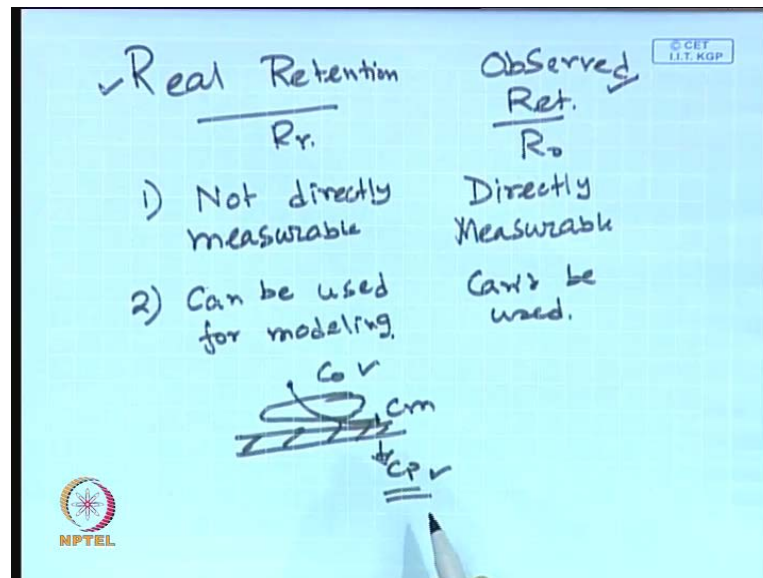


**Novel Separation Processes**  
**Prof. Sirshendu de**  
**Department of Chemical Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture No. # 03**  
**Membrane separation processes**

(Refer Slide Time: 00:28)

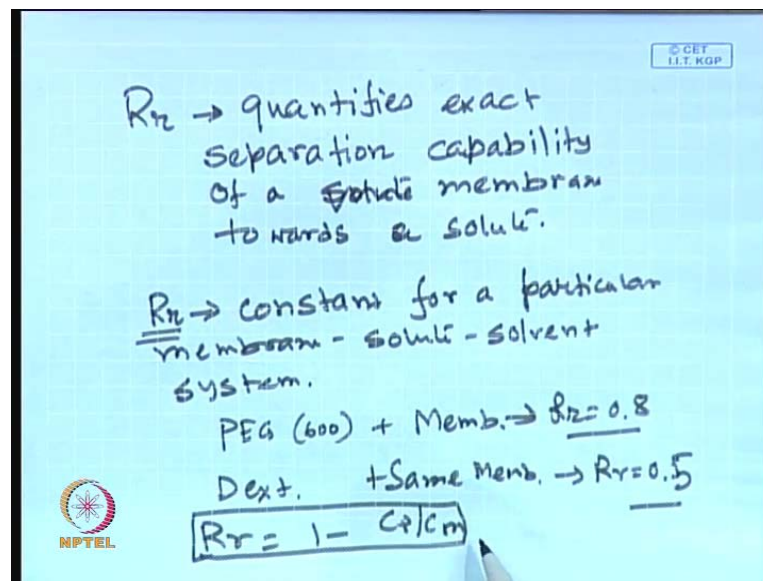


So, now will the second part of this lecture, we come we talk about, the difference between the real retention and observed retention. First of all, it is observed retention is directly measurable, experimentally measurable, this not directly measurable. Second one is that it can be used for modeling purposes cannot be used for modeling purposes. Simply, because actually between that, two things this is  $C$  naught bulk concentration, there is a concentration getting exist that is,  $C_m$  and you will be having a permeate concentration. In the permeate side  $C_p$  so, between  $C$  naught and  $C_p$  there, exist they concentration boundary layer or mass transfer boundary layer whole, which the observed retention. It does not include, because of this fact it known as observed retention. This is known as the real retention, so observed retention the definition one minus  $C$  not minus by  $C_p$  the permeate concentration will be entirely depending, on the operating condition right. What is the operating condition in typical membrane base processes, first is the feed concentration; if you fix your effluent feed concentration is fixed. So, forget about feed concentration. If, you fix your effluent because of concentration is fixed and what are the other operating condition, the other two operating conditions are one is the

pressure difference  $\Delta p$  across the membrane. Second one is the extent of turbulence that means, Reynolds number that, we are going to have in the system, you can have a stirred cell in the stirred cell if the because depending on the stirred speed.

You can have the turbulence or if, you have cross flow cell, you will discuss all the geometric electrons. If, you cross flow cell the linear velocity will defined, the Reynolds number. So, depending on the operating condition the permeate concentration will change therefore, permeate concentration include the variation of operating conditions. So, that cannot give the exact, you know separation extent of separation by the membrane that is, that has the affinity of the particular solute. So, real retention is in the definition, is it gives the actual separating separation capability of a particular membrane towards; the solute that is why is called real retention.

(Refer Slide Time: 03:30)



So,  $R_r$  quantifies exact separation capacity of a solute of a membrane towards solute. So, it gives it is basically nothing, but it is constant for a particular membrane solute solvent system. So, there is the beauty of real retention, so it is constant for a particular membrane solute solvent system for example, if you have an aqua solution of poly ethylene glycol lets a 600.

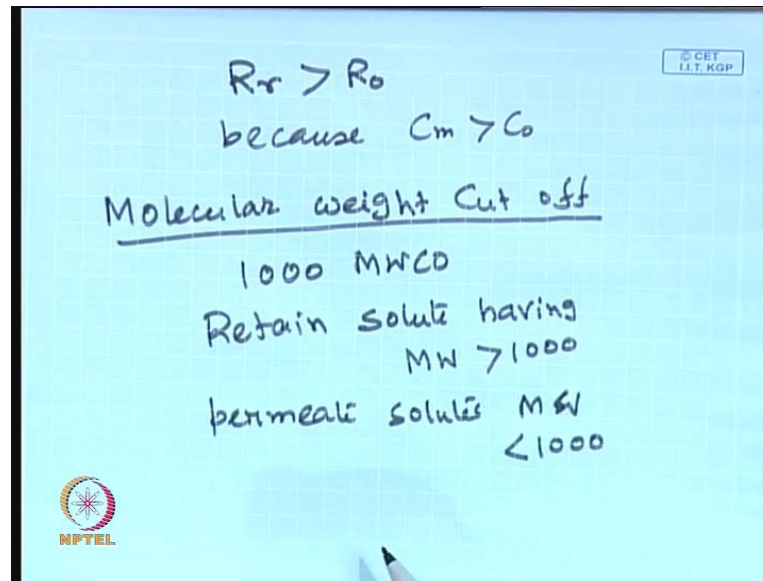
And if, you have membrane let say, some kind of membrane the real retention can be 0.8, but using the same membrane. If, you use a different solute for example, let say dextrans the real retention can be something else 0.1 or let us say 0.5, and this value of real

retention is fixed for a particular membrane solute system and mostly, we are talking about the aqua solution, the solvent is more or less water.

So, for different solute and membrane combination the real, the value of the real retention becomes constant, so it becomes an intrinsic property of the membrane towards particular solute. So, real retention is constant and if, you look into the definition of real retention, it keeps a its basically fundamental property of the polymer film of the solute and gives the partition coefficient, it just like a partition coefficient of the membrane it between the two phases right of the solute between the two phases in the upstream side, and in the downstream side, it gives a relationship or a partitioning value of the solute, in the upstream side of the membrane and in the downstream side of the membrane.

So, this is a very gross assumption one can so this real retention it. So, it basically a very gross assumption is very simplistic version of the pour flow modeling. If, you like to have a detailed, you know modeling or detailed, you know understanding of transport of the solute to the membrane pores, one has to go for the pour something called pour flow modeling which is beyond, the scope of this particular course may not be discussing it, but the easiest way to define the value of the solute upstream and downstream of the membrane, by defining a partition coefficient, and that partition coefficient is constant for a particular membrane solute system and the simple definition is real retention. So, real retention has great implication as well as the modeling of the system is concerned and design of the system is concerned and see,  $R_r$  is always greater than observed retention  $r_{naught}$ , because  $C_m$  is greater than  $C_{naught}$ .

(Refer Slide Time: 07:03)

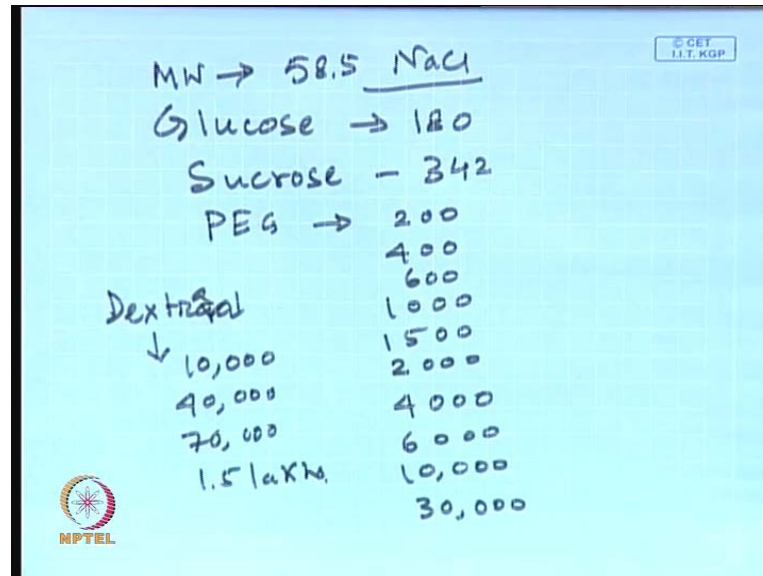


Now, next definition that we (O) cover gross the molecular weight cut off basically gross interpretation or gross minimum of pore size of the membrane. Now, it is very difficult to find out, the pore size of the membrane there are various method for example, standing electron micrographic and various other methods are available, for you know for carrying out the pore size distribution of the membranes. Now, membranes cannot be having a single pore deliver distribution of the pours and average pores is reported, if require sophisticated instrument costly instrument, which will be in the cost will be in the order of 40 to 55 lakhs for measurement of the pore size distribution of the particular polymeric membrane.

Now, that easiest way to find out the you know. Suppose, you are going to develop some membrane in laboratory, and try to characterized, what cut off the membrane is one will be rough polymer pore size in membrane, the that weight define is molecular cut off in membrane is having 1000 molecular cut off means, it will retain solutes which will be having molecular above 1000 above 1000, and it can permeate it will permeate solutes with molecular weight less than 1000. So, that means it is say 1000 molecular cut off now how to find out the molecular cut off curve, how to determine the molecular cut off our particular membrane is very important? And is quite crucial as for as the membrane (O) such as a concerned. So, what one has to do what has to prepare a solution of various molecular solution of solutes high different molecules weights for example, I prepare

suppose, I prepared in membrane, I prepare solution of lets a ten percent solution one percent solution of various solutes having molecular weight.

(Refer Slide Time: 09:34)



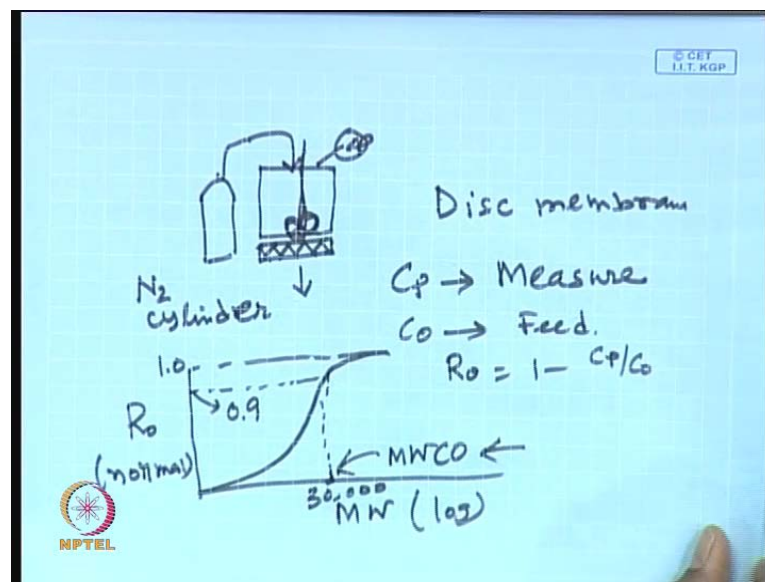
It is a first molecular weight again 58.5 that is sodium chloride right next, let say glucose having is molecular weight around 180 sucrose molecular weight of 342, PEG of various molecular weight are available 200 400 600 1000 1500 2000 like that p g 4000 6000 10,000 30,000 various molecular weight of polyethylene glycol fractions are available. Similarly, you can have Dextran fraction Dextran is again polysacride Dextran it has different molecules let say, 10,000 40,000 70,000 1.5 lakhs so what will do and generally this molecular cut off curves of generated by using neutral solute that is, quite important sodium chloride is not a neutral solute it is simplest possibly electrolyte.

You can think of but other molecules like glucose sucrose various molecular of polyethylene glycol Dextran all are neutral solute. they are not charged why they are using neutral solute ?and why not charge solute of doing the molecular cut off characteristic of simply, because some of the membrane whenever talking about the polymeric membrane some of the membranes are becomes membrane matrix becomes charged so, the pour cell becomes charged. So, the in the very large space mall distance the charger interaction and electro electric properties of these of between solute and the membrane matrix is become very important deviling thinks, like that will talk about, letter on always thinks, because very important charger interaction become very

important. (O) We talk about very small distances therefore, those can have an effect on the separation characteristics that, is why the charged molecules are generally avoided creating or characterizing the molecular weight cut off curves.

What we do generally? use neutral solutes like glucose sucrose polyethylene glycol Dextran various fraction of Dextran so, and so forth on now we prepare solution of these very small of let say, one percent solution of all these molecular weight of materials so, where let say we have seven or eight or ten of these polymers. So, you prepare one percent solution or all of them each of them, certain solution then using the test cell it typical test cell look like this.

(Refer Slide Time: 12:26)



It is a body as a body, it is a cylindrical cell shall has a body and in the in the in the bottom there is a know disc is there you can put the disc membrane the lower portion of these flinch is curved. It is basically know (O) so, that the it the permeate that is coming out it can go throw it may it is pressurized by nitride cylinder and you putting stirred if why putting stirred? Simply because to avoid the deposition over the membrane surface to minimize it, you cannot avoid it completely to minimize it now, using various molecular weights you measure the retail.

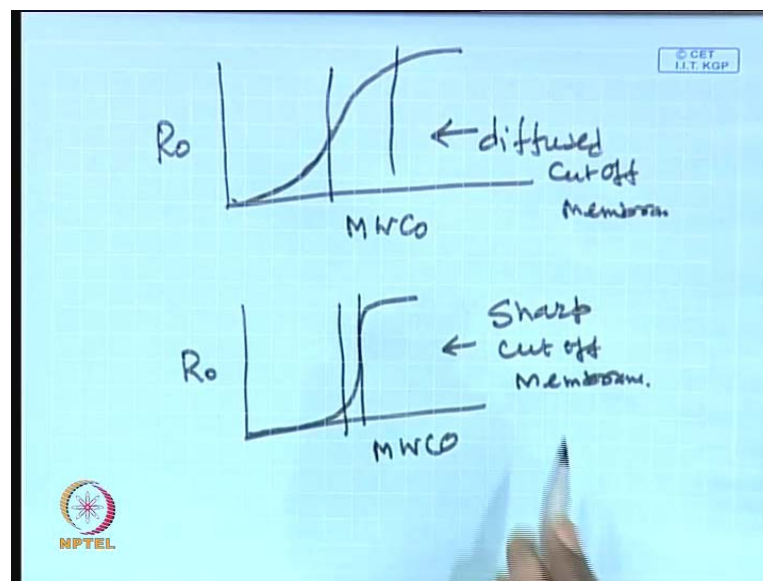
You carry out these experiment is called stirred cell experiment, you carry out these experiment is solutions of different molecular weights and measure the permeate concentration the filter, there is coming measure the permeate concentration once you

measure the permeate concentration. You know the feed concentration you can find out the observed retention  $1 - \frac{C_p}{C_f}$ . Now, in this case you have to have very high stirring speed to effort the polarization over the membrane surface so, that you can assume that over these cells over the volume of the feed the concentration is more or less uniform.

Now you plot the observed retention versus molecular weight in a similar scale this is logarithmic scale this is normal scale, so it is a similar scale now, if you plot the observed retention versus molecular of the solute of that membrane, you will you are going to get a curve something like this is one means hundred percent retention, there is no solute coming in the permeate and in these is separate curve the value corresponding to 90 percent separation is suppose to be a molecular weight cut of the membrane.

Suppose these molecular weight comes out to be let say 30,000 then we call the membrane has 30000 molecular cut off membrane that because what we have to fix some value 0.9 0.9 5 or 0.9 8 something like that, it is general norm it is a heuristic or it is a thump rule that when the value of the corresponding to the molecular molecule the value of the molecular corresponding to the observed retention of 0.9 will be termed as the molecular weight cut off.

(Refer Slide Time: 15:29)



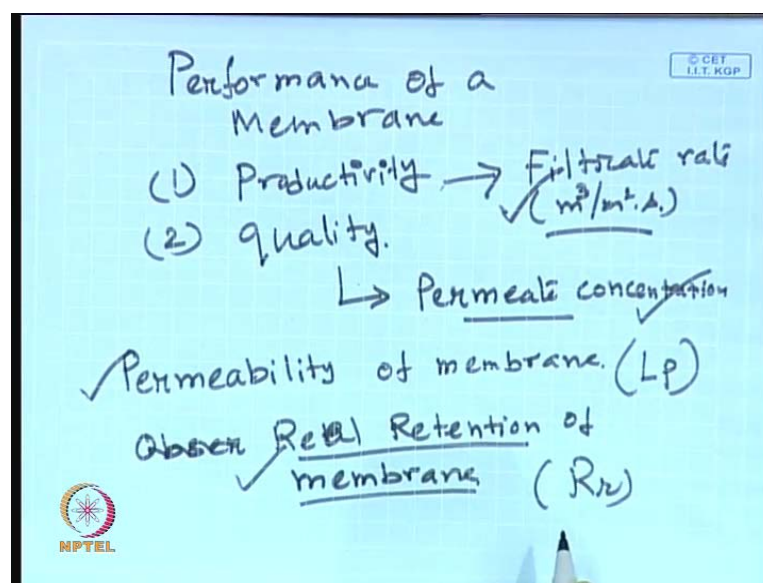
Now, these molecular weight cut off curves can be a two types, it can be diffused cut of membrane that means, this is spread out it is known as diffused cut off membrane, and



this is called a sharp cut off membrane permeate. If I talk about a molecular membrane have a molecular weight ten thousand cut off thus ten thousand may not be very sharp that means, it can retain molecular weights of have ten thousand or nine thousand even so that is called diffused cut off membrane and these cut off is a strif or it is a sharp cut off membrane.

Now, whether you will be getting the sharp cut off membrane or whether you will be getting diffused cut off membrane that will be entirely depending on the pore size distribution of the membrane. Now, one has to control the pore sizes distribution, how to control the pore size distribution? you can thinker the costing conditions of the membrane and can vary the pore size the membrane, and you can optimize the whole system can make it standardized rule. What are the various condition of the costing solution? costing composition solution composition type of solvent are using whether using a d m f or whether you are using a acetone, and that that depends concentration of that time of operation or time of exposes to the ice water bath temperature of aniline duration of aniline. All these are various conditions of the all it is the various costing condition by tinkering of by changing this costing condition, one can have various molecular weight pore size distribution, and one can land up with various molecular weight cut of the membrane now, there are for any modeling purpose for forget about modeling.

(Refer Slide Time: 17:43)





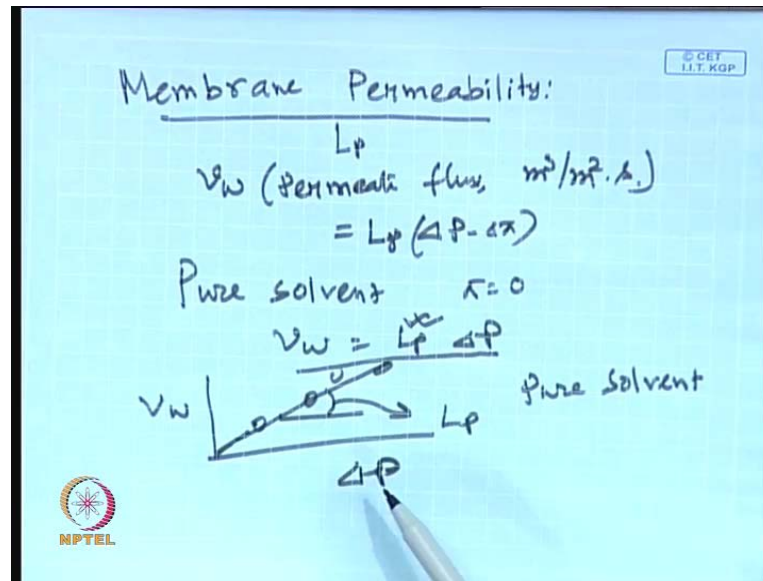
For if you look into the performance of a membrane, there are two things one has to look into these two things are one is productivity, another is the quality. Productivity means how much filtrate rate you are going to get, that is a volumetric filtrate rate meter cube per meter square second or liter per meter square hour, and quality is the permeate concentration these two are the performance indicating parameters of any membrane based separation processes.

Now, in order to get these values there are two definitions which are indicating which are two these definitions one is the permeability of the membrane, another is the observed retention or real retention observed retention is rate thing. So real retention is the actual one permeability indicates how porous your membrane is that means, if you talk about membrane higher permeability you are going to get high productivity, and real retention of the membrane means if the real retention becomes very high membrane is very selective to particular solute, that is means retention will be very high that means for real value of real retention one that means, a  $C_p$  is equal to zero that means, you are not going to you are not going to get any solute in the permeate stream.

The membrane retains everything for that value of for such case for the one it is pure water in the downstream in the permeate stream value of both real retention and observed retention are equal to one therefore, real retention is directly related to the quality of the membranes performance. So these two parameters productivity and quality are directly related to the two properties of the increasing properties of the membrane, one is the permeability another is the real retention therefore, whenever you are trying to design a membrane based process is or trying to predict the system performance.

One has to look into for independently determine these two properties one is the membrane permeability, another is the real retention the notation that is used for the real retention are or another is the for membrane permeability. It is known as  $L_p$  now there are there are you know independent methods by which one can determine the value of membrane permeability and determine the value of real retention separately.

(Refer Slide Time: 21:04)



Now, we are going to discuss these methods, membrane permeability is basically any fluxes proportional driving force that is called you know that there is a physical law so called permeate flux, that is the productivity of the process permeate flux is meter cube per meter square second or liter per meter square hour is proportional to the driving force. Driving force is  $\Delta P$  in this case and proportionally constant is permeability now, if you are talking about it about pure solution, pure water pure solvent there (()) pure solution if it is the pure solvent, then that is no osmotic pressure in it if there is a solution then there are osmotic pressure will be there, and you have to work come the osmotic pressure. So the effective operating transform pressure will be  $\Delta P$  minus  $\Delta \pi$  so, there will be minus  $\Delta \pi$  here, if you talk about talking about the presence of the solution for pure solvent  $\Delta \pi$  is equal to 0. This the question of  $\Delta \pi$  so  $V_w$  is equal to  $L_p$  time  $\Delta P$  this is known as phenomenal epical equation, phenomenal epical equation means any phenomena can look in this uni first can be explained that flux is proportional to driving force, that is a phenomena.

Now, it is very simple how to once you know the definition is very simple to find out the membrane permeability. What to do? basically use the same stirred cell and use water instead of use pure solvent instead of the solution and conduct the experiment under different operating pressure, and measure the permeate flux and plot the permeate flux versus  $\Delta P$ . You take pure water or pure solvent carry out the experiment under solvent carry out the experiment under various operating pressures for a particular

pressure, you measure the permeate flux and similarly, you can generate the data of permeate flux at various operating pressures, then plot these values let say they will be something like this and then draw a straight line among them.

It must be going through the origin then slope of this curve will give you the membrane permeability, so one has to conduct experiments using pure solvents and various operating trans membrane pressures measure the permeate flux plot the permeate flux versus  $\Delta p$ , it has to be a straight line through origin that slope of that straight line will give that permeate membrane permeability and you need not to give a stirred speed you need not to give anything because that is, no polarization here because there is no solute present there is very simple experiment one can get the permeability and most of the cases if you conduct their experiments will be inherently getting straight line absolutely no problem.

(Refer Slide Time: 24:34)

Experimental Determination of Real Retention: Concentration Polarization

Diagram showing a concentration gradient from  $c_0$  to  $c_m$ .

$R_r = 1 - \frac{C_p}{C_m}$

High Polarization:  $c_0 \uparrow$ ,  $\Delta P \uparrow$ ;  $\downarrow R_r$

Low Polarization:  $c_0 \downarrow$ ,  $\Delta P \downarrow$ ;  $\uparrow R_r$

$R_r \approx R_0 = 1 - \frac{C_p}{C_0}$ ;  $c_0 \approx c_m$

NPTEL logo is visible in the bottom left corner.

Now, the second definition the second quantity the real retention experimental determination real retention is big complicated in these case, what you do? is that the polarization it different of polarization and these polarization basically in these case concentration polarization. I talk about it more little later on a raised earlier the since there is a know in this in this process, the concentration of the solute over the membrane surface is higher compare to the concentration of the solute in the feed. so their exceeds say concentration gradient these phenomenon is known as the concentration polarization

accumulation of solute over the membrane surface is known as the concentration polarization.

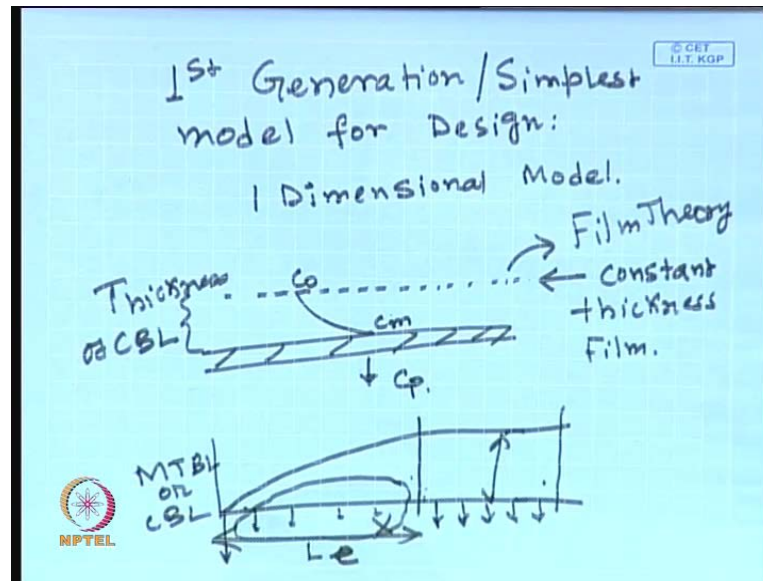
Now, there is no way to determine the membrane surface concentration right directly there is directly experimental measurement it is an active field of research people trying to find out the membrane surface concentration now, what is generally done we carry out the experiments at low polarization condition there, are two things one is high polarization condition and low polarization condition let us define high polarization and low polarization. these high polarization low polarization are directly related to the value of membrane surface concentration, if the membrane surface concentration is high we call that condition as a high polarization condition, high polarization condition will be obtained when you are talking about high concentration solute in the feed high value of  $\Delta p$  and low value of turbulence low Reynolds numbers, if you have high value of concentration in the feed under pressure gradient more solutes will be transported to the membrane surface, so increasing the membrane surface concentration if you talking about high  $\Delta p$  because the rate of more and more solute will be again coming to the membrane surface causing high increasing the surface concentration.

If we talk about the low Reynolds number that means the surface concentration will be it will not be taken away low Reynolds number means that it will be that the scarring of these things will be less solutes will be less so, they the membrane surface concentration will be pretty high so, low polarization condition means we are talking about low concentration low pressure reverse of this and high turbulence therefore, if we carry out an experiment in a small stirred cell under low operating low feed concentration, and lower  $\Delta p$  or trans membrane pressure drop and at very high stirring speed, what it does it mixes everything whatever dip whatever  $(C)$  through the membrane surface.

So, throughout the whole feed chamber the concentration more or less constant right in this case in such case, you conduct the experiment you carry out the experiment at very low at low polarization condition and measure permeate concentration what the permeate concentration will give that you observed retention right  $1 - \frac{C_p}{C_{Naught}}$ , but in these cases  $C_{not}$  is roughly equal to  $C_m$  because membrane surface concentration and bulk concentration are more or less same therefore, the  $(C)$  value observed retention value that you are getting under these conditions will be almost equal to real retention value therefore, the real retention of a membrane and a solute can be obtained.

If you conduct experiment under low polarization conditions and in that case the observed retention will be equal to real retention when you so it since, real retention is a constant property of the membrane of solute, it can be fully used once determined for the various other operating conditions. Now, there makes the full background of the first and first generation model for designing.

(Refer Slide Time: 29:23)



Let us; call it first generation model or the simplest model for design. So this model is nothing but a one dimensional model and what we assume a assumption is basically the solutes will produce a constant film over. The membrane surface solute depositing over the membrane surface will produce a film of constant thickness, so that will be so that will be concentration gradient present from  $C$  bulk to membrane surface concentration in the coming out to be permeate side.

Now, what is wrong with these? this is the first generation model what is wrong with these the error is found here is that mass transfer boundary layer are these basically thickness of concentration boundary layer concentration of boundary layer or mass transfer boundary layer. The assumption is that we are we are assuming a constant thickness of this boundary layer actually these boundary layer the mass transfer boundary layer or concentration boundary layer, if you plot as the function of  $x$  the channel length it will be something like this and we are talking about the little half portion where the thickness is constant, so in this assumption this is known as the film

these we consider in the case constant thickness of the film, we constant these theory known as the film theory is is the first and simplest model for I know attended towards the design of any membrane separation processes. Now, I will just try to give an idea, what is the first coming of this first generation model? The mass transfer boundary layer the developing region is called the you know the intense length for the mass transfer boundary layer and this is a fully developed mass transfer boundary layer. So, we are assuming therefore, in this case where the developing mass transfer boundary layer the thickness of mass transfer boundary layer will be small and is gradually increasing since, that thickness is small it will be offering low resistance against the solvent flux.

So, initially of the channel will be giving, will be getting high permeate flux or high productivity later on it will be decreasing gradually once you.

Once the mass transfer boundary layer grows and only become fully developed, it becomes constant that is how? Whenever you are using the film theory, you are not counting the initial portion of the permeate flux therefore, use of film theory will be alias giving, you under prediction of the permeate flux, because you are overlooking the developing mass transfer boundary layer. And that, region will be substantial if you remember that transport phenomena class that, can taken probably in the last year

(Refer Slide Time: 33:20)

Hydrodynamic B.L.

$$\frac{L_e}{d_e} = 0.05 Re$$

$\downarrow$   
2000

$$L_e \approx \underline{4-6 \text{ cm.}}$$
  

Mass Transf BL.

$$\frac{L_e}{d_e} = 0.05 Re \frac{Sc}{10^5}$$

$\downarrow$   
 $u/PD = 0(5) \approx 10^{11} \text{ m}^2/\text{s}$

$$L_e \approx 80 \text{ cm}$$

That if, you talk about the hydrodynamic boundary layer that means, velocity boundary layer the intense length is given as  $L_e$  by  $d$  will be equal to 0.05 Renaults number.

And since, in these case we are talking about, very clean channel and maximum Renaults number under lamina, flow condition will be a 2000 that twenty to hundred the  $L_e$  will be trans out to be 4 to 6 centimeter. Therefore, for the channels the channel membrane in the membrane separation units, we are talking about, in that thickness the equivalent diameter that intense length of the hydrodynamic boundary layer becomes 4 to 6 centimeter only. Here is the full channel length will be in meters if, the hundred centimeter, two hundred centimeter.

So, whether you will be having an intense length of 4 to 6 centimeter in a channel of length hundred centimeter does not matter and the other hand, if you look into the intense the length of mass transfer boundary layer what is the intense length it will be  $0.05$  Renaults smite number, where smite number  $\mu$  by  $\rho d$  this become very intense length. Now, we are talking about solutes which will be having higher molecular weight for separation in this thesis higher molecular means, they will be having lower diffusivity and that diffusivity will be in the order of  $10$  to the power of minus  $11$  meter square per second that is the typical the maximum diffusivity. We can think of any solute under in the in the present world is of mono valent salt that is sodium chloride  $10$  to the power minus  $9$  meter square per second, there is the maximum diffusivity if you increase the molecular weight diffusivity decreases of the proteins.

We can have diffusivity in the order of  $10$  to the power minus  $11$  meter square per second therefore, thus be and let say if you consider the viscosity will in the order of let us say water viscosity into the minus  $C$  Pascal second, and density roughly the water density so the  $\mu$  by  $\rho$  becomes the kindevity viscosity become  $10$  to the power minus  $6$  so it becomes  $10$  to the minus  $6$  into the  $10$  to the minus  $11$  so, it becomes  $10$  to the  $5$  so it will be smite number will be order of  $5$  if smite number will be in the order of  $10$  to the power  $5$  this becomes the intense length becomes substantial in this case the intense length can be as I as eighty centimeter of higher.

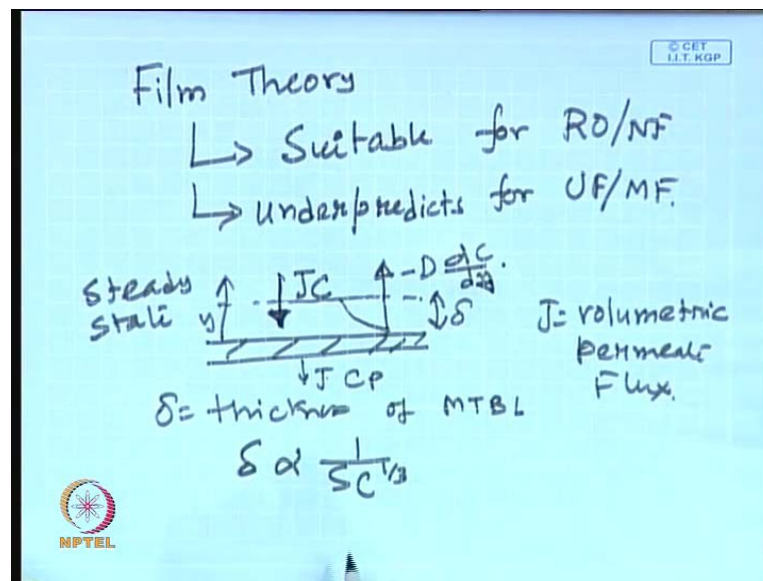
If, we talk about the channel of  $100$  centimeter  $150$  centimeter lets a two meters and half of it  $50$  percent is the intense length then, it is substantial. Now, if you do not calculate the incorporate, the permeate flux in the intense length that is coming out, which will be the maximum obviously, we are going to under predict the permeate flux. So, that is the there is the first almost, you know important drawback of film theory, because in the film theory. We are consenting the portion of the channel length of the mass transfer



boundary layer will be constant that, means fully developed region. Which will be only occurs at the letter of the channel now film , so film theory can be can be incorrect in that case of ultra filtration.

When we are talking about separation of protein, by it will be quite closed to the correct value. When we are talking about separation of the salt, which will be having very much high, you know diffusivity in the order of 10 to the power minus 9. So, film theory will be giving more or less correct estimation of productivity. In case of reverse osmosis, and nano filtration is case of ultra filtration micro filtration. We are talking about separation of the solutes, of proteins of the solute. Which will be having molecular weight quite high, and low diffusivity in the order of 10 to the power of minus 10 into the minus 11 then film theory will be giving you in a accurate prediction or under prediction.

(Refer Slide Time: 37:36)



So, let us defined the utility region of using of film theory suitable for R o NF under predicts for UF or MF ultra filtration, and micro filtration now having kitting these limitation of film theory and micro filtration now having kitting these limitation of the film theory. Let us, look into the film theory at the talking about, the steady state only because most of the operation done in steady state. There are three flux towards the membrane surface, one is the permeate flux one is the convective flux J type C J is the volumetric permeate flux, and this is the mass transfer boundary layer with thickness

delta is the thickness of mass transfer boundary layer and concentration boundary layer and volumetric fluoride is J and the concentration is C p.

So, volumetric fluoride time is concentration will give you the flux of the solute flux towards the membrane. Now, J is the volumetric permeates flux and we are assuming, the thickness the mass transfer boundary layer will be extremely small, if you remember from that, transfer phenomena delta is inversely proportion to smite number may be smite number 1 upon 3. Therefore, for these case is we are talking about, high smite number 10 it the power 5 therefore, the thickness of mass transfer boundary layer will be extremely small.

So, we can safely a assume that the velocity feed that, within the mass transfer boundary layer. In that transfer reaction that means, in the y direction will be more or less same, but in opposite sign to the permeate permission velocity in the permeates side. So, let as look into the identify various proxies at the steady state towards the membrane, one is the convective flux of the solute towards the membrane, because of the pressure gradient, another is the diffusive flux high from the membrane, because of the membrane surface concentration will be a higher, because of presence of concentration polarization concentration gradient will be in the boundary layer. There will be backward diffusion from, the membrane surface towards the (0). So, D is the second diffusion minus and dc by dy and in convective flux in the permeate side.

(Refer Slide Time: 40:38)

$J_C \rightarrow$  Convective solute Flux towards membr.  
 $J_{C_p} \rightarrow$  Conv. solute flux away from membr.  
 $-D \frac{dc}{dy} \rightarrow$  Diffusive solute flux away from Membrane.  
 At. <sup>Steady</sup> S.S.  $\rightarrow$  these three fluxes  
 $\underline{\underline{\sum J_s = 0}}$

© CET I.I.T. KGP  
NPTTEL

So, there will be three fluxes present and we just identify them,  $J$  times  $C$  convective solute flux this will be having a unit  $k g$  per meter square. Second solute or kilo mole per meter square, second solute flux towards membrane  $J C_p$  convective solute flux away from membrane and  $-D \frac{dc}{dy}$ . It is the diffusive solute flux away from membrane, now at the steady state these three fluxes summation of fluxes will be equal to 0  $J$  transfer. The solute flux summation of these 3 fluxes will be equal to 0 that, gives the governing equation of the productivity in the systems.

(Refer Slide Time: 42:11)

The image shows a handwritten derivation on a light blue background. The equations are as follows:

$$J C - \left(-D \frac{dc}{dy}\right) - J C_p = 0$$

$$J (C - C_p) + D \frac{dc}{dy} = 0$$

$$\frac{dc}{dy} = -\frac{J}{D} (C - C_p)$$

$$\int_{C_m}^{C_0} \frac{dc}{C - C_p} = -\frac{J}{D} \int_0^{\delta} dy$$

$$\ln \frac{C_0 - C_p}{C_m - C_p} = -J \delta / D$$

$$\ln \frac{C_m - C_p}{C_0 - C_p} = J / K \rightarrow \text{Film theory Equation.}$$

$$J = K \ln \left( \frac{C_m - C_p}{C_0 - C_p} \right)$$

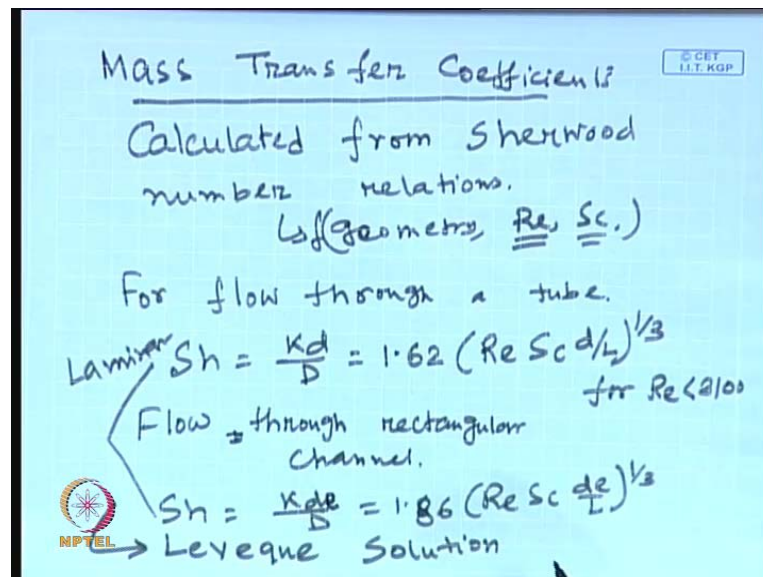
Logos for NPTEL and CEY I.I.T. KGP are visible in the bottom left and top right corners of the slide respectively.

What I that at the map  $J C$  minus of minus  $D \frac{dc}{dy}$  minus  $J C_p$  will be equal to 0, so  $J$  times  $C$  minus  $C_p$  plus  $d \frac{dc}{dy}$  will be equal to 0. So, it can get the governing equation of concentration  $dc \frac{dy}{dy}$  will be equal to minus  $J$  by  $dc$  minus  $C_p$  separate, the variables  $dc$  by  $C$  minus  $C_p$  will be minus  $J$  by  $d \frac{dy}{dy}$  and indicate over the thickness of mass transfer boundary layer from 0 to  $\delta$  is, the thickness of the mass transfer boundary layer and on 0 means, on the membrane surface the concentration  $C_m$  at the edge of the boundary layer is the  $\delta$   $C$  is equal to  $C_0$ , these gives you  $\ln \frac{C_0 - C_p}{C_m - C_p}$  divided by  $C_m - C_p$  is equal to minus  $J \delta$  by  $d$ . So, will be getting  $C_m - C_p$   $\ln$  of that is equal to  $J \delta$  by  $d$  is nothing, but the mass transfer film mass transfer coefficient. So,  $J$  is equal to  $K \ln \frac{C_m - C_p}{C_0 - C_p}$ , this is known as famous film theory equation.

So, if you somehow can estimate membrane surface concentration, if you can know the permeate concentration either by experiment or theoretically and you know the feed concentration and if you can; if you know; how to estimate mass transfer coefficient.

You can estimate the value of permeate flux, now before going into the little calculation, how to solve the equation? How to estimate the membrane surface concentration? Would like to discuss something about, the mass transfer coefficient, because the expression mass transfer coefficient is quite relevant, and quite important is caring out these type of calculation.

(Refer Slide Time: 44:32)



Mass transfer coefficient for estimation of mass transfer coefficient, there are several Sherwood number relation are available calculated or estimated not calculated estimated from Sherwood number relations and obviously, they will be depending on the geometric there function of geometric of the flow channel, and Renaults number smite number Renaults number basically, the turbulence smite number is the property of the feed, and for flow through a tube. The Sherwood number has given as  $K d$  by  $D$ , this  $K$  is the mass transfer coefficient small  $d$  is the diameter of the tube, capital  $d$  is the diffusivity of the solute in the system. These becomes  $1.62$  Renaults smite  $d$  by rest to the power  $1$  upon  $3$  for lamina flow region.

Laminar flow is less than  $2100$  and for flow through rectangular channel the Sherwood relation is  $Kde$  by  $D$  is equal to  $1.86$  Renaults smite  $d e$  by lrest to the power one upon  $3$

for the lamina flow region. So, both are is laminar and they are known as the Leveque solution under laminar flow condition, this relations are derived from the theory only they are not correlation, so they are and the solution if first present in the Leveque. So, it is known as, the Leveque solution, so they are not correlation. They are derived from the first principle and they are theoretically derived d e is the equivalent diameter, and I will give you the definition of d after giving the definition of Renaults number and smite number Renaults number and smite number.

(Refer Slide Time: 47:29)

$Re = \frac{\rho u_0 d_e}{\mu}$ ;  $Sc = \frac{\mu}{\rho D}$

$\rho \rightarrow$  Density  
 $\mu \rightarrow$  viscosity  
 $D \rightarrow$  Solute diffusivity

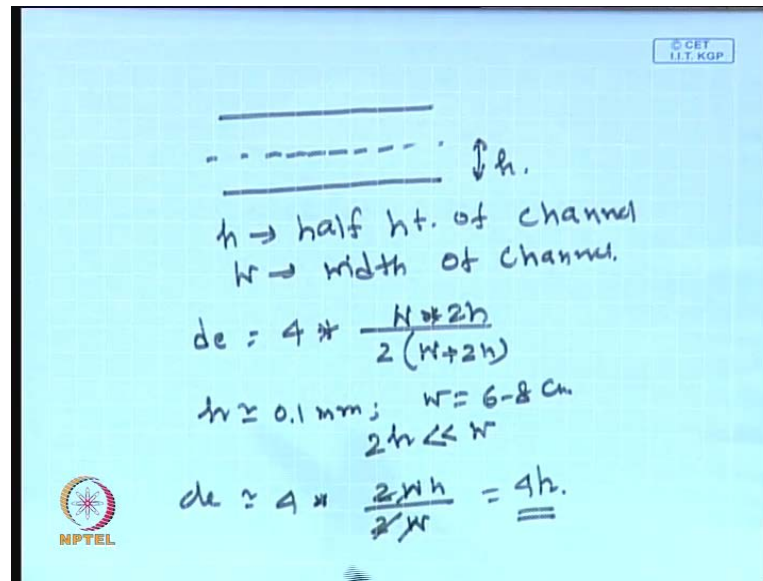
$L =$  Length of channel  
 $d_e =$  equiv. diam.  
 $= 4 \times \frac{\text{Wetted Area}}{\text{Wetted Perimeter.}}$

$N \rightarrow$  width;  $h \rightarrow$  half ht.

© CET I.I.T. KGP  
NPTEL

Renaults number is  $\rho u_0 d_e$  by  $\mu$  and smite number is  $\mu$  by  $\rho d$ . So,  $\rho$  is basically density  $\mu$  is viscosity of the solution density of the solution  $d$  is the solute diffusivity in the solution. So, these three are solute solvent property  $L$  is the relation is the length of the channel geometric length of the channel. What is equivalent diameter? Equivalent diameter if, you remember the definition of the equivalent diameter. If, is 4 times wetted area wetted perimeter for a tubular membrane for a tube geometric, the diameter will be diameter, in a diameter of the tube for other geometric like rectangular channel excreta. Let say,  $W$  is the width  $h$  is the small height, half height of the channel. So, we are talking about the equivalent diameter of a rectangular channel.

(Refer Slide Time: 49:15)



Rectangular channel means, there are two plates and flow is occurring between the two plates. This is the half channel height  $h$  full channel having  $2h$  and  $h$  is the half height of the channel and  $W$  is the width of the channel. So, equivalent diameter is 4 times wetted area, what is the wetted area  $W$  time  $2h$ , the cross section area  $W$  times  $2h$ . What is the wetted perimeter with width both side and full height on both side. So, 2 times  $W$  plus  $2h$  and these types of membrane channels very thin channel  $h$  will be in the order of 0.1 millimeter, and the other hand width will be in the order of 6 to 8 centimeter. Therefore,  $h$  is if the  $2h$  will be much less than  $W$ .

So, under this is condition  $d_e$  becomes 4 times  $2Wh$  and this become  $2Wh$  will be negligible small compare to the  $W$  itself. So, two  $Wh$  will be canceled out  $d$  will be roughly 4 times half height. So, if you know the half height of the channel that is the geometric specifications of the channel. You can find out what is the equivalent diameter, once you know the equivalent diameter will be in the prediction to calculate, the Reynolds number will be in a if you know the I know, physical properties of the solute and solvent will be able to know the smite number once you know, the smite number once you know the Reynolds number, once you know the geometric like half height they equivalent diameter of the channel or in the diameter of the tube and if, you know the length of the channel or length of the tube will be in the position to calculate the Sherwood number or mass transfer coefficient under lamina flow condition.

In fact, for the turbulence flow condition and for the stirred cells, because the most of the cases stirred cell is very important, as module for the stirred cell defined mass transfer relation Sherwood number relation available using, those relations will be able to final out the estimate, the mass transfer coefficient and it can utilize them in order to find out the in order to calculate. The permeate flux permeate concentration, if will just started it will take probably one more class to wind out, the how the first generation model is utilize to predict, the system performance that will see in the next class.