Novel Separation Processes
Prof. Dr. Sirshendu De
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture No: # 28
Gas Separation (contd.)

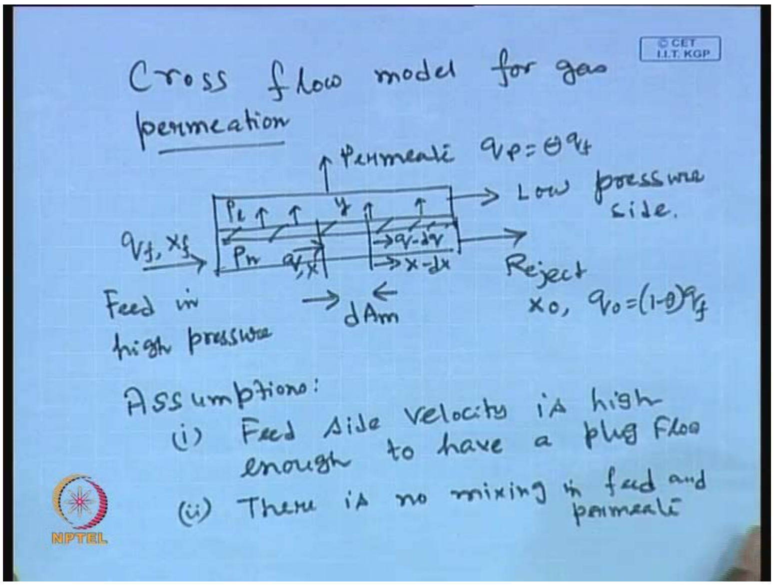
Good morning everyone, so we were looking into the Gas Separation by membrane based separation processes. And in the last class we have discussed about various modes of gas separation by membranes, depending on the type of flow or flow configuration, one can have a cross flow type of separation B cause system; one can have a counter current flow, one can have a (())current flow.

So, in the in the last class, we have talked about a super mixed system, that means a system that is really called, really acts like a CSTR that means, the gas velocity is so high that in fact, you one can put starrer, in the feed stream as well is in the permeate chamber; so that the streams are really mixed. So, one can one can assume a completely starred tank reactor model, in order to develop the design equation of this type of systems.

Now in today's, so and now we have we have we derived the design equations based on the starred tank reactor model, for a completely mixed system. And we have seen that there are around 7 variables that, one can have in such a system; now out of this 7 variables, 3 or 4 are known and rest of them can be one can determine, and there are several types of unknowns, may be they are the it may be that the fractional recovery in the permeate or the permeate concentration or reject concentration.

Mole fraction in terms of mole fractions, they are known or they may be unknown; now out of these 7 parameters, 4 are known and 3 are unknown, so one can use the designed equations and there are 3 numbers of equations and there are 3 unknowns. So, it is a totally solvable system, and one can predict the system performance.

(Refer Slide Time: 02:13)



Now in today's class, we talk about another mode of flow configuration in gas permeation that is the cross flow model for gas permeation through a membrane. So, let us let me draw the schematic of this, this is the feed is coming into the system at high pressure, feed in high pressure and the flow rate is q f, the mole fraction is x f, p h is the pressure in the feed side, is a higher pressure, p l is a lower pressure and the permeate is taken out from the other side. And these q p is a fraction of q f, so that fraction is the theta that is the cut of feed that is the percentage of feed, that is going into the permeate and it is a low pressure side.

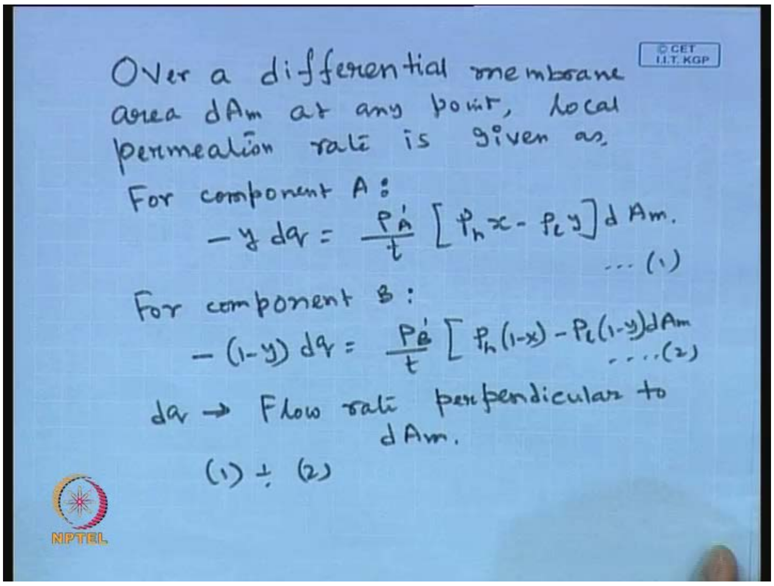
Often the permeate side is attached to a vacuum, so that a low pressure is maintained and there is a substantial pressure difference across the membrane that, drives the solute to go move from the feed side to the permeate side. Why is the concentration is a mole fraction of the required desired component in the permeate side, and the permeate is really moving into the permeate side by a cross flow, why it is a cross flow because, the reject is the feed stream is going into the, under the permeate is going in normal to the direction of the feed flow.

So, the reject side will be having a composition a mole fraction x naught, and it will be having a q naught which is nothing but, 1 minus theta times q f. Now, if you consider a differential element in the in the feed chamber that it that it is located between a membrane area of d A m. And at the location x, q is the, at the location l q q is the flow rate, and x is the mole fraction, and l plus delta l the flow rate, since some amount of permeate is going away to the permeate; so flow rate will be really decreasing, and at the exit of the differential element of flow rate will be q minus d q. And the mole fraction will be x minus d x, why these will be negative simply because, as you go along the length of the membrane, the feed flow rate will be decreasing because, some amount is already moving over to the permeate side. And similarly, some amount of the component, the desired solute that will be also moving into the permeate, so the mole fraction will also be decreasing.

So, x minus d x will be a mole fraction at the exit of this differential element, and q minus d q is the flow rate that is at the exit of the differential element. Now, what we are assuming we are assuming, let us list down the assumptions, assumptions are number one, the feed side velocity is high enough to have a plug flow. Secondly, the permeate there is no mixing there is no mixing in feed and permeate, so there is there lies the difference between this model and the earlier model.

In the earlier configuration, we have assumed that there is complete mixing of the feed side, as well as in the permeate side, so we assumed a completely starred time model or completely mixed up model. In this case it is the path flow therefore, at every location of the membrane length, the feed (()) flow rate as well as the composition will vary.

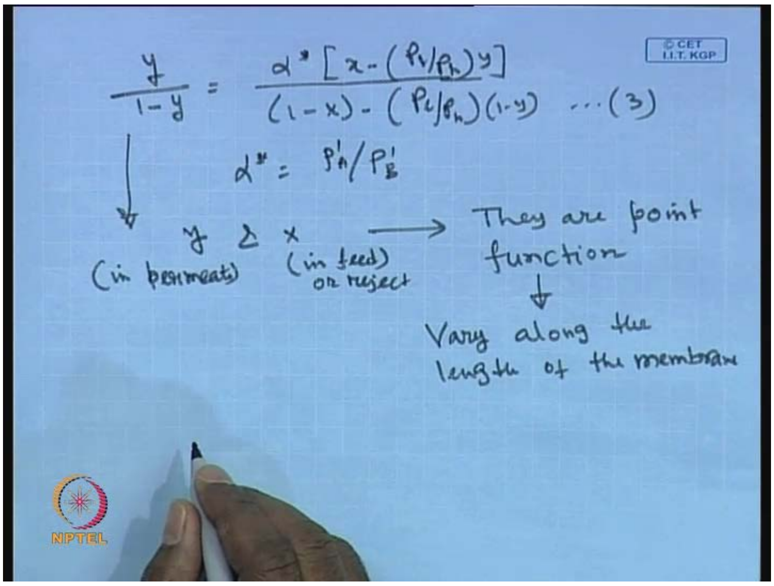
(Refer slide Time: 06:44)



Now, over a different differential membrane area d m, d A m at any point, local permeation rate is given as, for component A. And what is the local permeation rate, local permeation rate will be nothing but, the permeability of the particular species multiplied by the driving force. What is the driving force, driving force is the partial pressure of that particular component in the feed side minus the partial pressure of the particular component in the permeate side. So, minus y d q is equal to P A prime divided by t [P h x minus p l y] d A m.

So, t is the thickness of the membrane P A prime is the permeability of species a through the membrane, let us write down these equation number 1, for component B, you can have a similar balance, minus (1 minus y) d q is P B prime by t p h (1 minus x), so it is a, we are considering its a binary mixture of A and B minus p l (1 minus y) d A m, this is equation number 2. Now, these 2 and d q is the, what is d q, d q is the total flow rate perpendicular to the differential element d m, flow rate perpendicular to the differential element d A m.

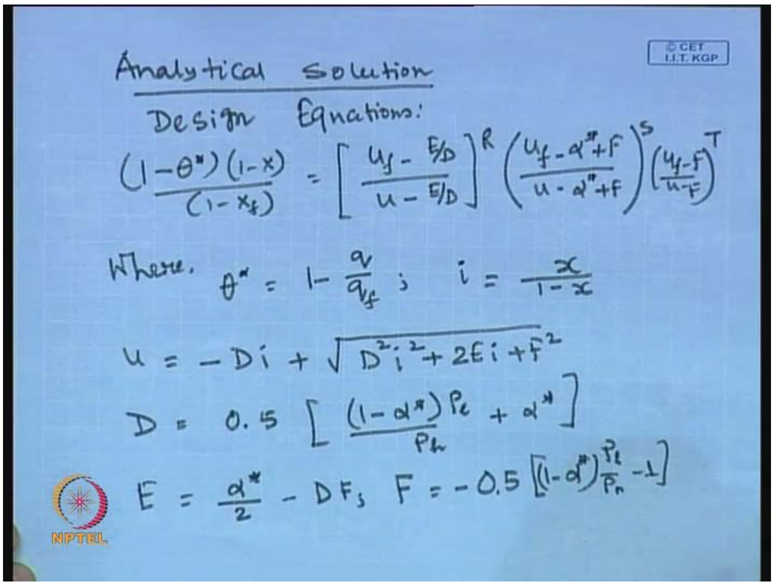
(Refer Slide Time: 09:15)



Now, you can divide this equation number 1 by 2, and see what you get, if you divide equation number 1 by 2, we will be getting as 1 minus 1 minus y is equal to alpha star [x minus (p l over p h) multiplied by y] (1 minus x) minus (p l over p h) into (1 minus y), let us say this is equation number 3. And what is alpha star, alpha star is nothing but, the ratio of P A prime of permeability of component 1 component A, and the that of the of the component B. The permeate composition y is vary and the reject composition x, mole fraction x that varies as a point function throughout the length of the membrane. So, y and x that is y is in permeate, and x in feed or reject, they are point functions, what do you mean by point functions because, they are they are varying as a function of length along the membrane, vary along the length of the membrane.

Now, I this if you if you really solve these differential equation now, I am not going to solve the differential equation in this class because, that is not the purpose of this big huge you know derivation. Now, I am just writing the analytical solution, and I will be finally, writing the design equation for this particular cross flow system. So, this system is solved I mean, you have to solve y d q is equal to P A prime by t p h x minus, the equation number 1 and 2, in the differential form that we have to write it down, that those those two equations have to be solved simultaneously.

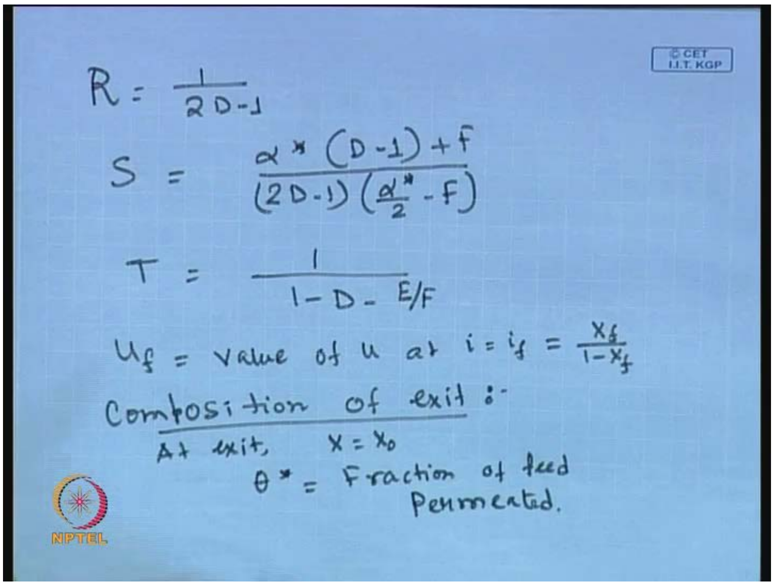
(Refer Slide Time: 11:29)



And and the analytical solution that, you are going to get after solution I am just writing the design equation, the design equations are given as (1 minus theta star) into (1 minus x) divided by (1 minus x f) is equal to [u f minus E over d divided by u minus E over d] raise to the power R (u f minus alpha star plus F divided by u minus alpha star plus F) raise to the power s multiplied by (u f minus F divided by u minus F) raised to the power T. This is a huge expression and all these you know, symbols they are different meanings let us write down, what are the meanings theta star is 1 minus q by q f, so q is the feed that is going into the permeate, and q f is the feed that is getting into the system, in the reject stream, we define a quantity called i, i is defined as x divided by 1 minus x. Basically, this is the mole fraction in the in the reject stream of a this is the ratio of the mole fraction or in the reject stream of component a and component b.

And u is given as minus D times e i plus under root D square i square plus 2 E i plus F square, and what is D, D is 0.5 (1minus alpha star) times p low divided by p h plus alpha star that is D. And what is E, E is alpha star by 2 minus D times F, and what is F, F is minus 0.5 into (1 minus alpha star) p l by p h minus 1. So, the basic parameters were the operating condition p l, p h properties are alpha star, so all these things will be can be can be can be expressed in terms of the known values.

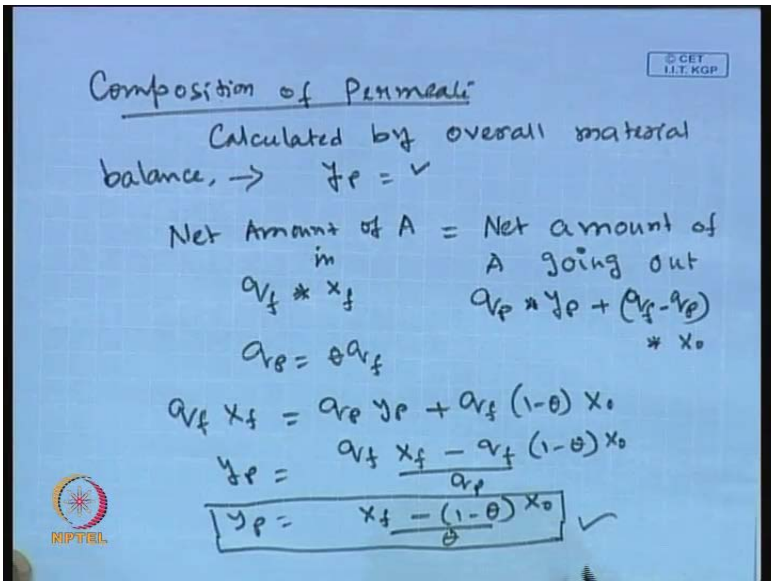
(Refer Slide Time: 14:32)



And there are some more parameters R, R is given as 1 over 2 D minus 1, and what is S, S is alpha star times (D minus 1) plus F divided by (2 D minus 1) (alpha star by 2 minus F). And what is T, capital T is nothing but, 1 over 1 minus D minus E by F, and what is u f u f is value of u u at i is equal to i f, and what is that, that is i f is equal to nothing but, x f divided by 1 minus x f that means, at the feed conditions.

Now, composition at the exit of the feed stream is given as, at exit x is equal to x naught that is the reject composition finally, that is going out, theta star is cut ratio fraction of feed that is permeated. And y p is the mole fraction at the exit of the permeate is estimated by the overall material balance.

(Refer Slide time: 16:20)

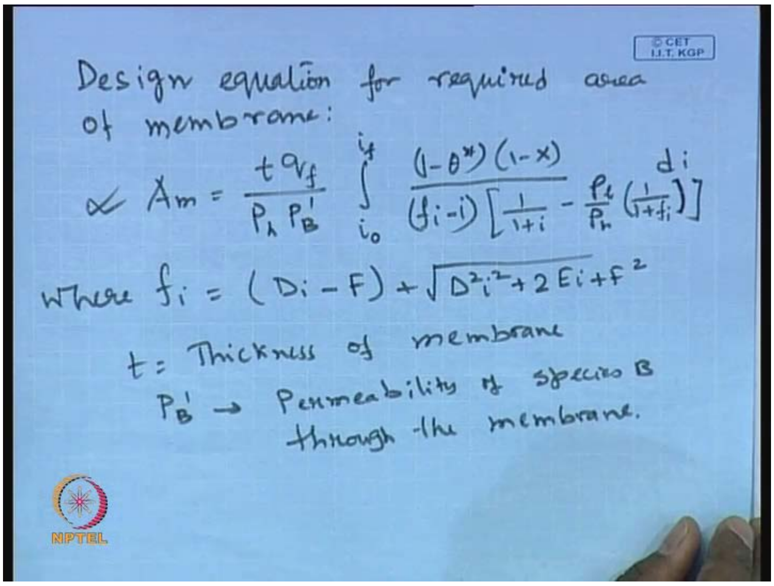


Composition of permeate is calculated by overall material balance balance, that is y p either you can calculate using that, what is overall material balance, net amount of A going into the system in and what is the net amount, q f multiplied by x f. Net amount of A going out at the steady state, A going out there are two components one is by the reject stream, another is by the permeate.

So, it will be q p multiplied by y p plus (q f minus q p) that is the reject stream multiplied by x naught, and and and q p is nothing but, theta times q f that theta is the cut ratio, the percentage of fraction of feed is going into the permeate. So, if you do that q f x f will be nothing but, q p y p plus q f (1 minus theta) x naught, x naught you have calculated just we have we have written earlier; so y p is nothing but, q f x f minus q f (1 minus theta) x naught divided by q p.

So, q p is is is again q p is nothing but, q f into (1 minus theta), so q f will be cancelled out, so x f minus (1 minus theta) times x naught divided by theta. So, that is the composition of the desired solute that is the solute A, in the permeate stream that can be obtained by overall material balance.

(Refer Slide Time: 18:56)

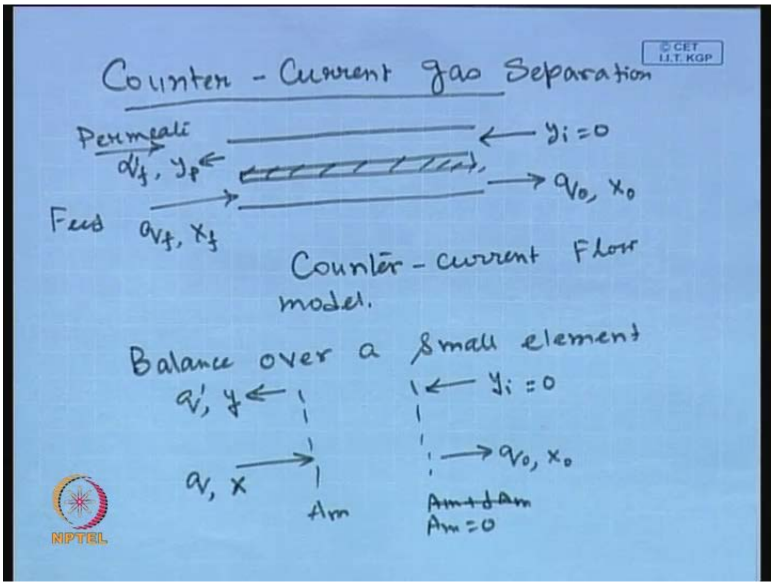


Now, how you obtain the membrane area, required the design equation for required area of the membrane for required area of membrane is given by this expression A m is equal to t times q f divided by p h P B prime, so this is the pressure at the higher higher in the feed side, from i naught to i f (1 minus theta star) (1 minus x) divided by (f i minus i) [1 over 1 plus i minus p l by p h (1 over 1 plus f of i f into i) d i. Where f of i is given as (D i minus capital F) plus under root D square i square plus 2 E i plus F square, and your t is the thickness of membrane, your P B prime is the permeability of of species B, through the membrane.

So, this the design equation for the membrane area required for such a system, so given the operating condition you can calculate, and and the target of removal you can calculate the various values; and these integral has to be evaluated numerically by using trapezoidal rule or Simpson's rule. So, that is the that is how the calculations are design of any cross flow system will be done, so in the last class we have seen the completely mixed up system, in today's class we have seen the cross flow system.

Now, I will go through one more system analysis that is for the counter current application or counter current configuration that is very important because, outer wall is you know configurations the counter current will be the most efficient. Because, in the case of counter current the your allowing a neutral neutral stream that is moving that that moves in the opposite direction of the feed in the permeate, that is called a carrier gas for example, nitrogen or helium or so whatever. Now, these carrier gas takes up the the species a and it moves in the opposite direction of the feed, so the concentration difference the partial pressure it it is maintained at at the maximum possible level. So, you will be getting the more permeation and the system efficiency will be enhanced significantly.

(Refer Slide Time: 22:32)

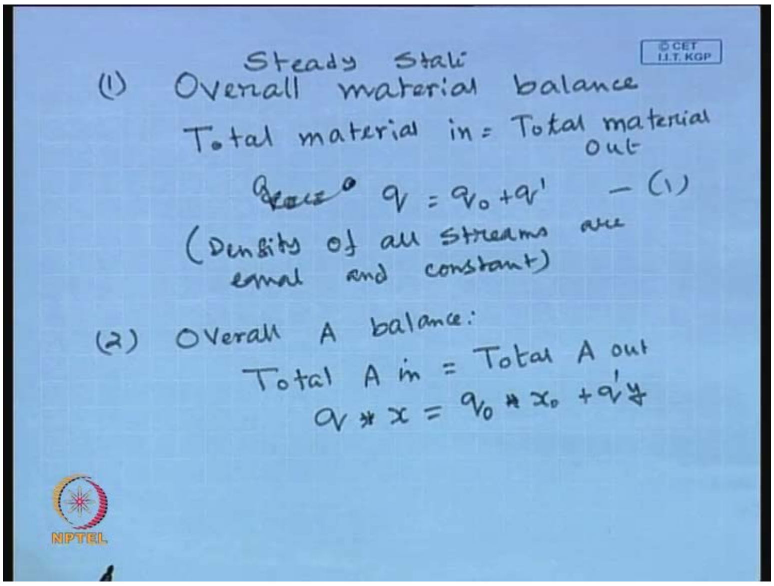


So, counter current configuration is quiet important, in this counter current gas separation, we will be having the two chambers separated by the membrane, and the feed is this is the feed, side this is the permeate side, q f is feed flow rate, x f is the feed concentration. And the mole fraction of the species A q naught is the flow rate of the reject stream, x naught is the composition of the reject stream. And in the case of permeate you will be having y i is equal to 0, it is going in the other direction that means, it is a one way if the carrier gas is entering into the system, there is no solute.

So, it is takes up the composition where the molecule species A and it carries it away, q f prime is the flow rate of reject in the, flow rate of the permeate that is going out, and y p is its composition; that is a typical schematic of a counter current flow model. Now, we do a balance over a small element, this is A m, this is A m plus d A m that differential element membrane area, and so q is the flow rate at this particular location, x is the composition at this particular, particular location; and we consider this end is located at the exit. So, A m will be equal to 0 that means, we start the calculation length from here, so membrane length will be 0 here, and it is going all the way up, and we will be getting the true full length of the membrane. So, A m will be the effective area of the membrane will be 0 there, it will be having some value A m at any x at at any any any x location.

So, y i so q naught is the flow rate of the reject stream, x naught is its composition and the it is coming as y i will be equal to 0, and at any location it is q prime is the flow rate of the permeate stream and the composition is y. So, that is the notation we are going to use, in this when we are doing a small elemental balance in the counter current flow configuration.

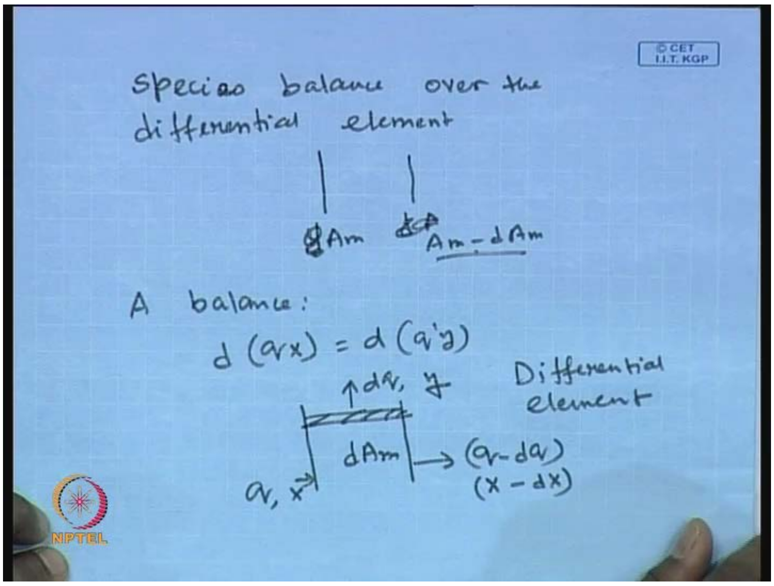
(Refer Slide Time: 25:57)



Now, let us write down various design equations, number one is overall material balance, in the overall material balance, and we we have assumed that it is a steady state process, total material in is equal to total material out. So, q 0 is equal to so q is equal to q 0 plus q prime, q is total material going into q q 0 plus q prime, and we are assuming that the density remains constant for all these streams; density of all streams are equal and constant, they do not vary over the length of the membrane (Refer Slide Time: 26:49), so this is one equation.

Then you do overall second equation, that you overall A balance total A in total A out, q is the flow rate multiplied by the mole fraction of A of A that is q times x that is going into the system; q 0 is the reject flow rate multiplied by the mole fraction of the reject stream plus q prime is the flow rate of the permeate that is going out with the composition y. So, that is the total A going out, now the overall A balance over the differential element, now if you do a that, is a overall balances, now if we do the differential balances, so whatever we have done till now, over the element we are doing the overall balance.

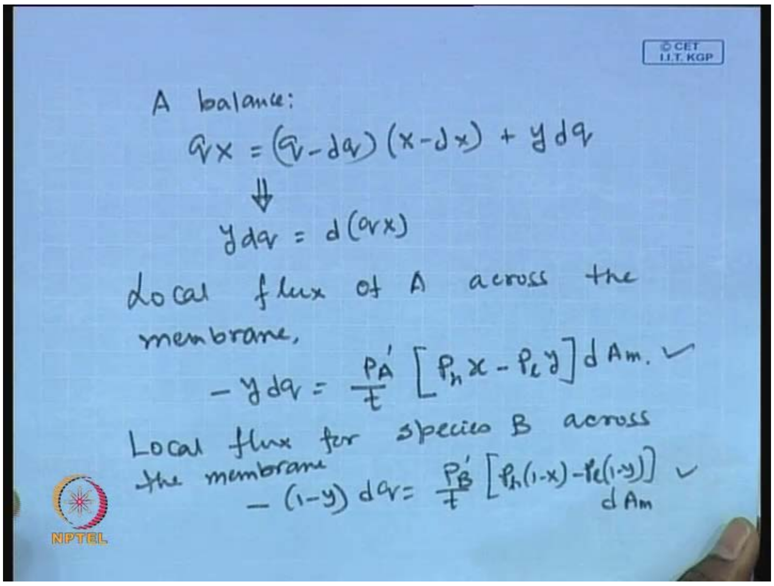
(Refer slide Time: 28:19)



Now, we can do the differential the species balance over the differential element over the differential element, what is your differential element, differential element is between d A m and d a this is A m and A m minus d A m in the earlier case, this was located at the exit point. Now, now if you do a species balance overall over the differential element, the A balance will give the following d (of q x) is equal to nothing but, d (of q prime y), so what is the depletion in the feed side that will be the gain in the permeate side.

So, the overall, this differential balance will be schematically can be shown like this, this is the area A m d A m that is nothing but, A m and A m minus d A m, so this area is d A m. Now, q and x is going into this system, and what is going out is (q minus d q) is the flow rate because, some amount of permeate is going out and since, it will be depleted in x, so x is the mole fraction of the component A, so x minus d x will be going out. And let us say, this is the differential element of the membrane and d q is the gain in the, flow rate in the permeate stream and y is its composition, so this will be the differential element differential balance over the differential element; and this schematic gives differential element with various streams.

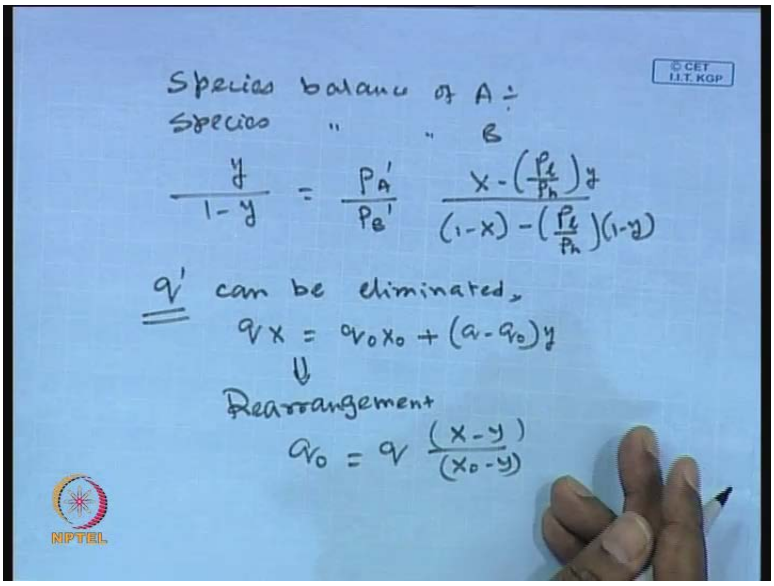
(Refer Slide Time: 30:29)



Now, if we really do that, the A balance gives you q times x is equal to (q minus d q) into (x minus d x) plus y times d q, and if we just simplify this equation, this gives you y d q is equal to differential of (q times x). And you can have a local flux of A across the membrane, the local flux will be the permeability of of A divided by the thickness multiplied by the delta p, delta p is the partial pressure of A in the feed side minus partial pressure of A in the permeate side.

So, if you would like to find out the local flux of A across the membrane, this gives you minus y d q is equal to P A prime by t, t is the thickness multiplied by [P h times x minus P l times y] multiplied by d A m. And local flux for species B across the membrane, can be given as minus of (1 minus y) d q is equal to P B prime over t [P h (1 minus x) minus P l (1 minus y) d A m]. Now, if you combine these two equations, like earlier that mean you divide these equation by these equation, you will be getting a relationship between y and x with the various P l and P h ratio, and the permeability ratio.

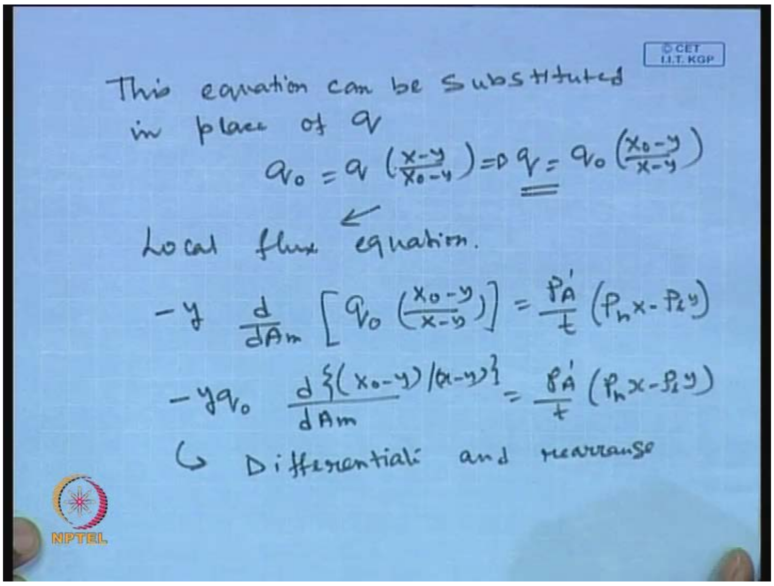
(Refer Slide Time: 32:40)



That means, if you if you divide species balance of A by species balance of B you will land up with the following expression 1 minus y by 1 minus y is equal to P A prime by P B prime x minus (P l over P h) times y (1 minus x) minus (p l over p h) into (1 minus y). Now, q prime can be eliminated, so in the earlier equations, I think we have we have if if you go through the sequence of equation, you can find out from which equation q prime can be eliminated from this q x is equal to q 0 x 0 plus q minus q naught times y 5.17 and 5.18.

So, this is the this is the from the overall material balance, and overall A balance from overall material balance and from overall a balance, one can eliminate q prime, and these equation can be rearranged, and this can be written as q naught is equal to q into (x minus y) divided by (x naught minus y).

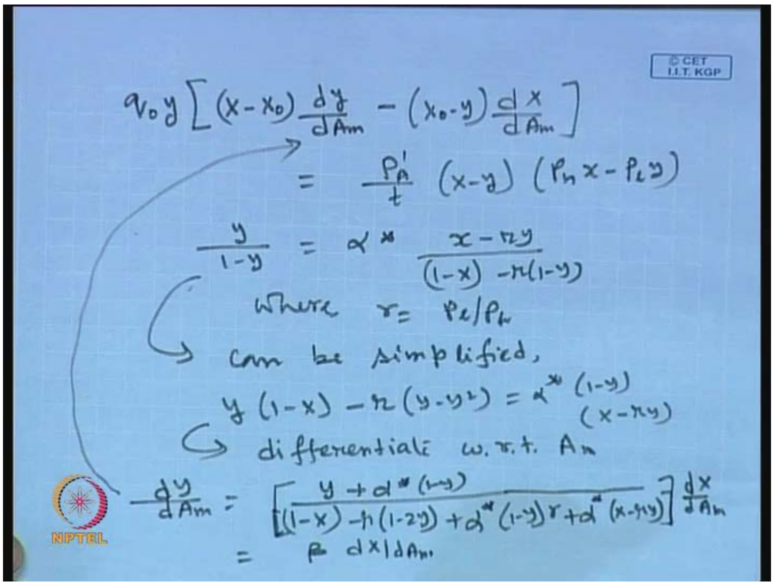
(Refer Slide Time: 34:38)



Now, this equation can be substituted in q, the the the other the these the earlier equation, this equation can be substituted in place of q that means, the the earlier equation was q naught is equal to q into (x minus y) divided by (x naught minus y), so you can get the expression of q in terms of q naught as (x naught minus y) divided by (x minus y). So, this can be substituted in the local flux equation which is nothing but, a differential equation.

If you do that you will be getting minus y d d A m times [q naught (x naught minus y divided by x minus y)] is equal to P A prime divided by t (P h x minus P l y). So, these can be further simplified as minus y q naught d (x naught minus y) (x minus y) within bracket d A m is equal to P A prime by t (P h x minus P l y). Now, we take the you know we differentiate this equation and rearrange, in this case both x and y are function of A and B because, they are varying as a function of the length.

(Refer Slide Time: 36:44)

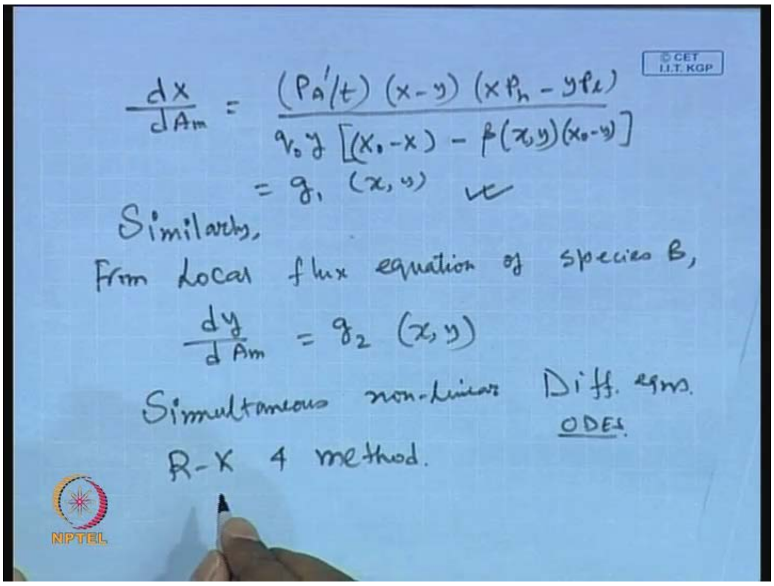


So, if you do that, if you carry out this differentiation, then what you will be getting and rearrangement, you will be getting q naught times y into [(x minus x naught) d y by d A m minus (x naught minus y) d x by d A m] is equal to P A prime by t (x minus y) (P h time x minus P l times y). And you have the relationship between y and x as the relationship is 1 minus y divided by y divided by 1 minus y is equal to alpha star x minus r y divided by (1 minus x) minus r into (1 minus y) where r is nothing but, the ratio of lower to higher feed side pressure.

Now, this equation can be simplified as y into (1 minus x) minus r (y minus y square) is equal to alpha star (1 minus y) (x minus r y), so you can differentiate this expression with respect to A m, these are mathematical (()). Differentiate with respect to A m what you will be getting is that, you will be getting an expression of d y d A m in terms of explicitly d x d y m that will be substituted here so, you will be having only one differential equation d x d A m.

So, d y d A m will be nothing but, [y plus alpha star (1 minus y) divided by (1 minus x) minus r (1 minus 2 y) plus alpha star (1 minus y) times r plus alpha star (x minus r y)] multiplied by d x d A m; so this can be substituted in this differential equation, so it will be basically in terms of d x d A m only. So, these can be written as beta times d x by d A m the whole term is a beta and beta is a function of both x and y, and the parameters were r and alpha star and i is nothing but, the ratio of the pressure in both the chambers.

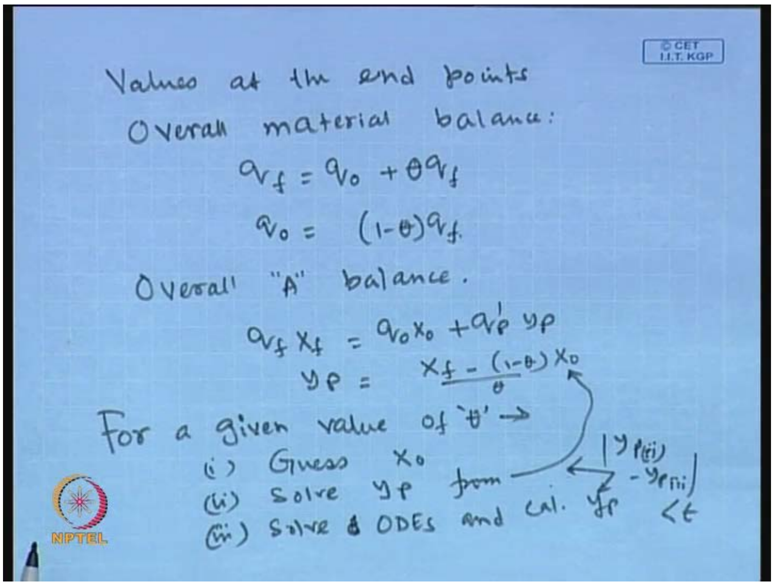
(Refer Slide Time: 39:43)



So so, one can express d y d A m in terms of d x d A m, and d x d A m can be rearranged as (P a prime over t) (x minus y) (x P h minus y P l) divided by q 0 times y [(x 0 minus x) minus beta which is a function of both (x and y) (x 0 minus y), now so this is the expression of of of d x d A m. Similarly, from the local flux equation of the species of of species B from local flux equation of species B, you can obtain an expression of d y d A m, let us say this is g 1 as a function of (x and y), it will be obtained as g 2 as a function of (x and y).

Now, these two equations are ordinary differential equations, and you know the what are the what are the composition at the inlet and they can be solved simultaneous differential equations, simultaneous non-linear. In fact, they are ODE's and you can merge forward, and using a Runge-Kutta 4 method, we can really solve it numerically.

(Refer Slide Time: 41:44)

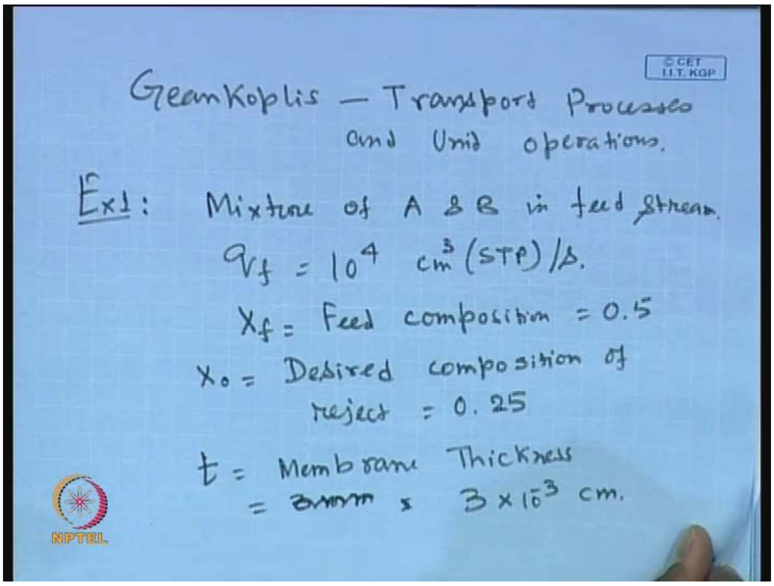


Now, you can get different values at the end points, the values at the end points can be obtained by overall balances, because that is important; overall material balance is given as q f is equal to q 0 plus theta times q f; so theta is the cut ratio and q 0, so therefore, q 0 is nothing but, (1 minus theta) times q f. And overall A balance will give you q f x f is equal to q naught x naught plus q p prime y p that we have derived earlier, y p is equal to x f minus (1 minus theta) x naught divided by theta.

Now, for a given value of theta theta, what you have to do you have to guess a value of x naught that is the composition at the reject stream at the outlet, then you have to solve y p from these equation. Now check the, now solve differential equations ODE's and calculate y p. And check the value of y p that you are getting from step number 3 is coming close to step number 2 or not, if not if if yes fine, so the so you you compare these two values y p from step two minus y p of step 3, mod of that whether that is less than epsilon or not.

If is fine otherwise, you have to guess a value of new value of x naught that is how, this this counter current flow configuration of gas separation will be using a membrane system will be conducted. Now, these will be definitely a a material, which cannot be done in a in a class, so one has to go for the numerical technique, and if you are really interested, you can formulate a problem and really solve this problem.

(Refer Slide Time: 44:27)

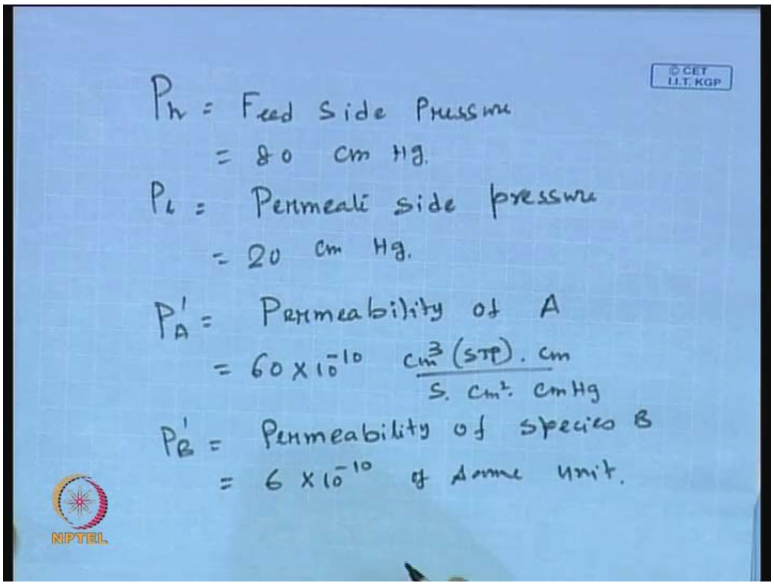


Now, this material has been taken from the book of Geankoplis transport processes and unit operations, so if anyone is interested you can go through this book, and just take up the typical values, they are they have given several problems of these type of things; so you just take up the relevant data, and try to solve these problem numerically. Now, next what we are going to do, we will be based on the principles that we have already said, I I think, if you can remember that the first two flow configurations, can be solved analytically.

So, I will be taking a couple of examples, and try to solve in the class to demonstrate how these design equations will be utilized to design a real gas separation system. So, the first problem is example one it is dealing with a separation suppose, you have a mixture of gas A and B, it is a binary mixture of A and B, in feeds feeds stream; the feed flow rate is given as q f as 10 to the power 4 centimetre cube at STP Standard Temperature and Pressure, that is one atmospheric pressure.

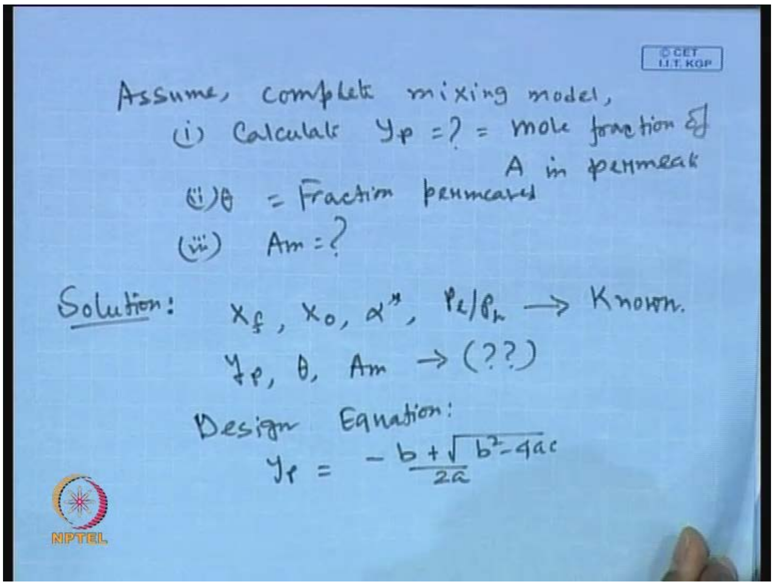
And 27 degree centigrade q f is 10 to the power 4 centimetre cube per second, and feed composition that is going into the system is 0.5 that means, you have a mole fraction of a 0.5, that is going into the system. And the desired composition of the reject is given x naught is desired composition of reject, that is given as 0.02 0.25, so you have set what will be my reject composition, so you have to design the system because, you have to find out what is the required membrane area for this particular application. The membrane thickness is given as t, membrane thickness is given as 3 millimetres that is that is not 3, 3 into 10 to the power it is it will be further less, so it is given as 3 into 10 to the power minus 3 centimetre.

(Refer Slide Time: 47:16)



P h the feed side pressure is given, 80 centimetre mercury, P l the permeate side pressure that is 20 centimetre mercury, and the permeability is of two species, they are given P A prime is permeability of species A, and this is given as 60 into 10 to the power minus 10 centimetre cube at (STP) multiplied by centimetre divided by second centimetre square centimetre mercury. And P B prime is permeability of species B, and this is given as 6 into 10 to the power of minus 10 of this unit, it is the same unit.

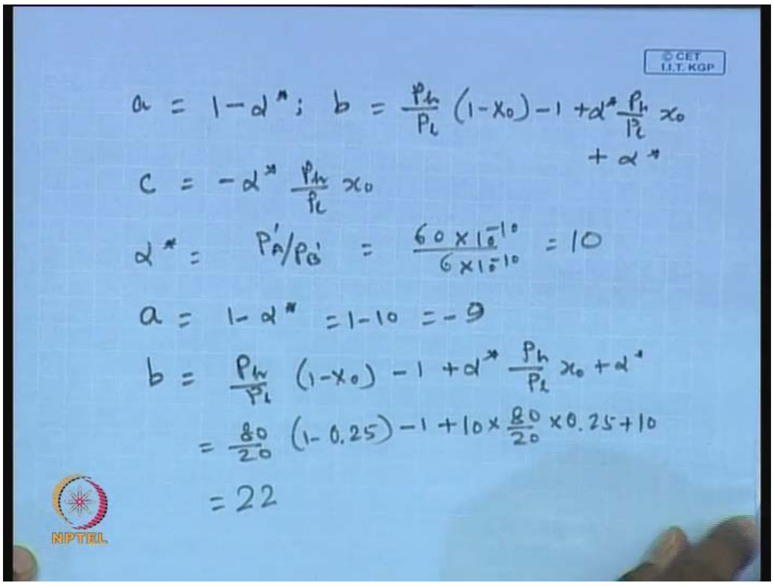
(Refer Slide Time: 48:58)



Now, you have to assume a completely mixed mixed up model, assume complete mixing model, calculate y p that is the mole fraction in composition in the permeate, mole fraction of A in permeate, theta is a fraction permeated; and third the membrane area required for this purpose. So, let us look into this problem, so it is a completely mixed up model, so we have all the design equations, we have we have let us let us say let us write down what are the various values of the parameters are known to us, there are 7 variables some 4 are known, so x f, x naught, feed composition is known to us x naught that we have we have, set that this is the composition of the reject.

Alpha star that is the ratio of permeability of A and B that is known P l by P h the operating conditions are given, so all these four parameters are known. And what we have to determine, we have to determine y p that is the composition in the permeate theta the cart ratio of the feed that is going into the permeate, and membrane area that is required, that we are going to find out from the solution. So, if you remember the design equation of permeate composition, the permeate composition was given as y p is equal to in the form of a quadratic minus b plus b square minus 4 a c divided by 2 a, and that the the we we talked about it say it was a it was a quadratic.

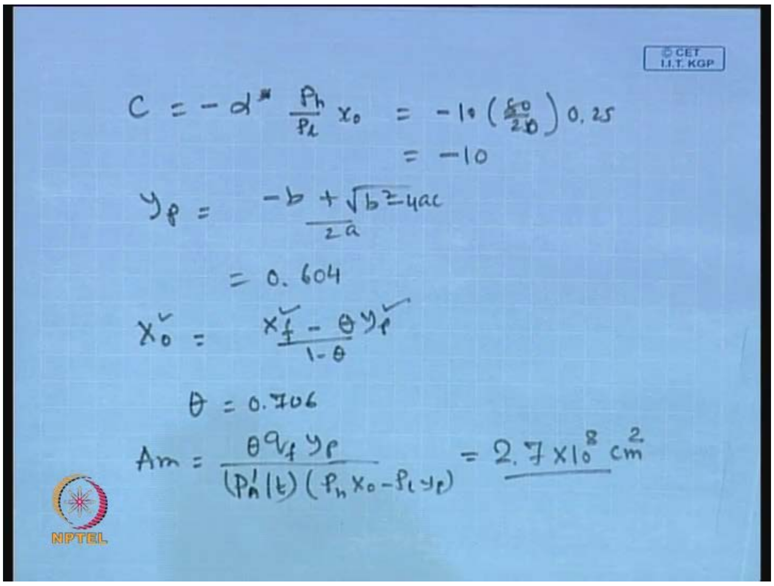
(Refer Slide Time: 51:19)



And these a, b, c are the various parameters which you are which relate the operating and known known values a is nothing but, 1 minus alpha star b is equal to your P h over P l 1 minus x naught minus 1 plus alpha star P h by P l times x naught plus alpha star, your c is minus alpha star P h by P l times x naught. And your alpha star is P A prime divided by P B prime, so this is 60 into 10 to the power minus 10, and this is 6 into 10 to the power minus 10, so this value will be 10 that means, permeability of a is 10 times larger than that of b, so more a is expected to be in the permeate compact to b.

Let us let us find out the other variables, so a is 1 minus alpha star, so it is 1 minus 10, so it will be minus 9, b is P h over P l 1 minus x naught minus 1 plus alpha star P h by P l x naught plus alpha star. So, this will be 80 divided by 20 into 1 minus 0.25 minus 1 plus 10 into 80 divided by 20 into 0.25 plus 10, and if you calculate this value it turns out be 22.

(Refer Slide Time: 53:09)



Now, if you put the value of c, c written minus alpha star times P h over P l over x naught, put the values these turns out be minus 10 (80 by 25 20) into 0.25, so it turns out to be minus 10, and now if you put the value of a, b, c in the expression of b square minus b minus b square minus 4 a c divided by 2 a, this turns out be 0.604. So, x 0 you have x f this relation x f minus theta times y p divided by 1 minus theta, so you you you you have calculated y p, you know x f, you know x naught, so you can calculate the value of theta and theta turns out to be 0.706.

So, for this particular problem 70 percent of feed can be recovered in the permeate. And expression of A m if you remember, it was theta times q f times y p divided by P A prime divided by t into P h x naught minus P l y p, now if you really put all the values these turns out to be 2.7 into 10 to the power 8 centimetre square. So, this is a typical order of magnitude of membrane area that you are looking for for this particular problem.

So, relevant you know configuration can be you know can be can be the module can be prepared by inserting by ensuring that, this much of membrane area, you will be getting in this system in order to to get this system feasible. So, in the next class, we will be I I have decided to solve one more problem, and for the this type of system, and then we will go move for the next topic, that is surfactant enhanced separation processes, thank you.