

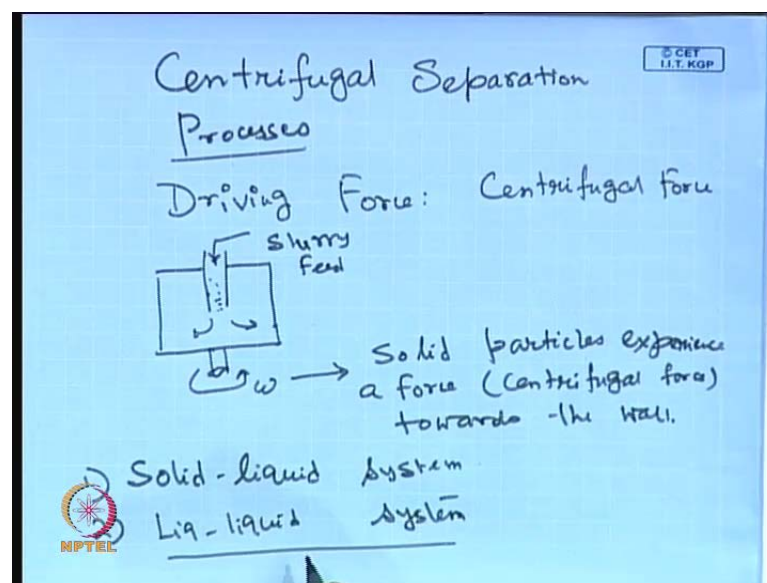
**Novel Separation Processes**  
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**Lecture No. # 35**  
**Centrifugal Separation Processes**

Good morning everyone, so we will be starting with our next separation process, it is the Centrifugal Separation Process. And these will be used whenever you will be having to, you will require to separate the slurries from solid particles from liquid, so they are not completely soluble, they are called slurries, when the suspended materials or the solid materials are there in the liquid, so it is called a slurry.

And in order to, of sizable size for example, they are of the size of the microns and so on, so forth, and whenever you will be applying a centrifugal force, these particles will be subjected to a body force which is nothing but, the centrifugal force  $\omega^2 r$  sort of thing. And they will be pushed towards the wall **of the** of the container, and it will be entirely depending on the density of this particle. So, heavier particles will be pushed more towards the wall so, if you have slurry of heavier and lighter materials, the heavier particles will be pushed towards the walls, and they can be separated out from the wall.

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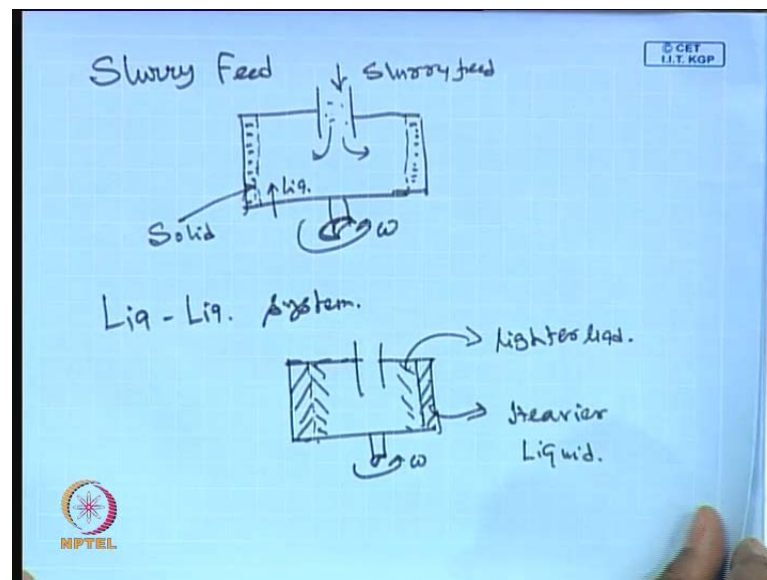


So, in centrifugal separation of processes, the main driving force is centrifugal force, the driving force is centrifugal force that induces separation, and it will be something like

this process can be described by the schematic. So, this is the slurry feed will be going into the system, and this the middle shaft will be rotating and the whole system will be rotating, and solid particles experience a force directed towards the wall.

Because, of this rotation solid particles in the slurry, they experience a force there is a centrifugal force towards the wall. Now, **there are** this centrifugal separation can be operated on two systems, one is the solid liquid system, there is a slurry system; second one is liquid-liquid system, where the two liquids will be having a difference in density. So, therefore, it is not necessary, that the centrifugal separation process **can be operating** can be operated on solid liquid system; only it can operate on a liquid-liquid system as well if there is a density difference of the two liquids that, we are talking about.

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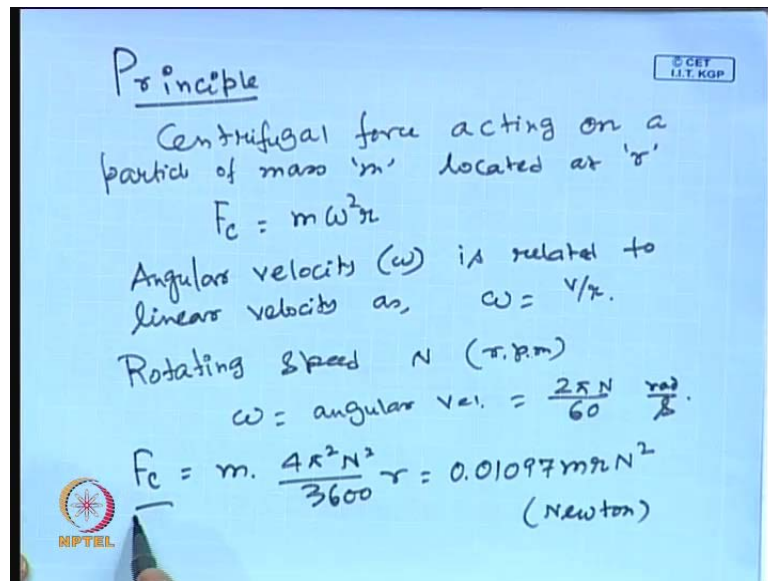
So, for slurry feed, you will be having, and the central the bowl will be rotating about the central axis with a speed  $\omega$ , and the slurry will be coming and the solid particles will be experiencing a force. So, centrifugal force towards the wall so therefore, they will be depositing over the walls. So, you will be getting a layer of solid particles there, and this will be the liquid and you will be having solid particles there.

Now, these solid particles can be separated out, in case of liquid-liquid system and we are talking of two liquids, one is heavier, another is lighter. Now, the heavier liquid that will be having higher density, it will be pushed towards the wall, and they will be existing close to the wall, and you will be having another liquid to the other side. So, you

will be having a heavier liquid towards the wall so, this is nothing but, a heavier liquid higher density, and this will be the lighter liquid.

So, the heavier liquid can be taken out from the wall, and the centrifugal separation process will be affected in this system. So, we can have either a solid liquid system or we can have a liquid-liquid system of different densities as well.

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Now, let us look into the principle of centrifugal separation process it acts, let us centrifugal force acting on a particle of mass m located at a radial distance r from the center, the force is represented by F c is equal to m omega square r. The angular velocity is related to the linear velocity by the relation, where omega is the, m is the mass, omega is the angular velocity, r is the radial distance. Angular velocity, omega is related to linear velocity as, omega is equal to v by r.

Now, if you have a rotating speed in revolution per minute RPM, RPM is nothing but, the Revolution Per Minute, rotating speed N this is (r, p, m) revolution per minute, then omega the angular velocity is given as 2 pi N divided by 60. So, it is rotation per minute so 60 is it confers into per second, and one full rotation or full revolution means 2 pi radian.

So, these angular velocity will be radian per second, then the centrifugal force in Newton can be calculated at as F c will be nothing but, m into 4 pi square N square divided by

3600 times  $r$  and these turns out to be  $0.01097 \text{ m times } r \text{ N square}$ . So, these the unit is Newton, the unit of the centrifugal force.

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Gravitational Force on 'm'

$$F_g = mg.$$

Ratio of centrifugal to gravitational Force

$$\frac{F_c}{F_g} = \frac{r\omega^2}{g} = \left(\frac{r}{g}\right) \left(\frac{2\pi N}{60}\right)^2$$

$$= 0.001118 \text{ } r \text{ N}^2$$

$g = 10 \text{ m/s}^2$

Centrifugal Force is times more than 'g'.

For example  $r = \text{rad. of centrifuge bowl}$   
 $= 0.1016 \text{ m.}$   
 $N = 1000 \text{ r.p.m}$

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So, on the other hand, if you compare this centrifugal force with the gravitational force the, then we can see how much more the centrifugal force is compared to the gravitational force. Now, gravitational force working on a mass  $m$  is given as  $F$  of  $g$  is nothing but,  $m$  times  $g$ , the ratio of centrifugal to gravitational force, then can be computed ratio of centrifugal to gravitational force, can thus we computed  $F_c$  by  $F_g$  is equal to  $r$  omega square divided by  $g$ . So, it will be  $r$  over  $g$  replace the expression of omega that is  $2\pi N$  over  $60$  square of that so, is turns out to be  $0.001118 \text{ } r \text{ N square}$ .

Now, in this expression we put  $g$  is equal to about  $10$  it is  $9.8$ , so you put it about  $10$  meter per second square, thus the force developed in the centrifuge is  $r$  omega square by  $g$  times that of the gravity, so this is expressed now, so centrifugal by this ratio what you can say that, centrifugal force is this many times more than  $g$  or the gravitational force. So, how it is expressed now, it is expressed we give an example, if we talk about a radius of centrifuge bowl **radius of centrifuge bowl**, if that is  $0.1016$  meter, and if you talk about a rotational speed about  $1000$  rpm, revolution per minute.

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$$\frac{F_c}{F_g} = 0.001118 r N^2$$
$$= 113.6$$

Centrifugal force is 113.6 times gravitational force

$$\equiv 113.6g$$

if  $r = 0.2032$  then  $\frac{F_c}{F_g} = 227.2$

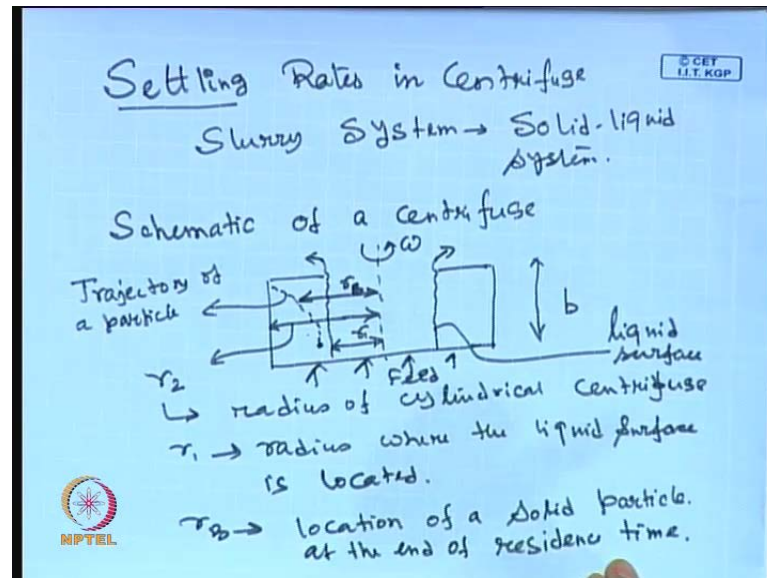
Centrifugal force generated is "227.2g"

Then  $F_c$  by  $F_g$  becomes, I will put the value  $F_c$  by  $F_g$  becomes  $0.001118 r N^2$  put the value of  $r$ , put the value of  $N$  square of that, so it turns out to be 113.6, so that means, we term that centrifugal force **force** is 113.6 times gravitational force. And it is expressed as that, as this and this whole statement is equivalent to 113.6  $g$  that is the nomenclature.

So, if generally sometimes, **we** you will see that the centrifuge is operating at 5  $g$  at 10  $g$  that means, it is 10 times the gravitational, the centrifugal force that is generated is 10 times the gravitational force. If your centrifuge is operating at 20  $g$  means, the centrifugal force that will be **generating** generated, it is 20 times more than the gravitational force. Though, if your  $r$  becomes 0.2032, then  $F_c$  by  $F_g$  will be 227.2, and we will call that centrifugal force generated is 227.2  $g$ .

That means, centrifugal force generated is 227.2 times more than the two times the gravitational force. So, that is the nomenclature, sometimes whenever you will be looking into the specification, that is written on the wall of the centrifuge you will see all these informations, so that means the centrifugal force generated  $x$  times the gravitational force.

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Now, let us look into the settling rates in centrifuge, now we are talking about let us say, talking about a slurry system, and always slurry system it is basically, solid liquid system. In that, so let us first draw a typical centrifuge, so you draw a schematic of a centrifuge, then all our nomenclature will be fixed; so we are having the liquid surface, and the whole system is rotating about its central axis with omega.

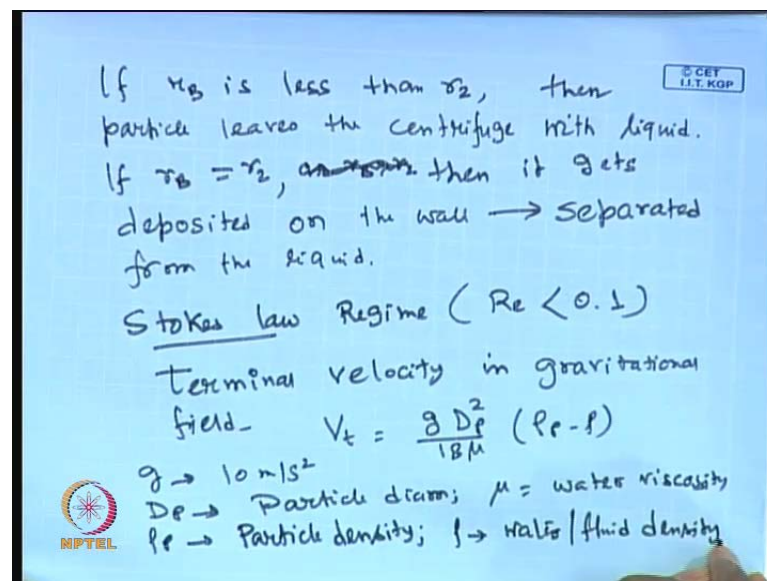
So, this height of the centrifuge is  $B$ , and let us this distance is  $r_2$ , so  $r_2$  is nothing but, the radius of cylindrical centrifuge, let us say this distance is  $r_1$ , so  $r_1$  is the radius, where the liquid surface is located, so this is nothing but, the liquid surface is located. And  $r_b$  let us any location of a solid particle  $r_B$  is location of a solid particle at any point of time let us say, and the feed is going like this, so you are, feeding the slurry from the bottom a whole thing is rotating.

And typical trajectory of a particle will be something like this, this is a typical trajectory of a particle, it is a parabolic in nature, trajectory of a particle, of a solid particle. Now, at the end of the residence time particle is at a distance  $r_B$  from the axis of rotation,  $r_B$  is location of the solid particle at the end of residence time. What is the residence time, residence time is total flow rate, the feed is being fed into the centrifuge at a particular flow rate, let us say that is  $q$  meter cube per second, and you have the you have a certain volume of the centrifuge, so let us say that is  $v$  meter cube, so the residence time is nothing but,  $v$  by  $q$ .



So, that is the average residence time of a **of a of a** particle that will be staying within the centrifuge, now if the **if the if**  $r_B$  is less than the  $r_2$ , the radius of the centrifuge, the particle will be going away it will not be settling on the wall, it will be going away in the **in the product stream**, so there will be no separation. So, our objective should be the flow rate should be maintained such that,  $r_B$  that is the location of the particle at the end of the residence time must be less than  $r_2$ , so that it will be hitting the wall and staying there, then only the separation will be possible.

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That means, if  $r_B$  is less than  $r_2$  then particle **leaves the bowl** leaves the centrifuge with liquid, if  $r_B$  is equal to  $r_2$  or greater than  $r_2$ , it cannot be greater than  $r_2$ , because there is a physical limit, if it is equal to  $r_2$ , then it deposits it gets deposited on the wall, once it gets deposited on the wall it will be effectively separated from the liquid. Now, in a stokes law regime **in stokes law regime**, what is that Stokes law regime, when the Reynolds number is less than 0.1 that is called the Stokes law regime. And what this Reynolds number will be defined on the diameter of the particle and the diameter of the particle will be extremely small in the order of micron and so, so it will be really small. The terminal velocity can be expressed a, terminal velocity under in gravitational field **field** becomes  $V_t$  is nothing but,  $g$  times  $D_p$  square  $D_p$  is the particle diameter  $18\mu$  into due into  $\rho_p$  minus  $\rho$ .

So,  $g$  is acceleration due to gravity 10 meter per second square,  $D_p$  is diameter of the

particle **particle diameter**,  $\mu$  is water viscosity or feed viscosity.  $\rho_p$  is particle density, and  $\rho$  is water or the fluid **fluid** density, whatever we are dealing with; this is the expression of terminal velocity under the gravitational field.

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Under Centrifugal force  
terminal vel.

$$V_t = \frac{\omega^2 r D_p^2 (\rho_p - \rho)}{18 \mu}$$

$$\frac{dr}{dt} = \frac{\omega^2 r D_p^2 (\rho_p - \rho)}{18 \mu}$$

$$\int_0^{t_T} dt = \frac{18 \mu}{\omega^2 (\rho_p - \rho) D_p^2} \int_{r_1}^{r_2} \frac{dr}{r}$$

$$t_T = \frac{18 \mu}{\omega^2 (\rho_p - \rho) D_p^2} \ln \frac{r_2}{r_1}$$

But, in the centrifugal fields the terminal velocity becomes, under centrifugal force the expression of terminal velocity is different; the terminal velocity **the**  $g$  will be replaced by the omega square  $r$ , at  $g$  will be replaced by the omega square  $r$ . And this terminal velocity becomes omega square  $r$   $D_p$  square  $(\rho_p - \rho)$  divided by  $18 \mu$ . So, that is the terminal velocity under centrifugal force and compared to the gravitational force, the  $g$  will be replaced by the omega square  $r$ , so acceleration due to gravity will be replaced by the centrifugal acceleration.

Now, this terminal velocity is nothing but,  $dr/dt$  if you **if you** look into the definition of velocity is  $dr/dt$  and acceleration is  $d^2r/dt^2$ . So therefore, this expression can be written as  $dr/dt$  that is the location at any point of time omega square  $r$  divided by  $18 \mu D_p$  square times  $\rho_p - \rho$ . Now, you can integrate it out, if you integrate it out see, it becomes just separate the variables  $dt$  equal to  $18 \mu$  divided by omega square  $\rho_p - \rho$   $D_p$  square integral  $dr$  by  $r$ , and this will be let us say from  $r_1$  to  $r_2$  from  $0$  to  $T$  terminal.

Now, these will give you an expression of the time  $t_T$  or the terminal **the** time becomes  $18 \mu$  divided by omega square  $(\rho_p - \rho) D_p$  square  $\ln r_2$  by  $r_1$ .



Now, if the volumetric flow rate becomes  $q$  and  $v$  is the volume of the liquid column in the centrifuge.

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if  $V$  is vol. of centrif. liq. column in centrifuge.

$q \rightarrow$  Steady state flow rate of liq.

$$t_r = \frac{v}{q}$$

$$\frac{v}{q} = \frac{18 \mu}{\omega^2 (\rho_p - \rho) D_p^2} \ln \frac{r_2}{r_1}$$

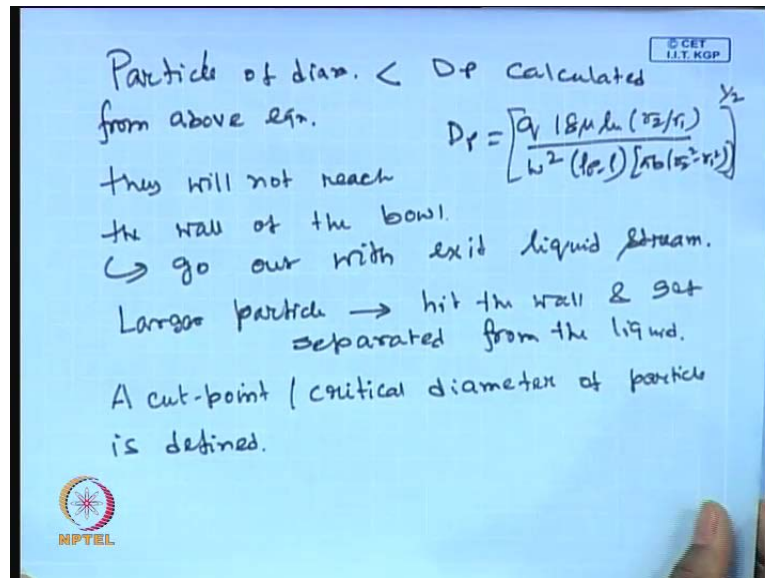
$$v = \pi b (r_2^2 - r_1^2)$$

$$q = \frac{\omega^2 (\rho_p - \rho) D_p^2}{18 \mu \ln(r_2/r_1)} [\pi b (r_2^2 - r_1^2)]$$

If  $v$  is volume of centrifuge volume of say of **of liquid column liquid column**, in centrifuge, so in fact, there will be two phases in the centrifuge one, the liquid phase another be the solid phase. And the solid phase volume will be the solid particle that will be residing on the wall, but that volume will be extremely small compared to the volume of the centrifuge. So, volume of the liquid **so volume of the liquid** be roughly equal to the volume of the centrifuge, and  $q$  is the flow rate, steady state flow rate of the liquid **of liquid**, so residence time is nothing but, as I told earlier it will be  $v$  by  $q$ , so it and this becomes **so** now, you can combine these two and  $q$  becomes  $v$  by  $q$ . So, you just replace **t by t in the** as  $v$  by  $q$  in the above expression. The earlier, expression this become  $18 \mu$  divided by  $\omega$  square  $\rho_p$  minus  $\rho$   $D_p$  square  $\ln r_2$  by  $r_1$ .

And what is the value of a volume becomes nothing but,  $\pi$  times  $b$  ( $r_2$  square minus  $r_1$  square), so  $q$  the expression of  $q$  is nothing but,  $\omega$  square ( $\rho_p$  minus  $\rho$ )  $D_p$  square  $18 \mu \ln(r_2$  by  $r_1)$  multiplied by  $\pi b r_2$  square minus  $r_1$  square. What I saying that this  $r_1$  will be very small, so it becomes the radius of the centrifuge itself, so the condition for this is nothing the flow rate should be matched by you should be given by this expression. Now, particles with diameter smaller then  $D_p$  calculated from this equation will not reach the wall.

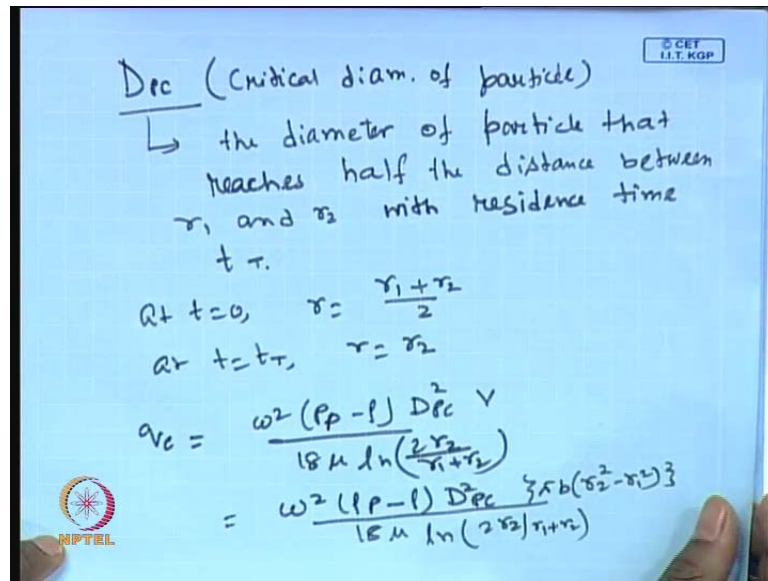
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So, the conclusion is the interpretation is particle of diameter **less than** smaller than  $D_p$  calculated from above equation, so what is the expression of  $D_p$ ,  $D_p$  is nothing but,  $[q \text{ times } 18 \mu l n (r_2 \text{ by } r_1) \text{ divided by } \omega^2 (\rho_p \text{ minus } \rho) \text{ times } \pi b (r_2^2 \text{ square minus } r_1^2 \text{ square}) \text{ raised to the power } 1 \text{ upon } 2]$ . So, if particle of diameter less than  $D_p$  calculated from this equation, then they will not **they will not** reach the wall of the bowl or they will go **they will go** with the effluent stream; they will be going out with the outgoing stream along with them.

And they will go out with exit liquid stream, now larger particles **larger particles larger particle** means particle diameter larger than  $D_p$ , they hit the wall and get separated from the liquid. Now, therefore, a cut point or critical diameter of the particle is defined **point or critical diameter of particle is defined** and that is defined this way that  $D_{pc}$ .

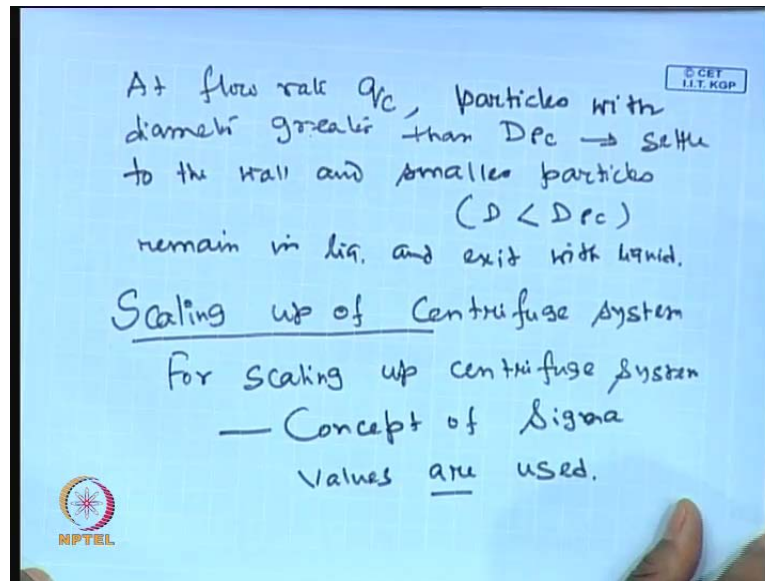
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So, it is denoted by nomenclature  $D_{pc}$  that is the Critical Diameter of the Particle **particle**, and this is defined as the diameter of particle, that reaches half the distance between  $r_1$  and  $r_2$  within residence time  $t_r$ . That means, at  $t$  is equal to 0,  $r$  equal to  $r_1$  plus  $r_2$  by 2 and at  $t$  is equal to  $t_r$ ,  $r$  is equal to  $r_2$  therefore, the critical flow rate can be calculated with this **with this** cut point.

Now, cut point diameter, what is the critical flow rate, the critical flow rate becomes in that case  $q_c$  is  $\omega^2 (\rho_p - \rho) D_{pc}^2$  multiplied by  $V$  divided by  $18 \mu \ln\left(\frac{2r_2}{r_1 + r_2}\right)$ . So, this **this** turns out to be  $\omega^2 (\rho_p - \rho) D_{pc}^2$  and volume can be replaced as  $\pi b (r_2^2 - r_1^2)$  and divided by  $18 \mu \ln\left(\frac{2r_2}{r_1 + r_2}\right)$ . So, at this flow rate that means, at the flow rate of  $q_c$  particles with diameter greater than  $D_{pc}$  will settle with the wall, and most of the smaller particle will remain in the liquid, and they will go out with the liquid.

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So, at flow rate  $q_c$  particles, with diameter greater than  $D_{pc}$ , they settle to the wall **the wall** and other small particle and smaller particles and what do you mean by these smaller particles, whose diameter is less than  $D_{pc}$ , they remain in liquid and exit with liquid. So, we can find out that the value of the particle diameter given a flow rate, what will be the typical value of the particle diameter, which will be settled on the wall or separated from the liquid and **( $\sigma$ )** diameter range that, will be going out with the liquid stream.

Now, let us talk about the sigma values and scaling up issues, scaling up of centrifuge system that is very important; because, you will be conducting all the experiments in a laboratory scale centrifuge. Now, if you would like to scale it up, you have for the industrial level centrifuge **which are higher** which are larger in size, so the scale up issues are very important. And for scaling up of **centrifuge system** centrifuge system a **concept of sigma value is used** concept of sigma values are used, then what is the concept of sigma **sigma** values.

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Volumetric Flow rate:

$$q_c = \frac{\omega^2 (\rho_p - \rho) D_p^2 c^2 V}{18 \mu l_n \left( \frac{2r_2^2}{r_1 + r_2} \right)}$$

$$= \frac{2g(\rho_p - \rho) D_p^2 c^2}{18 \mu} \cdot \frac{\omega^2 V}{2g l_n \left( \frac{2r_2^2}{r_1 + r_2} \right)}$$

$$= 2 V_{tg} \Sigma$$

$V_{tg} \rightarrow$  Terminal velocity under gravity

$$\Sigma = \frac{\omega^2 V}{2g l_n \left( \frac{2r_2^2}{r_1 + r_2} \right)}$$

Let us look into the volumetric flow rate, volumetric flow rate is given as  $q_c$  is equal to  $\omega^2 (\rho_p - \rho) D_p^2 c^2 V$  divided by  $18 \mu l_n (2r_2^2 / (r_1 + r_2))$ . Now you so, just separate these two out, so I will be taking the geometric and the operating conditions on the right hand side,  $(\rho_p - \rho) D_p^2 c^2$  divided  $18 \mu$  multiplied by  $\omega^2 V l_n (2r_2^2 / (r_1 + r_2))$ .

Now,  $r_1$  and  $r_2$  will be defined for a larger centrifuge, volume will be defined for a larger centrifuge and of course, the rpm will be may be separate or may be equal. So, I will just take this  $(\Sigma)$ , now multiply denominator and the numerator by  $g$ , so  $2g$ , so it becomes  $2g$  here, becomes  $2g$  here and what is this if you remember, this value is nothing but, two times terminal velocity due to gravity. So, this will be nothing but,  $2V_{tg}$  multiplied by and this is known as a sigma.

So,  $V_{tg}$ , what is  $V_{tg}$ ,  $V_{tg}$  is nothing but, the terminal velocity under gravity gravitational force and what is this sigma sigma is nothing but,  $\omega^2 V$  divided by  $2g l_n (2r_2^2 / (r_1 + r_2))$ . So, sigma value contains all the operating conditions those are required for a larger centrifuge, because the geometric parameters volume and  $r_1 r_2$  will be different for two different centrifuge, the rotational speed may be different for two different centrifuge. And  $g$  is of course, a constant, so one can take up a ratio of sigma values and and look into the you know, go for the scaling up

options, so we will just look into that.

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$$\Sigma = \frac{\omega^2 \pi b (r_2^2 - r_1^2)}{2g \ln \left[ \frac{2r_2}{r_1 + r_2} \right]}$$

↓      ↓  
[m<sup>2</sup>]

Σ → A physical characteristic of a centrifuge → not a characteristic for fluid-particle system.

Physical Interpretation

Σ → unit of area. It is area of a gravitational settler that will give same sedimentation rate as the centrifuge at the same feed rate.

So, **if you** if you put the dimension of sigma, the expression is already there omega square times pi b (r 2 squares minus r 1 square), so this whole term is basically nothing but, volume divided by 2 g l n (2 r 2 divided by r 1 plus r 2). This will be having a unit of meter square, so sigma is a physical characteristic of centrifuge, but not a feed particle system. So, this sigma is a physical characteristic of a centrifuge, it is not a characteristic for fluid particle system, it does not contain any property of the fluid, it does not contain mu it does not contain rho.

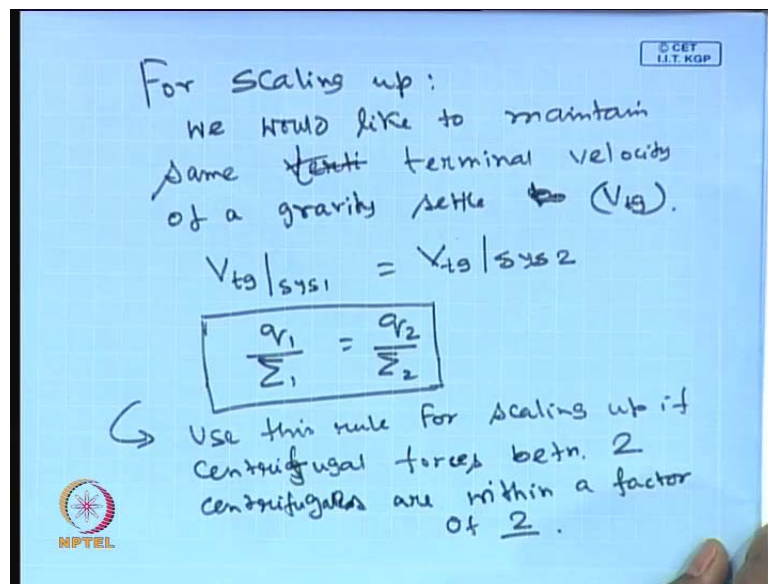
So, **it is** it is entirely, and it is not and it does not contain the particle, also it does not contain the radius of the diameter of the particle, **all this** all this fluid particle characterization that means, diameter and the density viscosity, they will be appearing in the V t g; that the terminal velocity under gravity. And this sigma value is nothing but, the characteristic of the centrifuge, so that is the difference between the two. So, physical interpretation of sigma is very important, what is the physical interpretation sigma, if you remember sigma has unit of area it means, it is area in meter square of a gravitational settler, what is the gravitational settler gravitational settler is nothing but, a tank where you just put the material slurry, and the solid particles will deposit or settle by under gravity and the terminal velocity is given by V t g.

So, sigma **is it is** it is area in meter square of a gravitational settler that **will leave that**



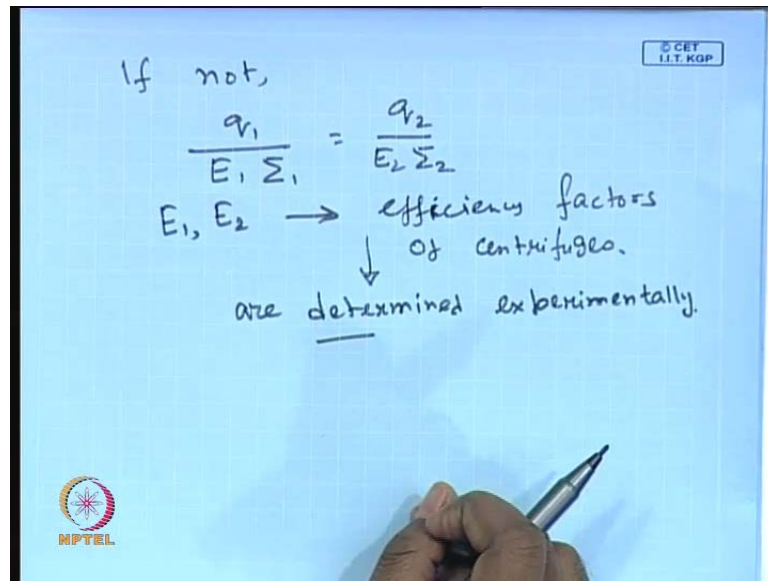
will leave same sedimentation characteristic, as the centrifuge at the same feed rate. So, basically, it gives an idea how this is compared with a gravity settler because, gravity settler is very common. So, it gives the area under which the sedimentation characteristic is almost equal or easily understandable, if you talk about a gravity settler because, **it is** it is easier to understand gravity settling method, compare to centrifugal separation process.

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So, therefore, in order to scaling up, for scaling up what we will do will try to maintain the same terminal velocity under gravity same terminal velocity, so we would like to maintain same terminal velocity of a gravity settler **t g**  $V_{tg}$ . So,  $V_{tg}$  for system one should be equal to  $V_{tg}$  for system two, that simply means  $q_1$  by  $\sigma_1$  is equal to  $q_2$  by  $\sigma_2$ ;  $q_1$  is the flow rate of smaller centrifuge  $\sigma_1$  is the  $\sigma$  value for the smaller centrifuge, and  $q_2$  is the flow rate of the higher centrifuge, so it may be 10 times of  $q_1$ . And  $\sigma_2$  is the characteristic  $\sigma$  values for the larger centrifuge, so **so** that is it, this **this** is dependable if centrifuge for centrifugal forces between the two or within a factor of two from each other that means, one can use **use** this rule for scaling up, if centrifugal **force** forces between two centrifuges are within a factor of two this is a thumb rule, this is a design thumb rule. That means, if the largest centrifuge, I will be **have will be** having centrifugal force, which is slightly less than 2 or less than or equal to 2, then you can use this rule, I just scaling up option.

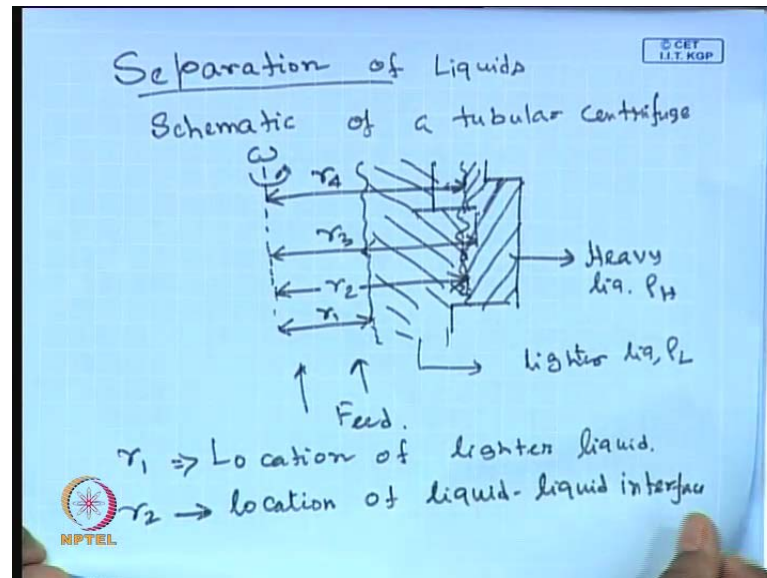
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And if not what happens, if not one has to use  $q_1 E_1 \sigma_1$  is equal to  $q_2 E_2 \sigma_2$  and what are  $E_1$  and  $E_2$ ,  $E_1$  and  $E_2$  these are basically, the efficiency factors of the centrifuge **of centrifuges**, these factors are determined experimentally. So, using this concept of sigma value, one can scale up a centrifuge from lower version to higher version, which can accumulate more flow rate, and more separation **will be** will be effected.

You will be in fact, we are going to we will be solving some other problems, example problems on this. Now, next we move on to the separation of liquid still now, whatever we have discussed we have talked about the separation of a slurry process that means, solid liquid system.

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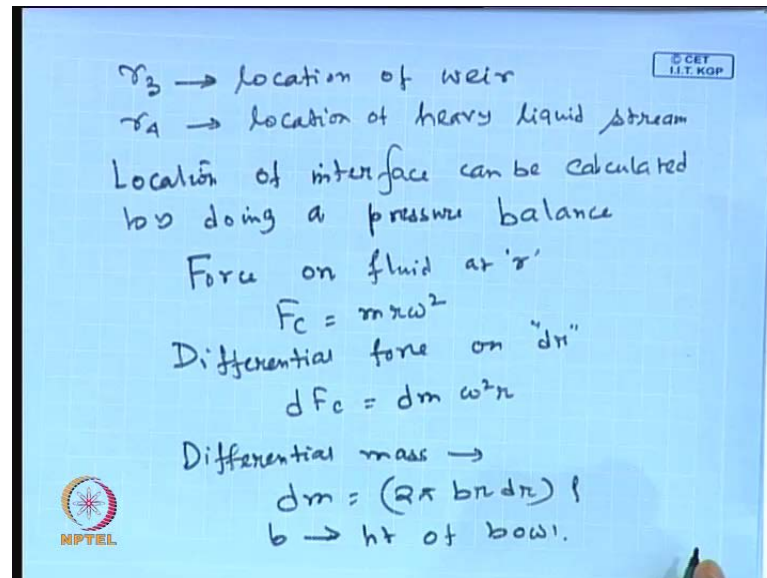


Now, let us move on to the separation of liquids that means, we are having two liquid stream of different density, so the schematic of a tubular centrifuge, let us draw a schematic of a tubular centrifuge. I am just writing a, I am drawing a symmetric part, so these have symmetry the axis about, which the centrifuge is rotating with a angular speed  $\omega$ , and let us say this is the portion where you are you will be having a liquid, these a now will be having a header here, **fine**, this is the lighter, lighter fluid, this is the heavier liquid.

So, the heavier liquid will push towards the wall, so it will be heavy liquid let us say  $\rho_H$  this will be a lighter one liquid  $\rho_L$ , and you are putting the feed from the bottom, and there is a separator here, that separator separates basically form the lighter to the heavier liquid other there will be mixed up. Now, let us put the dimensions, let us say the from the **from the** center the distance the level of the heavier liquid will be, let us say  $r_4$  and this distance is  $r_3$ , this separation is called, let us say  $r_2$ , and this level is  $r_1$  or it is the a feed is going there (Refer Slide Time: 43:10).

And let us say  $r_1$  is the core, they it is basically air core, so **in they** in this figure the geometry as a details are given like this,  $r_1$  is basically the location of lighter liquid **liquid**  $r_2$  is nothing but, the liquid-liquid interface; the interface **that the** that separates the two liquids location of liquid-liquid interface. And this part is called a weir  $w e i r$ , so this weir is basically separating out, physically the heavier liquid and a lighter liquid.

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$r_3$  is location of weir, and  $r_4$  is a location of heavy liquid stream or the product, so location of interface can be calculated by a balance of pressure in the two layers. So, let us and this location of interface can be calculated by doing a pressure balance between two layers, now the force on the fluid located at a radial location  $r$ , fluid at  $r$  is given as  $F_c$  is equal to  $m r \omega^2$ ,  $m \omega^2 r$  that is the mass into acceleration. Now, the differential force over a differential thickness can be expressed as differential force on differential element  $dr$  is expressed as  $dF_c$  time  $dm \omega^2 r$  the mass containing will be  $dm$ .

So, differential mass can be expressed  $dm$  is a differential mass, and differential mass can be given as  $dm$  is equal to  $2\pi b r dr$  times  $\rho$ , so this is the volume within the differential element and  $\rho$  is the density of the fluid. So, what is  $b$  is nothing but, the height of bowl, height of the tubular centrifuge.

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Handwritten derivation on a blue background:

- $dP = \text{diff. pressure}$
- $= \frac{dF_c}{A} = \omega^2 \rho r dr$
- $A = 2\pi r b$
- integrate between two points 1 & 2
- $P_2 - P_1 = \frac{\rho \omega^2}{2} (r_2^2 - r_1^2)$
- Equate pressure over  $r_2 \leftrightarrow r_4$  and  $r_2 \leftrightarrow r_1$
- $\frac{\rho_L \omega^2}{2} (r_2^2 - r_1^2) = \frac{\rho_H \omega^2}{2} (r_2^2 - r_4^2)$
- Location of interface:
- $r_2^2 = \frac{\rho_H r_4^2 - \rho_L r_1^2}{\rho_H - \rho_L}$

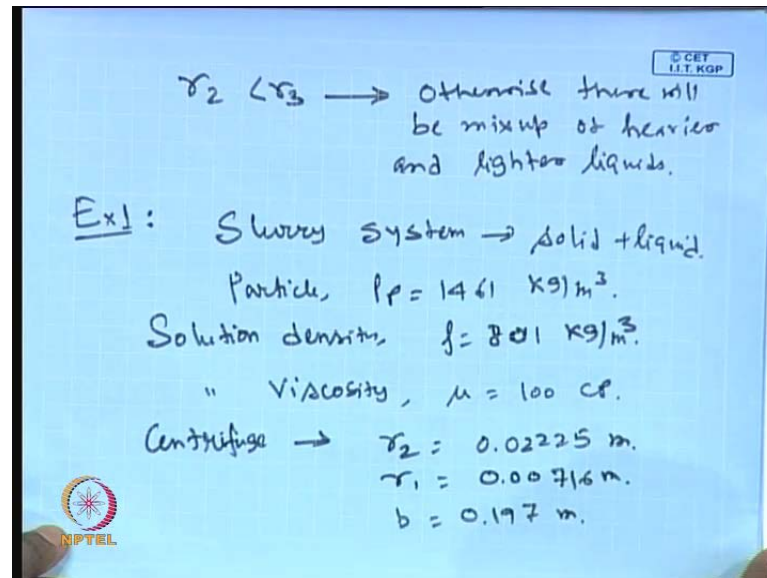
Logos: © CET I.I.T. KGP (top right), NPTEL (bottom left)

Now, you will be in a position to calculate the pressure force,  $d p$  is differential pressure force what is that force per unit area  $d F_c$  divided by area, so it will be  $\omega$  square  $\rho$   $r$   $d r$ , and what is  $A$ ,  $A$  is nothing but,  $2 \pi r b$ . Now, you can integrate this equation between  $r_1$  and  $r_2$  the pressure difference between two points So, integrate between two points, 1 and 2 we will be getting  $p_2$  minus  $p_1$  is nothing but,  $\rho \omega$  square by 2 ( $r_2$  square minus  $r_1$  square).

So, equating now **now** you can equate pressure on two sides, equate pressure over  $r_2$  to  $r_4$  and  $r_2$  to  $r_1$ , if you equate this pressures then you can get the your location of the interface  $\rho_L \omega$  square by 2 ( $r_2$  square minus  $r_1$  square) between  $r_1$  and  $r_2$  it was a lighter liquid. And  $\rho_L$  is equal to  $\rho_H \omega$  square by 2  $r_2$  square minus  $r_4$  square between  $r_2$  and  $r_4$ , it is only the heavier liquid.

So, the location of the interface, now you can you will be you will be able to calculate, location of interface, now **becomes  $r_2$  becomes**  $r_2$  square becomes  $\rho_H r_4$  square minus  $\rho_L r_1$  square divided by  $\rho_H$  minus  $\rho_L$ . Now, the interface  $r_2$  must be located such that,  $r_2$  is less than  $r_3$  otherwise no separation occurs between the heavy and the lighter fraction. So, this **this** gives you an expression of location of the interface, that has to be maintained.

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And  $r_2$  must be less than  $r_3$  otherwise, there will be mix up of heavier and lighter liquids and there is no separation possible, so that is basic principles of any centrifuge separation process, and we have discussed the principles for the slurry that is the solid liquid system, and the liquid-liquid systems, and various scales up issues as well. So, the idea is **if you** if you know or may in the laboratory scale that, given a centrifuge there its dimension etcetera, operating connections given, this is the extent of separation you are going to do, then if you would like to scale up you can you can do appropriately.

So, once after doing this let us look into some of the examples, so that the calculation procedures etcetera will be clear to us. Now, I have probably around three problems for you, so let us look in to the first one, first example of discuss solution containing particles with density, so let us talk about a slurry system, slurry system means **is** it is a solid plus liquid system. And **it has containing it as** it as containing particle of density 1461 Kg per meter cube, and it has to be and this particle has to be clarified removed by the centrifuge.

Solution density is given  $\rho$  is 801 Kg per meter cube, and viscosity of the solution **solution viscosity is also given** viscosity is given as  $\mu$  is equal to 100 centipoises. So, it is 100 time viscous than a water, and centrifuge bowl has the following dimensions  $r_2$  is given as 0.02225 meter, and  $r_1$  is given as 0.00716 meter your height is given as 0.197 meter.



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Calculate  $D_{pc}$  if  $N = 23000 \text{ rpm}$ ,  
 $q = 0.002832 \text{ m}^3/\text{hr}$ .

Soln.:  $\omega = \frac{2\pi N}{60} = \frac{2\pi(23000)}{60} = 2410 \frac{\text{rad}}{\text{s}}$

Bowl vol.,  $V = \pi b(r_2^2 - r_1^2)$   
 $= 2.747 \times 10^4 \text{ m}^3$

$\mu = 100 \text{ cP} = 0.1 \text{ Pa.s.}$

$q_c = \frac{0.002832}{3600} = 7.887 \times 10^{-7} \frac{\text{m}^3}{\text{s}}$

$\hookrightarrow D_{pc} = 0.746 \mu\text{m}$

Check  $V_t \rightarrow N_{Re} < 1.0$

Now, what you have to do you have to calculate the critical particle diameter, calculate  $D_{pc}$  such that,  $D_{pc}$  is the exist **exist** stream, if  $n$  is equal to 23000 rpm and  $q$  is given  $q$  is equal to 0.002832 meter cube per hour, so you have to calculate the critical particle diameter of the largest particle that in the exit stream. So, let us look in to the solution, the solution is straight forward you have two first you have to convert the rpm into radian per second.

So, it will be  $2\pi n$  divided by 60, so it will be  $2\pi$  into 23000 divided by 60 and it turns out to be 2410 radian per second, and bowl volume can be calculated  $v$  is equal to  $\pi b r_2^2$  minus  $r_1^2$ , and the if you put the values and these turns out to be  $2.747 \times 10^4$  meter cube. The viscosity is given as 100 centipoise that means 100 into 10 to the power minus 3, so it will be 0.1 pascal second.

So,  $q_c$  you can calculate 0.002832 divided by 3600 **this is** this is given in meter cube per hour, so it you can convert in to meter cube per second  $7.887 \times 10^{-7}$  meter cube per second. So, now, if you put all these values in your governing equation, in the equation of  $D_{pc}$  the  $D_{pc}$  turns out to be 0.746 micron, so **put** put all the values in the expression, so it turns out to be around 0.746 micro that means, the particles of diameter 0.746 and less, they will be going out with the liquid stream, and the particles with higher diameter, larger diameter they will be settled on the wall and they can be separated by this centrifuge. And you can check terminal velocity, and

terminal velocity and then check and you can check the Reynolds number, the Reynolds number will be less than 1 that means, you are within the stocks regime.

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Ex 2

$\rho_p = 1200 \text{ Kg/m}^3 \rightarrow \text{clarified by centrifuge}$

$\rho = 850 \text{ Kg/m}^3$

$\mu = 80 \text{ cP}$

$r_2 = 0.02 \text{ m}; r_1 = 0.01 \text{ m}$

$b = 0.25 \text{ m.}$

$D_{p0} = ?$

$N = 15000 \text{ rpm}$

$q = 0.002 \text{ m}^3/\text{hr.}$

5th

$\omega = \frac{2\pi N}{60} = 1570 \text{ rad/s.}$

$V = \pi b (r_2^2 - r_1^2) = 2.355 \times 10^{-4} \text{ m}^3$

$q = \frac{0.002}{3600} = 5.56 \times 10^{-7} \text{ m}^3/\text{s}$

So, I have **go go** move on move on for the next example, example number 2, again I were talking about a viscous solution it contains a particle of density, 1200 Kg per meter cube, it has to be clarified by the centrifuge, the solution density is given as 850 Kg per meter cube, the viscosity of the solution is 80 c p, and centrifugal bowl will be having dimension r 2 is 0.02 meter, and r 1 is 0.02 meter. The height is given as 0.25 meter, **you have to** you have to again find out the particle **you know**, critical particle diameter given n is equal to 1500 rpm, and flow rate q is equal to 0.002 meter cube per hour.

So, again you have the steps are straight forward, you have to calculate the angular velocity  $2 \pi n$  over 60, and this turns out to be 1570 radian per second and bowl volume becomes  $\pi b$  in to  $(r_2^2 - r_1^2)$ ; if you put the value it turns out to be 2.355 into 10 to the power minus 4 meter cube. And the flow rate is given as 0.002 meter cube by hour, so it turns out to be 5.56 into 10 to the power minus 7 meter cube per second.

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$$5.56 \times 10^{-7} = \frac{1570^2 (1200 - 850) D_{pc}^2}{18 \times 80 \times 10^3} \ln\left(\frac{2 \times 0.2}{0.01 + 0.2}\right) \times 2.35 \times 10^{-4}$$
$$\underline{D_{pc} = 2.57 \mu m}$$

Now, if you put all these values in your governing equation the equation of  $D_{pc}$ , this turns out to be  $5.56 \times 10^{-7}$  is equal to  $1570^2 (1200 - 850) D_{pc}^2$  divided by **80 into**  $18 \times 80 \times 10^3$ , that is  $\ln\left(\frac{2 \times 0.2}{0.01 + 0.2}\right)$  multiplied by  $2.35 \times 10^{-4}$ . And if you calculate  $D_{pc}$ ,  $D_{pc}$  turns out to be  $2.57$  micron, so just check the previous one it was  $0.746$  micron by changing the operating conditions and the dimensions slightly, you will be landing up with a by critical particle diameter of  $2.57$  micron.

So, there is a huge difference that is  $0.746$  micron, so that is almost  $1$  micron and **and** here **here** you are **you** getting around  $2.5$  times of that by changing, the flow rate and the other operating conditions. So, this gives an idea that over given the typical operating condition of a centrifuge, what is the typical diameter of the particles, those can be separated by using the centrifuge. And you know how to do the calculations, in the next class first, I will just solve one more example for scaling up of a centrifuge, and then I will move on to the next separation process that is the chromatography separation process, thank you.