Novel Separation Processes
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Lecture No. # 07
Membrane Separation Processes (Contd.)

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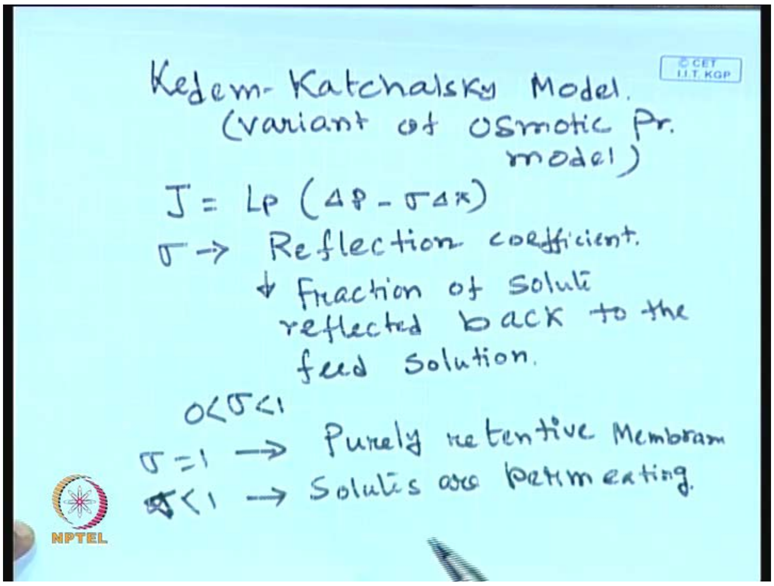

Kedem-Katchalsky Model.
(variant of osmotic Pr. model)

$J = L_p (\Delta P - \sigma \Delta \pi)$

$\sigma \rightarrow$ Reflection coefficient.
↓ Fraction of solute reflected back to the feed solution.

$0 < \sigma < 1$

$\sigma = 1 \rightarrow$ Purely retentive Membrane

$\sigma < 1 \rightarrow$ Solutes are permeating.

Well, another variant of osmotic pressure model is known as the Kedem Katchalsky model. This is a variation of osmotic pressure model, and the difference here is, we expressed the osmotic the solvent flux as L p time’s delta p minus sigma delta pi.

This sigma is the new thing that is incorporated here, and this is known as reflection coefficient. This reflection coefficient indicates how the amount of some amount of solute is always reflecting back, and some amount is going to the membrane system. If it is, that means fraction of solute reflected back to the feed solution or feed system, and rest are permeating through the membrane or passing through the porous membrane. So therefore, this sigma is always a fraction, sigma is equal to 1 indicates that purely retentive membrane.

Sigma less than 1 that means, solutes - some solutes are permeating through the membrane. So, this reflection coefficient is again an unknown parameter, and it has to be estimated from the optimization method I just mentioned earlier.

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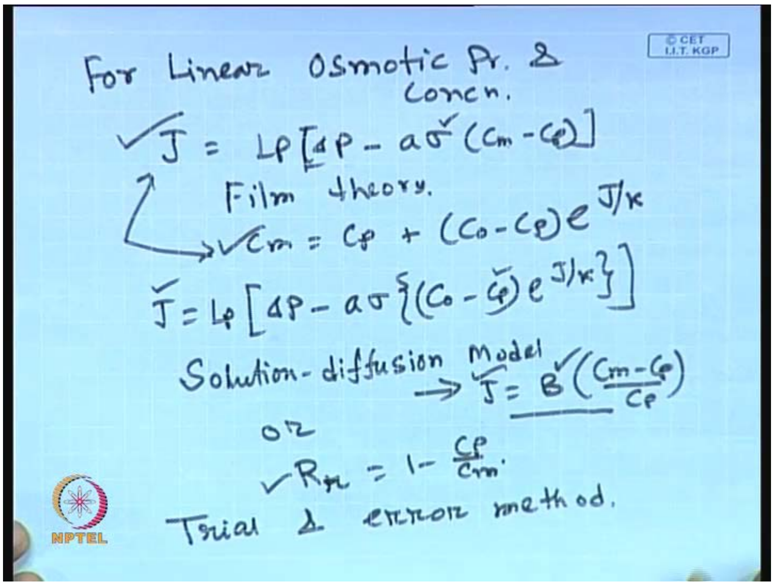

For Linear Osmotic Pr. & Concn.

$$\checkmark J = L_p[\Delta p - a\overset{\checkmark}{\sigma}(C_m - C_p)]$$

Film theory.

$$\checkmark C_m = C_p + (C_0 - C_p)e^{J/k}$$

$$\overset{\checkmark}{J} = L_p[\Delta p - a\sigma\{(C_0 - \overset{\checkmark}{C_p})e^{J/k}\}]$$

Solution-diffusion Model $\rightarrow \overset{\checkmark}{J} = B\left(\frac{C_m - C_p}{C_p}\right)$

or

$$\checkmark R_r = 1 - \frac{C_p}{C_m}.$$

Trial & error method.

Now, if you look into the expression, if you look into the linear relationship of osmotic pressure and concentration linear osmotic pressure and concentration relationship, the expression of J becomes L p delta p minus a sigma C m minus C p and 1 can combine these equation with the Film theory equation, C m is equal to C p plus C not minus C p exponential J by K and one can combine these 2 equation and replaced C m in terms of C p and this becomes J L P tents delta p minus a sigma C m minus C p. So, it will be C not minus C p exponential J by K. Now, so and the other equation you must you are having two equations two unknowns and 1 equation J and C p and other equation will be coming from the solution diffusion model.

Solution diffusion model is basically J is equal to B C m minus C p divided by c p, and or the definition of real retention R r is equal to 1 minus C p by C m. So, in your in your system there are 3 unknowns J, C m and C p, you must be having 3 equations to solve this 3 unknown. First one is the Darcy's law, second one is the film theory, third one will be the either solution diffusion model or definition of real retention.

Again 3 equations and 3 unknown have to be solved by trial, and error method solution and J and c p can be obtained. Now, if you use solution diffusion model you will be having 2 parameters to be estimated from the experimental data, one is solute permeability through the membrane another is the reflection coefficient. If you use the definition of real retention you will be having only one parameter, that is the reflection coefficient and real retention can be estimated from a separate set of experiment. So these are the variation of osmotic pressure models in terms of reflection coefficient this is known as the Kedem Katchalsky equation.

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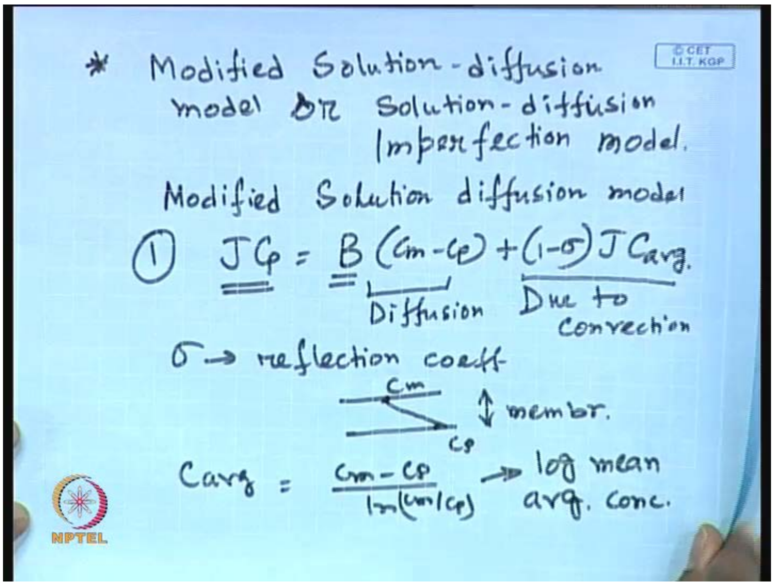

* Modified Solution-diffusion model OR Solution-diffusion Imperfection model.

Modified Solution diffusion model

① $\underline{\underline{J C_p}} = \underline{\underline{B}}\,\underbrace{(C_m - C_p)}_{\text{Diffusion}} + \underbrace{(1-\sigma) J C_{avg}}_{\text{Due to convection}}$

$\sigma \rightarrow$ reflection coeff

$C_{avg} = \frac{C_m - C_p}{\ln(C_m/C_p)}$ → log mean avg. conc.

Next equation, next model will be looking at it will be more realistic, this is known as the modified solution diffusion model or solution diffusion imperfection model. In this model we will be assuming that some amount of the solute will be transported through the membrane not only by diffusion but also by convection because of presence of higher pore size membrane in the membrane matrix.

So, J c p the modified diffusion model becomes J times C p is equal to B times C m minus C p plus 1 minus sigma J times c average, this is the solute flux through the membrane, B is the solute permeability through the membrane up to here, up to the first row of right hand side, it is the solution diffusion model, it is identical to the solution diffusion model. And this is 1 minus sigma J times C average is basically due to convection, this is due to diffusion. Sigma is the reflection coefficient; sigma is the amount of fraction of the solute that has been reflected or return by the membrane.

So, one minus sigma will be the fraction of the solute that will permeating through the membrane multiplied by C average, what is the C average? Because you are having a gradient of through the membrane, this is the membrane. At the membrane surface you are having C m, in the permeate side you will be having C p there will again some kind of gradient existent. Now, this is these concentration will be a huge concentration, so there is the problem, well C m and C p and the any concentration in between is not known to you. So, it is taken as log mean average temperature; average concentration, it is like that definition of LMTT, log mean average temperature it is the exactly like that. C average is taken as C m minus C p divided by l n, C m divided by C p, this is known as the log mean average concentration of the solute in the membrane system, in the membrane. Now, so this will be 1 equation.

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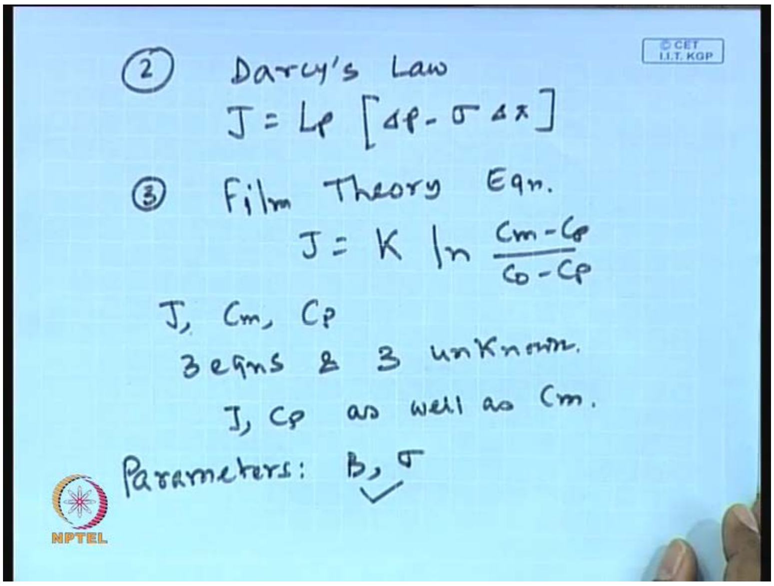

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② Darcy's Law

$J = L_p [\Delta p - \sigma \Delta \pi]$

③ Film Theory Eqn.

$J = K \ln \frac{C_m - C_p}{C_0 - C_p}$

J, Cm, Cp

3 eqns & 3 unknown.

J, Cp as well as Cm.

Parameters: B, σ ✓

The second equation will be the Darcy’s law, solvent flux through membrane. J is equal to L p time’s delta p minus sigma delta pi. And the third equation will be the film theory equation that is J is equal to K l n, C m minus C p divided by C not minus C p. Three unknowns J, C m and C p, three equation and 3 unknown is satisfied the degree of freedom can be solved, so one can get a system performance J and C p as well as C m.

Now, again in this case the parameters, one of parameters models. There are two parameters B and sigma; these two parameters will be estimated by the optima, by the comparing the calculated values of J and C p with the experimental data and minimizing the sum of square through the optimization technique. Now, depending on the situation one can have, actually these three all this methods are applicable

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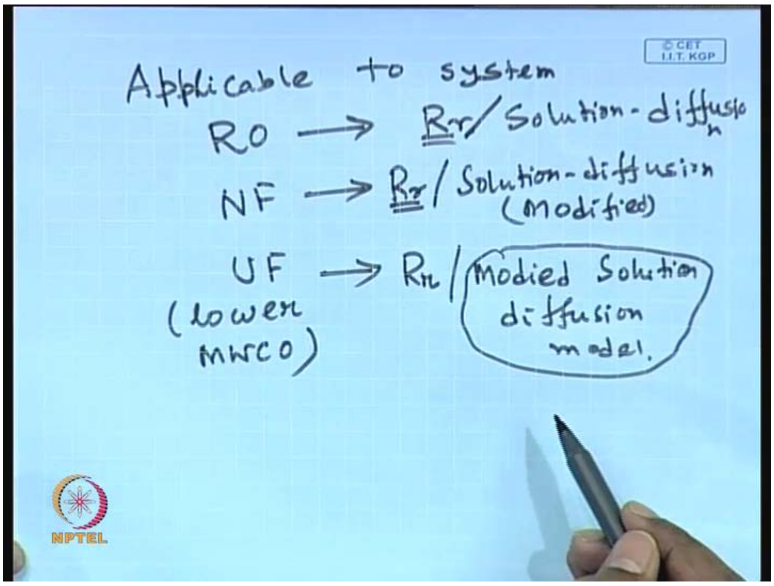

Applicable to system

RO → $\underline{R_r}$/ Solution-diffusion

NF → $\underline{R_r}$/ Solution-diffusion (modified)

UF (lower MWCO) → R_r/ Modied Solution diffusion model.

for, now let us look in to the applicability of this equations, applicable to system, which system? Reverse osmosis, nano filtration, and ultra filtration. The micro filtration is entirely different it is basically kick governing system and these ultra filtrations are lower molecular cut off ultra filtration. Lower molecular weight cut off ultra filtration for higher molecular weight cutoff we will be having kick governing mechanism.

Now, for R O you will be having the def R r method or solution diffusion models is good enough. Either R r or solution diffusion model is good enough because reverse osmosis contains very small pour size, so it is fair it is good enough to assume that the solute will be transfer by the diffusion through it. In nano filtration there are some more, there are pours which are basically, it nana filtration are open are membrane the pour size is slightly larger. So, either R r method or solution diffusion, modified solution diffusion model can, will be applicable. Why R r is applicable suitably in all this (( )) cases? Because it is basically a parameter which will be estimating from a separate set of experiment, so it will be dumping all the effects of solute transport to the membrane.

In the case of lower molecular cut off ultra filtration either R r method and modified solution diffusion model will be good enough, in fact this will be more appropriate for ultra filtration because ultra filtration pour size has much larger where the convection becomes place a dominant role as for as the transport of the solute through the membrane is concerned. Now, identity the various, you know as you have earlier said that during the modeling you have to have the transport outside the membrane and transport within the membrane.

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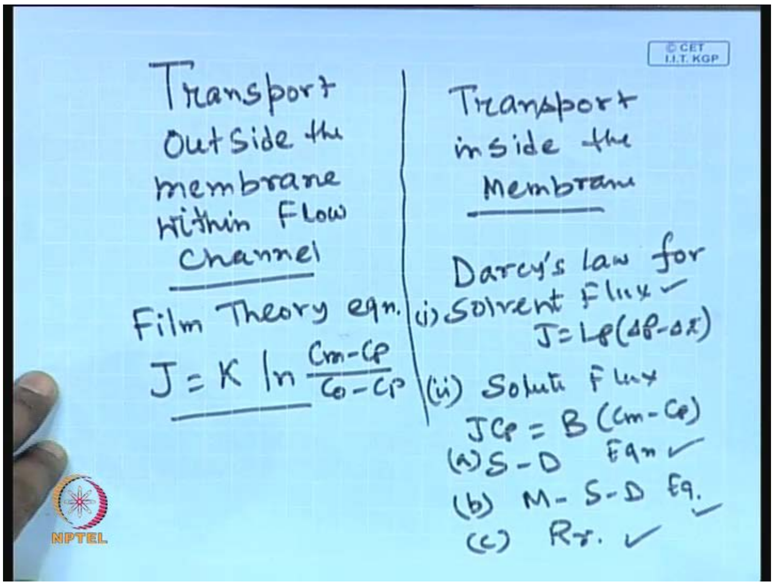

Transport Outside the membrane within Flow Channel

Film Theory eqn.

$J = K \ln \frac{C_m - C_p}{C_o - C_p}$

Transport inside the Membrane

Darcy's law for

(i) Solvent Flux ✓

$J = L_p(\Delta P - \Delta \pi)$

(ii) Solute Flux

$J C_p = B(C_m - C_p)$

(a) S-D Eqn ✓

(b) M-S-D Eq. ✓

(c) Rr. ✓

Now, let us identify the equations which will, which corresponds to the transport outside the membrane. Outside the membrane means within the flow channel and transport inside the membrane, transport outside the membrane is basically the film theory equation. If you remember that the derivation of film theory we solve the concentration boundary layer outside the membrane and we arrive at J is equal to K l n, C m minus C p divided by C not minus C p. K is the mass transfer coefficient or inverse of the resistance of mass transfer boundary layer or concentration boundary layer.

And transporting inside the membrane given by Darcy's law for solvent flux J is equal to L p del p minus del pi or sigma del pi, the variant of the that, that is one and second is the solute flux, solute flux is solution diffusion equation J C p is equal to B times C m minus C p, that is the solution diffusion equation, number one or it can be a modified solution diffusion equation or number three is simply the definition of real retention.

So, in case of solute flux either, one can use definition of real retention or one can use solution diffusion equation or one can use a modified solution diffusion equation, based on the system and accuracy of calculation you require for your system, and transport through the membrane can be defined by the Darcy's transport of solvent to the membrane can be defined by quantified, by the solvent flux, by Darcy's law or a modified Darcy's law by including the reflection coefficient. And transport law outside the membrane is solved by solving the concentration boundary layer equation in order to obtain the film theory equation.

Now, all this we have talked or discussed till now, it is the first generation model or the simplest kind of model, so you have to go for the next generation model for getting better accuracy, in order to go for the next generation model we have to identify what are the short comings of this present model.

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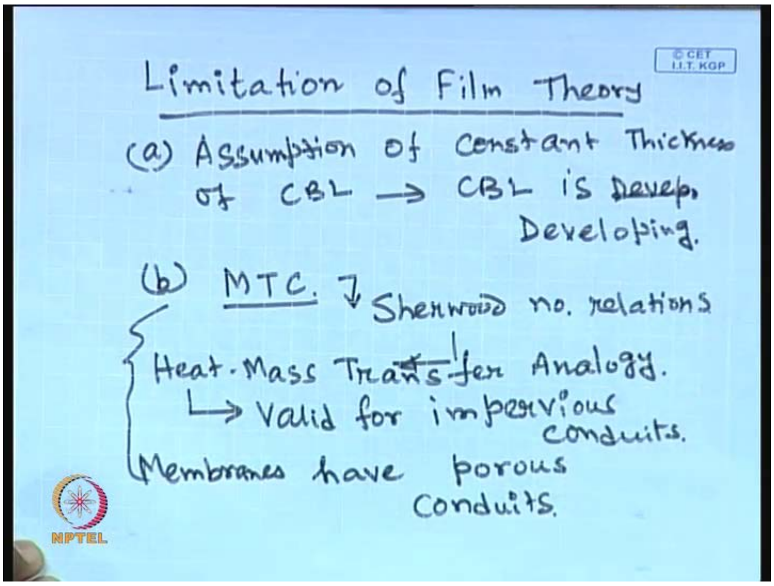

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Limitation of Film Theory

(a) Assumption of Constant Thickness of CBL → CBL is ~~Devep.~~ Developing.

(b) MTC. ↓ Sherwood no. relations

Heat-Mass Transfer Analogy.
↳ Valid for impervious conduits.

Membranes have porous conduits.

Now, let us write down the limitation of film theory or short coming of the film theory, if you remember or if you can notice that all the first generation models that we have discussed till now, one equation remains intact that is the film theory model. Film theory equation, film theory is obtained by looking into the concentration boundary layer that is existing outside the membrane, assuming that the thickness of concentration boundary layer or mass transfer boundary layer is constant.

So, the first limitation is constant assumption of constant thickness of concentration boundary layer, in fact we have discussed earlier the intense length required for the concentration boundary layer, will be much larger compare to the intense length required for the hydrogen boundary layer because of high smith number present of this system, high smith number will be reflected because this solute will be having very low diffusivity.

So therefore, if you use the film theory a substantial portion of the channel, where the boundary layer is developing is not encountered or not taken care of therefore, the permeate flux that you will be obtaining is using the film theory equation will be under predicting. So, there is one limitation the thickness of concentration boundary layer is assume to the constant, actually the concentration boundary layer is developing right. Second assumption, if you look into the expression of Sherwood number the mass transfer coefficient. Mass transfer coefficient where estimated from the expressions of Sherwood number, and this Sherwood number relations are all obtained from heat and mass transfer analogy, obtained from the Sherwood number relations are correlations for the turbulent flow and these are obtained from heat mass transfer analogy.

And if you remember the heat mass transfer analogy the cases for heat transfer or for the porous conduit, non pours conduit and impervious conduit. If the conduit is impervious those cases are solved or obtained by heat transfer experiments. They develop the expression for the nozzle numbers using heat mass transfer analogy, this number nozzle number expression are transformed into the Sherwood number relations. They are valid for impervious conduits, on the other hand our cases the conduit are porous therefore, the Sherwood number relationship that we are using for the mass for estimating the mass transfer coefficient are not correct, they should be corrected for the for taking into account the porosity of the wall or the suction parameter, the same amount of fluid will be sucked up as there will be porosity or suction is not taken care of by the film theory equation.

This is very important in case were the porous the wall is more permeable for example, micro filtration membrane the we will be having more porous wall, more porous membrane that means pore size very large, in case of reverse osmosis the pour size effect will be very small because the pour size itself be very small 2 to 10 armstrong. So, these in accuracy becomes very predominant incase of ultra filtration and micro filtration, they are not that it important or incase of reverse osmosis and nano filtration. These two are major assumption on limitation of the film theory in its standard form whatever we are taking.

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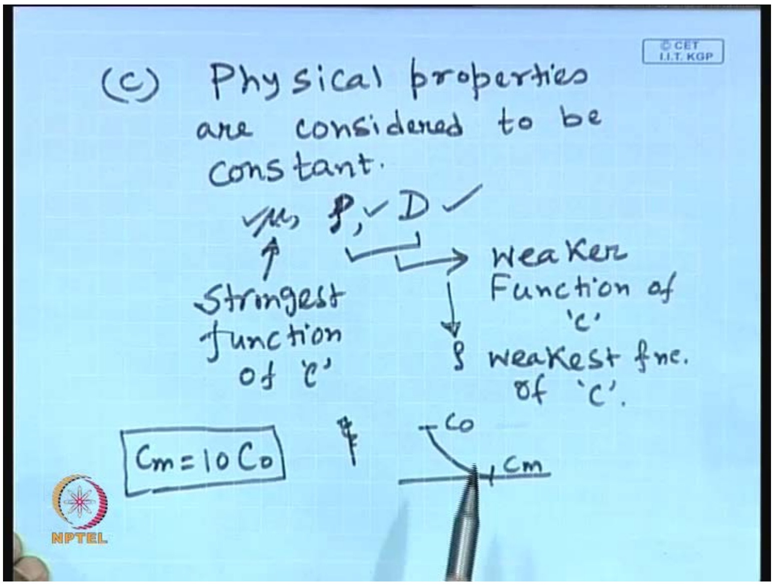

(c) Physical properties are considered to be constant.

✓μ, ρ, ✓D ✓

Strongest function of 'c'

Weaker Function of 'c'

ρ weakest fnc. of 'c'.

$\boxed{C_m = 10 C_o}$

The third limitation is very true that the physical properties are taken as constant. Now, let us look into the what are the physical properties are involve in such cases, one is mu there is a viscosity density rho and diffusivity D, these three transport coefficient are quite predominant in this case of mass transfer. Now, we know all this three properties thermo physical properties, physical properties, kinetic properties are functions of concentration in fact out of these three viscosity is the strongest functions of concentration.

Rho and D are weaker functions of concentration between these two; density is the weakest function of concentration. If you remember, in the polarization layer the variation of concentration is from C not to C m, in some cases C m, can be as high as 10 times C not therefore, their existing stiff concentration gradient in within the mass transfer boundary layer just outside the membrane. So, whenever there is such of tremendous change in concentration all these physical properties are suppose to be affective and they will be reflecting into the definition of mass transfer coefficient and mass transfer coefficient will be altered. Now, all this changes are limitations are not reflected in the film theory.

So, what will do in the in the next analysis is, will try to overcome the first two limitations that we have mentioned in this list of limitations, that is the constant thickness of mass transfer boundary layer. We do an analysis were we concentrating the developing mass transfer boundary layer with the porous wall. The third one, the variation of physical properties;

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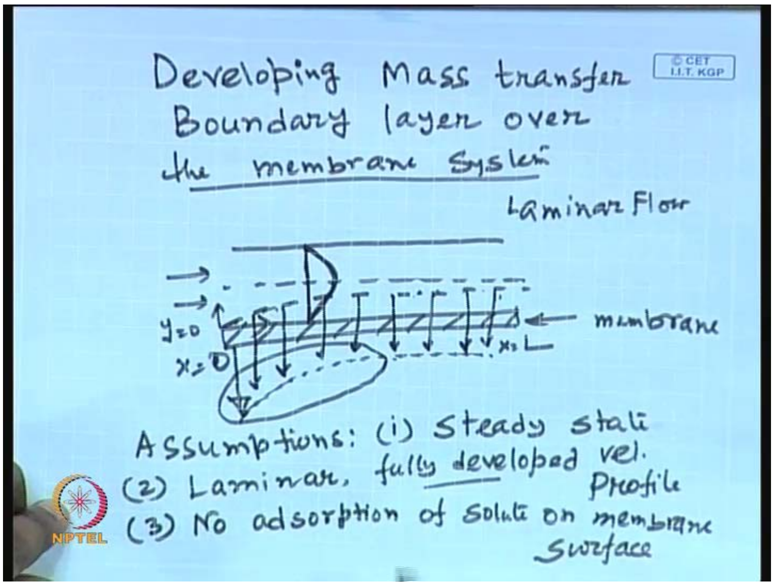

Developing Mass transfer Boundary layer over the membrane System

Laminar Flow

Assumptions: (i) Steady state
(2) Laminar, fully developed vel. Profile
(3) No adsorption of solute on membrane surface

it is very quite difficult to capture these features fundamentally, there are few paper that I will be telling you in the reference available in the literature in the next class, some of the reference done by my group and mostly these changes are incorporated in that form of correction factors included in the mass transfer coefficient like see that it correction factor in the nozzle number relationship. Let us look into how we can analysis a developing mass transfer boundary layer, so that the first two limitations of the film theory can be overcome. Mass transfer boundary layer and concentration boundary layer there are identical, so I am drawing a rectangular channel, the flow is occurring here and the coordinate system Y starts from the bottom and you will be having X axis, so 0 to L, x is equal to 0, (( )) X is equal to L and this is the membrane and we can assume a fully developed velocity profile, let us consider laminar flow. We can extend this analysis turbulent flow as well that I talk later on. We can assume a fully developed velocity profile that means a parabolic profile is existing simply because we have seen that hydrodynamic boundary layer the intense length required is l e by D e time is equal to 0.05 times Renault's number.

And for the laminar flow maximum Renault's number will be around 20 to 100, and the channel dimension that whatever we are talking about it will be in the order of one mile meter, the equivalent diameter will be roughly in order of 1 mile meter. So, with these dimensions the intense length for the hydrodynamic boundary layer because few centimeter, four centimeter or five centimeter or 6 centimeter like that, where as the channel length we are talking about in terms of meter, 100 centimeter. So, we can we can neglected the initial portion of the developing hydrodynamic boundary layer and constant that 99 personal channel, the hydrodynamic boundary layer is fully developed.

And on the other hand, mass transfer bounded layer is still growing because of the existing of very high smith number; mass transfer boundary layer will be growing something like this. Now, if you see that initially at the channel interest the mass transfer boundary layer thickness will be extremely small, so it will offer very less resistance against solvent flux. So, you have expected a very high solvent flux in the beginning and slowly it is growing, so it is offering more resistance against the solvent flux.

So, it will be decreasing slowly, now when this becomes almost constant, these becomes the permeate, the resistance also the constants and will be a constant permeate flux. So therefore, the productivity at the permeate flux is also obeying a kind of profile. Now in the film theory, whatever we have done earlier we are assuming, we are basically working on the litter of the portion of mass transfer boundary where the thickness becomes constants, here delta is growing delta is the concentration boundary layer thickness, there is a function X but here it is independent of X whatever the value of X you take it becomes constant.

So, in case of film theory we are not accounting this amount of flux, which are basically higher flux therefore, it gives an under predication. On the other hand in this case we will accounting these initial values of the flux as well in addition to the litter on values. So, let as write down the assumptions invert in this analysis, one is steady state flow, second is laminar flow, laminar fully developed flow. Fully developed in the sense of hydrodynamic boundary layer for developing in the sense of fully develop means, fully developed velocity profile or hydrodynamic boundary layer.

And third is no adsorption of solute on membrane surface, so the whole phenomenon should not be coupled with the adsorption phenomenon complications like adsorption, so the adsorptions is not there that means L p the permeability of the membrane intact.

Now, with these we are going to do an analysis developing mass transfer boundary layer towards the membrane surface. Now what you have to do,

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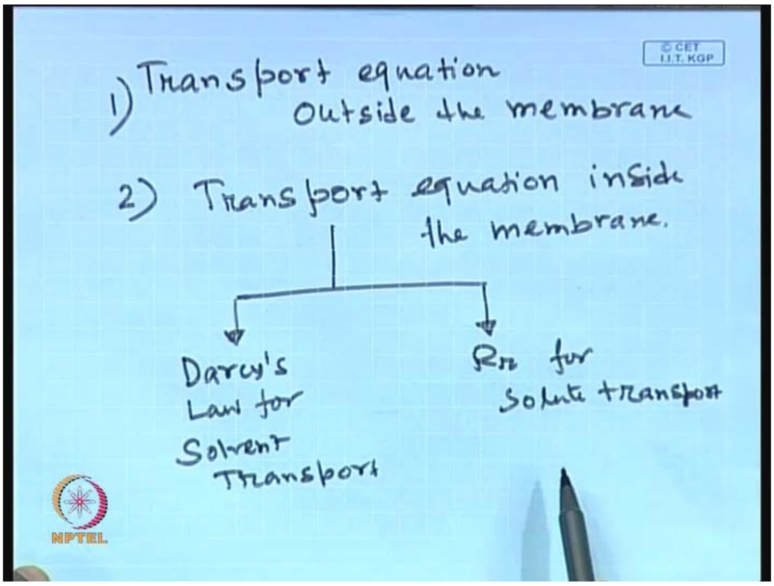

1) Transport equation Outside the membrane

2) Transport equation inside the membrane.

Darcy's Law for Solvent Transport

R_r for Solute transport

first you write down the transport equation outside the membrane that because very important now, we will write down transfer equation outside the membrane, will write down the transport equation inside the membrane, and among the transport equation we will be using Darcy’s law for solvent flux, for solvent transport and will be using that real retention for solute transport, this can be replace by any of the modified versions like solution diffusion model, modify solution diffusion model or things like that. Just for the sake of simplicity we will take the reality R r, definition of R r for the solute transport.

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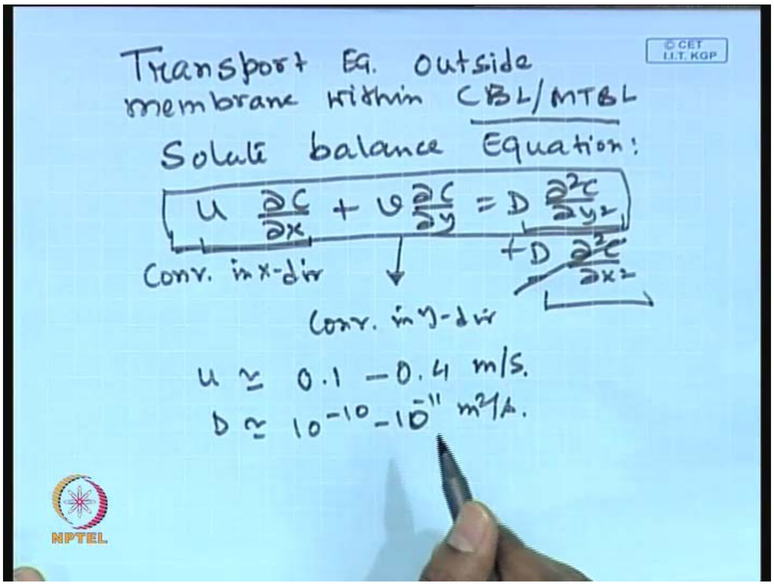

Transport Eq. outside membrane within CBL/MTBL

Solute balance Equation:

$$\boxed{\underbrace{u \frac{\partial c}{\partial x}}_{\text{Conv. in x-dir}} + \underbrace{v \frac{\partial c}{\partial y}}_{\text{Conv. in y-dir}} = D \frac{\partial^2 c}{\partial y^2}} + \cancel{D \frac{\partial^2 c}{\partial x^2}}$$

$u \simeq 0.1 - 0.4$ m/s.

$D \simeq 10^{-10} - 10^{-11}$ m²/s.

Now, let us write down the transport equation outside the membrane but within concentration bounded layer, within mass transfer boundary layer, because outside the concentration boundary layer or mass transfer law boundary layer everywhere concentration is same, the concentration it seen out everywhere, so there is no point in writing in balance equation there. So, what are the equations we are writing outside the membrane that is within the concentration boundary layer or mass transfer boundary layer?

Now, in these context I will just take a diversion an like to emphasis the importance of any boundary layer analysis, for any boundary layer analysis heat transfer, mass transfer boundary layer, boundary analysis, thermal boundary layer or hydrodynamic boundary layer the thickness of these boundary layers are extremely small, they will be typically in order of microns, 10 to the power minus 6 meter. On the other hand, the actual channel dimensions of the flow dimensions will be in terms of millimeter or centimeters but these boundary layers are very thin but within this boundary layers all the transport process will occur, moment transfer, heat transfer or mass transfer will occur within that thin boundary layer that why boundary layer analysis very important, all though boundary layer thickness is extremely small you cannot neglected the boundary layer analysis.

So, let us write down the transport equation outside the membrane within concentration boundary layer we write down solute balancing equation, the salute balancing equation will be simply u del C, del X plus V del c, del y is equal to equal D del square C del Y square plus D del square C del X square. Write the Z dimension is too long therefore, you write two dimensional equations it is a steady state therefore, is no del C del theta. X is along that along the membrane length Y is in the vertical direction in the transverse direction, so this is the convective term in the X direction, convection in x direction, convection in Y direction, diffusion in X direction, diffusion in Y direction.

Typically the actual velocity u and for the laminar flow will be ranging from point 1 to point four meter per sec 0.1 to 0.1 meter per second. On the other hand the diffusivity will be in the order of 10 to the power minus 10 to 10 to the power minus eleven meter square per second. So therefore, the convective terms will be the x direction will be much larger in magnitude compare to diffusive term in the x direction. So, we can safely neglect the diffusive term in the x direction, so these three quantities will constitute the solute much balance with in the mass transfer boundary layer.

Now, in order to get an exact estimated you should insert the profile of u, at the velocity profile u and V in the above equation.

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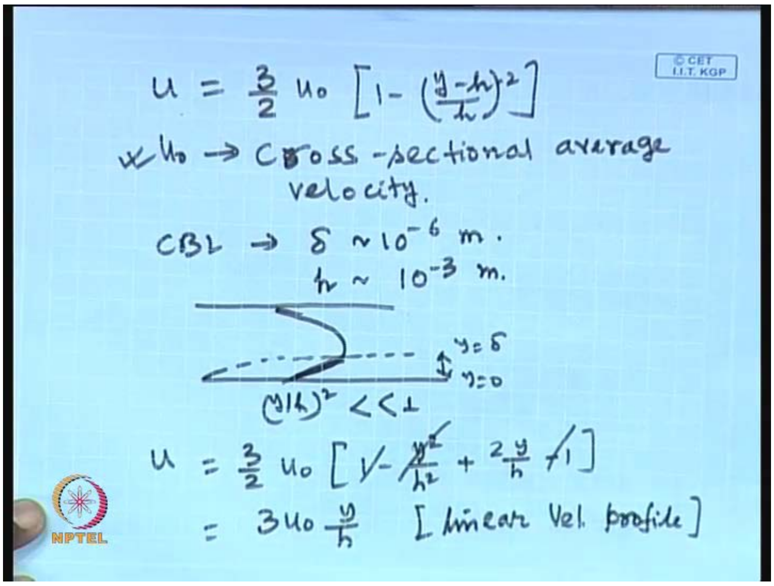

$$u = \frac{3}{2} u_0 \left[1 - \left(\frac{y-h}{h} \right)^2 \right]$$

$u_0 \rightarrow$ Cross-sectional average velocity.

CBL $\rightarrow$ $\delta \sim 10^{-6}$ m.

$h \sim 10^{-3}$ m.

$(y/h)^2 << 1$

$$u = \frac{3}{2} u_0 \left[\cancel{1} - \cancel{\frac{y^2}{h^2}} + 2\frac{y}{h} \cancel{-1} \right]$$

$$= 3u_0 \frac{y}{h} \quad [\text{linear Vel. profile}]$$

If you do that, what is the velocity profile in the x direction? we consider it is a fully developed velocity profile as for as the hydro dynamic is constant therefore, the parabolic velocity profile will exist, for a rectangular channel this will be function of y only and it is 3 by 2 mu naught into, 1 minus y minus h divided by h square of that, this is the velocity of the parabolic velocity profile in the rectangular channel. What is u naught? u naught is the cross section average velocity, this is a measureable quantity, and how will you measure it? You put a rot meter in a flow path and measure the flow rate you, know the geometric you divide cross sectional area, the velocity that you will be getting the value of u naught.

So, this is the velocity profile in the x direction and if you remember that concentration boundary layer thickness delta will be of the order of 10 to the power minus 6 meter, on the other hand the half channel height, this is the half channel height this will be in the order of millimeter, 10 to the power of minus 3 meter. So, if you just look in to the diagram this is the velocity boundary layer and this is the concentration boundary layer, so we do not require the full velocity profile to solve this equation because these equation is valid within y is equal to 0, to y is equal to delta, will required only these portion of the velocity boundary layer, and what is this portion? This is almost linear, and how can that we obtained? If you neglected y by h square on higher order terms, that means y by h square will be very less than 1. That means, h itself will be 3 order of magnitude higher compared to y therefore, y by h will be in the order of 10 to the power of minus 3, and square will be 10 to the power minus 6.

So, you can neglect that you will be getting the linear profile of velocity, that means u is equal to 3 by 2, u naught 1 minus y square by h square plus 2 y h minus 1, now 1, 1 will be cancel down and y square by h square that will be petty small that will be neglected, so this becomes 3 u 0 y by h. So, you obtained a linear velocity profile that is valid within the concentration boundary layer. So, we finalize the velocity profile in x direction.

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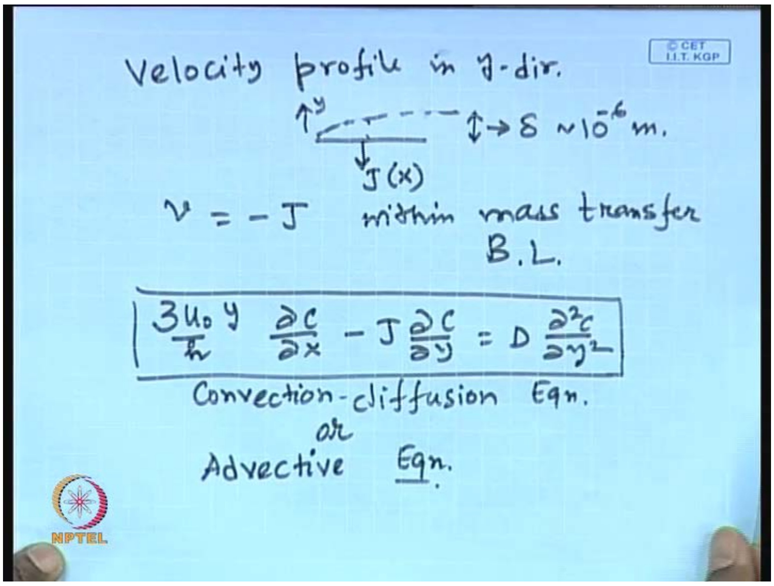

Velocity profile in y-dir.

$v = -J$ within mass transfer B.L.

$$\boxed{\frac{3u_0 y}{h}\frac{\partial c}{\partial x} - J\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2}}$$

Convection-diffusion Eqn.
or
Advective Eqn.

Let us look in to the velocity profile in the y direction, we are going to have a permeate flux, which will be a function of x at a particular location and where going to find out the y direction of velocity within these mass transfer boundary layer and since this mass transfer boundary layer, so there will be a velocity profile that will be existing and at y equal to 0 it its value will be minus J, t the velocity will be this positive y, so the velocity here will be equal to minus J at y equal to 0.

But, we assume that since these this thickness this delta thickness will be in the order of 10 to the minus 6 meter that will be extremely small, we assume the same velocity will prevail within that, so V is equal to minus J, within mass transfer boundary layer this is a valid assumption because the thickness of mass transfer boundary layer will be extremely small. So, let us write down as govern equation, our govern equation now becomes 3 u 0, y y (( )), del c del x, minus J del c del y is equal to D del square c del y square.

These equations are known as the convection diffusion equation or advective equation. You required to have one boundary condition on x, it required having two boundary condition on y because it is order to with respect to y and now let us looking in to the boundary condition so that we can solve them.

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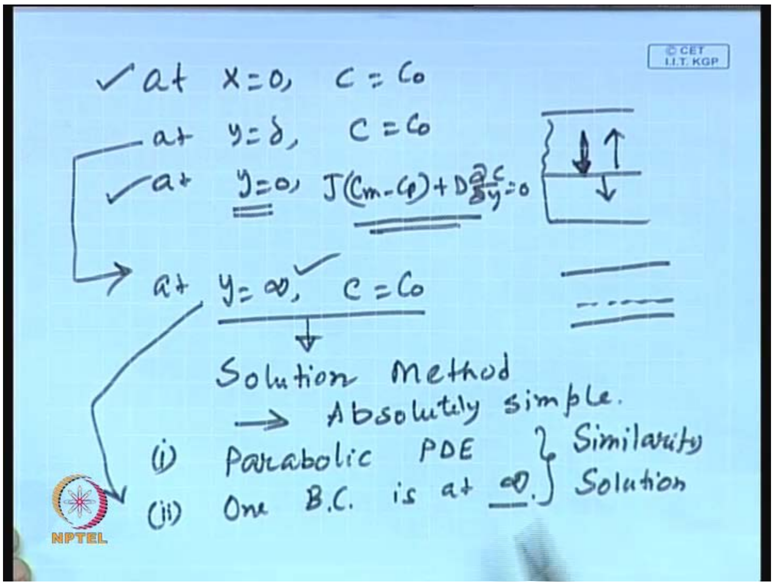

✓ at $x = 0,\ c = c_0$

at $y = \delta,\ c = c_0$

✓ at $\underline{\underline{y = 0}},\ \underline{\underline{J(C_m - C_p) + D\frac{\partial c}{\partial y} = 0}}$

→ at $y = \infty$, ✓ $c = c_0$

↓

Solution method

→ Absolutely simple.

(i) Parabolic PDE

(ii) One B.C. is at $\underline{\infty}$.

} Similarity Solution

At x is equal to 0 c is equal to c not, at y is equal to delta you will be having c is equal to c not, at y is equal to 0 at the steady state, all the flux is towards the membrane will be equal to 0, that means J times C m, minus C p, plus D del c, del y will be equal to 0, this is known as a Robin mix boundary condition. So, if you remember that, this is J time's c basically the convective power flux towards the membrane minus D del c, del y will be diffusive flux away from the membrane, J times it will be convective flux away from the membrane.

Since it is evaluated at y is equal to 0, I wrote c is equal to C m, it is at y is at 0 mean at the boundary, now if you remember, this is the governing equation for the one dimensional problem for the film theory equation, for the film theory equation we solve this equation in order to get the governing equation. Now, for a two dimensional problem therefore, for two dimensional problem the governing equation of one dimensional problem has become a boundary condition for a two-dimensional problem, we identity that.

Now, the point is y is equal to delta, now there is absolutely no problem regarding this boundary condition and this boundary condition, there is some problem regarding this boundary condition at y equal to delta c is equal to c naught, at y is equal to delta, delta is the edge of the mass transfer boundary layer but delta is not known to us it is very difficult to solve the problem with unknown boundary condition, your boundary condition as to be specified in order to solve a ordinary differential or partial differential equation. So therefore, what is happen, what is taken as that since this concentration boundary layer thickness will be 10 in the order of 10 to the power of minus 6, most of the channel it does not affect right, c becomes c naught.

So, we assume that instead of this we can safely use the boundary condition, that at y is equal to infinity, c is equal to c naught that the half channel height itself is always the real dimension is 1 millimeter, it is locally that infinite distance away from the mass transfer boundary layer because there the ratio of each and delta will be in the order of 10 to the power 3. So, that will basically thousand times more that it will be located thousand time the distance. Therefore, it is taken as y is equal to infinity; c is equal to c naught.

Now, let us what it does, this gives a tremendous you know the handling to the solution of the problem, the solutions technology becomes very simply, solution method becomes absolutely simple by taking this assumptions. What is so great about it? The greaten is of this boundary condition is that, if we look in to the governing equation the partial differential equation, we can identity this is a parabolic partial differential equation, and these boundary condition simply indicate that one of the boundary residing at infinitive.

Now, whenever these two thump rules are satisfied that these governing equations can admit a similarity solution, all it is known as a combined equation parameter solution or similarity solution or lumped parameter equation. What it does, in a partial differential equation you must having at least two independent variables, right the variables are x and y here.

Now, in case of similarity solution we define a combine variable of x and y, some form or functional form of x and y lets say eta is equal to y by x to the power half, eta is equal to the power y by x to the power 4 by 3, something some combine parameter can be, so whole governing equation and its boundary conditions can be expressed in terms of this combined parameter. So, that means you reduced number of parameters from two to one what does it imply, it implies we reduces the partial differential equation into a ordinary differential equation and the solution of ordinary differential equation is absolutely simple, so it simplifies the whole solutions. Now, the point is what will be the functional variation or what will be the similarities parameter.

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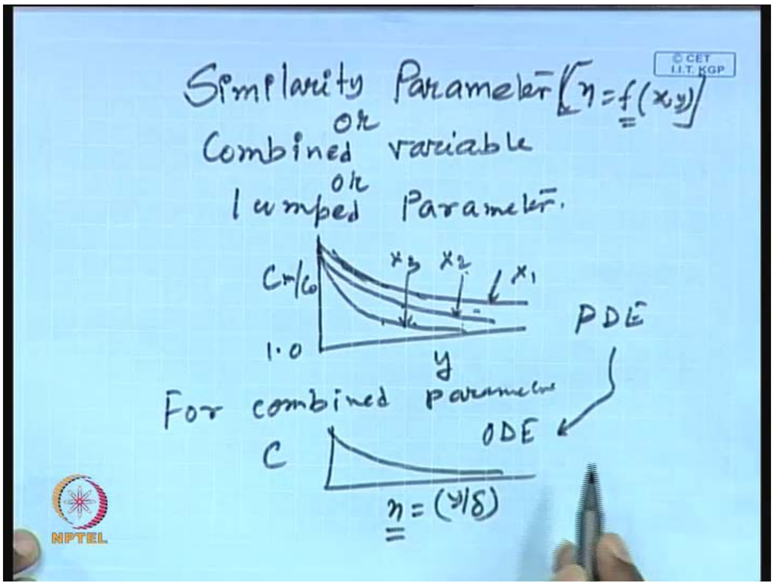

Similarity Parameter $[\eta = f(x,y)]$

or

Combined variable

or

lumped Parameter.

Similarity parameter is basically a combined parameter, which will be a function of x and y the independent parameter. What is this functional form; whether it is y by x to the power half or whether it is y by x to the power 1 third or y by x to power four third or whatever it is. This parameter is known as the similarity parameter or combined variable parameter or a lumped parameter. Now, geometrically before we going in to the actual calculation of evaluating the similarity parameter, I would like to tell you the geometry implication of this similarity parameter. If you plot c as a function of y for various x locations, the plots will be typically something like this, this is may be at location x 1, this is may be at a location x 2, this is may be at the location x 3. These are the various concentration profile across the channel, now this goes from c is equal to 1 to whatever, because this c is greater than 1 now c at membrane surface divided c naught, anyway.

Now, for combine parameter if you plot or similarity parameter, if you plot this concentration profile as a function of y by delta or there is there is how the similarity parameter is defined, eta is equal to y by delta, y by delta means the profile these profiles are known as the self similar profiles. So, if you divide by eta as y defined y eta as y by delta all these curves, all these individual curves will collapse on a single curve.

So, you can represent these, all these number of curves on a single curve therefore, the existence of two independent variables x and y, we no longer there it will be replaced by a combine parameter eta, and delta will be varying a particular form of x that will give you the variation relationship between y and x. So, instead of two parameters we will be talking about one parameter and PDE will be converted into ODE. So, that is how the similarity solution becomes very handy and it reduces the you know PDE to ODE and the solution of ODE are quite simple.

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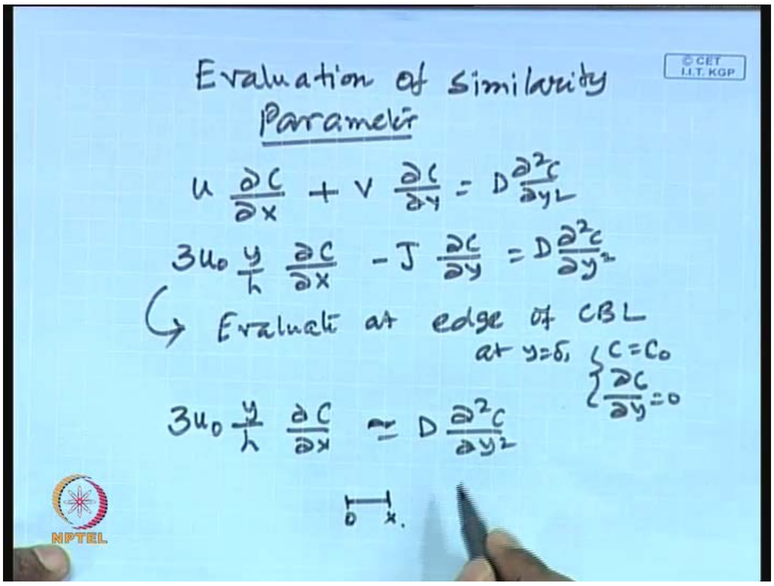

Evaluation of Similarity Parameter

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2}$$

$$3u_0 \frac{y}{h} \frac{\partial c}{\partial x} - J \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2}$$

Evaluate at edge of CBL

at $y = \delta_c$ $\left\{ \begin{array}{l} c = c_0 \\ \frac{\partial c}{\partial y} = 0 \end{array} \right.$

$$3u_0 \frac{y}{h} \frac{\partial c}{\partial x} \simeq D \frac{\partial^2 c}{\partial y^2}$$

Now, let us look in to how to evaluate the similarity parameter, the evaluation of similarity parameter will goes like this and similarity parameter solution or similarity solution is quite rampant, in case of boundary layer analysis because in all the boundary layer thermal boundary layer, concentration boundary layer, hydrodynamic boundary layer the profiles, the concentration profile or temperature profile or velocity profile all are self similar.

If you remember the governing equation was u del c del x, plus v del c del y is equal to D del square c del y square, you insert the velocity profile is becomes 3 u 0 y by h, h is the half height del c del x minus J del c, del y is equal to D del square c del, y square we evaluate the whenever we talking about the a governing equation, governing equation is valid throughout the whole control volume as well as on the boundaries but the boundary conditions are valid only on the boundaries but not within the control volume. So, these equation must be valid about the edge of the boundary layer, so evaluate this equation at the edge of boundary layer, at the edge of boundary layer, there are two condition will be satisfied compulsorily, mandatorily what are the mandatorily conditions? At the edge of boundary layer at y is equal to delta, c is equal to c naught, at y is equal to delta, del c del y will be equal to 0, these 2 are mandatory for any boundary layer whether it is thermal boundary layer, hydrodynamic boundary layer or concentration boundary. Layer in case of thermal boundary layer, at the edge of the boundary layer t is equal to t infinity or t not and del, t del y will equal to 0. In case of velocity boundary layer u is equal to u infinity and del u del I will be equal to 0, at y equal to delta.

So therefore, this term will not be at their at the edge of the boundary layer, so it is, it becomes 3 u 0 y by h, del c, del x is equal to D del square c, del y square now we do on order of magnitude analysis that means, we take a small distance let say from 0 to x this delta will be del 0 to x and delta y will be 0 to delta.

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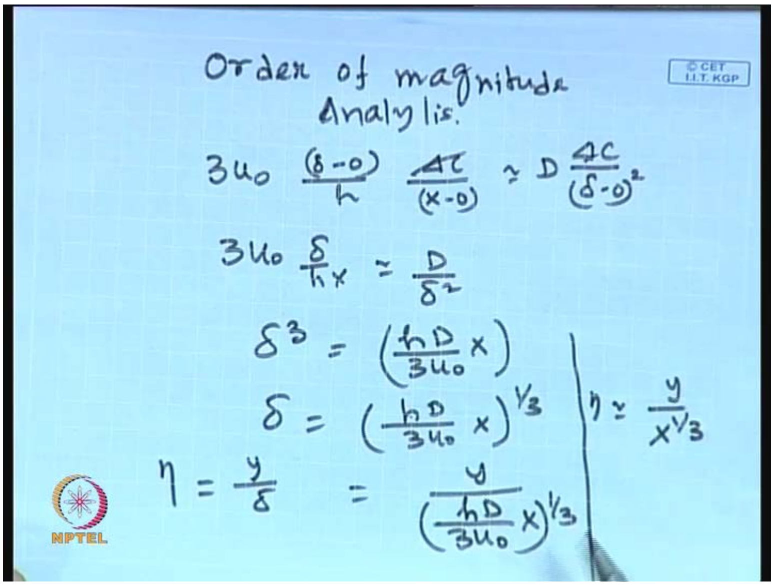

Order of magnitude Analylis.

$$3u_0 \frac{(\delta-0)}{h} \frac{\Delta c}{(x-0)} \simeq D \frac{\Delta c}{(\delta-0)^2}$$

$$3u_0 \frac{\delta}{hx} \simeq \frac{D}{\delta^2}$$

$$\delta^3 = \left(\frac{hD}{3u_0} x\right)$$

$$\delta = \left(\frac{hD}{3u_0} x\right)^{1/3}$$

$$\eta \simeq \frac{y}{x^{1/3}}$$

$$\eta = \frac{y}{\delta} = \frac{y}{\left(\frac{hD}{3u_0} x\right)^{1/3}}$$

So, we do an order of magnitude of this equation, that will give 3 u 0, y becomes delta minus 0 divided by h and delta c divided by delta x delta; x becomes x minus 0, roughly D del square c is nothing but delta c divided by delta y square is nothing but delta minus 0 square of that. If you ever do numerical differentiation, you can understand that delta square c is nothing but another difference in concentration, so if you do the numerical differentiation you can understand, so these becomes 3 u 0 delta by h this delta c delta c will be cancel, x is roughly D by delta square. So, this gives delta cube is equal to h D by 3 u 0 times x rest to x and delta becomes h D 3 u 0 x rest to the power 1 upon

3, if you put the dimension of h D 3 u 0 and x, this becomes rest to the 1 upon 3 becomes unit of distance.

So, what is the similarity parameter? Similarity parameter is eta equal to y by delta, so this becomes y divided y h D by 3 u not x rest to the power 1 upon 3, so the variation of the combined parameter will be in the form of y divided y x to the power 1 upon 3. So, this is the how the combined parameter is now evaluated and now, what we will do in the next class will be substituting this parameter in the governing equation and express our derivatives in terms of this combined parameter and reduce the PDE integral ODE and solve consequently. Thank you