Novel Separation Processes
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Lecture No. # 08
Membrane Separation Process

Good morning everyone; so we were looking into the limitation of the film theory, that is the one dimensional model, then we shifted to two-dimensional model in order to overcome the short coming that is the mass transfer boundary layer is still developing, then we try to we go on solving the governing equation to get the cross flow ultra filtration system and the mass transfer boundary layer is still developing.

So, what we have done? We wrote down the governing equation; we wrote down the velocity, it is basically a governing equation coupled with the velocity field as well as the concentration field for the solute. Now, this governing equation is nothing but parabolic partial differential equations, and we required to have three boundary conditions to solve these equations. Now, out of these three boundary conditions, we have seen that one of the boundary condition is residing at infinity, that makes it that you know there is a thumb rule that if a partial differential equation is parabolic and one of the boundary condition is residing at infinity, one can expect a similarity solution.

So, we have to identify the similarity variable, what is the advantage of getting the similarity solution? The advantage is if you have a similarity solution, then a partial differential equation will be reduced to an ordinary differential equation, and the solution of ordinary differential equation is always easier. So, we were in search of getting the similarity variable or combining variable, so what do you in similarity variable? What we in just in actual I just give an idea in similarity variable suppose, they are more than one variable in partial differential equation. Let us, a two variable then two variant independent variables. These two independent variables will be combined in a single variable and all that derivatives are now, expressed in terms of single variable that makes the reduction of partial differential equation into an ordinary differential equation.

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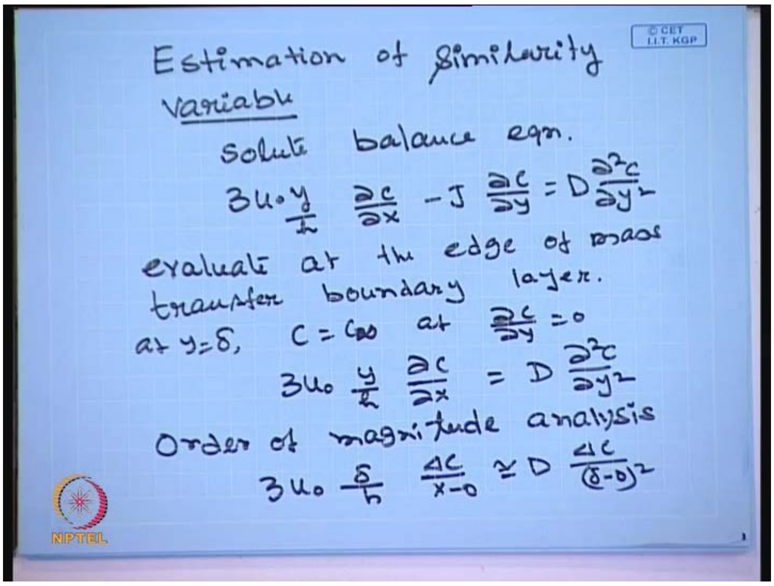

Now let us, look into the estimation of similarity variable is called similarity variable or similarity parameter or combine parameter and what you have to do? First, you write down the equation of motion, the solute balance equation; the solute if you remember the solute balance equation, we have written 3 u naught y by h del c del x minus J del c del y is equal to D del squared c del y squared. Now, since it is a governing equation, it must be valid on the boundary conditions also, so you evaluate these equations at the edge of mass transfer boundary layer.

If you do so now, since it is a boundary layer, we all known that on a boundary layer; the derivatives of the dependent variable will be equal to with respect to y at the normal variable will be equal to 0; that means, C is equal to c naught or c bulk and del c del y will be equal to 0, at y is equal to delta there is the at the edge of the boundary layer so therefore, these equation at the edge of the boundary layer now, becomes 3 u 0 y by h minus del c del x, the second term will manage and the term on the right hand side, that will remain there.

Now, you do an order of magnitude analysis; we do an order of magnitude analysis now, in this order of magnitude analysis, what do we do? We put the value of delta in terms of differences and y will be delta 3 u 0 delta by h and delta c will nothing but a change in concentration and delta x will be nothing but a delta x minus 0. We assume a small distance from the channel entrance, so it will be x minus 0 that will be 0 is equal to D and del squared c. If you familiar with the numerical differentiation it is nothing but a sort of delta c change in concentration, it will delta c and del y squared will be nothing but delta minus 0 whole squared. So, it will be delta minus 0 whole squared approximately. So, now you can simplify this equation and see what you get?

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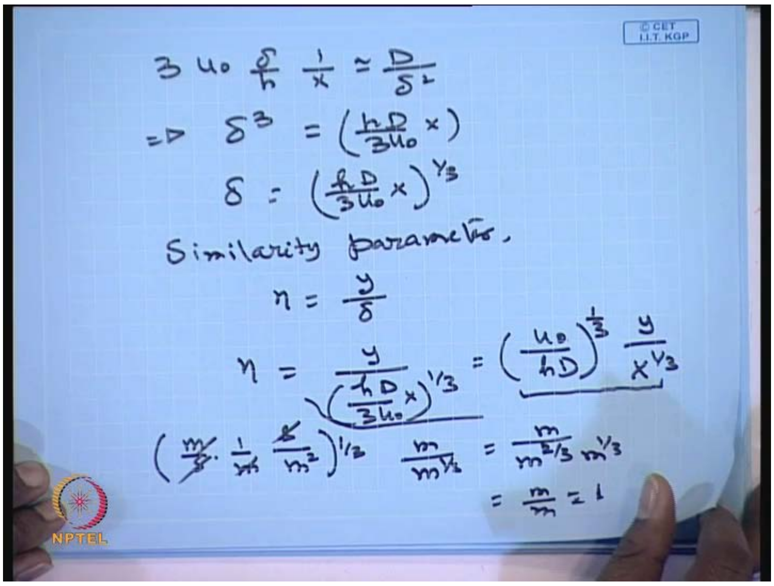

If you simplify this equation, you will be getting this 3 u 0 delta by h now, delta c will be cancelled from both the sides, it will 1 over x will be roughly equal to D delta c will be cancelled and will be delta squared and so, you will getting an estimate that delta cube will be nothing but h D by 3 u 0 x and delta will be nothing but h D over 3 u naught x rest to the power 1 upon 3. Now, the similarity parameter or combine variable parameter; similarity parameter is defined as eta equal to y by delta and why it is y by delta? I just gave a picture in the last class, if u express y by delta then, all the concentration profiles at different locations, we super imposes on single curves.

That means the variation; that independent variation in two variables will be combined or it will be cost on a single variable. So, now the similarity parameter; now put the expression of delta the becomes y divided by h D 3 u 0 x rest to the 1 upon 3 or it will be nothing but u 0 of h D rest to the power 1 upon 3 y over x to the power 1 upon 3. So, the variation of y e of eta of the similarity parameter, in terms of the independent variable y and x will be in the form of y divided by x to the power 1 upon 3. So, that is the variation; that is the functional variation or combination of the independent variables to formulate the definition of combine parameter or similarity parameter eta.

Now, the most interesting thing is that, if you look into the dimension of eta it trans out to be a dimensional list number. So, what is the let us have a dimensional check? Now, what is the dimension of u naught? Is meter per second, what is h? h is nothing but in meter or in diffusivity meter squared per second, rest to the power 1 upon 3 and we do an dimensional analysis and y is in meter and this is meter to the power 1 upon 3. So, second will be cancelling out; so meter will be cancelling out; so you will be getting meter in the denominator meter to the power 1 2 by 3 meter to the power 1 upon 3.

This will be nothing but meter. So, meter will be cancelling out and you will be having a dimension list number. So, the similarity parameter eta will be nothing but a dimension list number, where u 0 h D all are taken as constant and actual variation of eta with respect to independent variables y and x will be in the form of y x to the power divided by x to the power 1 upon 3. Thus, we identify the similarity parameter, so what we will do next? We will do; we will express all the partial derivatives of the governing equation in terms of eta. So, we carry out the partial derivative.

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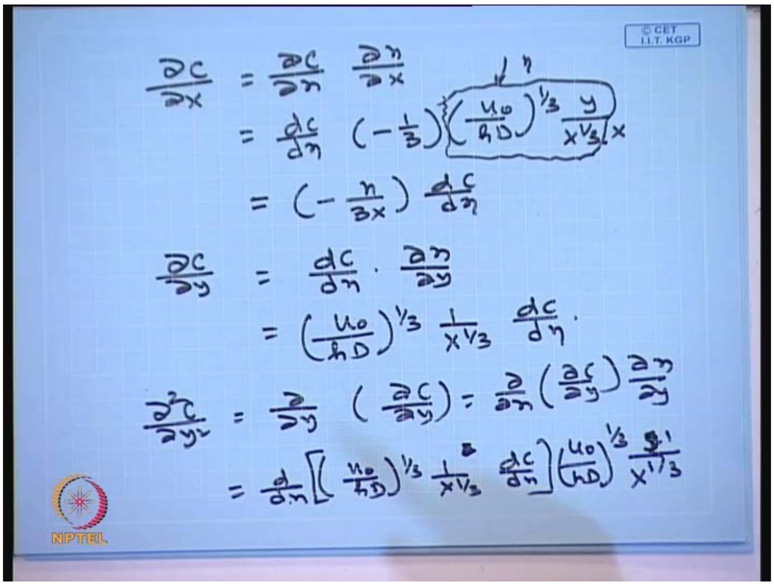

Let say del c del x; del c del x will be nothing but del c del eta del eta del x and since now, you are saying that eta takes here, of it now, there is only one parameter eta is combine variable, combine parameter. So, this del c del x del c del eta will be a total derivative d c D eta. On the other hand, del eta del x will be having some term, this trans out to be minus 1 upon 3 u 0 h by D rest to the power 1 upon 3 y x to the power 1 upon 3 multiplied by x, so there will be x to the 4 by 3 in the denominator and this terms out to be minus eta by 3 x d c d eta so, this whole thing u 0 rest u 0 y h t rest to the power 1 upon 3 y by x to the power 1 upon 3, this whole thing is eta.

If you look into the definition of eta, this whole thing becomes eta. So, I replace this by eta and x will be there in the denominator and you will be having minus eta by 3 x d c d eta. Now similarly, you do the other derivate; you express the other derivatives in terms of eta. If you do that, the rest of is very simple del c del y will be nothing but d c d eta and del eta del y. If you look into the definition of eta, del eta del y becomes u 0 h D rest to the power 1 upon 3 divided by x to the power 1 upon 3 d c d eta.

Now, if you take and so that is how del c del eta del c del y is written in terms of d c d eta and one more term on the right hand side is del squared c del y squared. In del squared c del y squared is basically, one more derivative of del c del y with respect to eta so, del y of del c del y, so here also we do del eta of del c del y del eta del y and we already expressed del c del y in terms of d c d eta.

So therefore, this becomes d d eta u 0 h D rest to the power 1 upon 3, let us put another bracket here 1 by x to the power 1 upon 3 d c d eta and deleted a del y is nothing but u 0 h D rest to the power 1 upon 3 y over x to the power. This y will not be here del c del eta del y, so you will be the 1 x to the power 1 upon 3 so now, if you carry out this differentiation with respect to eta, so all this terms is taken as constant, so it becomes d squared c d eta squared. So, I will be omitting couple of steps for simplification. You go ahead with the all steps on your go back to the hostel and carry out these derivations.

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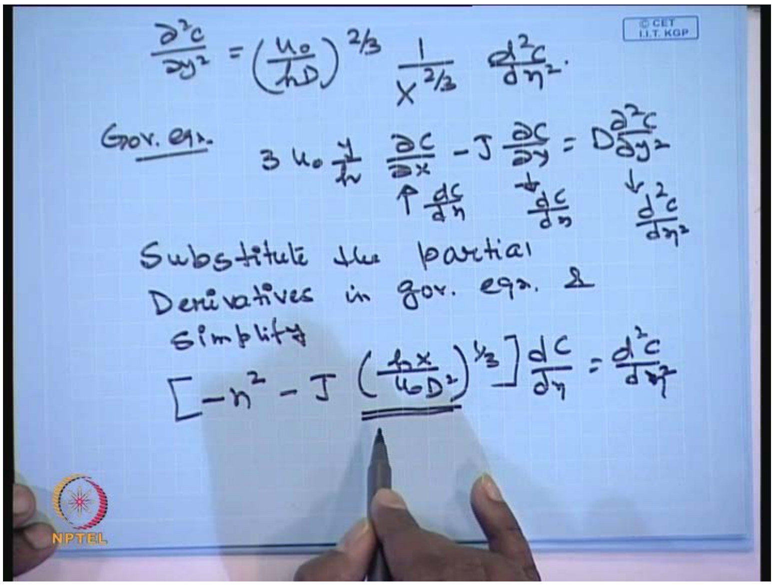

And the final form of this del squared c del y squared becomes del square c del y squared now, becomes u 0 y h D rest to the power 2 by 3 1 over x to the power 2 by 3 d squared c d eta squared. Now, what we have done till now, we have expressed all the derivatives, so let us, look in to the governing equation. The governing equation is not is 3 u 0 y by h del c del x minus J del c del y is equal to D del squared c del y squared. Now, what we have done? We expressed del c del x in terms of eta a combine variable since, it is a single variable so the partial derivative becomes or total derivative.

So, it becomes d c d eta now, this was also expressed in terms of d c d eta and this was expressed in terms of d square c d eta squared. Now, we express all the partial derivative and substitute in the governing equation, if you do that, then and we look up, we substitute the partial derivatives in governing equation and simplify it so that, I can write a compact form and simplify, it may be a couple of steps, and rearrangement basically after rearrangement whatever, you getting is minus eta squared minus J h x by u 0 D square raise to the power of 1 upon 3 d c d eta is equal to d squared c d eta squared.

This will be the form of your partial differential equation, in terms of x and y it boils down an ordinary differential equation of order two with respect to eta although, you have an independent variable x here. So, we cannot integrate it out in the present form for that you required to have some more you know rearrangement or mathematical manipulations, which will confirm the physical situation. Now, let us see what physical situation will allow us to kit or handle this x containing term.

I think we have discussed several times that, the concentration boundary layer; once the concentration boundary layer thickness grows it offers an extra resistance within the solvent flux. So therefore, the thickness delta more be the thickness delta less will be solvent flux or J; so J will be inversely proportional to delta.

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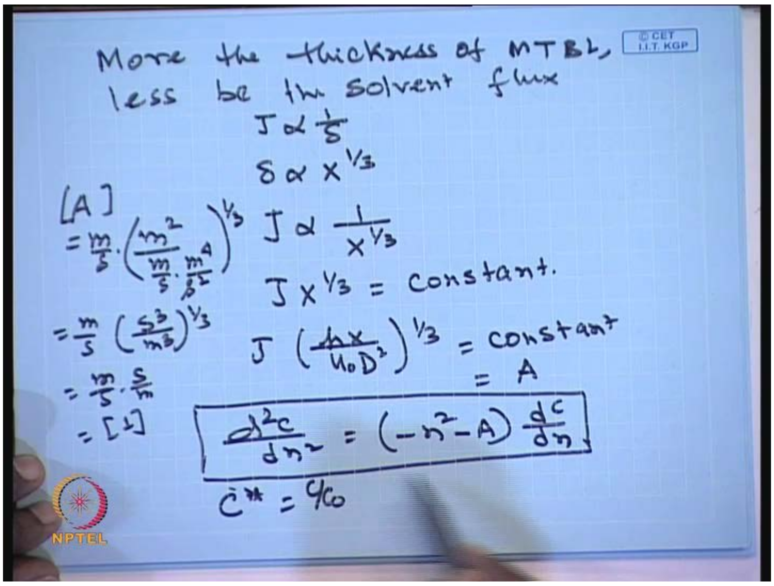

More the thickness of mass transfer boundary layer less be the solvent flux, because more will be the resistance again the solvent flux. So, that means, what is thickness of mass transfer boundary layer J delta? And what is the solvent flux J? So, J is inversely proportional to delta and if you remember that, what is the expression of delta? Delta will be proportional to x to the power 1 upon 3, that only we have derived delta is proportional to x to the power 1 upon 3.

So therefore, J is inversely proportional to x to the power 1 upon 3, so J times x to the power 1 upon 3 will be a constant. That means in this expression J times x to the power 1 upon 3 will be a constant and what is the role of this terms h by u 0 D squared they will be basically, when some dimension number they will make whole this things as non dimensional constant. Therefore, what I can do? I can write J times h x by u 0 D squared rest to the power 1 upon 3 is a constant, because h is a is a geometrical parameter which is constant; u 0 is a operating parameter cross per velocity that is the constant; D is the diffusivity of solute property that is a constant; so we can write, J h x divided by u 0 D squared rest to the power 1 upon 3 is our constant.

Let us say, that constant is A so under this situation, what is our governing equation now, boils to my governing equation now, becomes D square c d eta square is equal to minus eta squared minus A d c d eta. This is the form of governing equation now, in this equation let us see, we have already shown earlier, that eta is a non dimensional number and let us say, what is the dimension of A? Because of the presence of this constants h u 0 and D squared A will also becomes a non dimensional number.

Let us check that dimensionality of A that, A will be J will be having an unit of meter per second meter cube per meter per second. So, you will be having a unit of meter per second now, h will be having a unit meter and x will be having a unit meter. So, it will be meter square in the numerator, so meter squared divided by u 0 will be having an unit meter per second and D squared and D will be having an unit meter square per second.

So, it will be meter to the 4 per Second Square, this meter per second and meter to the 5 meter square. So, it will be meter cube in the denominator and second cube in the numerator. So, s cube 5 meter cube rest to the 1 upon 3, so it will be meter per second and this will second per meter it will be cancelling out. So, A will be a unit free number A is a unit free number eta is a unit free number so, and C is a dimensional number.

So let us, make it unit less. So, we define a C star as C by C not if you define a C star because the fit concentration is known to you if you define a C star non dimensional concentration as C by C naught then, express then this equation the replace C by C star and the whole equation become non dimensional equation so let us do that and write everything in the non dimensional form

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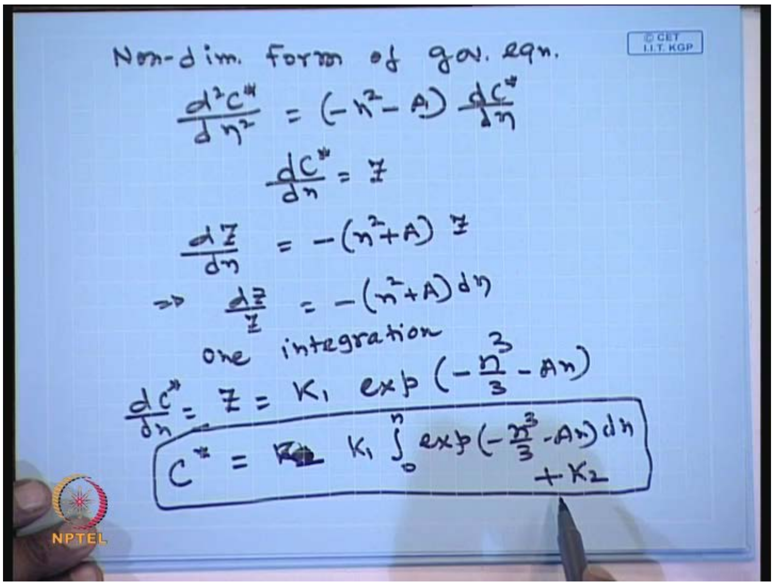

If you do that, the non dimensional form of governing equation now, becomes d squared c star d eta squared is equal to minus eta squared minus A d c star d eta. So, this becomes the whole equation because of non dimensional now, you are in position to integrate it out how to integrate this equation? Just assume d c star d eta is equal to let say a variable z, so this becomes d z d eta is nothing but minus eta squared plus a z, so this becomes d z by z is equal to minus eta squared plus A d eta. So now, just integrate it out it becomes on one integration this becomes l and z.

So, z is equal to K 1 exponential minus eta cube by 3 minus A eta and what is d z d eta and d z and what is z? Z is nothing but d c star d eta. So, let us put one more integration; carried one more integration with respect to eta if u carry out 1 more integration the expression becomes c star, a constant K 1 integration 0 to eta exponential minus eta cube by 3 minus A eta d eta plus another constant K 2.

This is the constant of integration for the second row; this becomes the concentration profile within the mass transfer boundary layer. Now, in order to evaluate the 2 constants K 1 and K 2, you must be requiring the boundary conditions; two boundary conditions on y and to evaluate the constants of integration K 1 and K 2. Now, we have two boundary conditions on y, we express these two boundary conditions in terms of combine variable eta.

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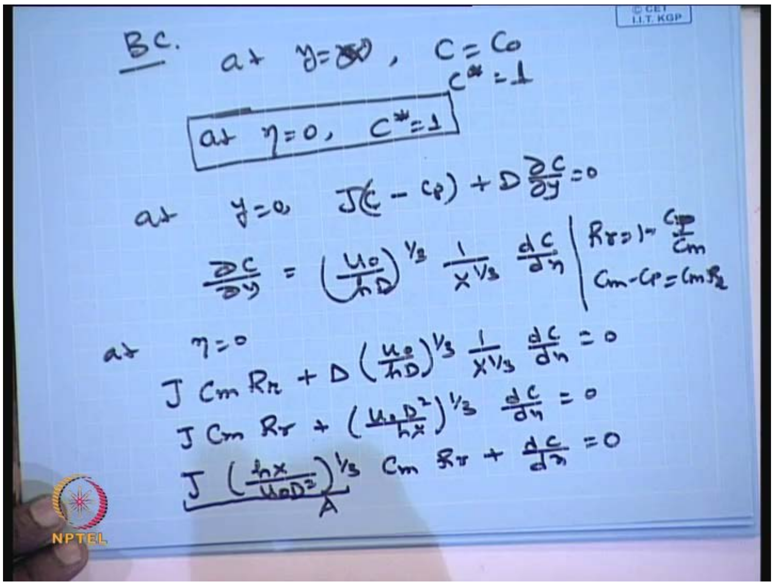

Let us do that, the boundary conditions at y is equal to infinity, you had C is equal to c naught, that means C star is equal to 1 and if you look into the definition of eta; eta is equal to nothing but y by x to the power 1 upon 3, so y equal to infinity means eta equal to infinity; so that means at eta equal to infinity c star is equal to 1, so this is the first boundary conditions that we are going to use to evaluate the integration constant K 1 and K 2. Now, at boundary a condition is at y is equal to 0. J C minus C p plus D del c del y will be equal to 0.

Now, if you get the expression of del c del y, let us look into that del c del y; what is del c del y? del c del y and y equal to 0 means eta equal to 0 and del c del y; if you remember del c del y will be nothing but u 0 divided by h D rest to the power 1 upon 3 over 1 by x to the power 1 upon 3 d c d eta and at y is equal to 0, what is c? c is nothing but C m that is at membrane surface because that you are coordinate starts from membrane surface towards the bulk.

So, you can write it as C m minus c p and what is C m minus c p? In terms of real retention it is nothing but if you look into; if you just remember the definition of real retention 1 minus c p over c m. So, C m minus C p is nothing but C m times R r. So, just put it there, so at y equal to 0 means at eta equal to 0 we had J times C m R r plus D times the whole thing u 0 divided by h D rest to the power 1 upon 3 1 over rest to the power 1 upon 3 d c D eta is equal to 0.

Now, D can be taken inside this bracket and it will becomes u 0 u 0 D square D cube, so it will be D square J C m R r plus u 0 D squared y h rest x rest to the power 1 upon 3 d c d eta equal to 0 and just multiply it, these becomes J h x by u 0 D square rest to the power 1 upon 3 C m R r plus d c d eta will be equal to 0. Now we have already proved that J times h x u 0 D square rest to the power 1 upon 3 is the constant A, then identified that the whole thing becomes a constant and that constant was A. So, you just substitute that so this is nothing but the constant A that we have defined earlier, you substitute that and let us see what we get?

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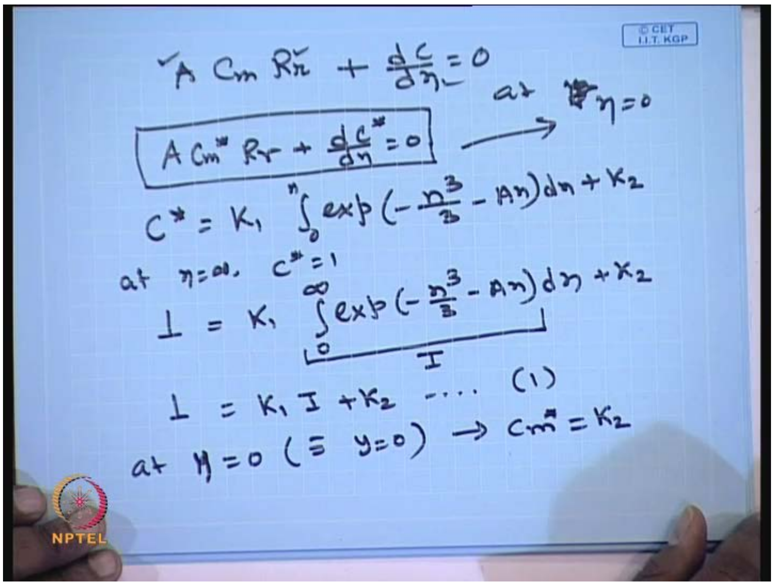

So, you will be getting A times C m R r plus d c d eta equal 0, at y equal 0 means nothing but eta equal to 0. Now, we A is an non dimensional number real retention is also a non dimensional number; eta is also a non dimensional, so you make c to a non dimensional by divided by c naught so, this becomes a C m star R r plus d c star d eta equal to 0, that is the mixed boundary condition that is prevailing at eta equal 0. Now, you will be using two boundary conditions; these boundary conditions and the other boundary conditions that is eta equal infinity, C star is equal to 1 in order to evaluate the two constants K 1 and K 2.

So, we have two boundary conditions and we will be utilizing them. Now, let us use the two boundary conditions and the let us, write the governing equations of c star; the governing equation C star is nothing but K 1 0 to eta exponential minus eta cube by 3 minus A eta d eta plus K 2. The first boundary condition, that is the eta equal to infinity C star is equal to 1, so just put it that 1 K 1. Now, 0 to eta will now become 0 to infinity exponential minus eta cube by 3 minus A eta d eta plus K 2. Now, let us look into these integral; these integral my eta is the double variable the integration is over eta and A is the constant now, what is 0 to infinity? 0 to infinity means it is a finite integral, that means it will knowing the value of A, but if you evaluate this integration you will getting a number, it is a finite integral, so let us call this finite integral as I.

If you can use any symbol any Simpsons rule or any trapezoidal rule to evaluate this integral and what is the value of infinity will be taking numerically, you will be taking left side 10. Let 10 11 12 something like that and just check, if you take 10 and numerically if you get 1 and let say in the next time you take 12 and say whether you are getting 1.2 or 1.01. If it is 1.01 10 is good enough as infinity; if it is 1.2 then you have to go for the higher terms, you have to assign higher value to infinitive.

So, that is all we have to do in numerically, so 1 is equal to K 1 I plus K 2, I is a definite integral, so let have this equation number 1 and we use the other boundary condition that is d c star d eta is equal plus A C m star R r. What is C m star? C m star is nothing but C star evaluated at y equal 0 and what is y equal to 0? y equal to 0 is nothing but eta equal to 0 and what is eta equal to 0? eta equal 0 implies on this integral, that means this integral is from 0 to 0. 0 to 0 means the integration will be vanished, so there is no contribution from here C m star is nothing but K 2. At eta equal to 0 that is equivalent to y equal to 0, your C m star is nothing but K 2 and what is d c star d eta? For that you have to get the expression of z.

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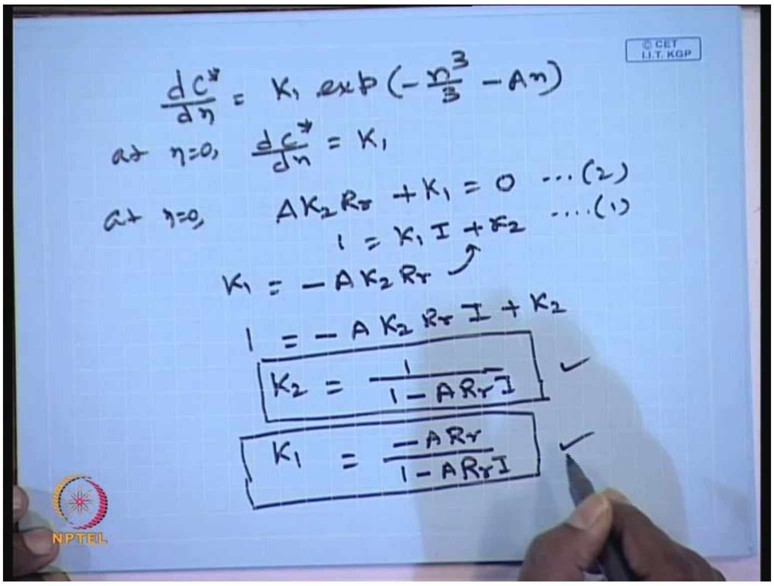

The d c star d eta if you look into expression of d c star d eta, this become K 1 exponential minus eta cube by 3 minus a eta. So, at eta equal to 0 what is the fate of d c star d eta? This will be nothing but K 1 where exponential 0 exponential 0 is 1, so let us put this expressions into the governing in the boundary conditions at y equal to 0. Whereas, A C m star at eta equal to 0; A C m star C m star means K 2 R r plus d c star d eta at eta equal to 0 that is nothing but K 1 will be equal to 0. So, this is equation number 2 and let us, writes down the first equation 1 is equal to K 1 I plus K 2 is equation number 1 now, the just two equations and two unknown K 1 and K 2.

Let us, put K 1 is equal to minus a K 2 R r from the equation number 2 and substitute over here, so what you get is minus a K 2 R r I plus K 2 from here, you can estimate the value of K 2; so K 2 will be nothing but 1 minus 1 minus A R r, I and what is K 1? K 1 is nothing but A R r, K 1 will be A R r divided by 1 minus A R r I. So, this will be two integration constant what is I? I is nothing but definite integral from 0 to infinity exponential whatever; it is so you are now; you can completely solve the concentration profile in terms of eta.

Now, eta being a combine variable that is basically a functional variation of y by x to power 1 upon 3 now, you can fix the value of x and for different y, you can carry out the concentration, so will be getting the concentration profile. Similarly, at a fixed value of y even carry out the concentration and various x values. So, you will be getting the concentration profile along the x at a constant, at a particular value of y so likewise one can get the concentration profile, but that will not solve our purpose simply because we do not require the concentration profile in the mass transfer boundary layer. What we require? We require the value of the concentration of membrane surface.

So that, we can connect it to the transport phenomenon loss which will be prevailing within the porous membrane and get the value of permeate concentration and permeate flux. Let us do that now, we have the if you remember the expression of A.

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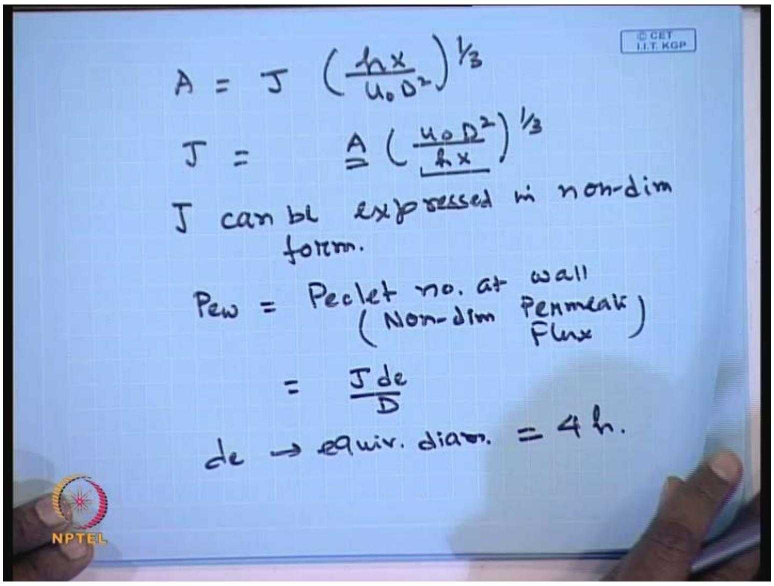

The expression of A is nothing but J h x by u 0 D squared rest to the power 1 upon 3, so we can express J as A u 0 D square y h x rest to the power 1 upon 3 and we define now, this thing is non-dimensional; the whole thing is non-dimensional, you will be having a dimensional, so express J are the solvent flux in its non-dimensional form. What is the non-dimensional form? P w this basically the peclet wall; this is called peclet number at wall or this is a non-dimensional permeate flux.

We define P w as J d e by D, what is de? de is equivalent diameter since, it is a rectangular channel diameter will be equivalent diameter and we have derived earlier, since it is a thin rectangular channel d e will be roughly equal to four times half of h, so this will be nothing but four times of h.

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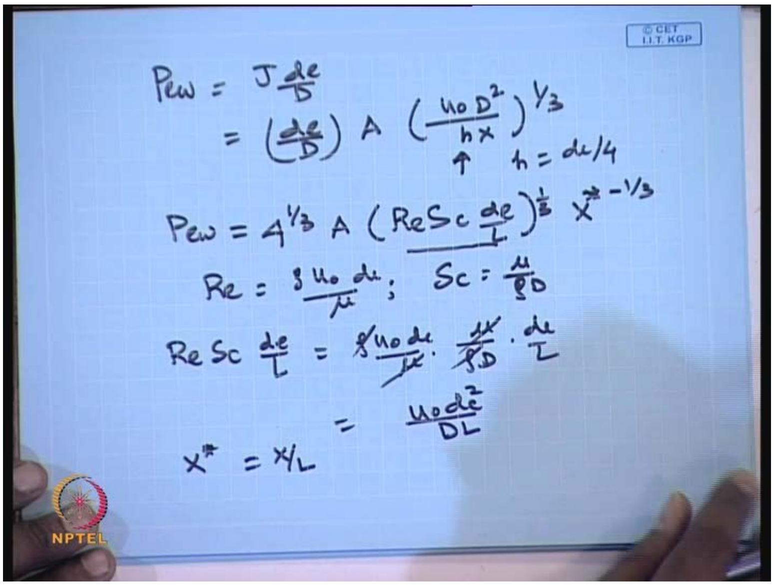

So let us, look into the form of non dimensional flux that will be P w is equal to J d e by D, if you substitute that what your will be getting is at now, you express J is equal to de by D; J is A times u 0 D squared over h x rest to the power 1 upon 3. You express h as D e by 4, so what you will be getting is that? You will be getting 4 to the power 1 upon 3 times A times, Renault smite de by l cubic rest to the power 1 upon 3 x to the power minus upon 3.

Again, I have omitted a couple of steps, what I have what is Renault number? Renault nothing but row u 0 d y mu and smite will be mu by rho D and Renault smite de by l will be what rho u 0 de by mu times mu by rho times de by l, is mu mu will be cancelled rho will be cancelled, so you will be getting u 0 de square y D l and what is x star? x is nothing but x by l so if you now, replaces x by h non dimensional version that is x by l then, the whatever is there within the bracket that becomes Renault times smite times de by l rest to the power upon 3 in a sense, you will be having this as u 0 de squared by D l. Once, we identified that then, we are in a position that A 1 will be can be expressed in terms of P w and other non dimensional variables.

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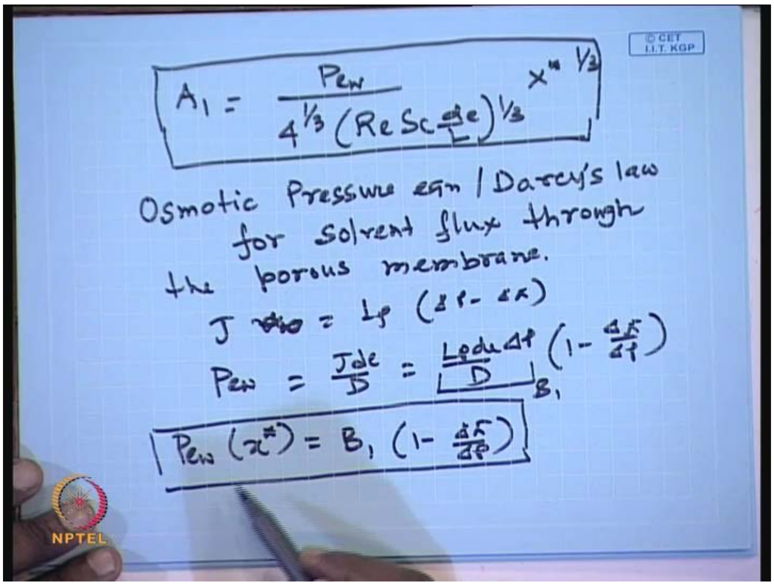

Premise A 1 will be nothing but P w divided by 4 to the power 1 upon 3 Renault smite de by l rest to the power 1 upon 3 x star x to the power 1 upon 3 that is the expression of A. Now, with this formulation and we have one more equation that is the osmotic pressure law; osmotic pressure equation or Darcy's law for solvent flux through the porous membrane. What is that P w is equal to J; J is equal to l p del p minus del pi; del pi is nothing but a function of concentration at the membrane surface.

Now, we can make it a non dimensional because everything in non dimensional, so we can express P w as J de by D. So, this becomes l p de by D and take delta p out so that, whatever we is within the bracket that becomes a non dimensional. So, l p delta p de by D becomes a non dimensional number and the whole thing in the first better becomes non dimensional and P w becomes non dimensional. So, this is valid for every x location.

Because at every x location now, my membrane concentration is varying, so C m only function of x. So, P w will be it will be a function of x depending on the C m because delta pi is the whole function of C m that you have already seen earlier'. So, P w will be nothing but a function of x and this becomes and this constant is known as this another constants, let say this b 1; so this becomes b 1 times 1 minus delta pi by delta p now, with this formulation one can calculate the value of membrane surface concentration at every x location and once, you get the value of membrane surface concentration every x location.

You will be getting a value of membrane the permeate concentration through the definition of real retention and every x location, you can integrate it over the length and can get the length average permeate concentration. Once you known the membrane surface concentration on every x location by using this expression; you can get the value of non dimensional permeate flux at every x location and can get integrated it out easy using Simpson rule or any rule. To get the length average permeate flux. How you will do that? Let us look into that algorithm.

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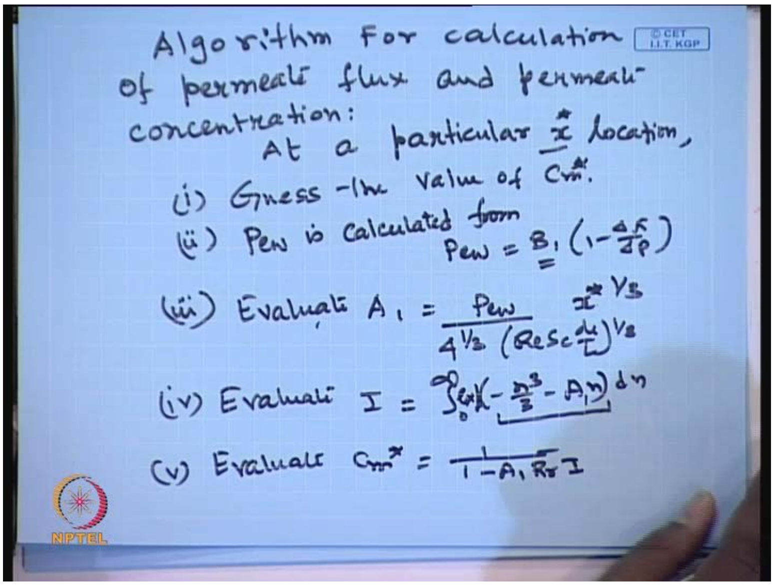

For calculation of permeate flux and permeate concentration. Let say, at a particular x location; particular x star location it is a non dimensional, guess the value of C rn; guess the value of C m star. Step number 2 is to calculate P w from Darcy's law that means from this equation P w is equal to b 1 minus 1 minus delta pi by delta p. What is b 1? b 1 is nothing but l p delta p d e by D; l p is known d e is the geometric factor delta p is operating condition D is the diffusive there is known; so b 1 will be known; delta p will be known; in delta pi, delta pi is basically a functions of membrane surface concentration and real retention and osmotic coefficients all these are known.

Since, you have guessed the value of C m star, this can also be calculated; so one can calculated P w from there now, third is you calculate evaluate A 1 from the equation P w 4 to the power 1 upon 3 Renault smite de by l rest to the power 1 upon 3 x star x to the power 1 upon 3. From this equation, we are talking about particular x star location.

So, at the particular x star location, you have already calculated P w substitute that Renault number is known to you; smite number is known to you; this is basically the solute and solvent properties are known de by l are the geometric factors so, you will be in a position to evaluate the value of A 1. Once you know the value of A 1, you can evaluate the definite integral I 1; evaluate I, what is I? If you remember is nothing but 0 to infinity minus eta cube by 3 minus A eta minus eta squared.

Now, eta cube by 3 minus A eta d eta. Now A 1; this A 1 is known to you, so you just substitute there, so there is nothing everything is known in the expression. So, you put a value of higher limit and evaluate the definite integral I. Once, you evaluate the definite integral I then, evaluate C m star from the expression 1 minus A 1 times R r times I.

This was the expression of C m star from the boundary condition, so I can evaluate C m star once again, because a 1 now estimated real retention is known to you; I will be estimated from this. Now, compare the guess value of C m star and evaluate 1 of the C m star compare step number 1 with step number 5. This is exponential does not matter, this is everything within the exponential it will be known to you.

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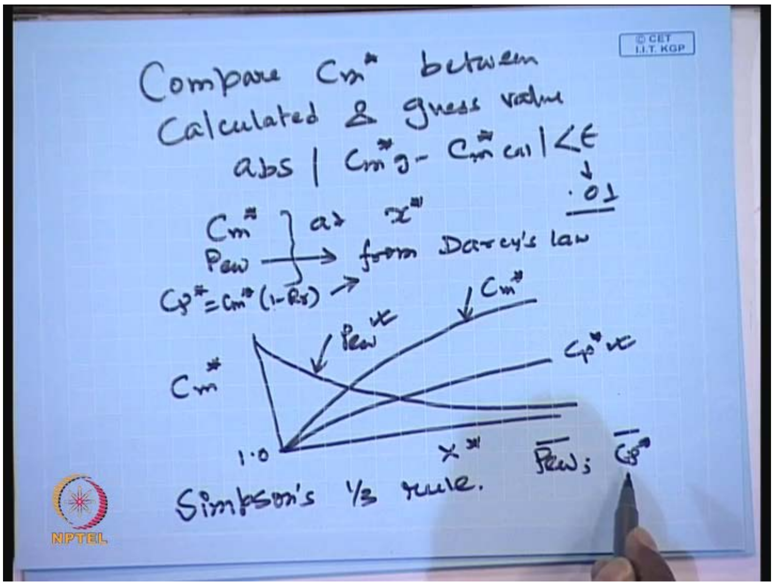

Now, you compare C m star between calculated value and guess value, and see whether these absolute value of C m star guess minus C m star calculated is less then epsilon or some small number, let say 0.01 or something. And if not then, you calculated the you have another guess of C m star and like that. So, that why you will be converging a value of C m star at a particular x star location; once you know the value of C m star, you can evaluate delta pi at a particular x location, because delta pi is a sole function of C m star. So, you can get from, you can go to the Darcy's law, and evaluate the value of permeate flux at that particular location.

So, once C m star is known P w is known from Darcy's law expression, once C m star is known c p at the particular X location is also known c p star is nothing but C m into 1 minus real retention it is known so, that is also known. Once you determine all this quantities for that particular x location then, go to the next x location x plus delta x. In that case, the easiest way to solve it the guess value should be the convest value in the previous step.

So, at the next step x star plus delta x, you have a guess of C m star that was the convest value at the location x star and do the same and calculated this carryout this iteration and get a convest value of C m star. So, likewise what you can get? You can get a profile of C m star, as a function of x star and C m star will always starts from 1 and its lowest value is C naught. So, it is starts from 1 and it will go up like that, your C p star will also follow the trend of C m star, but at a lower level because it is multiply by 1 minus R r; R r will be typically pointed m, so it will be 0.1, so this the profile of C m star as the function of x and this will be a profile of C p star as a function of x and the profile of P w will be reverse and it will be the profile of P w as x star. Now, one can now our final aim is to find the value of P w and to get the estimate of permeate concentration this gives the productivity or the quantity is gives the concentration or quality so therefore, you can do a length average by using Simpson 1 third rule and can get a length averaged permeate flux or productivity of the process and length average permeate concentration and to estimate the system performance.

Now, in this algorithm what this is more or less, so let us summaries whatever we have done. We have used the property of similarity transformation and reduce the governing partial differential equation into an ordinary differential equation and from that, we got again, you know set of algebraic equation and at different x location; we can solve these two expressions in iteratively, set of equation iteratively and can get the estimate of system performance at a particular x location.

We repeat this process at all the locations in the channel and can get the length variations of the decide quantities and then, we again do a numerical integration to get the average flux. Now, there is a another shortcut method so, this method so we have already shortcut the whole method come p D into o D and then we got the method, but still there is substantial numeric stuff invoked here. There is another shortcut method that I am going to describe you which will basically require solution of two coupled non-linear algebraic equation that can be obtained by defining a Sherwood number or length average mass transfer coefficient. So let us so we taught the simply this calculations and procedure.

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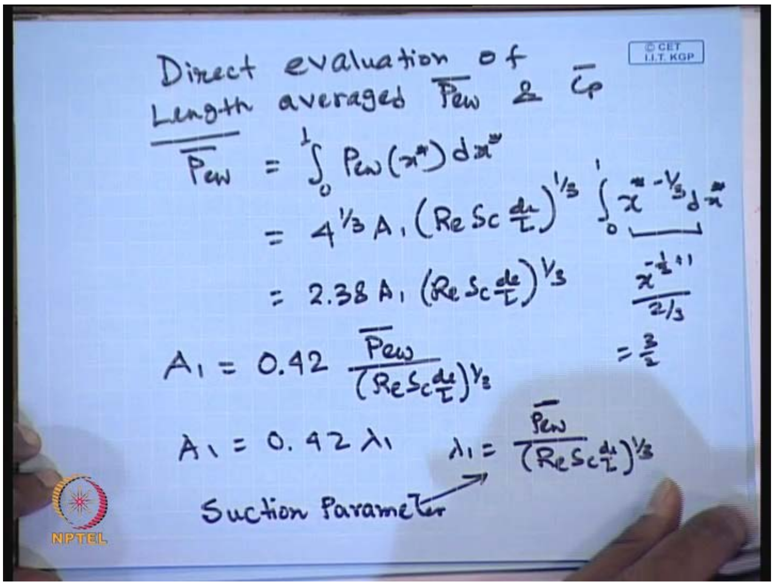

And directly we get the direct evaluation of length average permeate flux, so whatever we have done it is indirect evaluation, you first get the profile of permeate flux and permeate concentration then, do a length averaging by some numerical technique. Direct evaluation next, we do a further simplification for direct evaluation of length averaged permeate flux, this bar represents the averaged and permeate concentration C p bar.

Now, so we do a length averaging of expression of P w, so P w will be the length averaged means 0 to 1 P w x star d x star, so that is the length averaging of permeate ah flux, so this will be 4 to the power 1 upon 3 A 1 Renault smite de by l rest to the power 1 upon 3 0 to 1 x star to the power minus 1 upon 3 d x star and this will be, what this will be nothing but a value of 1.5 minus 1 upon 3 plus 1; so it will be 2 by 3; so minus 1 upon

3 plus 1 divide; so it is 2 by 3; so it is 2 by 3; so it will be 3 by 2 and only put the limit it becomes 1 so, you just the whole integral becomes 1.5 correct; so if you do that you will be getting 2.38 A 1 Renault smite de by l rest to the power 1 upon 3.

So, we can write down the expression of A 1, in terms of length averaged permeate flux what is that that will be nothing but 0.42 that is the inverse of 2.38 P w divided by Renault smite de by l rest to the power 1 upon 3 and this P w is nothing but this is a length averaged permeate flux and all these are operating conditions and the geometric factors all known to us. So, we write down A 1 as 0.42 lambda 1, what is lambda 1? lambda 1 is nothing but P w divided by Renault smite D by l rest to the power 1 upon 3, so this becomes a suction parameter, we can assume that the permeate flux nothing but the porosity of the wall and the whole thing is being sucked out and this is also known as the suction parameter.

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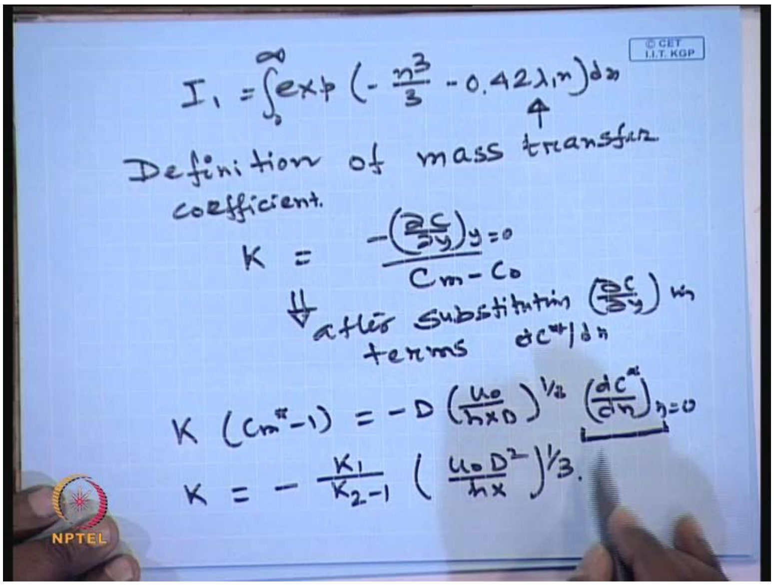

So, let us evaluate now, let us look into the I 1, the definite integral, this 0 to infinity exponential minus eta cube by 3 minus 0.42 lambda 1 eta d eta. Once we know the value of lambda, we can evaluate the definite integral I; now let us look into the definition of mass transfer coefficient. The definition of mass transfer coefficient will be K, if you remember del c del y minus of that y evaluate at y equal to 0 divided by C at y equal to 0, that is C m minus C at bulk that is C naught. These are the definition of mass transfer coefficient. Now, if you do that, what you should do? I can give you it is an assignment;

you express del c del y in terms of d c star d eta and make it non dimensional. So, it becomes C m star minus 1, because it is non-dimensional with respect to C naught. So numerator denominator become C m star minus 1; numerator to the expressed in terms of d c star d eta, and get the expression of d c star d eta from the solution, evaluate it at y equal to 0 means at eta equal to 0, and see what you get? What you get that means after substituting del c del y in terms of d c star d eta and if you substitute that, what you get is C m star minus 1, that is the denominator; it will multiply there minus D u 0 h x D rest to the power 1 upon 3 d c star d eta at eta equal to 0.

This becomes this now, what we can do? You can look into the constants of integration and find out what is the value of d c star d eta. If you do that, then finally the in fact this is these becomes a constant of integration, and C m star is nothing but one of the constant of integration. So, this K becomes minus K 1 divided by K 2 minus 1 into u 0 D squared by h x rest to the power 1 upon 3. So, that gives directly and expanded, if you replace the value of K 1 and K 2; this K 1 by 1 over K 2 minus 1 will become minus 1 over I. Anyway, we will stop here in today’s class, we will continue in next class, and get the expression of mass transfer coefficient, and see how it will simplify our course of calculations. Thank you.