional to x to the power 1 upon 3. Once you get the expression of K you can get the expression of Sherwood number as a function of x star. What is Sherwood number? Sherwood number is nothing but, K de by D so this will be 1 over I 1 de by D u 0 D Square divided by h x rest to the power 1 upon 3. Now we write the half height in terms of equivalent diameter h is equal to de by 4 and an express x star as x by L then.

We can easily find out you substitute all these here in the expression and you will be getting the expression of Sherwood number as a function of x star as 4 to the power 1 upon 3 divided by I the whole thing within the bracket becomes Renault smith de by L rest to the power 1 upon 3 x star rest to the power minus 1 upon 3. In fact in the last class we have derived that the whole thing that is within the bracket with a with a power 1 by 3 will turn out to be Renault smith de by L so those student who have not attended the last class please go through the lecture notes of the last class otherwise you will be lost. Now you are in a position once you do the get the expression of Sherwood number as a function of x star you will be in a position to get the length averaged Sherwood number.

What is it? 0 to integration 0 to 1 s h x star d x star. So, once you carried out this integration the whole integration turns out to be 2.381 divided by I Renault smith de by L rest to the power 1 upon 3. Now this very crucial and important relationship from this important relationship we can get various interpretation about the mass transfer coefficient of Sherwood number in a channel where the channel walls are porous. Unlike the channel walls are impervious in case of heat and mass transfer analogy. Now let us see how we can do it.

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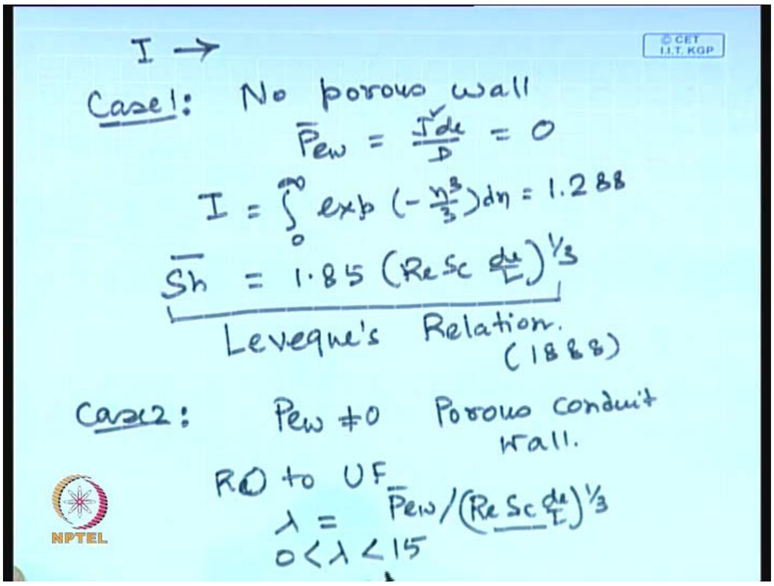

I →

Case 1: No porous wall

$$\overline{Pe}_w = \frac{J d_e}{D} = 0$$

$$I = \int_0^\infty \exp\left(-\frac{\eta^3}{3}\right) d\eta = 1.288$$

$$\underbrace{\overline{Sh} = 1.85 \left(Re\,Sc\,\frac{d_e}{L}\right)^{1/3}}_{\text{Leveque's Relation.}}$$

(1888)

Case 2: $\overline{Pe}_w \neq 0$ Porous conduit wall.

RO to UF

$$\lambda = \overline{Pe}_w / \left(Re\,Sc\,\frac{d_e}{L}\right)^{1/3}$$

$$0 < \lambda < 15$$

Now depending on the value of I or the physical situation there are several simple simplified cases that we arise. Case number 1 is, No porous wall that means we are talking about a flow through a impervious concrete, that means the membrane is not present the wall is impervious. In that case there is no question of permeate flux, if there is an question of permeate flux the peclet wall which is nothing but, non dimensional permeate flux J de by D there is J is equal to 0 that means p w equal to 0. So, in that case what happens you can evaluate the integral I and the fate of integral I becomes this exponential minus eta cube by 3 d eta. What is this? This is this you can evaluate numerically and this integration integral turns out to be 1.288.

You can use a trapezoidal rule and fix this expression and evaluate the integral put the infinite value as the upper limit of the integration may be 10 or 12 and you can find out that this integral integration results a value 1.288. If you substitute this in the expression of length averaged Sherwood number what it will give you? You will be getting 1.85 Renault smith de by L rest to the power 1 upon 3. Now if you remember we have talked about this uh relationship of Sherwood number of mass transfer coefficient earlier, while we are talking about the Sherwood the Sherwood the mass transfer in a wall which will be obtained from the heat and mass transfer analogy for a heat transfer in a non porous conduit this is known as Leveque’s relation.

This is the Leveque's relations it was in fact derived in the year 1888. Almost hundred and twenty years back. Now this expression so that is why told you when we talked about the Sherwood number relations under the lamina flow conditions, they are exactly the relations they are not correlations. They can will obtain from the fundamental theory like whatever we had derived right away, but on the other hand the dittos bolter relationship for the turbulent flow that is 0.023 Renault rest to the point smith rest to the 0.23. That is a correlation it is generated from the experimental data conducted and fitting the data into the particular form.

On the other hand Leveque's relation the nozzle number and Sherwood number relations for the flow through a tube or for the flow through a pipe or rectangular conduit, they are that the relations they are not correlations. So, this Leveque's relation and now if let us look into the expression of I. Case number 2; this case is of particular interest to us in this case my Pe w is not equal to 0. There is wall suction that is present in the system, that means you are talking about a porous conduit or conduit which will be having porous wall. Now for the case of a reverse osmosis to reverse osmosis to ultra filtration the reverse osmosis flux will be extremely small. It is almost non porous conduit.

So, J will be pretty small on the other hand, the ultra filtration flux will be higher and micro filtration flux will be highest. So, the effect of porosity on the wall will be much prominent when we more from the operation from the reverse osmosis to micro filtration. So, in the case ultra filtration and micro filtrations these effect of porosity will be of primary interest and if you plot the value of in we have we have seen we have looked into the quantity. we have defined a quantity lambda the suction parameter yesterday if you remember that is nothing but Pe w bar divided by Renault smith de by L rest to the power 1 upon 3 for a typical case these quantity Renault smith de by L will be ranging in the order of 10 to the power 3 to 10 to the power 5 under the lamina flow conditions.

And the value of Pe w that we will getting in case of micro filtration that will be in the order of 10 to the power of minus 5 meter cube per meter square second. In case of ultra filtration it will be 2 or 3 in to the power minus 6 meter cube per meter square second. So, you know what is the order of magnitude or range of values of this ah Pe w the permit flux. In the case of reverse osmosis and the case of micro filtration we know the typical value of Renault smith de by L under the lamina flow conditions. Under these cases for reverse osmosis to ultra filtration these lambda maximum it varies for 15. So, it varies from 0 to 15.

Which case will correspond to 0? The case 0 means Pe w equal to 0 that means, there is no 1 suction. so we get back this relationship non porous conduit for the case lambda equal to 0 we get back the non porous conduit for the for any other value of lambda which is not equal to 0 corresponds to the porous wall.

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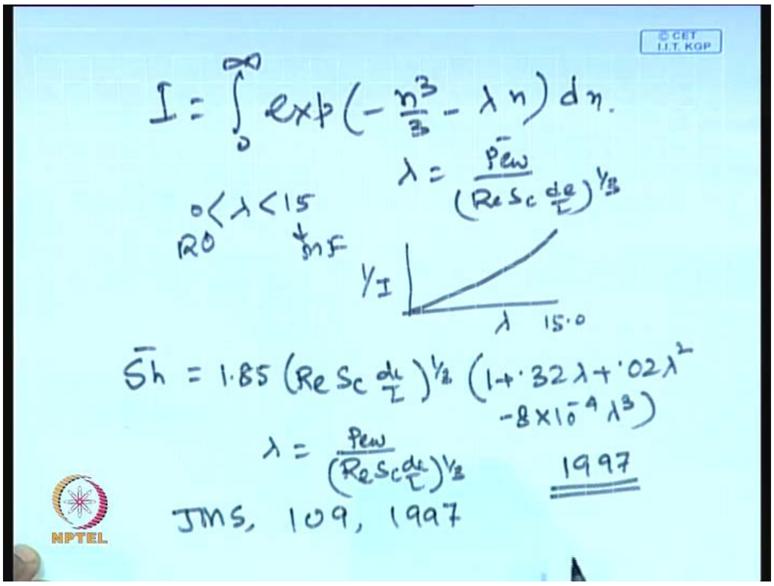

$$I = \int_0^\infty \exp\left(-\frac{\eta^3}{3} - \lambda\eta\right) d\eta.$$

$$\lambda = \frac{\bar{Pe}_w}{\left(Re\,Sc\,\frac{d_e}{L}\right)^{1/3}}$$

$0 < \lambda < 15$

RO $\uparrow$ MF

$$\bar{Sh} = 1.85\left(Re\,Sc\,\frac{d_e}{L}\right)^{1/3}\left(1 + .32\lambda + .02\lambda^2 - 8\times10^{-4}\lambda^3\right)$$

$$\lambda = \frac{Pe_w}{\left(Re\,Sc\,\frac{d_e}{L}\right)^{1/3}}$$

1997

JMS, 109, 1997

Now if you remember the expression of Ii becomes 0 to infinity exponential minus eta cube by 3 minus lambda eta D eta, this lambda is nothing but, Pe w by Renault smith de by L rest to the power 1 upon 3. So, we know the typically the lambda varies from 0 to 15. Now what we can do? We can take different values of lambda this will from this is for reverse osmosis and this from micro filtration. So, we can we can take different values of lambda between 0 and 15 and calculate these integral this definite integral is absolutely no problem and if you can plot 1 over I versus lambda it will show a plot something like this and this 15. So, this cause can be obtained by numerical integration of the integral definite integral I by putting various values of lambda.

Now what we can do? We can feet a polynomial of in terms of lambda. So, if you do that and substitute in a governing equation the Sherwood number expression becomes 1.85 Renault smith de by L rest to the power 1 upon 3 1 plus 0.32 lambda plus 0.02 lambda square minus 8 into 10 to the power minus 4 lambda cube. So, these lambda is nothing but Pe w divided by Renault smith D by L rest to the power 1 upon 3. If lambda equal 0 now so, it basically these expression derive from derived from the fundamentals. We are we are expressing we are doing numerical integrations of this integral I for the various values of lambda. And after doing a numerical integration we are fitting a craft a polynomial in terms of lambda further a polynomial and substitute in the governing equation.

Now if you see that if they the wall is not porous, if it is impervious then lambda equal to 0 if lambda is equal to 0 put this expression, you get back your (( )) relationship. if lambda is not equal to 0 you will be getting these expression and that is the correction to the (( )) relation for the non porous conduit and this is derived in 1997 and that is the first talk of my PhD thesis. So, if you if you look into the journal of membrane science volume number 109 and 1997 give a search of forward the page number, you will get this article. So, you have solve this problem for the rectangular convict for the tubular flow for the radial channel flow so similar analysis can be carried out for the tubular flow.

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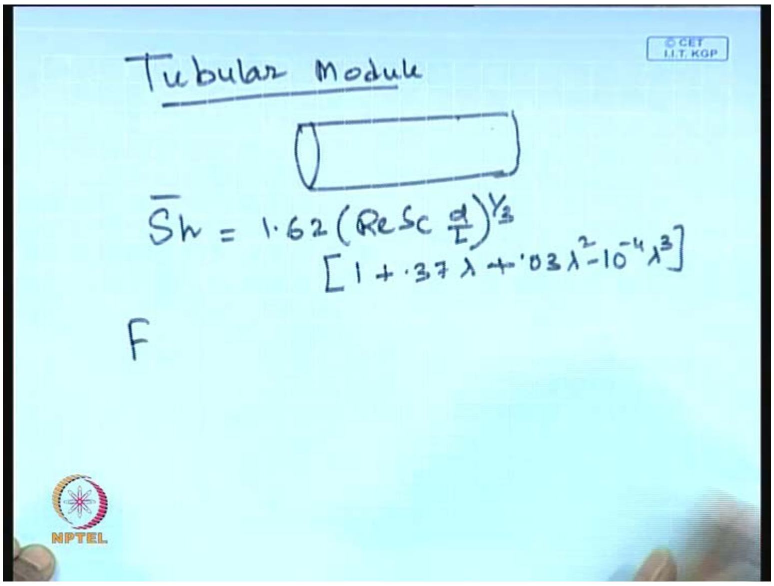
Tubular Module

$$\overline{Sh} = 1.62\left(Re\,Sc\,\frac{d}{L}\right)^{1/3}\left[1 + .37\lambda + .03\lambda^2 - 10^{-4}\lambda^3\right]$$

F

If the conduit is tube that means the flow is occurring creative and the wall is porous. So, the membrane is basically kept in the wall and the Sherwood number relationship can be obtained in the same fashion. You can look into that paper and see the derivate derivation in detail and the Sherwood number the length of the Sherwood number becomes 1.62 Renault smith d by L rest to power 1 upon 3 multiplied by 1 plus 0.37 lambda plus point 0 3 lambda square minus 10 to the power of minus 4 lambda cube. So, if lambda is equal to 0 for the non porous conduit you get back the (( )) solutions for the flow through a tube is 1.62 Renault smith d by L rest to the power 1 upon 3 and d is basically the inner diameter of the tube.

Now let us look into the calculation prep method, how it increases the speed of the calculation? How it simplifies our calculation what we are aiming for?

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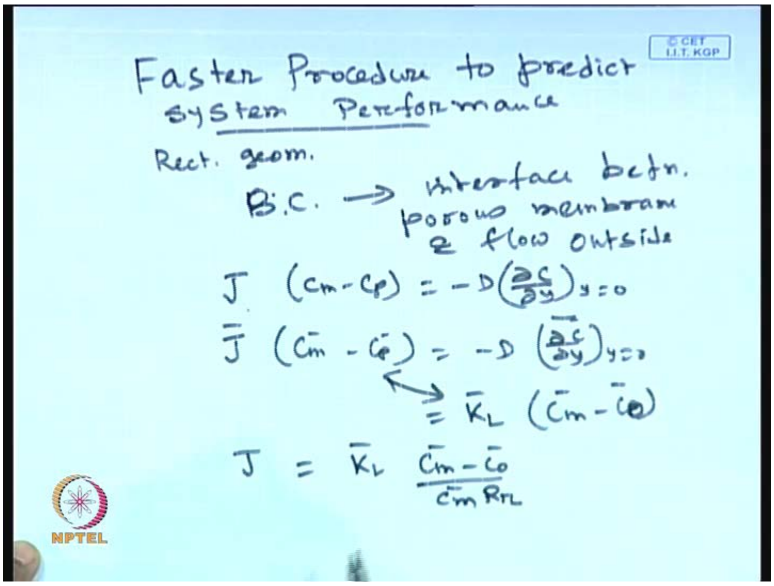

Faster Procedure to predict system Performance

Rect. geom.

B.C. → interface betn. porous membrane & flow outside

$$J\ (c_m - c_p) = -D\left(\frac{\partial c}{\partial y}\right)_{y=0}$$

$$\bar{J}\ (\bar{c}_m - \bar{c}_p) = -D\ \left(\overline{\frac{\partial c}{\partial y}}\right)_{y=0}$$

$$= \bar{K}_L\ (\bar{c}_m - \bar{c}_0)$$

$$J = \bar{K}_L\ \frac{\bar{c}_m - \bar{c}_0}{\bar{c}_m R_{rL}}$$

Let say faster method, faster procedure to calculate or to predict system performance. Now we are talking about the again the same rectangular geometry flow through the rectangular convict, if you remember the boundary condition the that is the on the that the condition the boundary condition that is the interface between the between porous membrane and flow outside is J times cm minus C p is equal to minus D del C del y at y equal to 0. Now this relationship is hide for every x location in the channel, that means if you talk about a control volume between x and x plus delta x this relationship will whole good premise it will be valid for every x location. Now instead of delta x now is if u consider this delta x is basically the full length of the membrane module.

In that case all this quantities will be replaced by the length averaged quantities. So, we can talk about a length averaged quantities C m bar minus C p bar is equal to minus D del C del y y equal to 0. The whole derivative will be bar, the length averaged bar and if you look into definition of mass transfer coefficient this will be nothing but K L bar C m minus C p C m minus C naught. The that we have we have just in the beginning of this class, we have read the definition of mass transfer coefficient K L is equal to minus D del C del y at y is equal to 0 divided by C m minus C naught. So, now you take this 2 relationship and see what you get you will be getting J is equal to K L bar C m minus C naught divided by C m R r. All these are length averaged quantities C naught bar there is no meaning for C naught bar because you know the feed concentration so that is the C naught. Now make it non dimensional that means we multiply both side by D equivalent divided by diffusivity of the solute.

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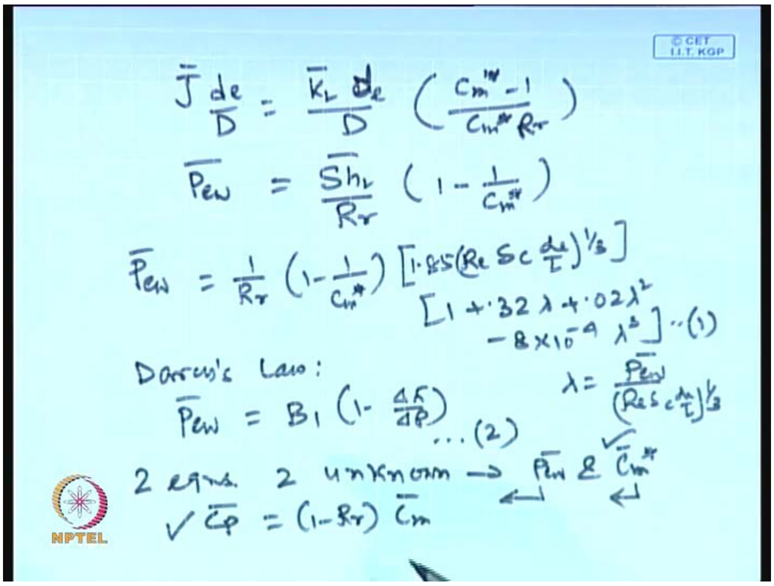

$$\frac{\bar{J}\, d_e}{D} = \frac{\bar{K}_L\, d_e}{D}\left(\frac{C_m^* - 1}{C_m^* R_r}\right)$$

$$\overline{Pe_w} = \frac{\overline{Sh_L}}{R_r}\left(1 - \frac{1}{C_m^*}\right)$$

$$\overline{Pe_w} = \frac{1}{R_r}\left(1 - \frac{1}{C_m^*}\right)\left[1.85\left(Re\, Sc\, \frac{d_e}{L}\right)^{1/3}\right]\left[1 + .32\lambda + .02\lambda^2 - 8\times10^{-4}\lambda^3\right] \cdots (1)$$

Darcy's Law:

$$\overline{Pe_w} = B_1\left(1 - \frac{\Delta\pi}{\Delta P}\right) \ldots (2)$$

$$\lambda = \frac{\overline{Pe_w}}{\left(Re\, Sc\, \frac{d_e}{L}\right)^{1/3}}$$

2 eqns. 2 unknown → $\overline{Pe_w}$ & ✓ $\overline{C_m^*}$

✓ $\overline{C_p} = (1 - R_r)\,\overline{C_m}$

So let us see what you get J de by D is equal to K L bar de by D make it non dimensional, so this becomes C m star minus 1 divided by C m star R r so this becomes nothing but the length averaged permeate flux non dimensional Pe w and this will be nothing but length averaged Sherwood number and this will be divided by R r 1 minus 1 by C m star. So, you will be getting the expression of permeate flux in terms of length averaged Sherwood number. I will know the expression of Sherwood number in terms of Pe w.

What is that? if you write that you will be getting this as 1 over R r 1 minus 1 by C m star and what is Sherwood number length averaged this will be 1.85 Renault smith de by L rest to the power 1 upon 3 multiply by 1 plus 0.02 lambda that was the expression guess a 0.32 lambda plus 0.02 lambda square minus 8 into 10 to the power of minus 4 lambda cube, lambda being a number which will be nothing but, p w bar divided by Renault smith de by L rest to the power 1 upon 3. So, these will be giving an expression of relationship an liberating relationship between C m star and Pe w where there are 2 unknowns Pe w and C m star and you have the Darcy’s law that is valid throughout the module that means Pe w is equal to B 1 times 1 minus delta pi by delta p this is equation number 1 this is equation number 2.

Now this we are defined in the last class that is the found a Darcy’s law if you make a non dimensional you will be getting these expression so what is this expression delta pi delta pi will be the function of C m star. If you remember so, i will get an algebraic equation between Pe w bar and C m star i-i will be getting another algebraic equation be between Pe w bar and C m star 2 equations 2 unknown. Now they can be solve by using an iterative technique by using some non-linear algebraic equation solver. So, 2 equations 2 unknown, what the 2 unknowns Pe w are bar and C m star. These 2 equations 1 and 2 will be simply solved and 1 can get an estimate of length averaged permeate flux and non dimensional length averaged permeate a membrane surface concentration.

Once you will be knowing the non dimensional membrane surface concentration you will be in the position to get the permeate concentration as well. That is related to C p is nothing but, 1 minus real retention times C m bar. So, once you get the length averaged permeate a membrane surface concentration you will be in a position to estimate the length averaged permeate concentration. So, this is a this is the quickest way to solve the in a in a the cross ways system rectangular conduit, without going into without solving the profile of the permeate flux and permeate concentration. And first of all without solving any partial differential equations so we got read of solution of partial differential equation.

In the last class we have seen, how we get the profiles of permeate flux and permeate concentration and from that we learnt it to the length averaged permeate flux and permeate concentration. In fact the length averaged quantities are of interest to you because you cannot measure ah these quantities at every x location. What you can measure? You can measure a length averaged permeate flux or permeate concentration. So, therefore that is a primary interest to you and these shortcut method via the

Sherwood number length averaged Sherwood number of mass transfers coefficient that will give you direct estimation of length averaged permeate flux and permeate concentration. There is why, this is the shortest method to module the system.

That goes for the cross force system in a rectangular conduit in a tubular module the same thing is valid, only difference is the de equivalent in the rectangular thing will be replaced by the inner diameter of the tube number 1. Number 2 the expression of Sherwood number will be different in case of tubular module that will be 1 point sixty 2 Renault smith de by L rest 1 upon 3 into a this expression will be this binomial will be something else that you already written down. So, 1 can solve the by the same way 1 can solve the get the system predict the system performance, in case of tubular flow without having any adjustable parameter,we can predict it from the fundamentals. Next important thing that will be doing a probably after couple of class i will be solving some of problem in that class itself, so that things will be very clear how to use these theories to re to have an actual prediction in a real problem in a real system.

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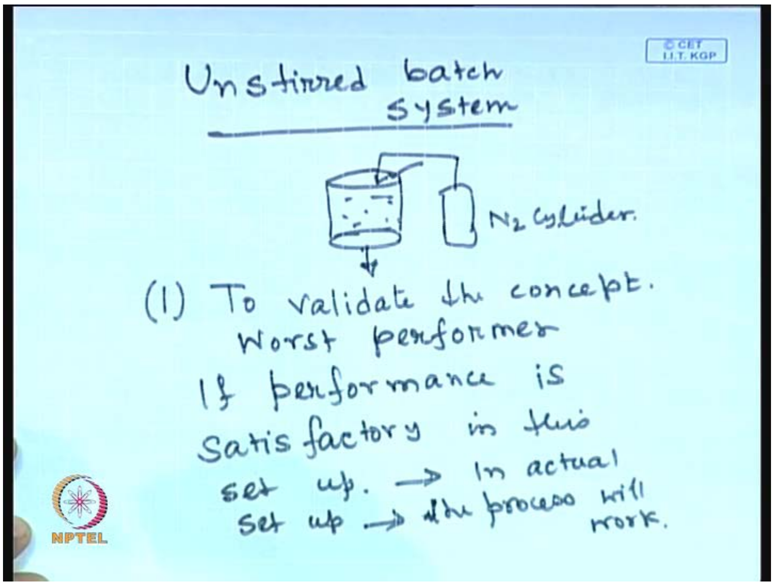

Unstirred batch system

(1) To validate the concept.
Worst performer
If performance is
Satisfactory in this
set up. → In actual
set up → the process will work.

The next thing that i will be teaching you is that unstirred batch system. What is unstirred batch system? Unstirred batch system is basically it is a small setup it can be large setup as well there are 2 flanges, in fact 3 flanges. There is a top flange there is a cylindrical volume and there will be a bottom flange. The top flange is basically this top and bottom flanges will be connected to the body is a circular body and in the bottom flange it will be it will be there will be concentrate groups on it we put the a solid surface a solid substrate that is very porous. Let say porous stain less steel support over that we put the membrane we cut a membrane piece from rectangular shape into a circular shape and put it over there and then the solution is feed here. Now the whole thing is closed and this solution is pressurized by nitrogen cylinder. Now you can under the nitrogen cylinder pressure the filtration will be carried out and you will be getting the permeate flux and you are not using any studying in the system.

So, when these unstirred batch system why these unstirred batch system is so important that we will be studying it. The application on unstirred batch system is twelve 2 4 first 1 is that, to validate the concept. What is the validation of the concept? Suppose you would like to a treat a some a fluent or some fruit juice or some kind of liquids stream. Now which membrane you have to select? Suppose the this suppose the membrane you will be having a wide varieties of membrane if you talk about the ultra filtration membrane, you will be having probably 10 15 molecular cut offs. Which membrane you have to select? There is very crucial aspect before doing any before conducting any experiment you have to identify which membrane is suitable for you particular purpose.

So, what we can do since it is a smaller apparatus and there is there is no need of any power and you just fill charge the you knowing cylinder charge the fit tank the charge the this balloon with the fit solution and pressurize it and leave it. So, what you will be getting? You will be getting the permeate flux so you just get the on you get permeate you just analyze the permeate quality. If you see the permeate quality is up to the mark and it is acceptable that membrane will be good enough for a higher scale of or higher module. Now number 1 number 2 is that if a concept is validated that you will be using 10000 molecular cut off that will (( )) to you know validate your system to bring down the concentration below a particular level.

That will be the best because this is a worst performer unstirred batch system is the worst performer. Why is the worst performer compare to the cross flow system? Why is worst performer? Because you are not using any stirring, you are not using any turbulence, if you increase the turbulence in an actual system for example, the cross flow system the system that we are jus just studied now on the cross flow system rectangular channel or in a spiralular module or in a tubular module the cross flow system will allows performs better than the unstirred batch system. Because in the unstirred batch system there is no turbulence and the solutes will be deposited over the membrane surface the effect of concentration polarization will be maximum there.

Since the polarization will be maximum the permeate quality will tent to diorite the maximum level. Even in that cell if it satisfies you the environmental regulation rule that means in a cross flow system or in actual system the turbulence is more then it will be a sure success there as well. So, that is why unstirred batch system are quite crucial and that give you that gives you the experiment contacted in such systems will give you a firsthand information or confirmative measured, that you what you are doing that is a thing that the step. Is the number 1 so its a worst performer so if performance is satisfactory here in this setup in an actual setup it will surely succeed the process will work.

And second thing is second advantage of using the unstirred batch system is that some times when you are talking about an industrial effluent, you are not very sure about the physical quantities. If you remember if you just ah whatever we have done the modeling you have done the modeling we have done, if you know the physical properties of the solute perfectly. If you know the diffusivity, if you know the density, if you know the viscosity all this things, if you know the osmotic pressure relationship how it varies with the concentration all the calculations can be done. But in an actual industrial effect is very difficult to estimate the osmotic pressure is very difficult to estimate the diffusivity of the system because it will be containing 10000 elements.

Instead of 1 spices there will be thousand of spices present in that system so it will very difficult. You do not know how what are the first of all it is very difficult to identify what are the spices present? Number 1 number 2 is even if you identify the spices present what are the major spices and it is to be very difficult to get the value of diffusivity for each and every spices. And it will be very difficult to get the value of osmotic pressure expression as a function of concentration with respect to each and every spices. So, basically you should know what will be the effective osmotic pressure. You should know what is the effective diffusivity? So, for that if you look in to the effective osmotic pressure if you look into the expression of osmotic pressure it is a (( )) polynomial in concentration that means, there are 3 constants invoked in it.

Because you do not know the molecular weight as well if you do not know the first constant even, because you do not know the molecular weight first constant is vents constant R T by m or you know T is the temperature in absolute scale m is the molecular weight you do not know the molecular weight because it very difficult to estimate the species itself. So, there are 3 unknown constants in rope fault in the expression was osmotic pressure and 1 more constant is unknown constant that that is you know difficult to estimate, that is the effective diffusivity. So, at least these 4 constants 1 or really unknown the viscosity density 1 can measure viscosity density 1 can measure and can put in a program or in your calculations.

So, 4 constants have to be determined. So, again you have to go for an optimization for the optimization if for determination of 4 constants using a theory you must be requiring the a huge experimental data bank. That bank must be consisting about twenty or more than twenty experimental points, then you can you will be probably thinking of to estimate 4 constants by doing an optimization in sub routine. Now getting twenty experiments means in a cross flow cell that a steady state experiment that we had talked about that mean that means 1 set of experiment will give you 1 set of data. 1 permeate flux and 1 permeate quality. But so you have to conduct huge number of experiments so at least 25 or 27 experiments 27 is 3 into 3 that means kit concentration operating pressure and cross flow velocity.

If you vary all these 3 operating parameters 3 three times then we will get 3 into 3 power 327 experiments. So, minimum 27 experiments need to be conducted to estimate this 4 unknown constants maximum 36 or more. Now conduction of these many experiments we will talk 2 huge amount of time, huge amount of man power, huge amount of cost. But if you conduct 1 unstirred batch experiment in 1 experiment you can generate more than 27 data. How will you do that?

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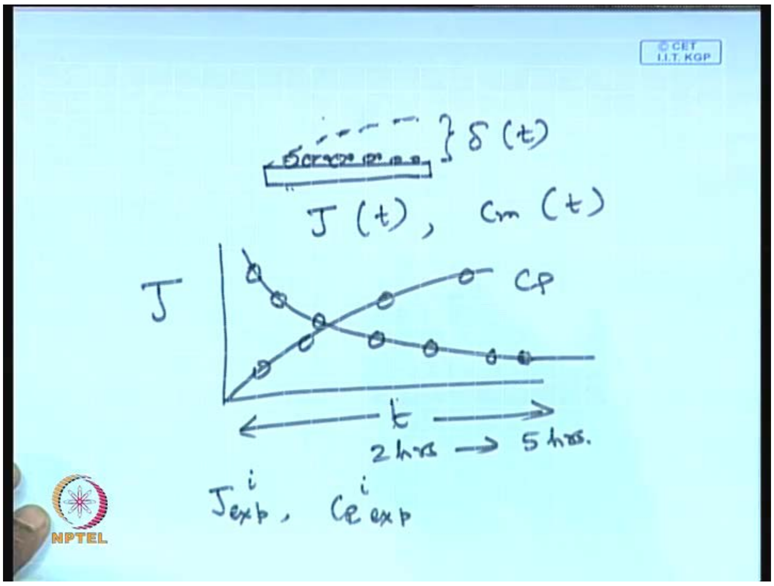

Now in a unstirred un stirred batch cell, what will generated what happen is that over the membrane the solutes will be getting deposited and it is the whole system is left to its naturally tendency. You are not controlling it, you are not putting any star, you are you are not inducing any turbulence in your system. So, it will be left to itself solutes will be depositing over the membrane surface. So, you will be getting a an in development of concentration boundary layer and this concentration of boundary layer will be now a function of time. This delta becomes a function of time. Now if delta becomes function of time then, What you can do? You will be you will getting a permeate flux which will be a function of time and you will be getting a membrane surface concentration which will be a function of time.

So, therefore if you have a measurement the permeate flux verses time will be having a trend something like this. So, you can have various experimental points on this curves, now you can conduct this experiment let say for 2 hours you can conduct this experiments let say for 5 hours, every 10 minutes you can take a measurements or every 5 minutes or every 15 minutes it’s up to you. You can take a measurements so you can get the permeate flux and permeate concentration data. This C p as a function of time so you can collect as many data as possible. Now every time measurements every time point this measurements will be J experimental and C p experimental the I x point will be corresponding to J e x p I and C p e x p I so you can have huge number of this data, on the other on the other hand if you would like to have a steady state side cross flow system of 1 experiment of let lets a duration of 2 hours or 1 hour will be giving you 1 data.

So, you have to conduct so many experiments and the other hand if you do a unstirred batch cell you can generated so many data from a from lets a 2 hours or 3 hour by conducting 1 single experiment. Then you can fit these data in your optimized sub routine lets a 25 data or 27 data and can estimate the 4 unknown constants. That is why the study of unstirred batch cell is very important. So, let us write down the so again the problem for the modeling the problem remains same

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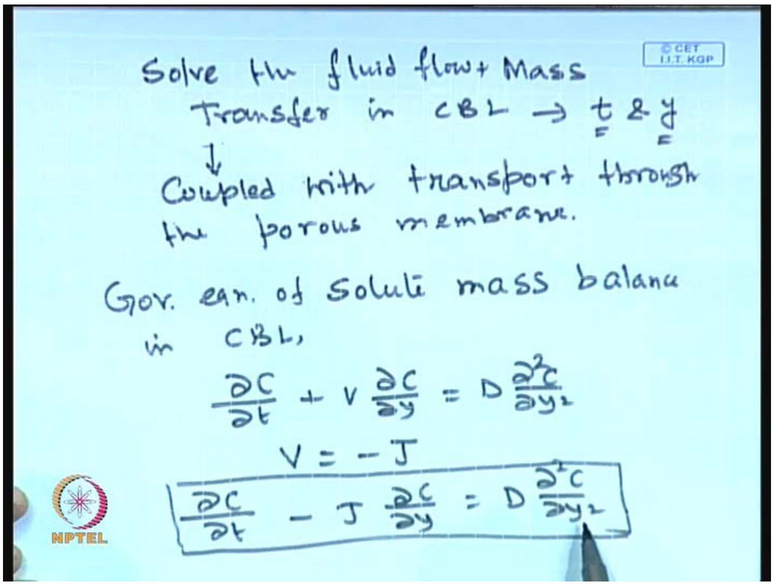

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Solve the fluid flow + Mass Transfer in CBL $\rightarrow \underline{t}$ & $\underline{y}$

$\downarrow$

Coupled with transport through the porous membrane.

Gov. eqn. of solute mass balance in CBL,

$$\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2}$$

$$v = -J$$

$$\boxed{\frac{\partial c}{\partial t} - J\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2}}$$

We have to solve the fluid flow, Plus mass transfer in concentration boundary layer that will be function of time and y. so, it grows in the y direction so it is a function of time and y. And it has to be coupled with transport through the porous membrane in the it was the function of it was a steady state process it was a function of x and y and in this case it is a function of T and y. If you write down the governing equation of solute mass balance in concentration boundary layer, it will be nothing but del C del T plus v del C del y is equal to D del square C del y square, this is the governing equation of concentration of solute mass balance within the concentration boundary layer.

And since there is no other no flow in the x direction or whatever so v will be nothing but minus J. What is J? J is the permeate flux at y equal to 0. So, the governing equation now becomes del C del T minus J del C del y is equal to D del square C del y square.

These equation has to be solved along with its initial and boundary conditions in order to get C within the concentration within the mass transfer boundary layer as a function of y and T. Then that has to be evaluated at y is equal to 0 that is at the membrane surface and that has to be hooked up with the transport law or with the porous membrane or the Darcy's law it will be giving you a prediction of the permeate flux how it varies with time so that is the idea.

In order to solve the concentration at y is equal to 0 or C m at the membrane surface you have to solve this equation within the mass transfer boundary layer in order to solve this equation

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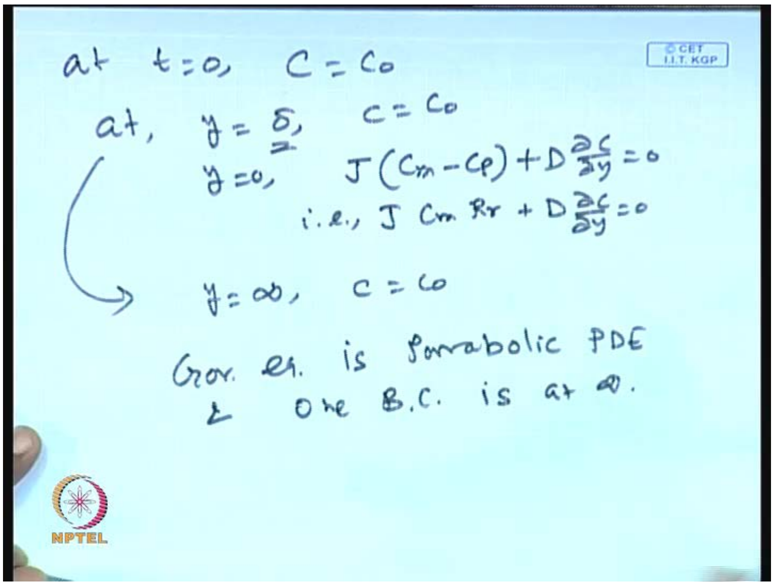

at $t=0,\quad C=C_o$

at, $y=\delta,\quad C=C_o$

$y=0,\quad J(C_m-C_p)+D\frac{\partial C}{\partial y}=0$

i.e., $J\,C_m R_r + D\frac{\partial C}{\partial y}=0$

$y=\infty,\quad C=C_o$

Gov. eq. is parabolic PDE

& one B.C. is at ∞.

You require to have 1 initial condition with respect to T and 2 boundary conditions with respect to y so let us write down the conditions. At T is equal to 0 C is equal to C naught. Because it was same concentration everywhere at y is equal to delta C is equal to C naught at y is equal to 0 you will be having the convective diffusive the convective diffusive boundary condition, mixed boundary condition present J C m minus C p plus D del C del y is equal to 0 as earlier. So, this becomes J C m R r plus D del C del y is equal to 0 this equivalent. Now again the mass transfer boundary layer will be extremely thin it will be in the order of micron I is the actual you know dimension of the of the system will be probability of 4 time higher order of magnitude.

So, we can replace this replace this boundary conditions by y is equal to infinitive. So, we can do a modification here for the ease of mathematical treatment, we can replace this boundary condition y equal to infinity C is equal to C naught. Again as we have discussed earlier the governing equation is a parabolic partial differential equation and 1 boundary condition is at infinity. That means it can have a similarity solution and these case it will be having a in detail similarity solution and this partial differential can be reduced to an ordinary differential equation. But before that we have to identified what is the similarity parameter and what is the functional variation of the similarity parameter with respect to the independent variable y and T, but before that we should make this equation non dimensional and that will that will make our life simple

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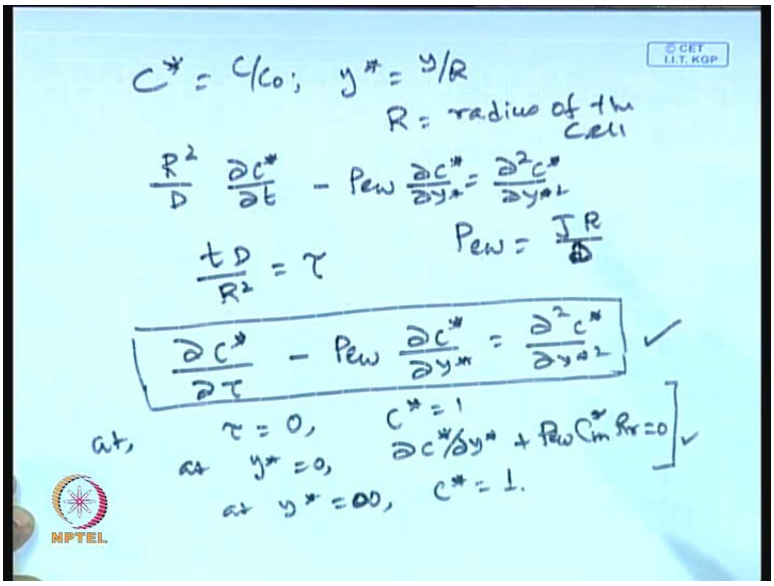

$C^* = C/C_0; \quad y^* = y/R$

R = radius of the cell

$$\frac{R^2}{D}\frac{\partial c^*}{\partial t} - Pe_w \frac{\partial c^*}{\partial y^*} = \frac{\partial^2 c^*}{\partial y^{*2}}$$

$$\frac{tD}{R^2} = \tau \qquad Pe_w = \frac{JR}{D}$$

$$\boxed{\frac{\partial c^*}{\partial \tau} - Pe_w \frac{\partial c^*}{\partial y^*} = \frac{\partial^2 c^*}{\partial y^{*2}}} ✓$$

at, $\tau = 0$, $c^* = 1$

at $y^* = 0$, $\partial c^*/\partial y^* + Pe_w C_m^* R_r = 0$ ✓

at $y^* = \infty$, $c^* = 1$.

We write C star is equal to C by C naught write y star is equal to y by R where R is the radius inner radius of the cell of the ultra filtration of the cell, it is a cylindrical cell. So, it is the inner radius of that cell and just put it there so it becomes R square ah R square by D del C star del T minus Pe w del C star del y is equal to del square C star del y star square. What is Pe w? It is the non dimensional flux. That is J R by solute diffusivity the physical the dimensional becomes R here instead of D equivalent in the earlier case. So, it is J R times D divided by D so hold it as so in the C by C naught. So, right hand side is completely non dimensional the second term on the left hand side is non dimensional. The first term of the left hand side has to be non dimensional quantity.

That means T D by R square must be having non it must be defined as non dimensional time. It will be having a unit of time if you replace the unit of T diffusivity and R square it will be giving a it is a unit free number del C star. In fact that is how 1 has to find out what is the non dimensional form of any independent quantity? If he do not know, if it is not apparent that what is the form of 1 dimensional time or something some independent parameter, if you know the others so just substitute there and do a simplification and make other terms non dimensional.

So, the that the term the third term which what will be the non dimensional version that will appear automatically so del C star del tau minus p w del C star del y star is equal to del square C star del y star square so this is the governing equation and the initial conditions the non dimensional conditions are at initialing boundary conditions at tau equal to 0 C star is equal to 1 at y star is equal to 0 del C star del y star plus Pe w C m star R r equal to 0 and at y star is equal to infinity C star is equal to 1. So, that is the complete problem statement of this unstirred batch cell. These are the governing equation this 3 are the various initial and boundary conditions, we will be having 2 boundary conditions in y and 1 initial condition in x, in tau, in time.

Now you are in a position to you should find out what are will be the similarity parameter in this case and we do the same type of analysis as earlier. That means the governing equation must be valid at the edge of boundary layer at the edge of boundary layer del C del y will be equal to 0. Because we are we utilize this property because the governing equation is valid within the control volume as well as on the boundary on in contrast the boundary conditions are valid only on the boundary but, not within the volume of the system. So, we evaluate this boundary this governing equation at the edge of the boundary layer.

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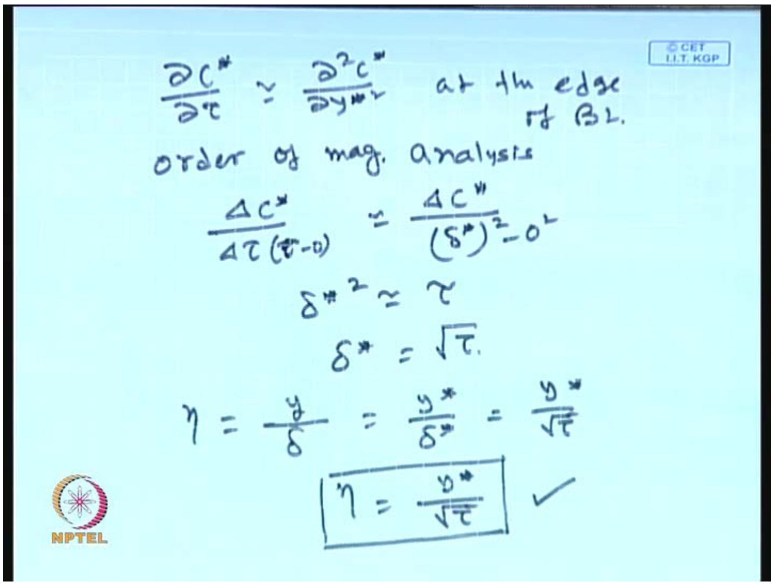

$$\frac{\partial C^*}{\partial \tau} \simeq \frac{\partial^2 C^*}{\partial y^{*2}}$$ at the edge of BL.

order of mag. analysis

$$\frac{\Delta C^*}{\Delta \tau (\tau - 0)} \simeq \frac{\Delta C^*}{(\delta^*)^2 - 0^2}$$

$$\delta^{*2} \simeq \tau$$

$$\delta^* = \sqrt{\tau}.$$

$$\eta = \frac{y}{\delta} = \frac{y^*}{\delta^*} = \frac{y^*}{\sqrt{\tau}}$$

$$\boxed{\eta = \frac{y^*}{\sqrt{\tau}}} \quad ✓$$

So, what you will be getting is del C star del tau will be roughly del square C star del y star square at the edge of boundary layer. Now do an order of magnitude analysis as earlier del C star is written by delta C star and this delta change in delta C star occurs within the time tau from the beginning. So, delta tau delta tau will be nothing but T tau minus 0. So, this becomes tau right and that will delta square C star is again some kind of delta C star change in concentration and delta y star square will be will be delta star delta star minus 0 square minus 0 square. So, that is the so you will be basically this delta C star delta C star will be cancelled and delta tau will be replaced by a tau and you will be landing up with delta star square will be equal to tau delta star will be nothing but root over tau.

Now if you define the similarity parameter the similarity parameter is defined as y by delta that means y star y delta star divided by radius on numerator as well as denominator so this becomes y star by root over tau. So, these is the form of similarity parameter in this particular case so the independent variables y and tau will be related as y star into tau to the power minus half that particular combination is the similarity parameter in this case. It is exactly like the similar earlier 1 what it was it was equal to y star divided by x star to the power 1 upon 3, in this case it is y star divided by tau to the power half. Now using this symbol similarly parameter now it what we can do we can put the derivatives of the governing equation in terms of similarity parameter only 1 parameter therefore, you can reduce so we reduce the because now there are the 2 independent parameter no longer exist only 1 parameter will be sufficient that is the similarly, parameter or the combine parameter so the whole governing equation can be written in the form of combine parameter and now it becomes an ordinary differential equation which can be solved as we have solved already in the case of cross ways system and we look in to the solution in detail in the next class. Thank You.