Novel Separation Processes

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Lecture No. # 04

Membrane Separation Processes (Cond..)

Good morning, every one; so, we looked into we were continuing, with membrane base separation processes and we have looked into several aspects of membrane base processes various definitions and you know basic modeling and one dimensional modeling, on so forth.

Now in this class, we look into some more definitions, and various transport. When those will be in hope in the membrane base processes. So, the first aspects of that will be looking, in to today is class fluxes; and driving forces, involved in membrane based processes.

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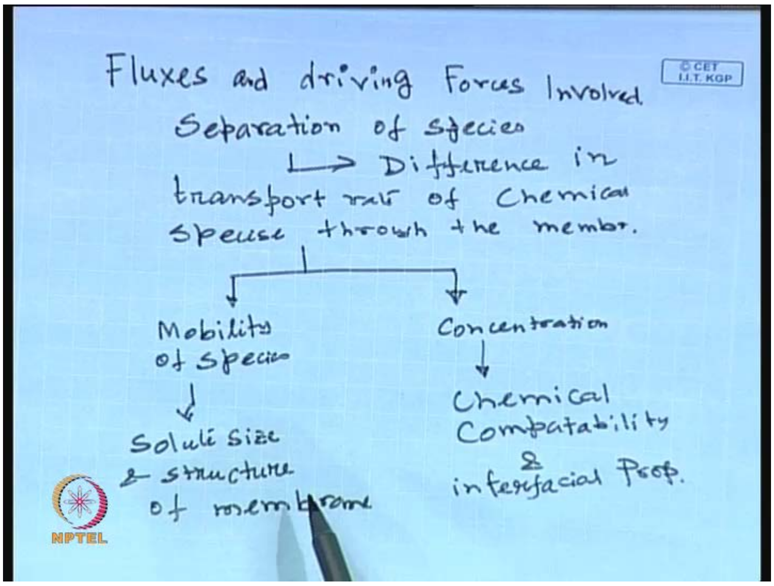

Fluxes and driving Forces Involved

Separation of species

↳ Difference in transport rate of Chemical speuse through the membr.

Mobility of species

↓

Solute size & structure of membrane

Concentration

↓

Chemical Compatability & interfacial Prop.

Now, we have to already looked into the definitions that separation of species, occurs; because of difference in transport rate; of chemical species through, the membrane these depends; on two factors one is the Mobility of the species, another is the concentration. Now, Mobility will depend; on the solute size, and structure of membrane and concentration of course, will depend on the chemical compatibility: and interfacial properties. So let us, elaborate on this now, Mobility of the species, will depend on the solute size, and structure of the membrane.

If, the solute size is small compare to the membrane size for example, you are talking about let us, say salt solution salt species: in reverse in microfiltration membrane; in microfiltration membrane. The pore size is extremely, large on the other hand. The salt content the salt is produce small, So, there will be almost convection of the salt through the membrane pores on the other hand. If, the species size is compare comparative, come to compare to the size of the membrane pores, then there will be hindered diffusion.

So, there may be hindered diffusion: there may be free diffusion that means; it is a free convection case in case of depending on the solute size and the size of the ratio of the solute size, to the pore size the mobility of species: because very important and concentration of the species through; the membrane phase will entirely depend on. The chemical compatibility and interfacial properties for example, if you have a solute which is hydrophilic in nature and if, you have the material of the polymer. Which is hydrophilic, that will be more amenable, to move through the membrane phase.

If, the solute is hydrophobic, then it will be not be a like: it will not be liking the hydrophilic in while invite of the polymer and similarly, other things also, chemical compatibility interfacial properties hydrophilicity hydrophobicity, will become very important as for as the specific transport rate of the species all constant through the membrane phase. Now, let us, look into the basic forces on the transport that means: there is a flux means there is a driving force unless until; there is a driving force, it could not have flux of any material flux means volumetric fluoride permit cross section area; normal to it.

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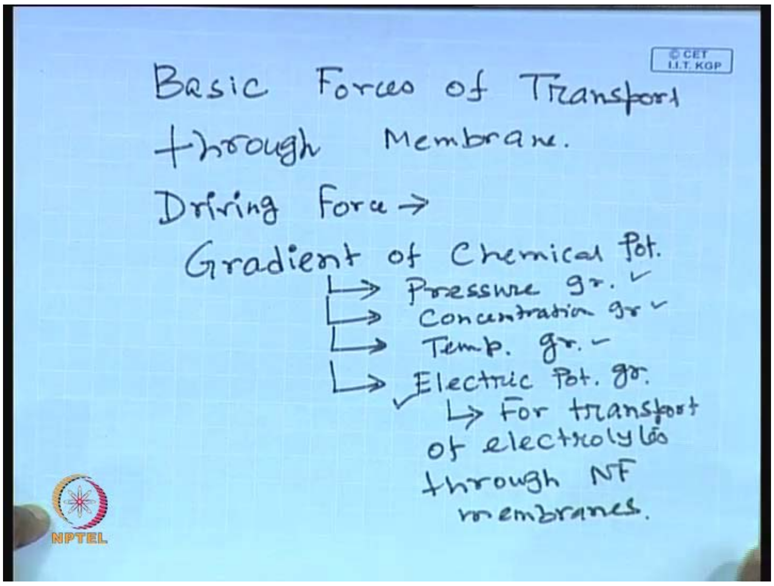

Basic Forces of Transport
through Membrane.

Driving Force →

Gradient of Chemical Pot.
- ↳ Pressure gr. ✓
- ↳ Concentration gr ✓
- ↳ Temp. gr. ✓
- ↳ Electric Pot. gr. ✓
  - ↳ For transport of electrolytes through NF membranes.

So let us, look into the various basic forces of transport through membrane phase. Now, what is the driving force? I think you are discussed; it earlier driving force will be nothing, but the Gradient of chemical potential. Now, this gradient of chemical potential will be having the three components one is the pressure difference pressure, gradient concentration gradient, and temperature gradient. You can have and electrochemical electric potential gradient.

Now, typically; chemical potential will be having three components pressure concentration and temperature for a nutria solute and if, you have a choice species for example, if, you are talking about; the transport of a salt. Which is a choice species? That means electrolyte through the nano filtration membrane; then electric potential gradient, becomes very important for transport of electrolytes; through nano filtration membranes.

Then, electric potential gradient becomes important otherwise; there will be three components, pressure concentration, and temperature: all these three will constitute; the chemical potential gradient. Now, among these three for a neutral solute now, among these three there may be one driving force, may be present there may be pressure gradient, may be present there may be concentration; gradient may be present become a temperature gradient present individually or may be two of them or three of them all together. Now, the various transport to loss through; a membranes are classified into several know classification categories.

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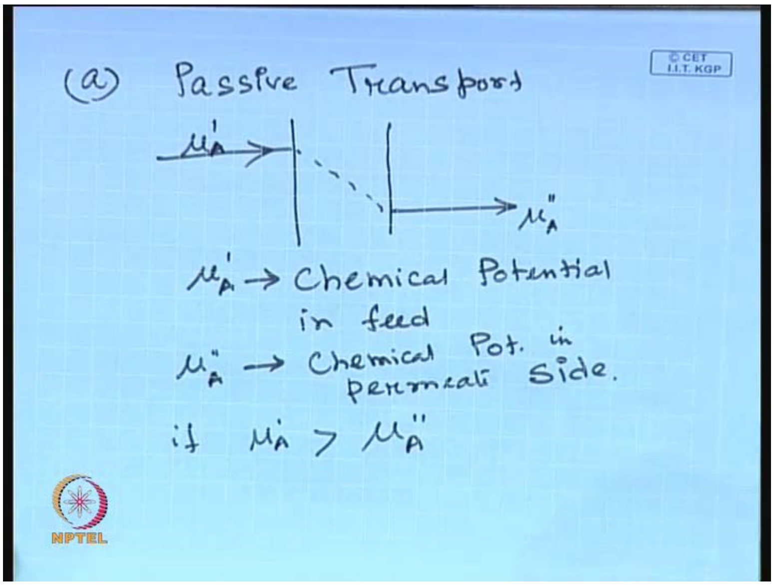

(a) Passive Transport

$\mu'_A \rightarrow$ Chemical Potential in feed

$\mu''_A \rightarrow$ Chemical Pot. in permeate side.

if $\mu'_A > \mu''_A$

The first one is Passive Transport in passive transport; the solute will be transport it through, the membrane phase by electrochemical by by by by gradient of chemical potential, and mu A is mu A prime is chemical potential in fixed: it and mu A double prime is chemical potential in permeate side, on downstream. If, mu A prime is greater than mu A double prime then; you call it a passive transport these are normal: case the driving force the driving force is more in the upstream; compare to a downstream. So, there will be a transport of species from the upstream to the downstream. So, that is called the passive transport and it is one and more: circumstances.

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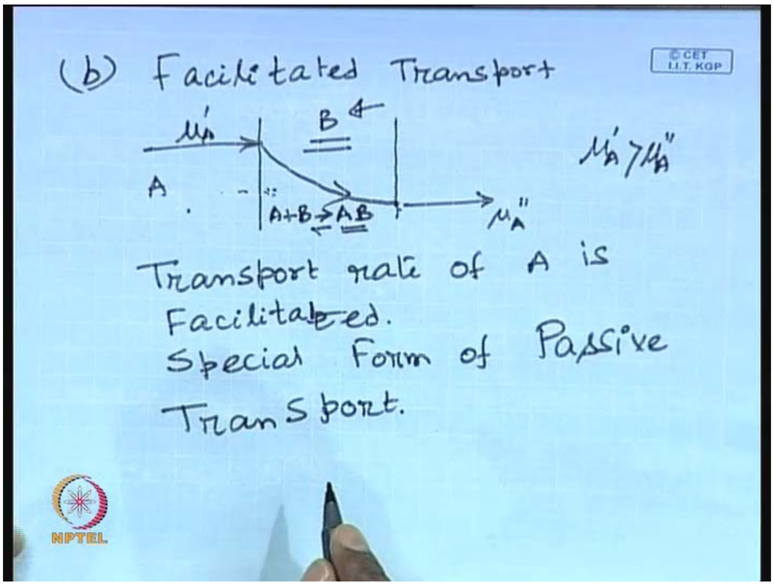

(b) Facilitated Transport

$\mu'_A > \mu''_A$

Transport rate of A is Facilitated.

Special Form of Passive Transport.

Now, second one is facilitated transport in this case also the transport is from higher chemical potential; to lower potential chemical potential, but the transport is facilitated by an external agent or third agent present, in the: membrane phase for example, the species a has to be transported; from the feed side to the permeate side. The chemical potential in the feed side is mu A prime and that is, permeate side is mu A double prime and of course, mu A prime is greater than mu A double prime, but the point is there is a an agent present agent B is present, in the membrane phase.

Which forms A complex A plus B with the permeating species A in the form A complex A B and that will be transported; So, hear the transported; will be more why because when A is coming into the membrane phase; it forms A complex A B therefore, the concentration of the species a in the membrane phase will be immediately 0. So, it maintains a maximum concentration, difference between the membrane phase and feed phase therefore, more a will be coming into the membrane phase; the transport rate will be enhanced. Now, in the downstream there is no a present at all in the membrane phases.

So, there exist state stiff concentration gradient of species a from; the from the in the membrane phase in the polymer phase, from the fixed side to the permeate site. Because of this gradient this concentration driving force; the species will be transported to us: the permeate side, and in the permeate side there is no a present. So, again it will be discard basically; A B the complex A B again, breaks down be the reversible 1 the complex A B breaks down and the species a get transported on the permeate side; because of the presents of this external agent B in the membrane phase.

The transport rate transport rate of a is facilitated, the selection of the species B is such that it has a higher it cannot go into the feed stream or it cannot go, into the permeate stream. It will be within the polymer, phase. So, this is known as the facilitated transports. So, it is a special form of passive transport the third transport mechanism is known as the Active Transport Mechanism.

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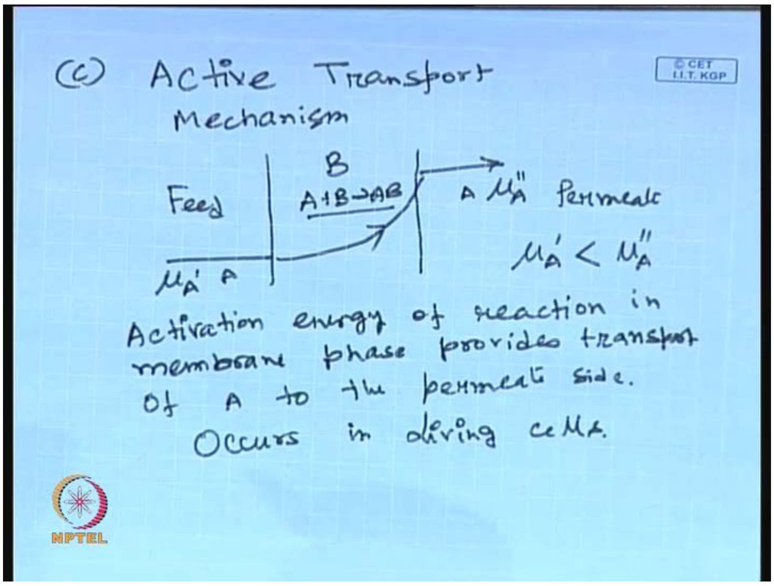

(c) Active Transport
Mechanism

$\mu_A' < \mu_A''$

Activation energy of reaction in membrane phase provides transport of A to the permeate side.
Occurs in living cells.

In this case the species is getting transported against; it is driving force. So, that means the in the feed side the chemical potential; will be lower compare to the chemical potential in the permeate side. So, mu A prime is much less compare to the mu A double prime, how can it happening, the species a getting transported from the feed to the permeate side here, also there is a an agent present within the membrane phase and there is a reaction going on A B A plus B going to A B. Now, activation energy, of the chemical reaction; provides the you know it provides, the driving force to transfer activation energy of reaction in membrane phase provides.

You know energy to transport of a to the permeate side these occurs in most of the living cells living cells. So, once we identify the so, these are the various transport laws. Which will be mechanism which will be encountering in membrane base processes in our course will be basically; looking in to the first two transport laws; one is the facilitated transport; another is the passive transport. Now, we can describe the transport process by phenomenological equation.

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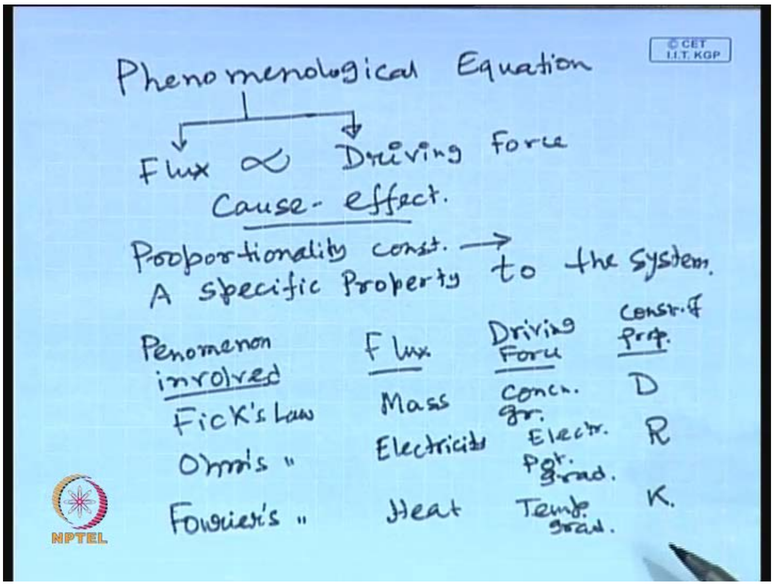

Phenomenological Equation

Flux $\propto$ Driving Force

Cause - effect.

Proportionality const. → A specific Property to the system.

Penomenon involved	Flux	Driving Force	Const. of Prp.
Fick's Law	Mass	Concn. gr.	D
Ohm's "	Electricity	Electr. Pot. grad.	R
Fourier's "	Heat	Temp. grad.	K.

Now, phenomenological equation is very important this equation gives a relation between the flux and driving force. This is also known as, cause effect relationship these; cause effect relationship only says, that there is a flux of the species. If, there is a driving force present in the system say it is basically, the natural phenomena that is, why these equations are called phenomenological equations and there are there basically; proportional flux is proportional; to the driving force. Now, the proportionality; constant becomes very important and it is a property it is a specific property to the system that we are studying, now you know already; studied very as phenomenological equations.

I will just give some examples, and try to summarize, these phenomena phenomenon involved flux driving force constant or property of the system. The first phenomenon you know is fick's law the flux is mass flux and what is the driving force concentration? difference concentration gradient and what is the constant or proportionality? Is diffusivity second one is ohm's law the flux is basically, electricity charge or unit time. So, it is basically; current and driving force is basically, electric potential difference gradient and one is the constant and proportionality, it is the resistance, third one is Fourier's law the flux is heat flux driving force, is temperature gradient and constant of properties thermal conductivity.

Now, these properties of the proportionality constant becomes the property of the particular system. It becomes the constant property, the particular system diffusivity resistivity and thermal conductivity. In membrane based separation process, is we have the several driving forces for example, main driving force is constant is chemical potential. What chemical potential is an abstracting it has generally, three components for a neutral solute, as we have to already; discussed earlier concentration pressure and temperature. Now, one of these two the driving forces may be interdependent or they may be absolutely; independent.

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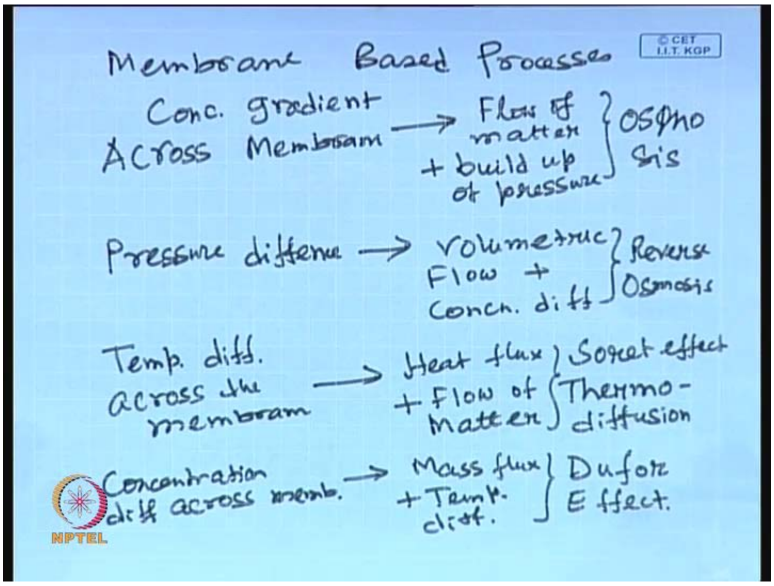

Membrane Based Processes

Conc. gradient Across Membram → Flow of matter + build up of pressure } Osmosis

Pressure differene → volumetric Flow + Conch. diff } Reverse Osmosis

Temp. diff. across the membram → Heat flux + Flow of matter } Soret effect Thermo-diffusion

Concentration diff across memb. → Mass flux + Temp. diff. } Dufor Effect.

Now, let us, looking to the cause effect relationship in membrane based processes concentration gradient across membrane, gives to flow of matter. That has be it will be getting a mass flux if, there is a concentration gradient, but at the same time it gives the it results into a buildup of pressure. It, results the buildup of pressure and there is, a difference in pressure of pressure gradient will exist because whole the pressure gradient will exist because of the concentration gradient more matter more species will be transported to us, the membrane it into a high concentration of the solutes on the membrane surface compare to the bulk.

Now, high consent high concentration of the solute the membrane surface means it is, osmotic pressure will be more because being a caligative property, that there will be difference in pressure between the bulk and the membrane surface. So, these phenomenon is known as and the pressure is known as the osmotic pressure and this phenomena is known as osmosis a pressure difference leads to volumetric flow plus concentration difference, this is the case of reverse osmosis that we have already; discussed temperature difference across the membrane obviously; the temperature difference will give a heat flux plus as well as flow of matter. That simply; means; if, you do not do anything, if, you just maintain two different temperatures in the feed side as well as, the permeate side, there will be a flow of matter.

The solvent will be moving from the feed side to the permeate side along with heat flux there is a temperature difference that, will be the heat flux because by the Fourier's law this combined effect is known as the thermal diffusion or effect, is known as the thermal diffusion or effect similarly, concentration difference across the membrane will gives a mass flux plus. The temperature difference these is, opposite to the so, ret effect an it is known as du for effect. So, in membrane base process; is there may be, is several concentration, in a driving force is present, and they will lead to several phenomenon; which may be interdependent or dependant.

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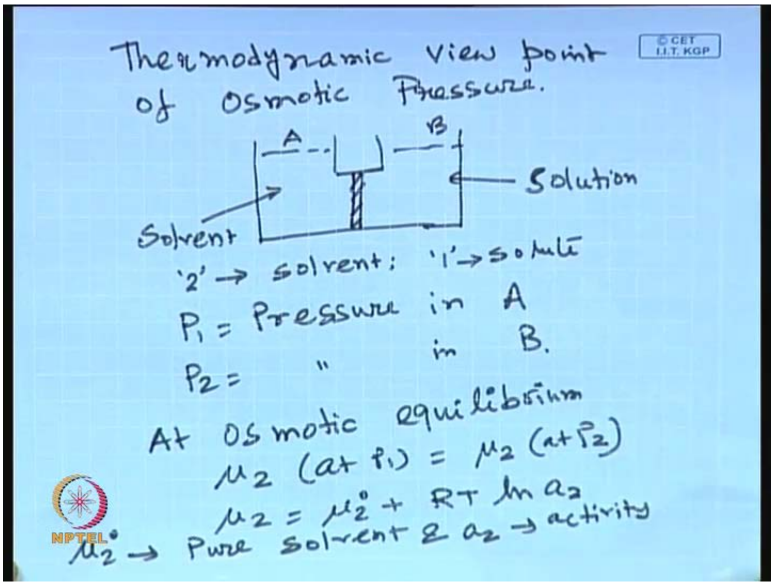

Thermodynamic view point of Osmotic Pressure.

'2' → solvent: '1' → solute

P_1 = Pressure in A

P_2 = " in B.

At Osmotic equilibrium

$\mu_2 \text{ (at } P_1) = \mu_2 \text{ (at } P_2)$

$\mu_2 = \mu_2^0 + RT \ln a_2$

μ_2^0 → Pure solvent & a_2 → activity

So let us, look into the osmotic pressure thermodynamic. The from the thermodynamic angel the definition of osmotic pressure. We will do the same example, we will take a same example, that we have to take a same example, that we have to taken in the last two classes that will be having two chambers separated by a semi permeable barrier or a semi permeable membrane the side a is pure solvent and chamber. B will be having a solution.

So let us, use this notation 2 for solvent and 1 for solute P 1 is equal to pressure in chamber a and P 2 is equal to pressure in chamber B. Now, at osmotic equilibrium the chemical potential of the solvent is equal on both the chambers. Now, these equation can be written as mu 2 is equal to mu 2 not plus R T l n a 2 well mu 2 not is for pure solvent chemical potential for pure solvent and a 2 is solvent activity. Now, we will do a hypothetical experiment, what do you do in the one of the in one of the cells; we puts some sodium chloride and see what happens.

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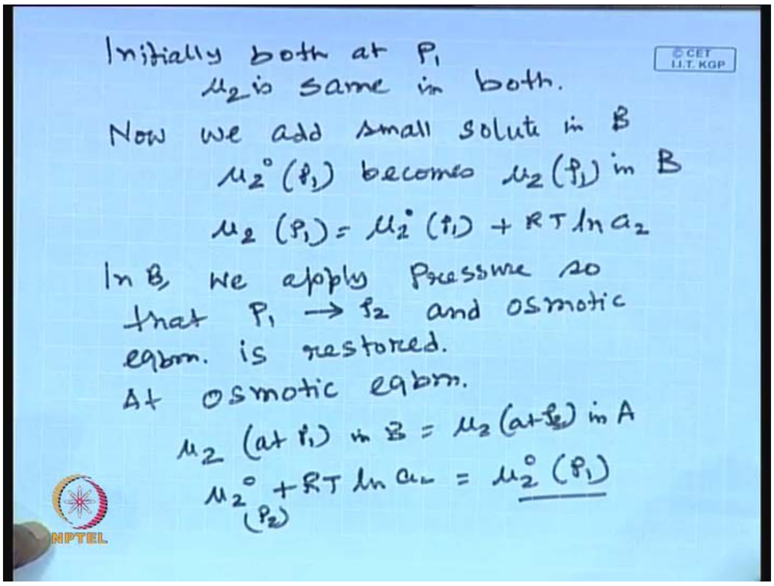

Initially both at P_1

μ_2 is same in both.

Now we add small solute in B

$\mu_2^{\circ}(P_1)$ becomes $\mu_2(P_1)$ in B

$$\mu_2(P_1) = \mu_2^{\circ}(P_1) + RT \ln a_2$$

In B, we apply pressure so that $P_1 \rightarrow P_2$ and osmotic eqbm. is restored.

At osmotic eqbm.

$$\mu_2 \text{ (at } P_1\text{) in B} = \mu_2 \text{ (at } P_2\text{) in A}$$

$$\underset{(P_2)}{\mu_2^{\circ}} + RT \ln a_2 = \underline{\mu_2^{\circ}(P_1)}$$

So, initially both the chambers both, at pressure P 1 and mu 2 is same in both the chambers, because there both, pure solvent pure water mu 2 is same in both that means initially. We add pure solvent on both the chambers, now we add a small amount of solute in chamber B. So, B becomes a solution now, it remains along the solvent it becomes a solution and mu 2 0 P 1. It was pure solvent; so, it is mu 2 0 at P 1 becomes mu 2 at P 1 in chamber B is still keep the pressure P 1 in chamber B. Now, mu 2 at P 1 becomes mu 2 0 P 1 plus R T l n a 2. Now, in B we apply pressure that so, that P 1 becomes P 2 and osmotic equilibrium is restored.

While we are applying pressure, because in when you are applying when you are adding salt in chamber B. More solvent will be coming from because solvent concentration is less their more solvent will be coming from the solvent side to the solution chamber from a to B. Now, your are applying pressure in chamber B and raise it pressure. So, that the solvent flux will be arrested and equilibrium of the an equilibrium will be restored that is, known as the osmotic equilibrium therefore, at osmotic equilibrium. You can write mu 2 at P 1 in B is equal to mu 2 at P 2 in a so, these becomes mu 2 not plus R T l n a 2 mu 2 not at P 2 at l n a 2 is equal to mu 2 not at P 1 because in chamber in chamber P 1 we had a pure solvent.

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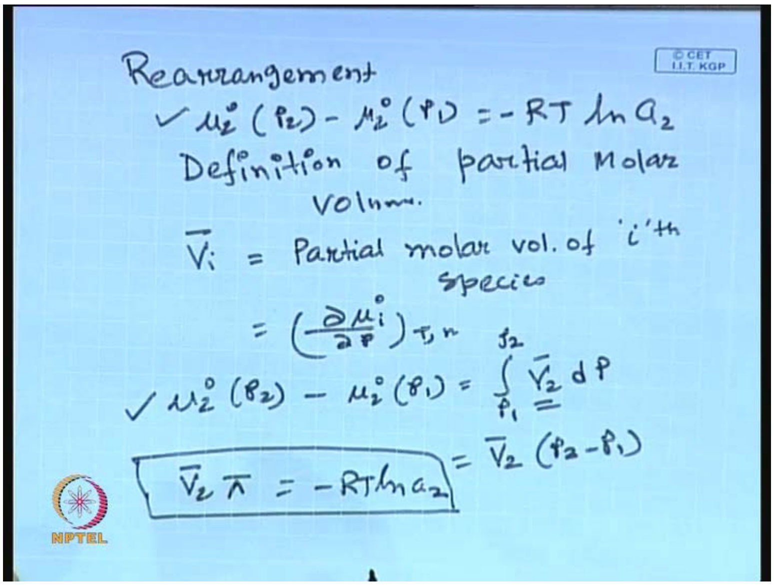

Rearrangement

✓ $\mu_2^\circ(P_2) - \mu_2^\circ(P_1) = -RT \ln a_2$

Definition of partial Molar volume.

$\bar{V}_i$ = Partial molar vol. of 'i'th species

$= \left(\frac{\partial \mu_i^\circ}{\partial P}\right)_{T,n}$

✓ $\mu_2^\circ(P_2) - \mu_2^\circ(P_1) = \int_{P_1}^{P_2} \bar{V}_2 \, dP = \bar{V}_2 (P_2 - P_1)$

$$\boxed{\bar{V}_2 \pi = -RT \ln a_2}$$

Now, we rearrange this equation and rearrangement results mu 2 0 at P 2 minus mu 2 0 at P 1 will be equal to minus R T l n a 2. Now, use the we invoke the definition of partial molar volume what is the definition V i? Bar is basically; partial molar; volume of I the species there nothing, but del mu i not del p at constant temperature and concentration. So, these equation can be written as mu 2 not P 2 minus mu 2 not P 1 is equal to P 1 indication from P 1 to P 2 V 2 bar d p.

Now, we assume that 2 that the molar volume does not change over the range of pressure P 1 to P 2 that means V 2 bar can be treated as constant. So, it will be nothing, but V 2 bar at a P 2 minus P 1.If, it is treated as constant within the small region therefore, now we can combine these equation and P 2 minus P 1. This is nothing, but the osmotic pressure because we are restoring the osmotic equilibrium. So, that combine these equation and these equation will be getting V 2 bar time pi is equal to minus R T l n a 2. So, these will gives a direct relationship of osmotic pressure with the activity of the solvent and temperature and now, of course, the molar volume of the solvent.

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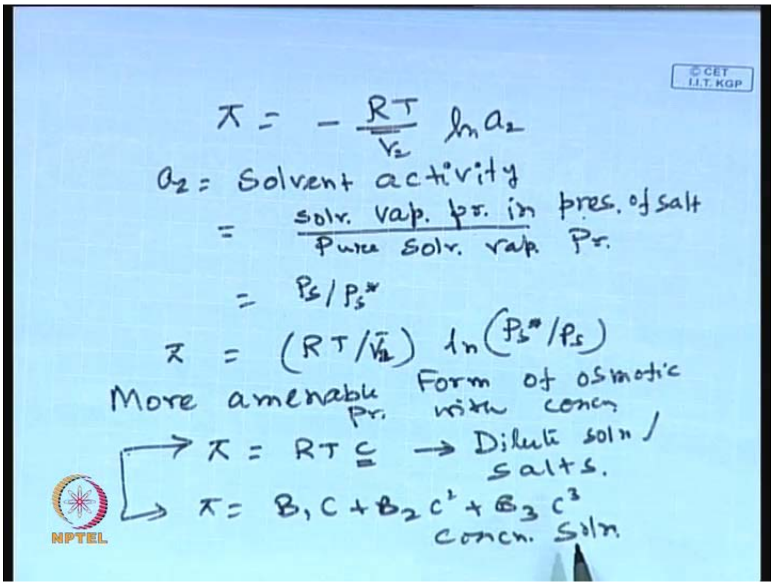

$$\pi = -\frac{RT}{\bar{V}_2} \ln a_2$$

a_2 = Solvent activity

$$= \frac{\text{solv. vap. pr. in pres. of salt}}{\text{Pure solv. vap. Pr.}}$$

$$= P_s / P_s^*$$

$$\pi = (RT/\bar{V}_2) \ln (P_s^*/P_s)$$

More amenable form of osmotic pr. with concn

$\to \pi = RT\underline{\underline{C}} \to$ Dilute soln / salts.

$\to \pi = B_1 C + B_2 C^2 + B_3 C^3$ concn. Soln

And pi becomes minus R T by V 2 bar l n a 2 so, what is a 2 a 2 is basically; solvent activity these nothing, but solvent vapor pressure in presence of salt and these pure solvent vapor pressure. So, it will be having p s divided by p s star and pi becomes R T by nu 2 bar l n p s star by p s .Now, it is not all is you know desirable are you will be getting the data of solvent vapor pressure in presence of salt and pure solvent vapor pressure difference temperatures and it will entirely depending on salt concentration.

So, and forth therefore, more amenable form of osmotic pressure with concentration because solvent vapor pressure in presence of salt is entirely; a function of the salt concentration. So, it becomes very difficult, so, directly, there is some amenable form of osmotic pressure with concentration or given pi is equal to R T C for here, C is the concentration in molar unit molar concentration. These for the dilute solution or for the salts and pi becomes B 1 C plus B 2 C square plus B 3 C cube for concentration solution of proteins. So, and so forth and these collaborates, the fact that osmotic pressure is a caligative properties and it is a function of concentration: and we have discussed in detail how know what are the range of applicability: of linear relationship of pi reverse C like R T C and non-linear relationship in as a function of polynomial with concentration in probably in last class. Now, once we identify, that osmotic pressure becomes a vital factor or parameter, in case of reverse osmosis. Where the pore size is extremely; small the smallest possible pore size. We can talk about in membrane base separation processes: is for example, sometimes.

It if the pore sizes are so, small it is 2 to 10 Armstrong the pore size. So, small sometimes the reverse osmosis membranes are called non porous impervious membranes non porous membranes. Sometimes you can come across in literature, non porous membranes means is basically you talking about reverse osmosis membrane or gas permeation membrane. Now, will look into several, osmotic pressure becomes very important: as far as the transport of reverse osmosis is concerned.

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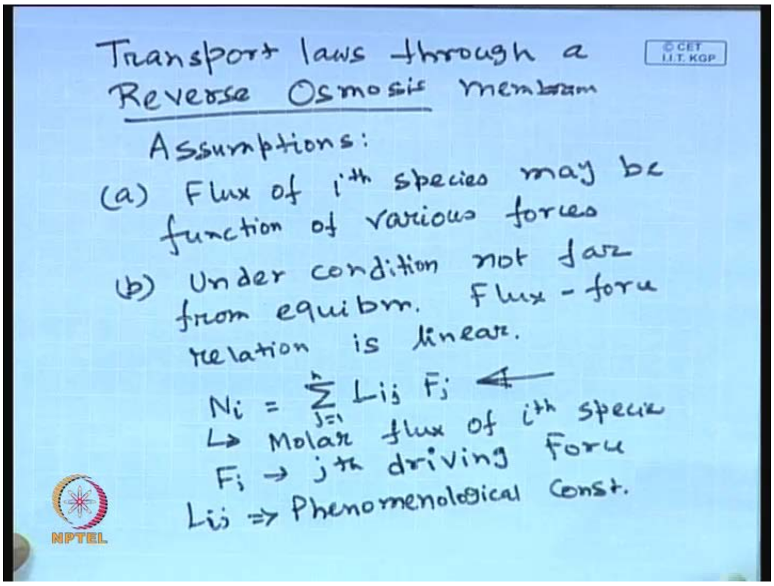

Transport laws through a
Reverse Osmosis membrane

Assumptions:

(a) Flux of i^{th} species may be function of various forces

(b) Under condition not far from equibm. Flux - force relation is linear.

$$N_i = \sum_{j=1}^{n} L_{ij} F_j \longleftarrow$$

N_i → Molar flux of i^{th} species

$F_j \rightarrow j^{th}$ driving Force

$L_{ij} \Rightarrow$ Phenomenological Const.

Now, we will look into the transport rates or transport, laws through reverse osmosis membrane first, we talk about, the transport of solvent and solute both. Now, there are certain assumptions equally; start looking into the theory. What are the assumptions flux? of i th species what are the i th species difference species?. One is the solvent another is the solute i th species may be a function, of various forces present in the system number 1 number 2.

Under condition not far from equilibrium, flux force relationship is linear therefore, we can write N i is the flux of molar flux of N i species will be a summation of L i j and F j N i is flux molar flux of i th species F j will be the j th driving force and these summation is over j 1 to N. Let us, a an number of driving forces are present may be concentration difference pressure difference so, and both and L i j L i j are basically; the proportionality, constants: and more often they are known as phenomenological, constant because these relationship is derived from the phenomenological equation that, I discussed in the start of the class.

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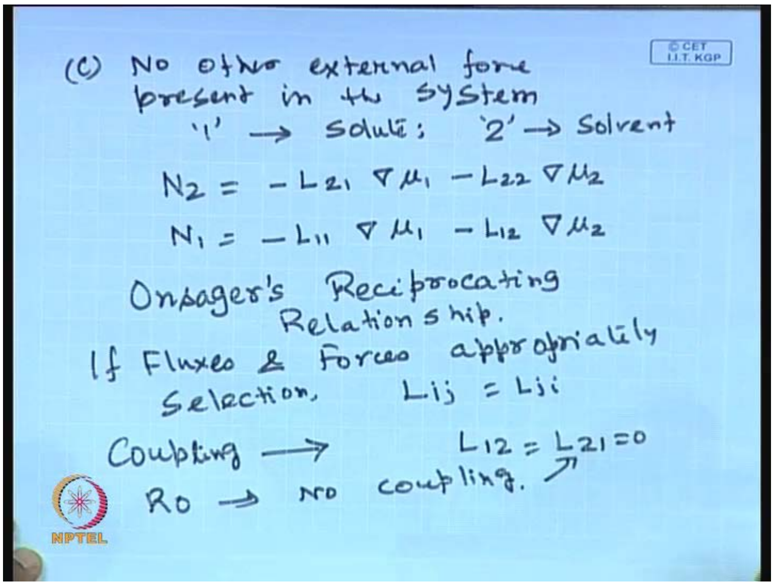

(C) No other external force present in the system

'1' → solute; '2' → solvent

$N_2 = -L_{21}\nabla\mu_1 - L_{22}\nabla\mu_2$

$N_1 = -L_{11}\nabla\mu_1 - L_{12}\nabla\mu_2$

Onsager's Reciprocating Relationship.

If Fluxes & Forces appropriately selection, $L_{ij} = L_{ji}$

Coupling → $L_{12} = L_{21} = 0$

RO → no coupling.

And we another assumption is that, we assume no other external force present; in the system to influence the transport of the species. Now, let us, write one for the solute and 2 for the solvent therefore, the solvent flux can be written us minus L 2 1 gradient of mu 1 that is, the potential gradient minus L 2 2 radiant of mu 2 the force is nothing, but minus of gradient, right therefore, N 1 can be written us minus L 1 1 gradient of mu 1 minus L 1 gradient of mu 2. Now, there is something called Onsager's reciprocal principle reciprocating; relationship this is that if, fluxes and forces; are approximately; selected if, fluxes and forces; are appropriately;.

Selected per means for proper selection of the flux and force L i j is equal to L j i that means; case coefficient are equal and will assume that and there is, another definition is for coupling says, coupling is basically; the influence of a driving force on the species of the other; That means the influence of a driving force that is, acting on species: to that may influence. The transport of species one that is called coupling are relationship between the two and we assume that in case of reverse osmosis: there is no coupling that means, L 1 2 and L 2 1 will be equal to 0 in this case. So, if, you assume that is no coupling or coupling will be extremely small.

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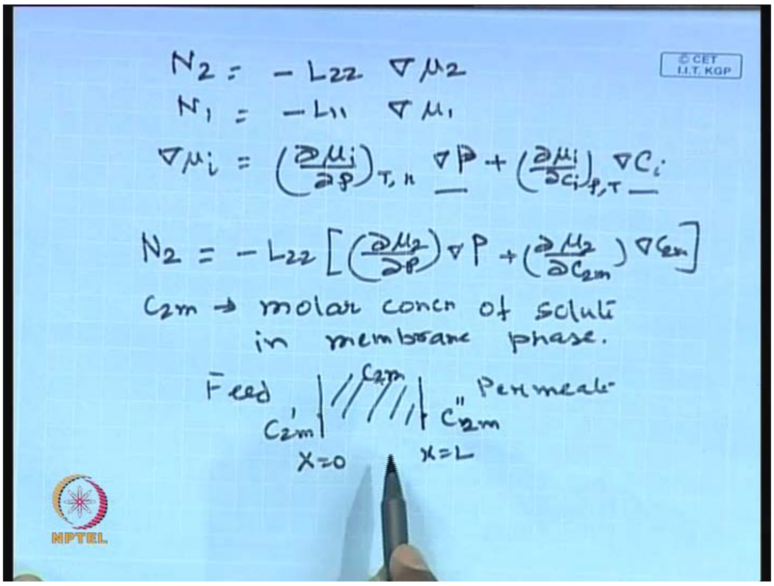

$$N_2 = -L_{22}\ \nabla\mu_2$$

$$N_1 = -L_{11}\ \nabla\mu_1$$

$$\nabla\mu_i = \left(\frac{\partial\mu_i}{\partial p}\right)_{T,n} \underline{\nabla p} + \left(\frac{\partial\mu_i}{\partial C_i}\right)_{p,T} \underline{\nabla C_i}$$

$$N_2 = -L_{22}\left[\left(\frac{\partial\mu_2}{\partial p}\right)\nabla P + \left(\frac{\partial\mu_2}{\partial C_{2m}}\right)\nabla C_{2m}\right]$$

$C_{2m} \rightarrow$ molar concn of solute in membrane phase.

In that case the solvent and solute fluxes becomes N 2 is nothing, but minus L 2 2 gradient of mu 2 N 1 becomes minus L 1 1 gradient of mu 1 and if, you look into the definition of chemical potential. Definition of chemical potential gradient, of mu 1 is nothing, but del mu i del p constant temperature and species concentration: gradient of p plus del mu i del C i p and t gradient of C i. Now, there should be one more term there is the gradient of temperature, but most of the reverse osmosis and membrane based processes they occur under isothermal conditions.

So, temperature gradient will be negligible is small are there will not be present at all these there will be there will be may be another term that is, for the electrochemical potential gradient. If, you talk about motion on the transport: of the choice solute in a charged in viralment for example, motion of the transport or electrolyte in case of charged nano filtration membrane otherwise, there are two main driving forces at present is delta p and delta gradient of pressure and gradient of concentration.

Now, in terms of this the solvent gradient can be written as minus L 2 2 del mu 2 del p gradient of p plus del mu 2 dell C 2 m gradient of C 2 m. Now, what is C 2 m C 2 m is basically; the molar concentration of solute, in barrier that means in membrane phase? If, you remember we are writing the transport; law of the solute and solvent within the membrane phase in our system will be having a membrane phase, a feed side these a permeate side. So, these x is equal to L x is equal to 0 and x is equal to L. What is L l is the thickness of the membrane or barrier and C 2 m is the concentration over hear in the membrane phase that means; concentration of the solvent in the membrane phase and you will be having a concentration in the upstream C 2 m prime and concentration of solvent in the downstream C 2 m double prime that is, at the interface of the bulk at the membrane.

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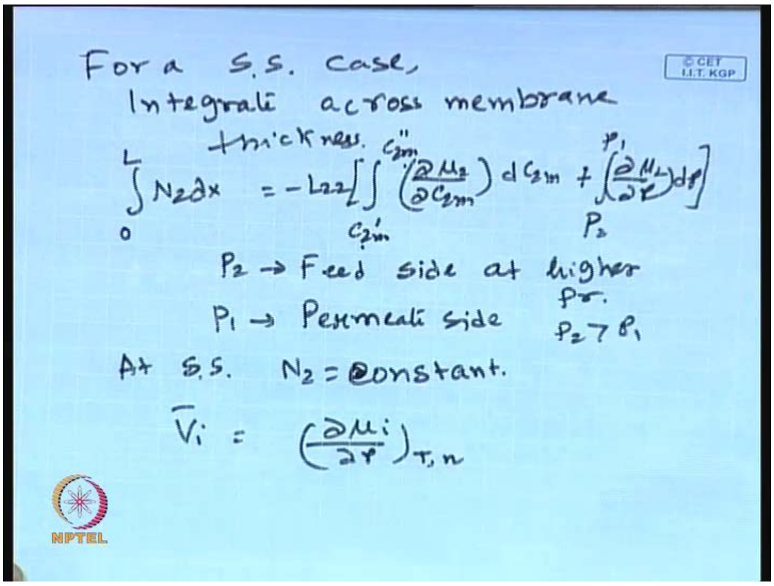

For a S.S. case,
Integrate across membrane thickness.

$$\int_0^L N_2 \partial x = -L_{22}\left[\int_{C_{2m}'}^{C_{2m}''}\left(\frac{\partial \mu_2}{\partial C_{2m}}\right) dC_{2m} + \int_{P_2}^{P_1}\left(\frac{\partial \mu_2}{\partial p}\right) dp\right]$$

$P_2 \rightarrow$ Feed side at higher pr.

$P_1 \rightarrow$ Permeate side

$P_2 > P_1$

At S.S. $N_2 =$ Constant.

$$\bar{V}_i = \left(\frac{\partial \mu_i}{\partial p}\right)_{T, n}$$

So, this is, basically; studied case the equation can be integrated across the integrate the aulor’s equation across membrane thickness. What you will get N 2 d x is equal to minus L 2 2 gradient of mu mu 2 del mu 2 del C 2 m d C 2 m from C 2 m prime to C 2 m double prime from the feed to the permeate side that means, from x is equal to 0 to x is equal to L plus these should be bracket here plus del mu 2 del p d p P 2 to P 1 P 2 is the feed side at higher pressure; these reverse osmosis membrane; and P 1 is pressure these reverse osmosis membrane, and P 1 is pressure; in the permeate side permeate side pressure and P 2 is greater that P 1. Now, we invoke at the steady state N 2 will be equal to N 2 is constant and now, invoke the definition of partial molar volume V i bar is equal to del mu i del p for constant temperature; and species concentration. So, what will be doing will be integrated, out and see the result what we get.

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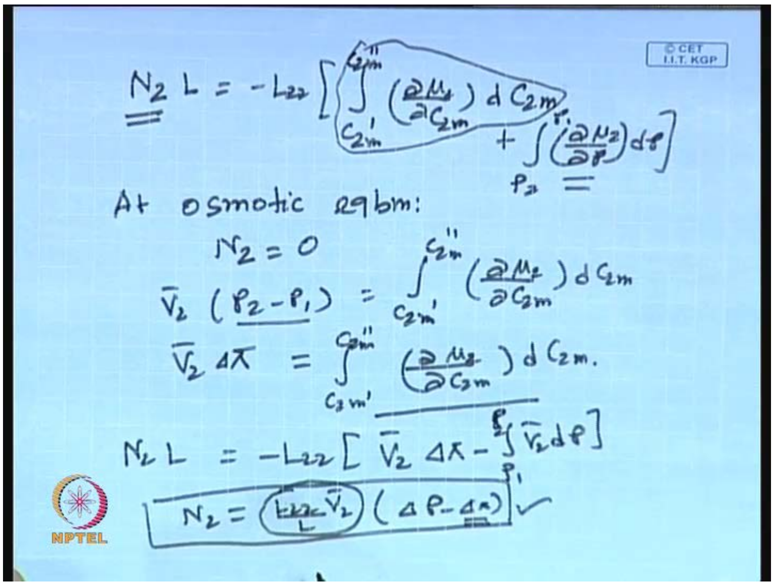

$$\underline{\underline{N_2}}\, L = -L_{22}\left[\int_{C_{2m}'}^{C_{2m}''}\left(\frac{\partial \mu_2}{\partial C_{2m}}\right) dC_{2m} + \int_{P_2}^{P_1}\left(\frac{\partial \mu_2}{\partial P}\right) dP\right]$$

At osmotic eqbm:

$$N_2 = 0$$

$$\bar{V}_2\,(\underline{P_2 - P_1}) = \int_{C_{2m}'}^{C_{2m}''}\left(\frac{\partial \mu_2}{\partial C_{2m}}\right) dC_{2m}$$

$$\bar{V}_2\,\Delta\pi = \int_{C_{2m}'}^{C_{2m}''}\left(\frac{\partial \mu_2}{\partial C_{2m}}\right) dC_{2m}.$$

$$N_2 L = -L_{22}\left[\bar{V}_2\,\Delta\pi - \int_{P_1}^{P_2}\bar{V}_2\, dP\right]$$

$$\boxed{N_2 = \left(\frac{L_{22}\bar{V}_2}{L}\right)(\Delta P - \Delta\pi)} \checkmark$$

NPTEL

We get after integration N 2 L minus L 2 2 integration C 2 m prime to C 2 m double prime del mu 2 del C 2 m d C 2 m plus del mu 2 del p d p from P 2 the P 1 bracket in .Now, let us, look into the saw the relationship; at osmotic equilibrium, what happens at osmotic equilibrium. We have found out that the solvent will not be passing through permeating from the feed side to the solution side that means; the solvent flux will be equal to 0 because it is a equilibrium the solute is not allowed, to pass at all the solvent flux will be equal to 0.

So, what does that gives that will give as? If, you put the if you put the value of del mu 2 del p as know V 2 bar the molar volume and we are assuming that pressure difference P 1 and P 2 molar volume does not change from this our equation, what will be getting is that V 2 bar P 2 minus P 1 will be nothing, but integral C 2 m prime to 2 m double prime del mu 2 del C 2 m d C 2 m. So, and what is this P 2 minus P 1 this is nothing, but the osmotic pressure difference.

So, this is basically; the osmotic pressure difference So, this is basically; the osmotic pressure difference between the permeate and feed C 2 m prime to C 2 m double prime del mu 2 del C 2 m d C 2 m. So, these equation gives the closure; to the our to the to the integration. Now, you can substitute the value of these quantities, in the governing equation and C. What you get now, if, you do that that means, I am going to substitute these you know the whole term these equation by V 2 bar times delta pi that gives a closure. So, N 2 L will be nothing, but minus L 2 2 V 2 bar times delta pi minus integration P 1 to P 2 V 2 bar d p. Now, we assume that V 2 bar is constant about the pressure range P 1 to p1 to P 2. So, it is becomes L 2 2 V 2 bar divided by l n 2 becomes these nothing, but delta p minus delta pi just inserted.

So, which will gives a within a algebraic manipulation will be getting the these expression. So, what these expression presents these expression gives a relationship of the solvent flux through the membrane. It is proportional to delta p to minus delta pi what is this delta pi delta? pi is basically; the osmotic pressure build up that will be that has to be that has to be overcome; by using the your operating pressure and whole this constants there whole thing this becomes, a constant for particular, membrane for a particular membrane the thickness is constant L 2 2 is the phenomenological constant V 2 bar is the molar volume of the solvent and if, you so, what is the what is the unit of N 2

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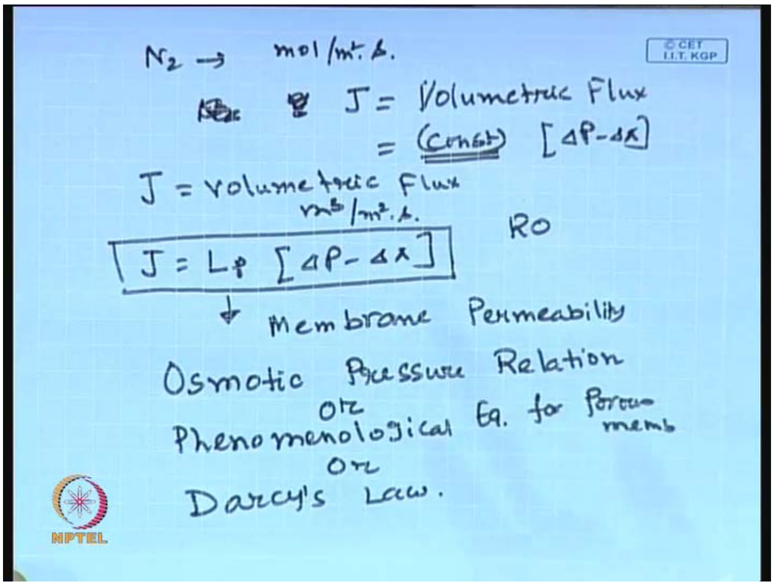

$N_2 \rightarrow$ mol/m^2.s.

$J =$ Volumetric Flux

$= \underline{\underline{(\text{const})}}\ [\Delta P - \Delta\pi]$

$J =$ volumetric flux

m^3/m^2.s.

RO

$$\boxed{J = L_p\ [\Delta P - \Delta\pi]}$$

$\downarrow$ Membrane Permeability

Osmotic Pressure Relation

or

Phenomenological Eq. for Porous memb

or

Darcy's Law.

The unit of N 2 will be mole per meter square per unit time, it is the molar flux. You can convert it into the into volumetric flux by multi prime the density of molecular weight. So, these becomes N 2 N 2 becomes, let say J is nothing, but volumetric flux and a group of constants. So, it will be another constant times, delta p minus delta pi these constant is known as the permeability, if, J is expressed in as a volumetric flux that means meter cube per meter square per unit time. You can easily, convert mole per meter square second into meter cube per meter square second by in invoked in the factors of density molecular.

So, and forth J becomes L p times delta p minus delta pi these L p is known as the membrane permeability, So, these relation gives the solvent flux incase of reverse osmosis through, the membrane, and this is, known as osmotic pressure relationship, or phenomenological logical equation for porous membrane. It has another name Darcy’s law. So, volumetric flux of the solvent is proportional; to driving, force the proportionality constant; is the membrane permeability. So, we add the proportionality, constant as diffusivity incase of fick’s law ne add the proportionality; constant, as thermal conductivity incase of Fourier’s law. We add the proportionality constant; as resistance in case of the, ohm’s law. We add the proportionality constant; as membrane permeability incase of Darcy’s law fine.

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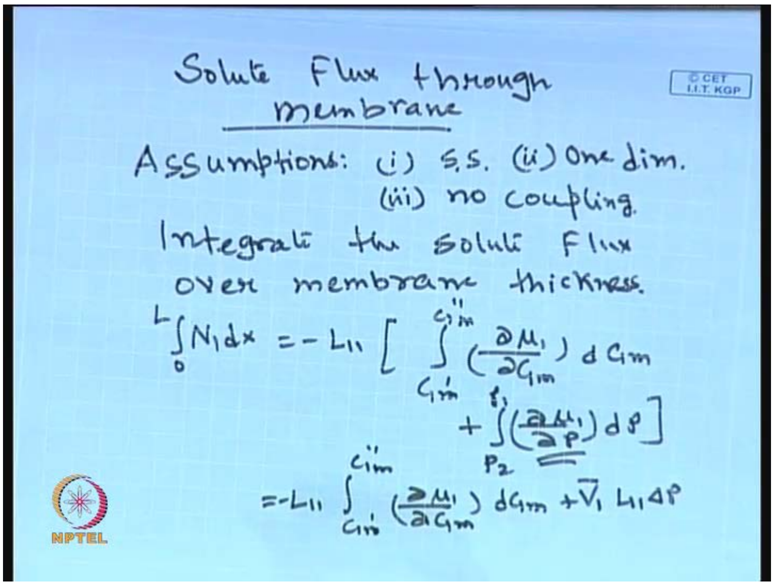

Solute Flux through membrane

Assumptions: (i) S.S. (ii) One dim. (iii) no coupling.

Integrate the solute flux over membrane thickness.

$$\int_0^L N_1\,dx = -L_{11}\left[\int_{C'_{1m}}^{C''_{1m}} \left(\frac{\partial \mu_1}{\partial C_{1m}}\right) dC_{1m} + \underline{\underline{\int_{P_2}^{P_1} \left(\frac{\partial \mu_1}{\partial P}\right) dP}}\right]$$

$$= -L_{11}\int_{C'_{1m}}^{C''_{1m}} \left(\frac{\partial \mu_1}{\partial C_{1m}}\right) dC_{1m} + \bar{V}_1 L_{11}\Delta P$$

Next we look into the solute flux through the membrane, the assumptions involved; are same steady state. One dimensional flow one dimensional transport and there is no coupling. So, we go back to the equation; of the solute flux and integrated. It the solute flux expression, over membrane thickness, if you do that, what will be getting is N 1 d x o to L is minus ,L 1 1 C 1 m prime to C 1 m double prime del mu 1 del C 1 m d C 1 m plus del mu 1 del p dp from P 2 to P 1 .So, will be getting as L 1 1 C 1 m prime to C 1 m double prime with minus sign here, del mu 1 del C 1 m d C 1 m plus V 1 bar L 1 1 delta

p. So, we assumed we invoke the definition of molar volume of the of the solute and include the and replaces del mu 1 del p in terms of the molar volume. So, will I think we requires a some more time to discussed and issue of get along the derivations we will discussed; in the next class and will and expression of solute flux through; the reverse osmosis membrane.

So, you get the expression of the solvent flux the reverse osmosis membrane will be getting the expression of solute flux. The reverse osmosis membrane for the first case the solvent flux, You will get an idea handle with the productivity; of the system, how much meter cube meter per litter per hour. You can process in the second case, one will be getting the solute flux to the membrane will be getting an idea and handle with the quality, of the permeate; first one I talked about the productivity; is a quantity and next one it will be the quality. What is the rejection? What is the quality? what is the amount of permeate concentration? Of the solute present in the permeate stream why the that is, visible or not those decisions one can made.