Novel Separation Processes
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Lecture No. # 06
Membrane Separation Process

Good morning, everyone so, we are looking into the various, you know the modeling aspects of membrane based separation processes and yesterday, we last in the last class, we talked about the concentration, polarizations. The problems associated with concentration, polarization various manifestation of concentration polarization and how it you know? It hinders the productivity of the process; and every aspects of manifestation of, concentration, polarization will lead to a decreases in productivity of the permeate flux of the system.

Now, in today class, what we will see in the first? We will see that, how we can minimize? Actually we cannot avoid; concentration, polarization, How we minimize concentration polarizations? And including the concentration polarization, what are the various models? How to provide the performance of the system?

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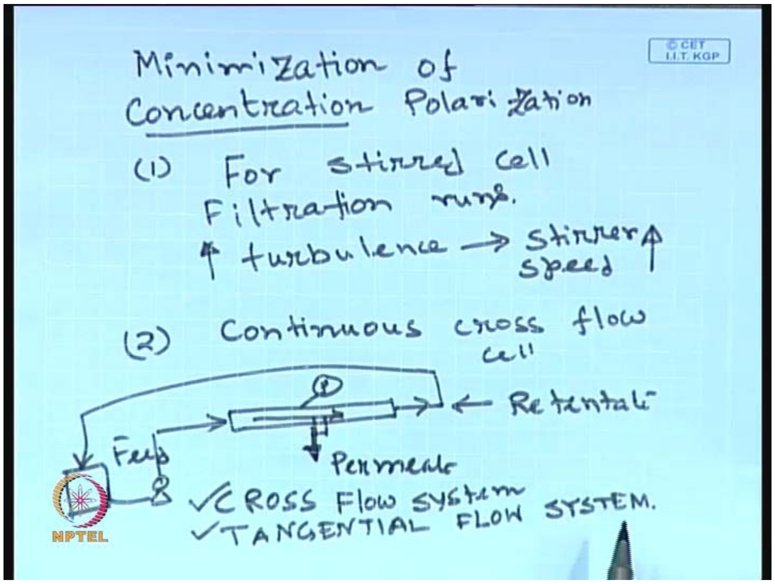

Minimization of Concentration Polarization

(1) For stirred cell Filtration runs.
↑ turbulence → stirrer speed ↑

(2) Continuous cross flow cell

✓CROSS Flow system
✓TANGENTIAL FLOW SYSTEM.

So, first we talk about; minimization of concentration polarization. The so, first aspect to minimize concentration polarization is, to increase turbulence in the system such that; the deposition over the membrane surface will be disturbed or it will decrease. So, there are if we have a stirred cell for, stirred cell filtration experiments of filtration runs 1 can simply minimize concentration, polarization, by increasing turbulence in the system increasing turbulence and turbulence can be, increased by increasing the stirrer speed.

That is the one option, in case of continuous cell continuing filtration cell, we called continuous cross flow. I will come to why it is cross? Why it is termed as cross flow continuous? Cross flow cell then, what happen? Suppose, there is the membrane filtration unit, the feed is being pumped from a fit tank and it is pushed through the cell, in the bottom and the top, there will be membrane surface and there will be pressure gauge here; and the returned it can be recycled back to the feed stream and 1 can get the permeate from the bottom and these stream is called retentate.

Now, by increasing the feed velocity, over the membrane surface, what it will do? It will decrease the deposition of the solute over the membrane surface. So, one can increase the feed velocity over the membrane surface and decrease the concentration, polarization thickness of the concentration polarization layer.

In a sense what will be getting? Since the thickness of concentration polarization will decreases, the permeate flux will increase, because it offers less resistance against the solvent flux. So, one can get increase increased productivity; Now, feed is flowing over the membrane surface tangentially, so, the surface of the membrane and the direction of, the feed flow, the parallel and the other hand the permeate that is coming; that is 90 degree with the direction of that flow that is way it is called a cross flow system. This angle is 90 degree; this system is called a cross flow system are sometimes it is called a tangential filtration system as well, either it is called cross flow system, because of these cross flow of the permeate with respect to it dentate of the feed are it is called tangential flow, because feed is allowed to flow tangentially over the membrane surface.

Therefore, one can by inducing such flow, in the system one can decrease the thickness of concentration boundary layer, and can reduce the concentration polarization layer and consequently one can have an increase the permeate flux, of the productivity, of the system.

Now, once we therefore, we should admit we cannot avoid concentration polarization, we can minimize it maximum so, with including concentration polarization, let us; see how we can do? We can have a system for prediction of the system performance

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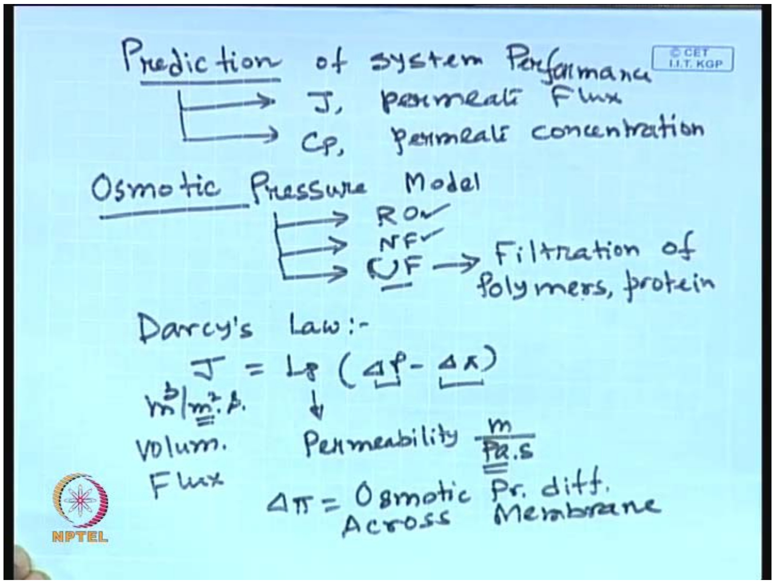

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Prediction of system Performance

→ J, permeate Flux

→ Cp, permeate concentration

Osmotic Pressure Model

→ RO ✓

→ NF ✓

→ UF → Filtration of polymers, protein

Darcy's Law:-

$$J = L_p(\underbrace{\Delta P} - \underbrace{\Delta \pi})$$

$m^3/m^2.s$

volum. Flux

L_p ↓ Permeability $\frac{m}{Pa.s}$

$\Delta\pi$ = Osmotic Pr. diff. Across Membrane

Now, prediction of the system performance basically; there are two aspects in the prediction, one is the productivity of the process that means, permeate flux another is, permeate concentration C p. Now, let us, first take the osmotic pressure model, Now, osmotic pressure model is quite important, in the case of reverse osmosis nano filtration of even is to some extend ultra filtration, when we are talking about filtration of polymers and proteins polymers, protein extra. Now, we have already seen in case of reverse osmosis and nano filtration, since, the pore size will be extremely small, the osmotic pressure is important role and the other hand, in case of ultra filtration. When we are talking about filtration of polymers and proteins osmotic pressure of the solute becomes significant, because of several aspects like charter interactions, membrane polymer, interactions thinks like that.

So, therefore, osmotic pressure becomes very important so, one in the case of membrane separation system almost 70 percent of the filtration, that you can think of ranging from; reverse osmosis to ultra filtration. The osmotic pressure controlled therefore, the osmotic pressure model becomes of at most important and you should discussed about; the osmotic pressure model. So, the first equation, that you will be getting about, the solvent flux of the permeate flux will be the Darcys Law. So, as we are discussed, in the last class any membrane base model will be having two distinct part, one is the model that will be simulating the flow outside, the membrane channel that will be including, the feed flow mass transfer and another will be that will be coupled with the feed flow phenomena that is; occurring or happening within the porous membrane so these 2 will be coupled and 1 will be getting system performance.

So, darcys law will give you the equation for the solvent flux to the porous membrane and if you remember J is equal to L p into delta p minus delta pi why a J is the solvent flux or the permeate flux it will be having it is the volumetric permeate flux. It will having a unit of meter cube per meter square second so, let us, make this think very clear meter cube per meter square, second is known as the volumetric flux and what is L p? l p is the membrane permeability and it will be having a unit of flux is meter cube per meter square, second is basically having a unit of meter per second these meter square is basically permeate membrane area.

So, will be having meter per second divided by Pascal; these Pascal, comes out from pressure on the other side delta p, is the operating pressure difference across the membrane feed pressure and minus the permeate pressure in most of the cases permeate pressure is atmospheric pressure, there therefore, gauge pressure in the feed side, we give you the delta p and what is delta pi? delta pi is the osmotic pressure difference towards, the membrane now, let us try to find out what is osmotic pressure difference across the membrane.

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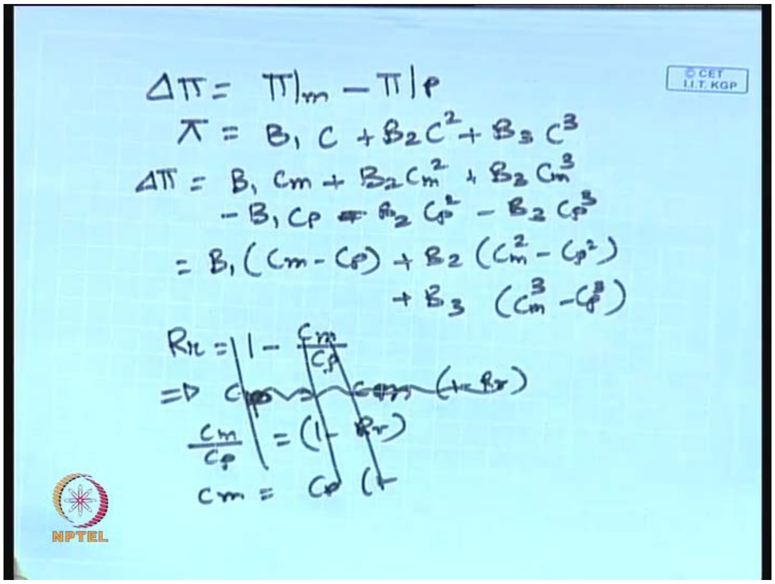

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$$\Delta\pi = \pi|_m - \pi|_p$$

$$\pi = B_1 C + B_2 C^2 + B_3 C^3$$

$$\Delta\pi = B_1 C_m + B_2 C_m^2 + B_3 C_m^3$$

$$- B_1 C_p - B_2 C_p^2 - B_2 C_p^3$$

$$= B_1 (C_m - C_p) + B_2 (C_m^2 - C_p^2)$$

$$+ B_3 (C_m^3 - C_p^3)$$

$$R_r = 1 - \frac{C_m}{C_p}$$

$$\Rightarrow C \cancel{C_p} \cancel{C_m} (1 - R_r)$$

$$\frac{C_m}{C_p} = (1 - R_r)$$

$$C_m = C_p (1 -$$

So delta pi, will be pi on the membrane surface in the membrane feed interface minus pi of the permeate stream so, and we know that osmotic pressure is the; qualitative property it is ever increasing function of concentration. Therefore, pi can be expressed as B 1 C plus B 2 C square plus B 3 C cube cm, therefore, delta pi will be B 1 C m plus B 2 C m square plus B 3 C m cube minus C 1 C p plus minus B 2 C p square minus B 3 C p cube. So, at membrane surface the concentration solute basically C m that is, the concentration of the solute at the membrane feed interface and C p is the permeate concentration. Therefore, you can write these things in compact form C m minus C p B 2 C m square minus C p square plus B 3 C m cube minus C p cube now we can have the a an equation of solute transport.

So, what we have, we are talked about is basically Darcys law. Darcys law is nothing, but the solvent transport through the, porous membrane now, we must having some kind of law which will dictate the solute transport across the membrane and ultimately; it will give a relationship of the solute concentration in the upstream of the membrane and downstream of the membrane. The simplest on that we have discussed, in the last class the real retention let us defined a real retention R r which is constant for a particular membrane solute system is, nothing, but C m minus C p. Therefore, you can eliminate C p from these equation of delta pi in favor of C m, because all are becomes constant, all are constant; so, what is C m? C m is nothing, but what is C p? C p is nothing, but C m into 1 minus R r now, I will just derive it C m minus divided by C p will be 1 minus R r so, C m will be C p into 1 minus R r I i. I think there is a mistake R r will be 1 minus C p by C m, if you remember the definition of R r equation.

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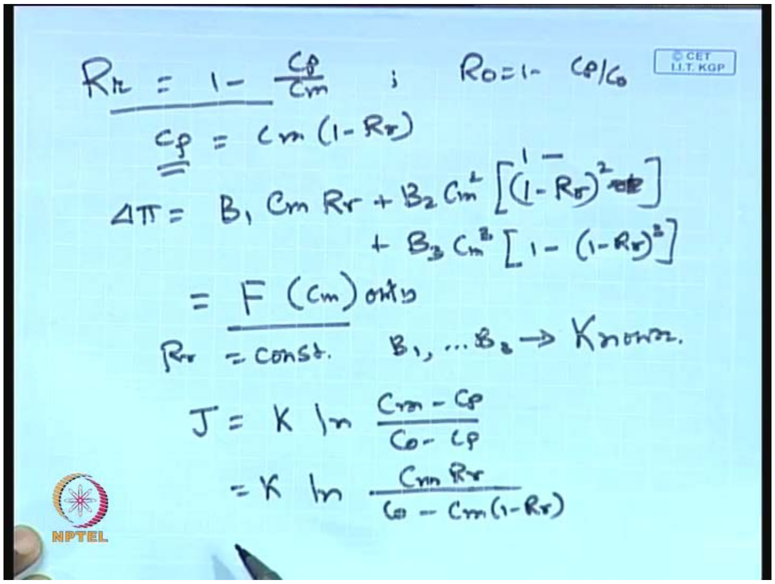

$$\underline{R_r = 1 - \frac{C_p}{C_m}} \; ; \quad R_0 = 1 - C_p/C_0$$

$$\underline{\underline{C_p}} = C_m(1-R_r)$$

$$\Delta\pi = B_1 C_m R_r + B_2 C_m^2\left[\overset{1\,-}{(1-R_r)^2}\cancel{-1}\right] + B_3 C_m^3\left[1-(1-R_r)^3\right]$$

$$= \underline{F(C_m)} \text{ only}$$

$$R_r = \text{const.} \qquad B_1, \ldots B_3 \rightarrow \text{Known.}$$

$$J = K \ln \frac{C_m - C_p}{C_0 - C_p}$$

$$= K \ln \frac{C_m R_r}{C_0 - C_m(1-R_r)}$$

Real retention, will be nothing, but 1 minus C p by C m, if you just look into the definition R r and observed retention what we have defined R not equal to 1 minus C p by C 0 so if we look into the definition of R r C m can be C p can be written as, C m into 1 minus R r. Now, we substitute the expression of permeate concentration, in the in the expression of delta pi osmotic pressure difference in favor of C m, if you do that the expression of C delta pi will nothing, but B 1 C m R r plus B 2 C m square 1 minus R r square minus 1 plus B 3 C m cube. I think it is 1 minus 1 minus R r and this will be C m cube 1 minus 1 minus R r whole cube; so, once you get this expression.

So, you can so what is delta pi? Delta pi is nothing, but a function of C m only, because R r is constant and the coefficients B 1 to B 3 are known to you. Now, we write down the expression of film theory J is equal to K is l N C m minus C p divided by C 0 minus C p. So, K is mass transfer coefficient let us, write C m minus C p nothing, but C m times R r and C p can be replaced in favor of R r as in favor of C m 1 minus R r; so, you have now you can e it the equate darcys law, the expression of the solvent flux from the darcys law and expression of the solvent flux in the film theory and what will obtain that will be getting an algebraic equation in term of C m only.

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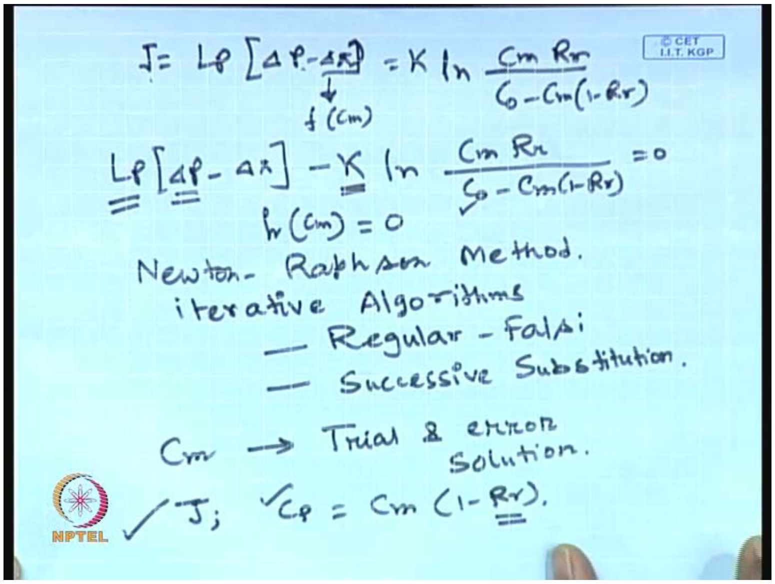

$$J = L_p\Big[\Delta P - \underset{\substack{\downarrow \\ f(C_m)}}{\underline{\Delta\pi}}\Big] = K \ln \frac{C_m R_r}{C_0 - C_m(1-R_r)}$$

$$\underline{\underline{L_p}}\big[\underline{\underline{\Delta P}} - \Delta\pi\big] - \underline{\underline{K}} \ln \frac{C_m R_r}{\underset{\checkmark}{C_0} - C_m(1-R_r)} = 0$$

$$h(C_m) = 0$$

Newton- Raphson Method.

iterative Algorithms

— Regular - Falsi

— Successive Substitution.

$C_m \rightarrow$ Trial & error solution.

✓ J; ✓ $C_p = C_m(1 - \underline{\underline{R_r}})$.

So J is equal to L p delta p minus delta pi is equal to K l n C m R r divided by C 0 into C m minus 1 minus R r and delta pi is nothing, but a function of C m in terms of real retention of coefficient expression of the osmotic pressure. So, basically you will be getting an algebraic equation L p, delta p minus delta pi, which is a function of C m that we have already same minus K l n C m R r divided by C 0 minus C m 1 minus R r equal to 0.

So, theses equation is nothing, but some non linear algebraic equation, in terms of C m, because permeate membrane permeability is known to you can find out from the separate set of experiment delta p is the, operating pressure that is known to you in the expression of delta pi, we have coefficient of osmotic pressure those are known to you and value of real retention, that can be stimulate from separate set of experiment from a that we have discussed earlier; K is the mass transfer coefficients there correlation of theoretical relation available for, mass transfer coefficient depending on the flow geometric and the, flow region one can find out the appropriate equation and evaluate the value of mass transfer coefficient main.

That properties of the feed and the solute and real retention is known C 0 is the, feed concentration that is nothing, but the operating condition so, by using channel error solution one can use Newton raphson method, there are other iterative algorithm can also, give available algorithm like regular- falsi successive substitution. So, using any iterative technique one can evaluate the value of C m from our expression it is basically trial and error solution, once you know the value of C m 1 can go back to the expression of permeate flux; either if the two expression and put the value of C m one can estimate, the value of permeate flux of the productivity of the process and one go to the expression C p. If you remember C p is nothing, but C m into 1 minus real retention so, 1 can get the value of permeate flux to productivity of the, process 1 can get the value of permeate quality of the permeate concentration; now, whole thing depends how accurate? You are in the in getting the definition of mass transfer coefficient how could that correlation? That we are using for your system number 1, number 2 how could your estimation of real retention?

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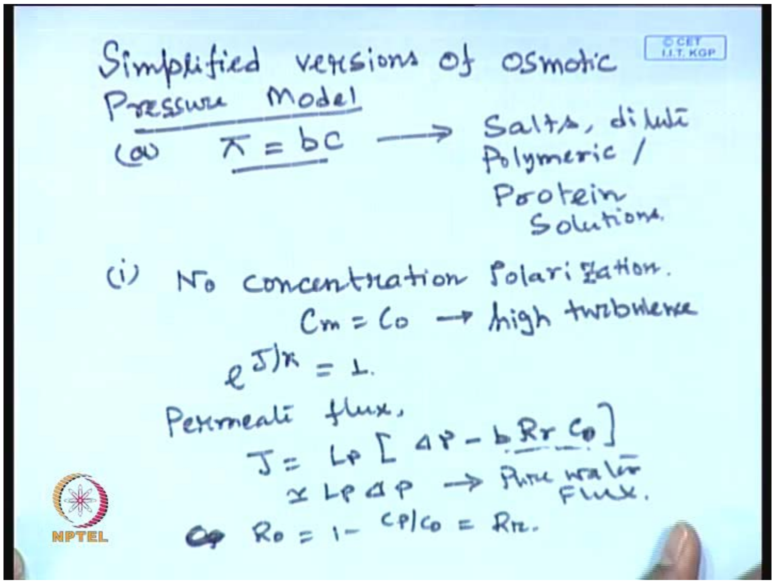

Simplified versions of osmotic Pressure Model

(a) $\pi = bc$ → Salts, dilute Polymeric / Protein Solutions.

(i) No concentration Polarization.

$C_m = C_0$ → high turbulence

$e^{J/k} = 1.$

Permeate flux,

$J = L_p [\Delta P - b R_r C_0]$

$\simeq L_p \Delta P$ → Pure water Flux.

$R_0 = 1 - C_p/C_0 = R_r.$

Now, we look into the some of the simplified version of this error solution, there is no analytical expression or solution that that becomes, that is there in, front of you. So, we can get some simplified version of the osmotic pressure model; the first simplification that we will be doing for all this calculation is that we are considering, pi is not a polynomial of concentration, but it will be linear in concentration. Let say pi is equal to b c this will be occurring for the case of salts monovalent and divalent salt and dilute salt of proteins polymers, under these concentration condition. Now, there can be several cases, the first case is no concentration polarization, when that can happen that can; that means C m is equal to C not that can happen when you have very high stirred speed or turbulence in your feed channel this can happen high turbulence in this case the concentration is uniform in the feed chamber that is more polarization it is the concentration of the solute throughout the feed chamber is always C 0. In that case e to the power J by K will be equal to 1 and the permeate flux becomes J is equal to L p delta p minus B times R r times C 0.

Therefore, the osmotic pressure corresponding to the feed concentration, will be basically giving you the contribution from the osmotic pressure expression; and that will be so, in fact you delta p is probably quite high compare to the osmotic pressure, of the feed solution so, we can easily neglect it.

So, it will a almost giving you the pure water flux expression this is known as the pure water flux, because for pure water there is no osmotic pressure contribution there is number 1 and what will be the permeate concentration? Permeate concentration will be the yet the C p will be 1 minus C p by C 0. So, you observed retention becomes real retention, in that case, because there is no consent, there is into the C m is equal to C0.So, observed retention becomes real retention and the concentration of permeate can be obtained by 1 minus R r times C0.

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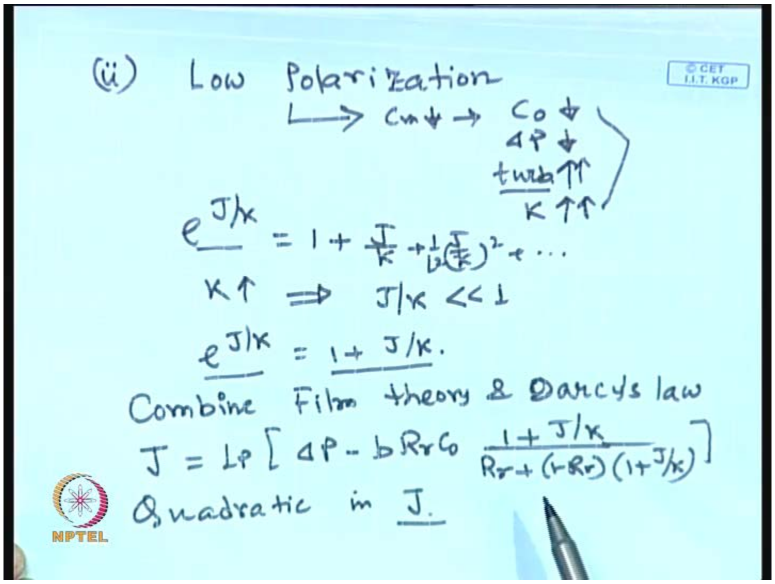

(ii) Low Polarization

$\longrightarrow C_m \downarrow \rightarrow C_0 \downarrow, \ \Delta P \downarrow, \ \underline{\text{turb}} \uparrow\uparrow, \ K \uparrow\uparrow$

$$\underline{e^{J/K}} = 1 + \frac{J}{K} + \frac{1}{\lfloor 2}\left(\frac{J}{K}\right)^2 + \cdots$$

$$K \uparrow \Rightarrow J/K << 1$$

$$\underline{e^{J/K}} = \underline{1 + J/K}.$$

Combine Film theory & Darcy's law

$$J = L_P \left[\Delta P - b R_r C_0 \frac{1 + J/K}{R_r + (1-R_r)(1+J/K)} \right]$$

Quadratic in $\underline{J}$.

The second simplification that; we can think about is low polarization. I think we have discussed the condition of high polarization and low polarization earlier in your class, low polarization means the condition where C m will be low C m will be low, if you have a low feed concentration, if you have low delta p the Trans membrane operating pressure and very high turbulence will be very high. For example, the turbulence very high means, mass transfer coefficient has to be very high, these are the condition which we favor the low polarization condition really in the membrane channel, under these situation e to the power J by K can be can be expanded is in a series infinite series 1 by J by K plus J by K square factorial 2, because higher termed since, that mass transfer coefficient are pretty high it can happen that J by K will be much less than 1.

Under these case, we can neglect the higher terms and these expansion exponential expansion is valid and one can have e to the power J by K as 1 plus J by K, under these condition one can combine film theory and darcys law, in the expression of the film theory we just put C m minus C p divided by C not minus C p equal to e to the power of J by K and in state e to the power J by K equal to 1 plus J by K and we the have darcys law and combine these two final and combining after the combination of two equation one can get a final expression something like this; L p times delta p minus b R r C not 1 plus J by K divided by R r plus 1 minus R r 1 plus J by K.

Now, if you look into these expression in these expression only one unknown is there expression permeate flux of J every parameter are known to as J and these gives a quadratic in J now these quadratic equation can be solved, I am not writing that solution of the equation these quadratic equation in J can be solved and 1 can get the prediction of the system performance.

In terms the productivity of the process and once you get, J you can easily find out what is the membrane surface concentration? That is C m and once you know the C m you can get the value of permeate concentration from the definition, real retention that C p is equal to C m into 1 minus R r so, that is how these can be solved?

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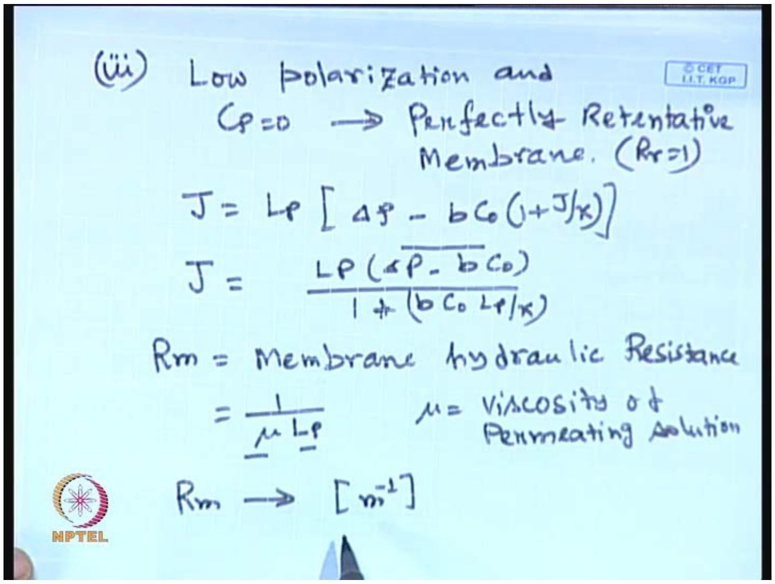

(iii) Low polarization and

$C_p = 0$ → Perfectly Retentative Membrane. ($R_r = 1$)

$$J = L_p \left[\Delta p - b C_0 (1 + J/k) \right]$$

$$J = \frac{L_p (\Delta p - b C_0)}{1 + (b C_0 L_p / k)}$$

R_m = Membrane hydraulic Resistance

$$= \frac{1}{\mu L_p}$$

μ = Viscosity of Permeating solution

$R_m \rightarrow [m^{-1}]$

Now, case number 3 will be low polarization, the earlier 1 and C p is equal to 0 that means the perfectly retentive membrane as for as, the solute is concerned perfectly retentive membrane. So, in these case, the expression becomes simplified further J is equal to L p delta minus b c 0 into 1 plus J by K. Under these conditions, perfectly retentive membrane here, real retention becomes 1 that means real retention becomes 1 and expression becomes simplified and one can have simplification of these as L p into delta p minus B times C 0 divided by 1 plus b c 0 L p over K. Now 1 so, basically what is this? This is the permeability, this is the driving force delta p minus b c 0 and the osmotic pressure contribution will come only, from the feed concentration and we discussed.

Now, we can and what is the term in the denominated? They will give some kind of resistance so one can define a membrane resistance R m R m is membrane resistance it is called membrane hydraulic resistance and the definition it is inversely proportional to the membrane permeability like, the definition of conductivity and resistance in electrical transport it is inversely proportional to the membrane permeability so it is defined as R m is equal to 1 over mu times L p what is mu? mu is the viscosity of permeating solution in most of the cases permeating solution is very dilute so therefore, the viscosity will be closed to the viscosity, of water and if now, if you look into the definition, the units of mu Pascal second and unit of L p 1 can come to the conclusion that R mu will be having unit of meter inverse. So, that is the definition of membrane hydraulic resistance and membrane permeability and unit of membrane resistance. If I substitute L p in terms of membrane resistance the whole terms in the denominator can be divided into 2 resistive terms let us let us substitute that and see what we get?

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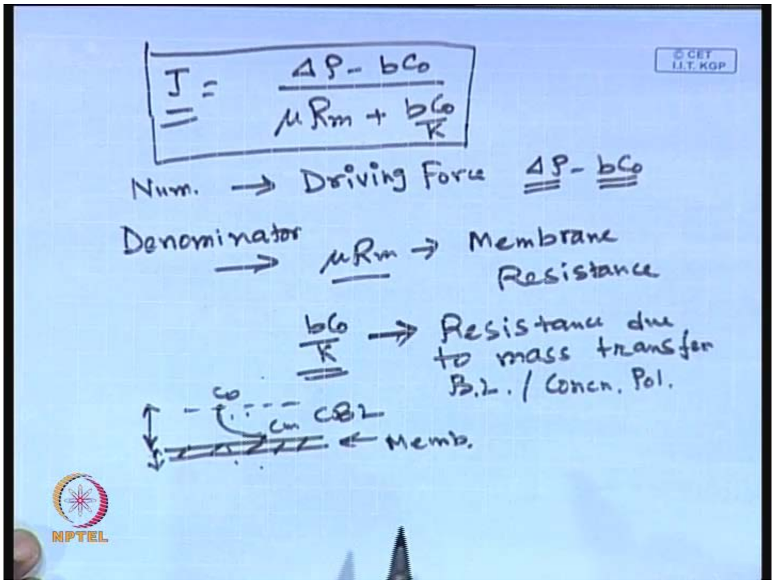

$$\boxed{\underline{J} = \frac{\Delta p - bC_0}{\mu R_m + \frac{bC_0}{k}}}$$

Num. → Driving Force $\underline{\underline{\Delta p}} - \underline{\underline{bC_0}}$

Denominator → $\underline{\mu R_m}$ → Membrane Resistance

$\underline{\underline{\frac{bC_0}{k}}}$ → Resistance due to mass transfer B.L. / Concn. Pol.

So, if we do that will be getting J equal to delta p minus b c not divided by mu times R m plus b c not by K. So, these expressions give the numerator gives the driving force and what is driving force? delta p minus b c not the driving force actually delta p it will be reduce by osmotic pressure contribution from the feed concentration, the first term of the denominator, the first term of the denominator gives the membrane resistance membrane hydraulic resistance, the second term if you remember, what is mass transfer coefficient? It is inverse of resistance mass Trans resistance offered by the mass transfer boundary layer or concentration boundary layer these gives resistance due to mass transfer boundary layer or polarizations concentration polarizations boundary layer or concentration polarizations.

Now, if you remember there these 2 resistances are in series this is the membrane. This is the concentration boundary layer or mass transfer boundary layer here, it is C not here it will be C m polarization the polarization layer will be here so, these resistance is given by these term and these resistance are given by these term they are in series. So therefore delta p by the driving force divided by summation of 2 resistance is will be giving you the solvent flux at the productivity if the process so, under these simplified cases one can have various simplified expressions of solvent flux or and permeate concentration by in an actual case were, you do not have such simplification then these two equation have to be solved by challenge error by using some appropriate iterative algorithm as we have discussed earlier now, if you remember that and all in all these discussion in the models.

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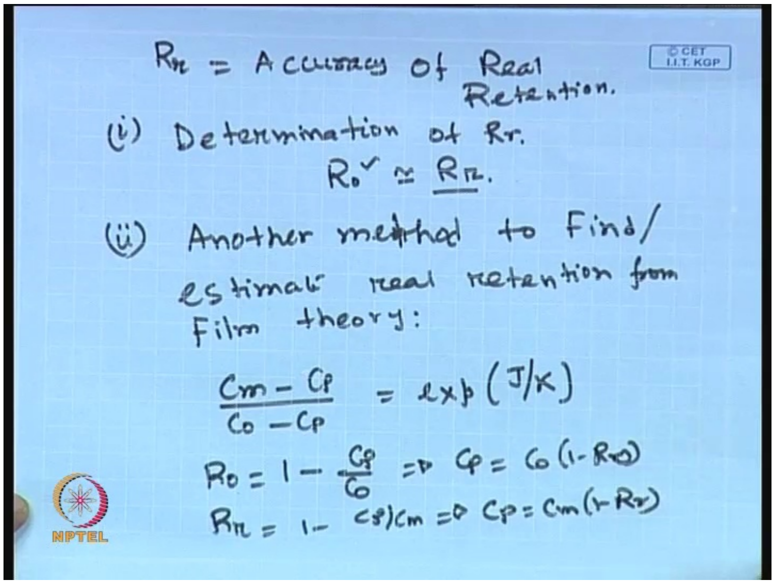

R_r = Accuracy of Real Retention.

(i) Determination of R_r.

$R_o \checkmark \simeq \underline{R_r}$.

(ii) Another method to Find/ estimate real retention from Film theory:

$$\frac{C_m - C_p}{C_o - C_p} = \exp(J/k)$$

$$R_o = 1 - \frac{C_p}{C_o} \Rightarrow C_p = C_o(1-R_o)$$

$$R_r = 1 - C_p/C_m \Rightarrow C_p = C_m(1-R_r)$$

One term becomes very important, that is the determination of the accuracy of real retention, then we have defined one method determination of real retention is basically partition coefficient, between C p and C m; how we determine? The real retention if you remember, we conduct the experiment in a in a in a stirred back cell is very high so that C m will be roughly equal to C not and in that case what is the observed retention will be getting that means you measure the permeate concentration, you know the feed concentration so observed retention will be known to observed retention will be almost equal to real retention so these since real retention is constant for a particular membrane solute system will utilize, that then another method find the real retention is from the film theory, find or estimate real retention from film theory.

If you looking to the expression of film theory, C m minus C p divided by C not minus C p is equal to exponential J by K now, what will be doing? Will be substituting the definition of real retention and observed retention and eliminate these C m and cp. So, if you remember the definition of observed retention observed retention will be 1 minus C p divided by C 0, so therefore C p is nothing, but C not into 1 minus R r and, what is real retention? That is 1 minus C p by C m so C p is nothing, but C m into on minus 1 minus R r is 1 minus R O.

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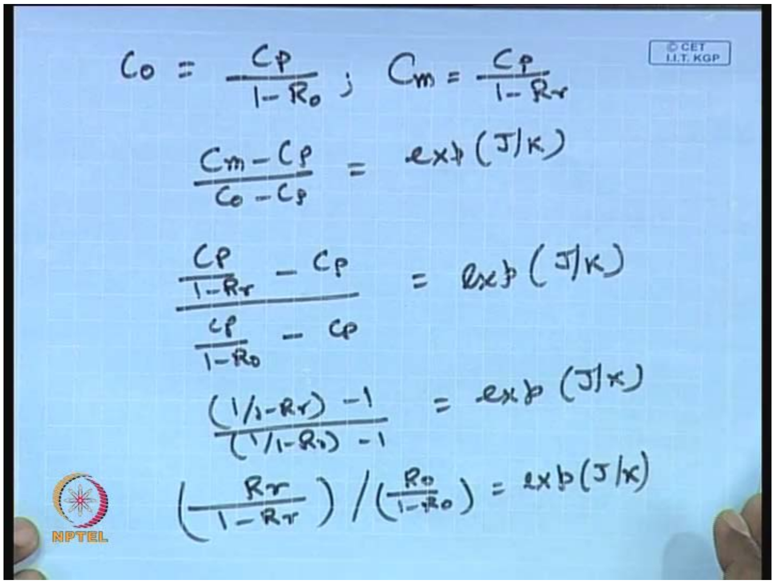

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$$C_0 = \frac{C_p}{1-R_0} \; ; \; C_m = \frac{C_p}{1-R_r}$$

$$\frac{C_m - C_p}{C_0 - C_p} = exp\,(J/K)$$

$$\frac{\frac{C_p}{1-R_r} - C_p}{\frac{C_p}{1-R_0} - C_p} = exp\,(J/K)$$

$$\frac{(1/1-R_r) - 1}{(1/1-R_0) - 1} = exp\,(J/K)$$

$$\left(\frac{R_r}{1-R_r}\right) / \left(\frac{R_0}{1-R_0}\right) = exp\,(J/K)$$

Now, we substitute this, in the expression of that means we evaluate C 0. What is C0? C0 is nothing, but C p divided by R O and what is C m? C m is nothing, but C p divided by 1 minus R r, Now, substitute these in the expression of the film theory that is C m minus C p divided by C 0 minus C p is equal to exponential J by K. If you do that what will be getting C m nothing, but C p divided by 1 minus R r minus C p C 0 will be C p divided by 1 minus R not minus C p is equal to nothing, but exponential J by K. Now, C p will be cancelled now all the terms so, what will be getting is 1 by 1 minus R r minus 1 divided by 1 by 1 minus R O minus 1 is equal to exponential J by K so, just simplify it will be getting R r divided by 1 minus R r is equal to that is in the denominator in numerator and denominator will be the simply R 0 divided by 1 minus R 0. If you take logarithm on both sides they will be logarithm and the exponential will be off.

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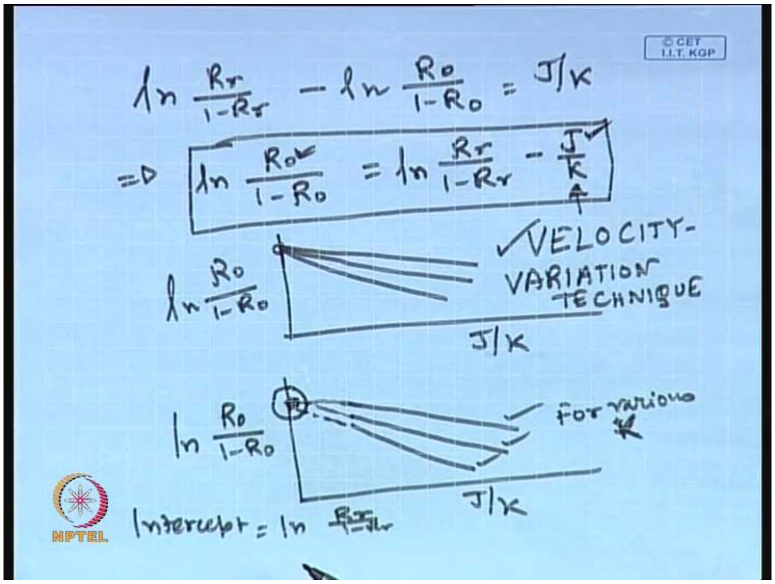

$$\ln \frac{R_r}{1-R_r} - \ln \frac{R_0}{1-R_0} = J/k$$

$$\Rightarrow \boxed{\ln \frac{R_0}{1-R_0} = \ln \frac{R_r}{1-R_r} - \frac{J}{k}}$$

If you take logarithm on both side will be l n R r 1 minus R r minus l n R 0 by 1 minus R 0 is equal to J by K so, l n R 0 divided by 1 minus R 0 will be nothing, but l n R r 1 minus R r minus J by K. Once you get the expression now, let us; see what in these expression what are the various terms? That can be experimentally measurable we can experimentally, find out the value of observed retention, because you know the feed concentration you know the you can experimentally measure, the permeate stream you can analyze the permeate stream at the permeate concentration so, you know value of observed retention real intention that is your aim to find out you know the experimental value of permeate flux, you can measure the solvent flux that is that will be known you can get the value of mass transfer coefficient; by adopting and a appropriate mass transfer relationship based on the geometric and flow region.

Now, if you vary the now what we can do? Vary the conditions what are the vary various conditions? The condition is that, you vary the cross flow velocity and the turbulence in the system either, if it is a stirred cell you vary the stirred speed, you conduct the experiment with one particular stirred speed thousand rpm measure, the permeate flux measure the permeate concentration. So, you know the R 0, you know J for the particular stirred speed now under the same delta p you change the stirred speed to let say, 15 hundred rpm conduct the experiment you know the value of J and R not like wise, you can generate a series of data and various levels of turbulence and can get the value of J and R 0.

In the cross flow system through the rectangular geometric, are a tubular 1 one can vary the the velocity cross the velocity by changing the bump setting so one can have generate, one can generate various values of you know levels of turbulence in the cross flow system as well and measure the value of J and real retention.

So, now, if you plot if you do so, that means, if you change the turbulence or velocity in a linear rectangular cross flow cell you can generate various values of R 0 and various values of J by K and if you plot l n R 0 divided by 1 minus R 0 versus J by K. You can expect the various state line depending declining slope the straight lines will be something like this; R in fact you want be getting these lines the plots look something like this l n R 0 by 1 minus R not divided by J by K it will be something like this and these curves are basically various, values of mass transfer coefficient various values of mass transfer coefficient now if you extend this curve.

If you extend this curve, you see that all these curves will basically merge to particular point, on the y axis the intercept and from the intercept what is the intercept the intercept is nothing, but l n R r divided by 1 minus R r so from that one can estimate the value of real retention since, these method will be and this method will be the accuracy of these method will be extremely depending on the how accurately will conduct the experiment.

So, because you are varying the cross flow velocity that means; you are changing the mass transfer coefficient and we are estimating the value of J and real observed retention. So, how accurately you can conduct? The experiments depending on that one can get the estimation of real retention from the intercept of this plot since these methods depend on the variation of the cross flow velocity these methods is known as the velocity variation technique.

So, by using velocity variation technique one can estimate; the real retention in this case the since, you are drawing the mean curves the in accuracy the experiments will be average doubt so, by using velocity variation technique 1 can estimate the real retention one can estimate the real retention the discussed earlier using these two methods, one can have, the value of real retention and can do the system, modeling all described earlier now will look into the various complicated systems for example if you remember we talked about the during the modeling we talked about two things are two aspects one is modeling of the flow outside of membrane another is the modeling of the flow through the porous membrane and through the porous membrane the one thing is that you have the solute transport law solution.

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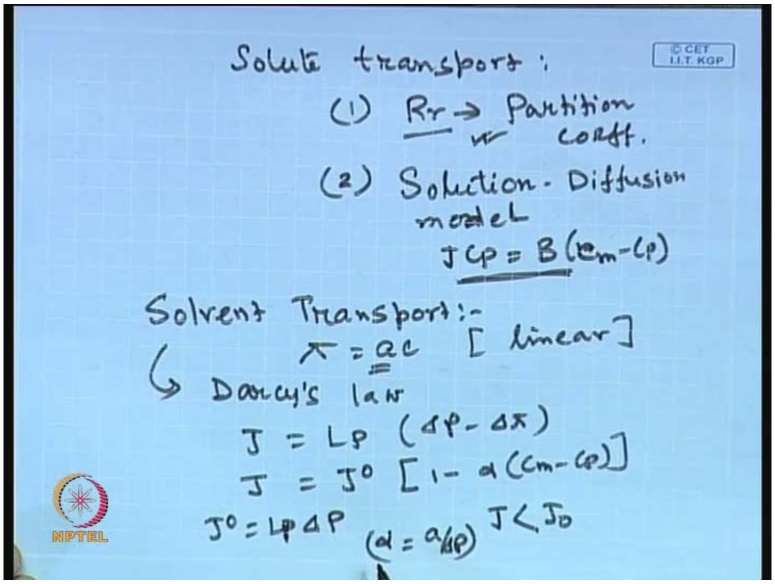


Solute transport :

(1) $\underline{R_r} \rightarrow$ Partition coeff.

(2) Solution - Diffusion model

$\underline{J C_p = B (C_m - C_p)}$

Solvent Transport :-

$\pi = \underline{\underline{a}} c$ [linear]

$\hookrightarrow$ Darcy's law

$J = L_p (\Delta P - \Delta \pi)$

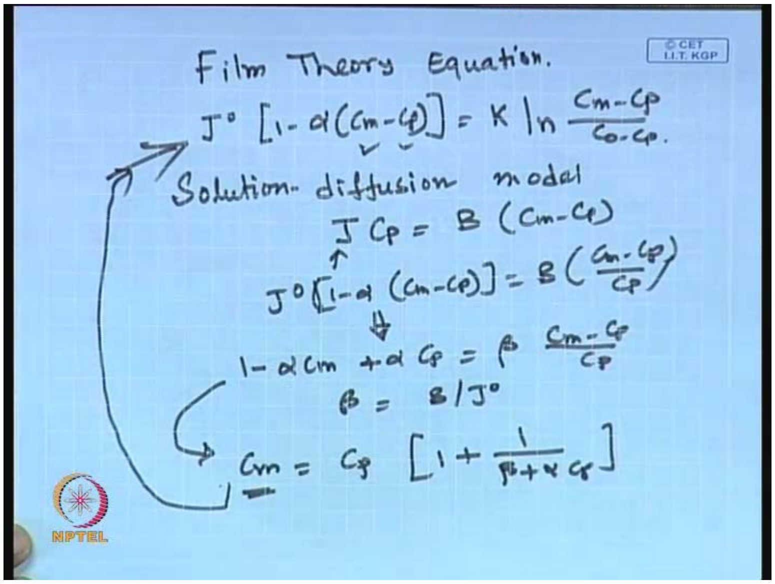
$J = J^0 [1 - \alpha (C_m - C_p)]$

$J^0 = L_p \Delta P$ $(\alpha = a/\Delta P)$ $J < J_0$

Transport law of the solvent transport laws, given by the darcys law the solute transport can be defined by two ways one is the real retention that is nothing, but a partition coefficient between the solute concentration across the membrane, surface or more realistic (()) described this is solution diffusion model, if you remember model that is J times C p is nothing, but B times 1 min C m minus C p this is solution diffusion model. So, basically either use the real retention that is very gross or are use the solution diffusion model this is really very gross, because it gives only partition coefficient it is it is adoqe definition of real retention relationship between C m and C p, but these definition is based on the physics of the system.

So, it is more realistic now, will see if we incorporate these a solution diffusion model to describe the solute transport through the membrane; how the equation will be solved? the solvent transport is still given by darcys law and for the case of shake of simplicity, assume pi is equal to a times C that is linear the same thing can be the same method can be utilized for, if you have an a polynomial relationship of osmotic pressure with concentration for the sake of demonstration and just taking a linear relationship of pi, as a function of concentration now, use the solute solvent transport darcys law J is equal to L p times del p minus del pi these gives an expression of in often non dimension analaisation J not 1 minus alpha C m minus C p what is J 0? J 0 is nothing, but L p times del p I take delta p common.

So, L p times delta p is nothing, but J not J 0 has a physical significance what is the physical significance L p? times delta p is the permeate flux when we use the use pure solvent in pure solvent that is no question osmotic pressure so, J 0 is L p delta p so, therefore, for any membrane separation system which is operating under delta p, you should expect a maximum fluoride or maximum productivity L p times delta p if we use the solution that the productivity of the permeate flux will be always less then L p times delta p. So, J 0 is the maximum permeate flux when you have expect the maximum permeate flux in your system at the very first instants before on set of polarization therefore for any J it will be less than J 0 and what is alpha? Alpha is nothing, but a times delta p what is a osmotic coefficient since, you taken delta p so, alpha is nothing, but a by delta p so alpha is non dimensional osmotic coefficient. (Refer Slide Time: 45:08)



Film Theory Equation.

$$J^0\left[1-\alpha(C_m-C_p)\right] = K \ln \frac{C_m-C_p}{C_0-C_p}.$$

Solution- diffusion modal

$$J C_p = B(C_m-C_p)$$

$$J^0\left[1-\alpha(C_m-C_p)\right] = B\left(\frac{C_m-C_p}{C_p}\right)$$

$$\Downarrow$$

$$1-\alpha C_m+\alpha C_p = \beta\,\frac{C_m-C_p}{C_p}$$

$$\beta = B/J^0$$

$$C_m = C_p\left[1+\frac{1}{\beta+\alpha C_p}\right]$$

Now, we couple these equation if the film theory equation so, what you get? is J 0 1 minus alpha C m minus C p is equal to K l n C m minus C p divided by C not minus C p now, will do not know so there is one equation that contains two unknown C m and C p and we do not know the relationship between C m and C p, in these case we are not going to use the definition real retention that will; connects C m and C p you will be using a more fundamental equation that is the solution diffusion model. So, if use solution diffusion model we get another equation that will be connecting C m and C p J times C p equal to B times C m minus C p. Now, we can combine and what is J is J can use the darcys law and we can get J not 1 minus alpha C m minus C p is equal to B times C m minus C p divided by C p. Now, these equation can be simplified to 1 minus alpha times C m plus alpha times C p is equal to beta times C m minus C p divided by C p where is beta is nothing, but B by J w J 0 so, one can get the expression of C m out of feed in favor of C p that will be, we simple C m will be equal to C p into 1 plus 1 by beta plus alpha C p.

So, in fact you can drive, all the relation in your own and can conform and now, you C m into these governing equation the film theory equation and the darcys law and can get an expression of and non linear algebraic equation on C p only now, if you substitute the expression of C m here.

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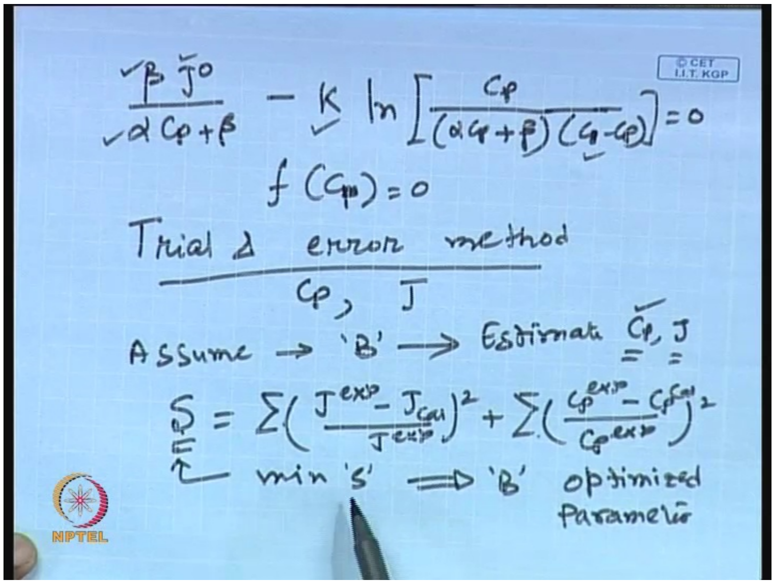

$$\frac{\checkmark\beta \, \overset{\checkmark}{J}^{0}}{\checkmark\alpha C_p + \beta} - \underset{\checkmark}{K} \ln\left[\frac{C_p}{(\alpha C_p + \beta)(\underset{\checkmark}{C_0} - C_p)}\right] = 0$$

$$f(C_p) = 0$$

Trial & error method

C_p, J

Assume → 'B' ⟶ Estimate $\overset{\checkmark}{\underline{\underline{C_p}}}$, $\underline{\underline{J}}$

$$\underline{\underline{S}} = \sum\left(\frac{J^{exp} - J_{cal}}{J^{exp}}\right)^2 + \sum\left(\frac{C_p^{exp} - C_p^{cal}}{C_p^{exp}}\right)^2$$

min 'S' ⟹ 'B' optimized Parameter

Let us, see what you get all? You get is B J w J 0 by alpha C p plus beta minus K l n C p alpha C p plus beta into C 0 minus C p is equal to 0. So, you will be getting these beta will be known alpha will be known J 0 will be known mass transfer coefficient, known feed concentration will be known so, again it will be give you give you a non linear algebraic equation in terms of C p. So, you can use the trial and error method to obtain the value of C p, once you know the value of C p you can substitute any of the equation you can get the estimation of J; if the value only one parameter if you remember, the parameter is there solute permeability through the membrane that B known to you most of the cases these B or the solute parameter for a particular membrane, is not known so, in this in .that case whatever; what is done is that? We assume a value of the assume a value of the B.

So, the now, everything known now, you estimate C p and J and compare these calculated value of C p and J for with the for various operating condition with the experimentally measured values and you create a an objective, function is as summation of J experimental minus J calculated this will be J calculated divided by J experimental square of that plus summation C p experimental minus C p calculated this will be C p calculated divided by C p experimental square.

Now, we compare with the experimental result that means, we compute the mean square difference some of all square differences now, why this is divided by J experimental? and this divided by C p experimental we can understand the numerator the basically, the difference between the experimental value and the calculated value in both the cases C p as well as J it is divided by J, because of the ordered magnitude of 2 two quantities typically the value of the solvent flux will be in the order of ten to the power of minus four to ten to the power of minus six from micro filtration to reverse osmosis on the other hand permeate concentration will be some ppm. Let say, 1 ppm 2 ppm 10 ppm something like; so, if you compute these square if you compute these square there will be mismatch in the order and will be grinding of with the wrong conclusion calculation, but if you divided by J experimental the whole thing becomes the order of 1 and that thing within these bracket will be the order of 1.

Now, there will be comparable in magnitude and one can do the minimization of these objective function s by basically you do a the this trial error method will be the actual algorithm or actual program (()) you look up with the optimizer for using, I m s library are any are matter just look up that optimizer and compute the and minimize these some s minimize s in that means you guess the value of B compute s you guess another value of B and compute s if s is further lower in case another value of B compute s likewise, you just find out minimize value of s and the corresponding value of B will be the optimized parameter what is the role of optimizer the instance a say you guess the value of B, that is the guess value compute the value of s and for the whether it is minimum or maximum of whatever it is you have another guess or another calculation.

So, guess next value of B and computes the optimizer there is in build algorithm, in the optimizer itself the next what will be the next value of the optimizer calculate itself and 1 can iterative 1 can do several iterations can land up with minimum value of s by which will be said by a tolerance value, you said a tolerance value 10 to the power of minus 1 to 10 to the power minus 2 1. It will be made to the program will be stopped and s will be giving the minimum and the value of B will be the optimized parameter so once you understand algorithm.

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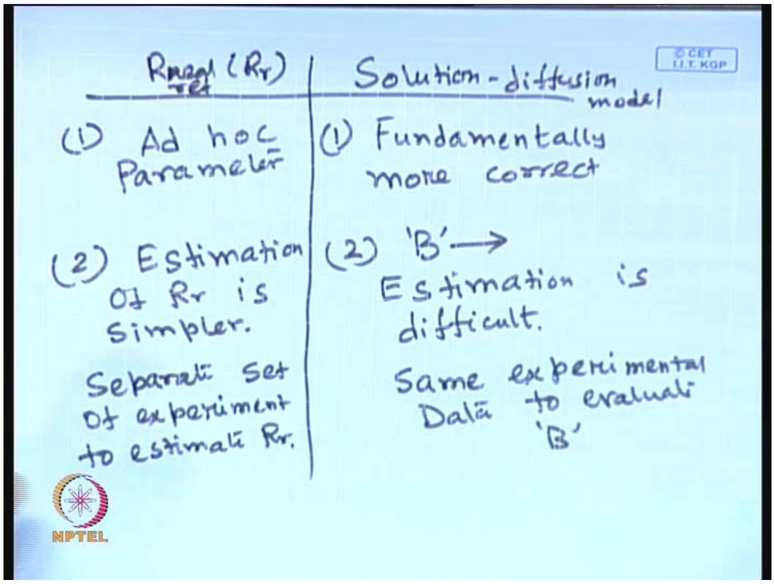

Rreal (R_r)	Solution-diffusion model
(1) Ad hoc Parameter	(1) Fundamentally more correct
(2) Estimation of Rr is simpler.	(2) 'B' → Estimation is difficult.
Separate set of experiment to estimate Rr.	Same experimental Data to evaluate 'B'

Now, let us, try to comparison between the two methods, one is the real retention R r another is the solution diffusion model this is fundamentally more correct, because it is derived from the thermodynamics, that we have derived earlier other hand this is an ad hoc parameter it is like partition coefficient which is constant, which is assumed to be constant for a particular membrane solute system on the other hand estimation of real retention is simpler and most importantly it can be estimated from a separate set of experiment, on the other hand the estimation of solution permeate solute permeability through the membrane that parameter will be, estimation is difficult why it is difficult? Because it involves the set of the experiment involve in the same solution same solute and the feed solution under the same operating condition, because you require the same experimental data to evaluate B on the other hand real retention, you require separate set experiment data to estimate real retention so these are the major difference between real retention and the solution diffusion model.

So, one can use any of these models equivalently, depending upon the system one can have the next variation, will look in to the some more variation of solution diffusion model which are more fundamentally correct, because in solution diffusion model, we have assume that the solute will be transported across the membrane by the concentration gradient only by diffusion only it may not happen; as the told in the last class that there will be imperfection in the membrane known, membrane will be having pore size of uniform distributions, there may be higher pore size is where the convective flux will be more dominant. So, there will be the extra term is the trough, will be occurring in the solution diffusion model and coupling them ,we can coupling those equation the film theory equation one can have better performance that will be discuss in the next class.