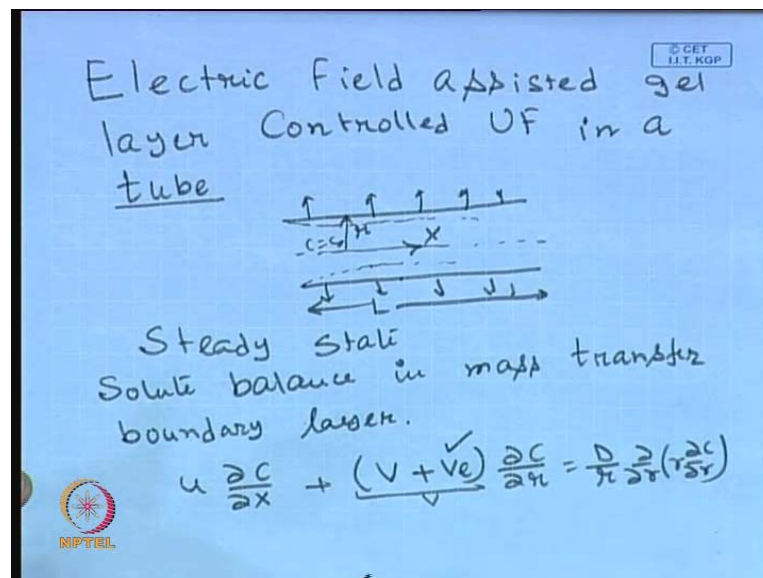


Novel Separation Processes
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Lecture No # 26
External field induced
Membrane Separation processes

Well in this class what we are going to do we are going to analyse a system, for we are going to have a electric field enhanced, gel layer control filtration in a tube. And we have done a corresponding part, if you remember in case of laminar in earlier without electric field, whenever there was a flow to a rectangular channel. So, in order to get a difference between that analysis and this analysis what I have decided to carry on an analysis in a tube number one. Number two, you are going to have a turbulent flow instead of a laminar flow. And see the, how see the analysis will be different in this particular case it is a gel layer controlling ultra filtration and it is being on asserted by external electric field.

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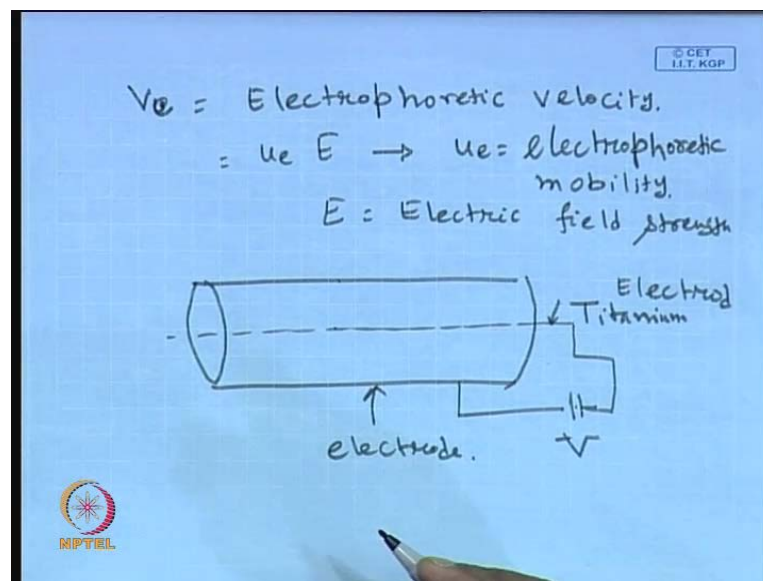


So, the problem that we are going to solved for is electric field assisted gel layer controlled ultra-filtration in a tube. So, there is the, this is the tube, this the length of the tube and there is the location and going to have the, you know? There is the delta and there is the permeation from the wall uniformly for the all the surface is and there is the

development of mass transfer coefficient boundary layer on the wall. So, basically what we are doing? So, here it is C equal to C_{naught} and it will be having C_{naught} up to C_g on the membrane surface so, within the mass transfer bound region.

So, what we.. If you remember, what we have done in the earlier analysis? We did a solute balance in concentration bound area or mass transfer bound area under the steady state, this is the steady state operation. Solute balance in mass transfer boundary layer or it will give if you write down in radial coordinates, it becomes $u \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial r}$ is equal to $D \frac{\partial^2 C}{\partial r^2}$, it should be basically V . What is this V ? So, if which is? Now u and V at the velocity component respectively in the X and r direction, X and r direction in the case y direction is nothing but r direction. Now, the velocity will be having two component, one will be the electrophoretic velocity another will be the; it is basically relative velocity.

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Now, the electrophoretic velocity can be reader to the electrophoretic mobility, V_e will be and these becomes u_e times E , where u_e is nothing but electrophoretic mobility and E is the electric field strength. How got the external electric field in the particular case? Because you are going to have a membrane on the surface of the tube so, that will be one electrode other is the other electrode. So, it will be something like, this is the wall and in the centre line you insert a tube of titanium. Now, connect the external power volt supply where this?

So, in the middle of the I in the middle plate is basically one electrode and the body of the set up are the cell is another electrode, this to electrode are connected by the external power supply, voltage power supply. Then it will create and external it will create an electric field between the wall and the centre line, thus how? The electric field will be generated in this particular set up unlike the rectangular plate. In a rectangular coordinates system, if it the 4 3 rectangle channel, it is very easy to identify the electrode the top plate, in the bottom plate you can connected to the external power supply. But in this particular case, you have to insert special you have to do it special arrangement by inserting detecting a part or electrode in a middle of the whole set up tube.

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Protein $\rightarrow D \approx 10^{-11} \text{ m}^2/\text{s}$

$Sc = \frac{u}{\rho D} = \frac{10^{-6}}{10^{-11}} = 10^5$

$\delta \propto \frac{1}{Sc^{1/2}} \rightarrow$ MTBL thickness small.

Diagram showing a tube with x and y axes.

$u \frac{\partial c}{\partial x} + (V + v_e) \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2}$

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NPTL

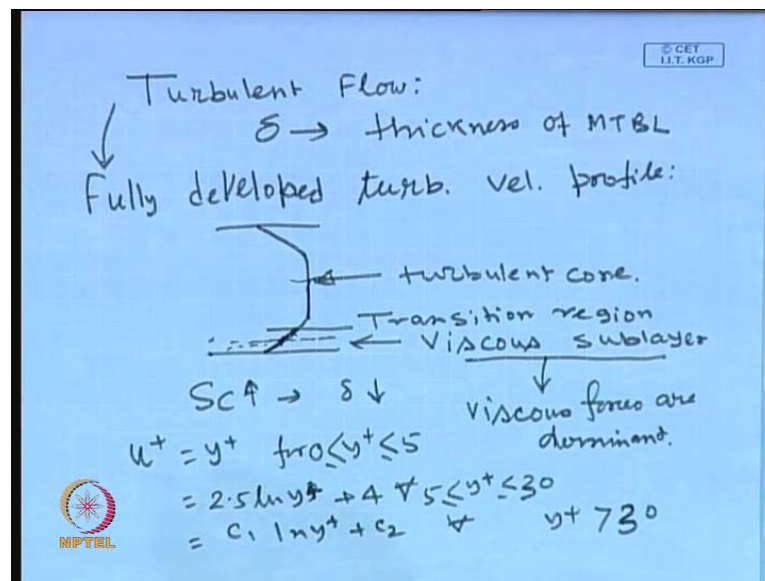
Now, if you remember? You have talking about the separation of the protein so, protein diffusivity will be in the order of 10 to the power of minus 11 meter square per second. So, you are talking about a smit number mu by ρ oe D mu is 10 to the power of minus 3 so, ρ oe is π so, 10 to the power of minus 6 D is 10 to the power of minus 11 so, it will be 10 to the power 5. So, the talking about very high smit number in has already seen earlier the thickness mass transfer boundary layer is inversely proportional to smit number.

So, the mass transfer boundary layer thickness or concentration boundary layer thickness will be extremely small. That means in the tube, the generation of the mass transfer boundary layer will be extremely small compare to the radius of tube. Now, our

concentration is varying within that thin boundary layer not outside, outside is C is equal to C naught. Therefore, is since the boundary layer it is very small we can neglect the carbonator affect, the same I just explaining in the higher steady class, that you stand in front of a big sphere, it will use the curvature affect, it will be observing as the flat plate.

Therefore, you need not to concentration this geometry as radius polar coordinates system; it will be simply a rectangular polar coordinates system in this particular case. So, if you can setup your bi coordinate from the at the wall goes into the centre and the X coordinated is as before. So, under this condition the solute mass balance equation in radius polar coordination stable boiled on to the rectangular coordinates system. So, this becomes you del C del X plus V plus V e del C del y is equal to d del square C del y square where? y is the coordinates system away from the wall.

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Now, we assume a turbulent flow, previously we assume a lamina flow, but in the case we are assume a turbulent flow analysis. Now, in this case as well as discussed earlier the thickness of mass transfer coefficient boundary layer delta is extremely small. Therefore, if you remember that, the considered about a fully develop turbulent velocity profile the fully develop turbulent velocity profile will be having the profile will be something like this, there will a thin region there is known as viscous sub layer. Then you will having a transition region, then will be having a turbulent core.

Now, since we are talking about the flow of high smit number, the delta will be very small that means delta will be within the delta it is every possibility that delta will be lying within the viscous sub layer. In the viscous of the sub layer, the viscous force is will be dominant. The viscous force is fully dominant and if you remember the velocity profile was it looks like $u^+ = y^+$, for y^+ line between 0 and 5 is equal to some constant $2.5 \ln y^+$, plus another constant of 4, that is for y^+ lying between 5 and 20 and there will be another constant let say $C \ln y^+$, plus another constant is 2 for y^+ greater than 30.

That means there is the particular profile and this profile linear in the viscosity sub layer, u^+ is equal to y^+ all is the non dimensional term is the come to that those to the one dimensional term. u^+ is equal to y^+ , that is for y^+ 0 to 5 from the wall to particular distance, there is the viscous sub layer it is the linear. And here, it is $2.5 \ln y^+$ plus 4 and beyond that it is the turbulent core. Now, if we assume that is term mass terms for boundary layer lies within the viscous of the sub layer of the fully develop turbulent velocity profile, then are we talking about the linear per of the velocity profile.

That means I am talking about u^+ is equal to y^+ , inner to border about the other two parts because the concentration different is nothing there, C is equal to C not everywhere. So, and these will occur only the, because you are talking about only high smit number solute. High smit number solute means, very small diffusivity since, the typical solution the filtration you know? Particles that are talking about proteins colloids things like that, which will be having pretty low diffusivity? Therefore, we are going to have a very high smit number and thickness will be pretty small and you assume in this particular analysis, there is high values in the viscous you know viscous sub layer.

If you do not assume that, obviously you it may not being there, but we can do it analytically in the class, it has a you have to go for the numerical methods further.

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Thickness of mass transfer boundary layer is such that it lies well within the viscous sublayer of the turbulent velocity profile.

$$u^+ = y^+$$
$$u^+ = \frac{u}{u^*}$$
$$y^+ = \frac{y u^*}{\nu}$$
$$u^* = \text{Friction Velocity}$$
$$= \sqrt{\frac{\tau_w}{\rho}}$$

Wall shear stress
Density of Solution

$\nu = \frac{\mu}{\rho}$
Kinematic viscosity.

Valid for $0 < y^+ \leq 5$

NPTL

So, for an assumption is thickness of mass transfer boundary layer is such that it lies well within the viscous sub layer of turbulent velocity profile. So, what is the velocity profile? u^+ is equal to y^+ now, let us come to the different you know condition etcetera. What is u^+ 1 dimensional term? u^+ is equal to, u by u^* and what is u^* ? u^* is known the friction velocity. And this is nothing but $\sqrt{\tau_w / \rho}$ what is τ_w ? τ_w is the wall shear stress; ρ is the density of the solution of the medium. And what is y^+ ? y^+ again non dimensional distance, it is $y u^* / \nu$ there is the what is ν ? ν is the this is the kinematic viscosity μ / ρ .

So, μ is the viscosity, ρ is the density there is known as the kinematic viscosity and this relationship is valid for y^+ lying between 0 and 5.

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Wall shear stress.
For smooth wall,
$$\tau_w = 0.03325 \rho u_0^2 \left(\frac{\nu}{R u_0} \right)^{0.25}$$

 $u_0 = \text{Cross sectional avg velocity.}$
$$= \frac{\text{Flow rate}}{\text{Area of C.A. of the tube.}}$$

$$\frac{\nu}{R u_0} = \frac{\mu}{\rho u_0 R} = 2 \frac{\mu}{\rho u_0 d} = \frac{2}{Re}$$

$$Re = \frac{\rho u_0 d}{\mu} = \frac{u_0 d}{\nu}$$

$$u = Ky$$

Next we look in to the correlation of τ_w or τ wall shear stress for smooth wall I think is known as the spanning friction of factor this τ_w becomes given as $0.03325 \rho u_0^2 \left(\frac{\nu}{R u_0} \right)^{0.25}$. You can look into any standard text book of it flow will be getting this expression for the smooth wall or smooth tube wall or tube with the smooth wall. And ρ is the density, u_0 is the cross section average velocity, this u_0 is the cross sectional average velocity.

How we can estimate the u_0 cross sectional average velocity? That we can estimate there you can measure, you know the flow rate, you put the rota meter and know the flow rate, you know the area of the cross section and the flow rate divided by area of the cross section will give the cross sectional average velocity. Cross section of the tube, what is the area cross section? πr^2 in this particular case.

Now, if it is there now, we can the that is thing within the bracket $\frac{\mu}{\rho u_0 R}$ is nothing but; u_0 is basically μ is nothing but μ by ρ , $\rho u_0 R$ you multiplied by the multiply and divided by 2 so, it is become $2 \frac{\mu}{\rho u_0 d}$ is the tube diameter there is 4 3 geometric so, $\rho u_0 d$ by μ is nothing but the Reynolds number so, this is nothing but 2 by Reynolds number where defined is Reynolds number is $\rho u_0 d$ by μ or $u_0 d$ by ν other manner. So, you identify that and let us get the expression of u in the form of Ky so that is our idea.

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$$u^+ = \frac{u}{u^*}; \quad y^+ = \frac{y u^*}{\nu}$$

Vel. profile:

$$u^+ = y^+$$

$$\frac{u}{u^*} = \frac{y u^*}{\nu}$$

$$u = \frac{u^{*2}}{\nu} y$$

$$= \frac{\tau_w}{\rho \nu} y$$

$$u = \frac{1}{\rho \nu} \cdot 0.03325 \rho u_0^2 (2)^{0.25} Re^{-0.25} y$$

$$u = \frac{0.0395}{\nu} u_0^2 Re^{-0.25} y$$

$$= K y$$

If you do that, then define u^+ as u by u^* and y^+ is equal to y u^* by ν . Our velocity profile in this particular case, y^+ is equal to y^+ just put the value of u^+ by u^* is equal to y^+ u^* by ν . So, it becomes u is equal to u^{*2} by ν multiplied by the y and if you remember? u^* was under root τ_w by ρ so, u^{*2} will be τ_w by ρ so, it will be τ_w by ρ times ν multiplied by y . And now, you put the expression of τ_w that we have to going in earlier. So, this become $\frac{1}{\rho \nu} \cdot 0.03325 \rho u_0^2 (2)^{0.25} Re^{-0.25} y$.

You just put the earlier expression and what will be getting is like that u is equal to $0.0395 \frac{u_0^2}{\nu} Re^{-0.25} y$. This is the velocity profile in the dimensional time and it is u is equal to K times y know we can right it in the form of u is equal to K times y .

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$u = Ky$
 Where, $K = \frac{0.0395 u_0^2}{\nu} Re^{-0.25}$
 $v_e = -J$
 Solute mass balance equation,
 $u \frac{\partial C}{\partial x} + (v_e - J) \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}$
 $u \frac{\partial C}{\partial x} - (J - v_e) \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}$
 B.C. \rightarrow at $x=0$, $C = C_0$
 at $y = \delta$, $C = C_0$
 at $y=0$, $D \frac{\partial C}{\partial y} + (J - v_e)C = 0$

Where K is 0.0395 u_0^2 divided by ν to the power minus 0.25. And we are assuming the mass transfer boundary area is really very thin and velocity component V in the definition of mass transfer boundary area will be same as that will be in the wall and the wall it was permeate permeation velocity. So, V will be approximated by minus J at the wall. Therefore, mass the solute mass balance equation now, becomes $u \frac{\partial C}{\partial x} + (v_e - J) \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}$. That means the electrophoretic velocity is in the opposite direction, J in the opposite direction.

So, $u \frac{\partial C}{\partial x} - (J - v_e) \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}$ if you remember that? The governing equation of solute same mass balance with in the mass transfer boundary area without any external electric field, if you put the value of v_e is equal to 0, this you get core governing equation when there is known electric field. Now, as look into the boundary conditions, we required to have one boundary condition with respect to X, 2 boundary condition with respect to y as earlier, the boundary condition where at X is equal to 0, C is equal to C_0 at y is equal to delta or edge of the boundary area, C is equal to C_0 and at y is equal to 0, $D \frac{\partial C}{\partial y} + (J - v_e)C = 0$.

So, these is the this boundary condition discussed earlier, that net flux net plus membrane at the steady state will be equal to 0. This is the diffusive flux high membrane $J \times C$ is the convective flux towards the membrane, minus $v_e C$ is the diffusive flux

away of the membrane due to electrophoresis. Now, this equation this boundary condition will set the full governing equation, the boundary condition of the governing equation.

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Gov. Eqn.

$$u \frac{\partial C}{\partial x} - (J - Ve) \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}$$

$$x^* = x/L; \quad C^* = C/C_0; \quad Pe_e = \frac{Ve d}{D}$$

$$Pe_w = \frac{J d}{D}; \quad y^* = y/R.$$

Final Form.

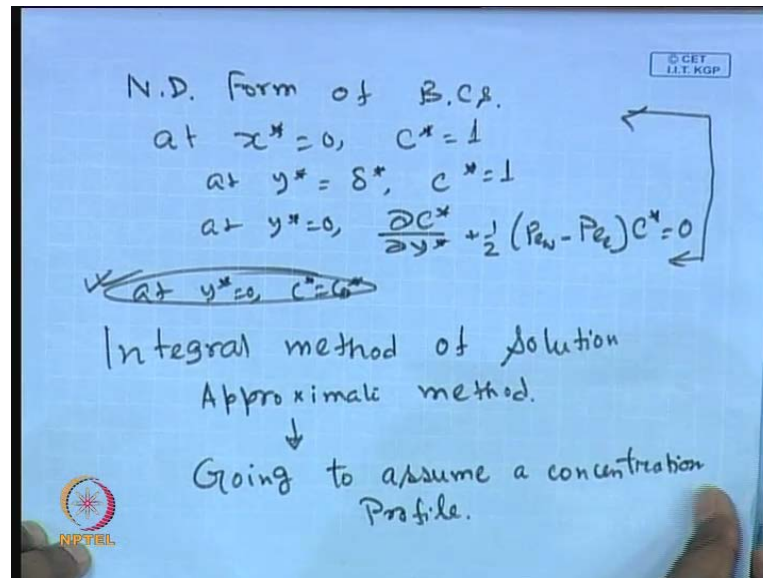
$$A y^* \frac{\partial C^*}{\partial x^*} - \left(\frac{Pe_w - Pe_e}{2} \right) \frac{\partial C^*}{\partial y^*} = \frac{\partial^2 C^*}{\partial y^{*2}}$$

$$A = \frac{0.0395}{8} (Re^{1.75} Sc \frac{d}{L})$$

Now, let us one dimensional, the one dimensional governing equation becomes $u \frac{\partial C}{\partial x} - (J - Ve) \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}$. Let us make it 1 dimensional by x^* as x/L , the length is known C^* as C/C_0 , Pe_e a clay number because of the electric field, becomes $Ve d/D$, Pe_w the peclay number because of the y section, that $J d/D$, D is the diffusivity and small d is nothing but diameter of the tube. And y^* is equal to y/R , R is the radius it will be non dimensional against that number.

So, if you really do all this, you know non dimensional equation, the final form of the equation becomes and writing the final form, you can match it $A y^* \frac{\partial C^*}{\partial x^*} - \left(\frac{Pe_w - Pe_e}{2} \right) \frac{\partial C^*}{\partial y^*} = \frac{\partial^2 C^*}{\partial y^{*2}}$. And the constant A is $0.0395/8$, Reynolds raise to the power 1.75 smit d by L . I am omitted number of steps probably two step, you please express all the quantities in the non dimensional form and do one step and the straight to derived the equation and check whether you getting same thing or not. If you remember, got the same formula in case of without electric field and lamina flow condition in the that case Pe_e term was not there it was 0 and this A was $3/16$ Reynolds smit d by L .

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Now, the non dimensional governing equation it must have the non dimensional form of boundary condition, these are at x^* is equal to 0, C^* equal to 1, at y^* is equal to δ^* , δ^* is the edge of the boundary area that means y by $R \delta^*$ delta by R , C^* equal to 1 and at y^* equal to 0, you have $\frac{\partial C^*}{\partial y^*} + \frac{1}{2}(P_{ew} - P_{ee})C^* = 0$. And since, it is the gel layer controlling filtration at y^* is equal to 0, C^* is nothing but C_g^* . But anyway this is the physical boundary condition, that will be utilising electron, but these are the three boundary condition for to solve the governing equation of the partial differential equation concentration in the mass transfer boundary area.

Now, these set of equation what we are going to do going to use and integral method of solution as earlier. It is an approximation method that means, you are going to approximate going to assume a concentration profile within the mass transfer boundary area. Since the concentration profile your assuming it is known as the known as integral method is known as the approximate method solution. So, we are going to use and integral method of solution, it is an approximate of the method, why did an approximate method? Because the going to assume a concentration profile.

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$$C^* = \frac{C}{C_0} = a_1 + a_2 \left(\frac{y^*}{\delta^*}\right) + a_3 \left(\frac{y^*}{\delta^*}\right)^2$$

$$\delta^* = \delta/r.$$

$$\left. \begin{array}{l} \text{at } y^* = \delta^*, \\ \text{at } y^* = 0, \end{array} \right\} \left. \begin{array}{l} C^* = 1. \\ \frac{\partial C^*}{\partial y^*} = 0 \end{array} \right\}$$

$$\text{at } y^* = 0, \quad C^* = C_0^* \checkmark$$

$$\text{Quadratic } C_0^* = a_1$$

$$C^* = C_0^* + a_2 \left(\frac{y^*}{\delta^*}\right) + a_3 \left(\frac{y^*}{\delta^*}\right)^2$$

$$1 = C_0^* + a_2 + a_3 \quad \dots (1)$$

$$0 = \frac{a_2}{\delta^*} + \frac{2a_3 y^*}{\delta^{*2}} \Big|_{\delta^*} = \frac{a_2}{\delta^*} + \frac{2a_3}{\delta^*} \dots (2)$$

Now, the concentration of profile that we have to going to select are the going to assume is C by C naught is equal to a 1 plus a 2 y star by del star plus a 3 y star by del star square. The delta star is nothing but delta y r at y star is equal to now the boundary condition the condition this concentration whole must satisfy is that at y star equal to delta star C star equal to 1 that is the definition. At age of the boundary area the concentration is at 99 percent of the field stream concentration.

So, it will be otherwise is delta star equal to 1, at the same time another boundary condition must be satisfied the boundary area, that is del C star del y star must be equal to 0 this 2 boundary condition must be satisfied the edge of the boundary area in a type of boundary area if it is the thermal boundary area at y star equal to delta star thermal boundary area this step equal to 1 and del C star del y star will be equal to 0. Now, since there are 3 constant you must having 1 more boundary condition, the boundary condition that is quite appendix at y star equal to 0 a C star equal to C g star.

Now, this 3 are bear minimum concentration, bear minimum condition known for this particular problem concentration profile to satisfied. Therefore, you must having 3, you know? Unknown constant in a concentration profile that mean, you can go up to a quadratic approximate solute means profile of the concentration. Now, using this 3 condition we can evaluate the constant a 1 a 2 a 3, let us do that and y star equal to 0 means; 0 is equal to a 1 plus: y star equal to 0 mean C star equal to C g star C g star

equal to a 1 the next term is equal to 0 so, the next term is also 0. Therefore, C star is equal to C g star plus a 2 y star del star plus a 3 y star by del star square.

Now, out of this 3 condition you already utilise this condition now we utilise this two condition at y star equal to delta star C star equal to 1 that mean, 1 is equal to C g star plus a 2 plus a 3 there is condition 1 condition and other one is del C star del y star is equal to 0. You take the derivative of the equation and put it equal to 0 at y star equal to delta star. So, since is gel layer concentration is constant so, though first term derivative the first term will be equal to 0. So, 0 equal to a 2 by del star plus 2 a 3 y star by delta square this y star has whole thing has to evaluated at del star so, that means del star del star be cancelling so, will be getting a 2 by delta star plus 2 a 3 by delta star there will be equal to 0 there is C star by del star.

Now, will be going to have the second condition so, using this two condition one you can get the concentration profile.

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Handwritten mathematical derivation on a blue background:

$$\frac{a_2}{\delta^*} + \frac{2a_3}{\delta^*} = 0$$

$$a_2 = -2a_3$$

$$1 = C_{g^*} - 2a_3 + a_3$$

$$a_3 = (C_{g^*} - 1)$$

$$a_2 = -2(C_{g^*} - 1)$$

$$C^* = C_{g^*} - (C_{g^*} - 1) \left(2 \frac{y^*}{\delta^*} - \frac{y^{*2}}{\delta^{*2}} \right)$$

$C_{g^*} = \text{constant}$, $\delta^* = \delta^*(x^*)$

So, a 2 by del star plus 2 a 3 by del star that will be equal to 0 so, a 2 is equal to minus 2 a 3. And other 1 is 1 plus C g star plus a 2 a 2 is minus 2 a 3 plus a 3. So, will be getting a 3 is equal to C g star minus 1 and a 2 is equal to minus 2 a 3 minus 2 C g star minus 1. So, the concentration profile going to get C star is equal to C g star minus C g star minus 1 2 y star by del star minus y star square by delta a square this is square. So, this is the

concentration profile you are going to use in order to solve this equation. Now, what we are going to do?

Now C_g star is constant to the C_g is constant and δ star is a function of x only. Now, what we are going to do? We are going to get the derivative of with respect x , get the derivative with respect to y and $\frac{\partial^2 C^*}{\partial y^2}$ and substitute in the derivative governing equation, that we are going to do.

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$$\begin{aligned} \frac{\partial C^*}{\partial x^*} &= -(C_g^* - 1) \left(-2 \frac{y^*}{\delta^{*2}} + 2 \frac{y^{*2}}{\delta^{*3}} \right) \frac{d\delta^*}{dx^*} \\ &= -2(C_g^* - 1) \left(-\frac{y^*}{\delta^{*2}} + \frac{y^{*2}}{\delta^{*3}} \right) \frac{d\delta^*}{dx^*} \\ \frac{\partial C^*}{\partial y^*} &= -(C_g^* - 1) \left(\frac{2}{\delta^*} - \frac{2y^*}{\delta^{*2}} \right) \\ \frac{\partial^2 C^*}{\partial y^{*2}} &= (C_g^* - 1) \left(-\frac{2}{\delta^{*2}} \right) \\ A y^* \frac{\partial C^*}{\partial x^*} + \frac{(P_{ew} - P_{e2})}{2} \frac{\partial C^*}{\partial y^*} &= \frac{\partial^2 C^*}{\partial y^{*2}} \end{aligned}$$

So, $\frac{\partial C^*}{\partial x^*}$ will be nothing but minus C_g star minus 1 minus $2 y^*$ by δ star square plus $2 y^*$ star square by δ star cube $d \delta$ star dx star. This becomes minus $2 C_g$ star minus 1 minus y^* star by δ star square second 2 out plus y^* star square by δ star cube $d \delta$ star dx star $\frac{\partial C^*}{\partial x^*}$, $\frac{\partial C^*}{\partial y^*}$ that becomes minus C_g star minus 1 2 by δ star minus $2 y^*$ by δ star square. And $\frac{\partial^2 C^*}{\partial y^2}$ will 1 more derivatives is this equation that means only this term will be there so, minus minus plus.

So, it will be C_g star minus 1 2 by δ star square so, these are various derivatives and that we are going to put in the governing equation. So, if you remember it was $A y^*$ $\frac{\partial C^*}{\partial x^*}$ plus $\frac{P_{ew} - P_{e2}}{2}$ it was minus, $\frac{\partial C^*}{\partial y^*}$ equal to $\frac{\partial^2 C^*}{\partial y^2}$ so, that was the governing equation was probably. Now, what we are going to do? We should this derivative in terms of y^* and

star and delta star in this equation and see what you get? If you substitute this all this derivatives is here.

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Handwritten derivation on a blue background:

$$A \frac{d\delta^*}{dx^*} \left(\frac{y^{*2}}{\delta^{*2}} - \frac{y^{*3}}{\delta^{*3}} \right) + \left(\frac{P_{ew} - P_{ec}}{2} \right) \left(\frac{1}{\delta^*} - \frac{y^*}{\delta^{*2}} \right) = \frac{1}{\delta^{*2}}$$

Take y^* zeroth moment

$$A \frac{d\delta^*}{dx^*} \int_0^{\delta^*} \left(\frac{y^{*2}}{\delta^{*2}} - \frac{y^{*3}}{\delta^{*3}} \right) dy^* + \left(\frac{P_{ew} - P_{ec}}{2} \right) \int_0^{\delta^*} \left(\frac{1}{\delta^*} - \frac{y^*}{\delta^{*2}} \right) dy^* = \frac{1}{\delta^{*2}} \int_0^{\delta^*} dy^*$$

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The final form that we are going to get is that $A \frac{d\delta^*}{dx^*} \left(\frac{y^{*2}}{\delta^{*2}} - \frac{y^{*3}}{\delta^{*3}} \right) + \left(\frac{P_{ew} - P_{ec}}{2} \right) \left(\frac{1}{\delta^*} - \frac{y^*}{\delta^{*2}} \right) = \frac{1}{\delta^{*2}}$. Now, what we are going to do? There is the it is the standard bounded area 3 in multiplied by both side dy^* that is the integrate 0 is moment. Take 0th moment means, multiplied by both side $y^* dy^*$ to the power of 0 dy^* and integrate across the bound area thickness 0 to delta star.

So, it will be $d\delta^* dx^* \int_0^{\delta^*} \left(\frac{y^{*2}}{\delta^{*2}} - \frac{y^{*3}}{\delta^{*3}} \right) dy^* + \left(\frac{P_{ew} - P_{ec}}{2} \right) \int_0^{\delta^*} \left(\frac{1}{\delta^*} - \frac{y^*}{\delta^{*2}} \right) dy^* = \frac{1}{\delta^{*2}} \int_0^{\delta^*} dy^*$. So, once you do that and just carry out this integration you know this becomes y^{*3} by 3 so, it will be $\frac{\delta^{*3}}{3}$ and $\frac{\delta^{*4}}{4}$ here and similarly, will be getting $\frac{1}{\delta^*} y^* - \frac{y^{*2}}{2\delta^{*2}}$ so, wholes things become $\frac{\delta^{*2}}{2} - \frac{\delta^{*3}}{3}$ so, you can simplified this thing and can do this analysis.

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$$\frac{A \delta^{*2}}{12} \frac{d\delta^*}{dx^*} + \left(\frac{P_{ew} - P_{ee}}{4} \right) \delta^* = 1$$

At $y^* = 0$, $\frac{\partial C^*}{\partial y^*} + \frac{1}{2} (P_{ew} - P_{ee}) C_{g^*} = 0$

$$-\frac{2}{\delta^*} (C_{g^*} - 1) + \frac{1}{2} (P_{ew} - P_{ee}) C_{g^*} = 0$$

$$\left(\frac{P_{ew} - P_{ee}}{4} \right) \delta^* = \frac{C_{g^*} - 1}{C_{g^*}}$$

$$\frac{A \delta^{*2}}{12} \frac{d\delta^*}{dx^*} + \frac{C_{g^*} - 1}{C_{g^*}} = 1$$

$$\frac{A \delta^{*2}}{12} \frac{d\delta^*}{dx^*} = 1 - \frac{C_{g^*} - 1}{C_{g^*}} = \frac{1}{C_{g^*}}$$

And finally, will be getting the need form integration as $A \delta^2$ by $12 d \delta^*$ plus P_w minus P_e divided by 4 times δ^* is equal to 1 this is the form will you get after you know why averaging the equation of solute mass balance equation. Now, if you remember the boundary condition you have at y^* equal to 0 it was $\frac{\partial C^*}{\partial y^*}$ plus half P_w minus P_e times C_{g^*} equal to 0 . Now, what we can do? We already got the derivative $\frac{\partial C^*}{\partial y^*}$ in terms of δ^* and y^* and then evaluate that derivative at y^* equal to 0 .

If you evaluate the derivative at y^* equal to 0 I am just talking about this term $\frac{\partial C^*}{\partial y^*}$ evaluate at y^* equal to 0 that means, the system will be off and will be getting minus to $C_{g^*} - 1$ divided by δ^* you put that into this equation and what will be getting minus 2 by $\delta^* C_{g^*} - 1$ plus half P_w minus P_e and C_{g^*} it becomes C_{g^*} there at y^* will be equal to 0 C_{g^*} equal to 0 . So, will be getting P_w minus P_e divided by 4 times δ^* is equal to $C_{g^*} - 1$ divided by C_{g^*} .

So, what we are going to do? Where going to substitute this into this equation? If a going to if you really substitute it what we are get is? $A \delta^2$ by $12 d \delta^*$ plus $C_{g^*} - 1$ divided by C_{g^*} equal to 1 . So, it take on the right hand side it will be getting $A \delta^2$ divided by $12 d \delta^*$ is equal to $1 - \frac{C_{g^*} - 1}{C_{g^*}}$

star minus 1 divided by C g star it was simplify thing C g star in the numerical cancelling out minus minus plus 1 so, you know simplify 1 by C g star.

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$$\frac{A \delta^{*2}}{12} \frac{d\delta^{*2}}{dx^{*2}} = \frac{1}{C g^{*2}}$$

$$\int_0^{\delta^{*2}} \delta^{*2} d\delta^{*2} = \frac{12}{A C g^{*2}} \int_0^{x^{*2}} dx^{*2}$$

$$\boxed{\delta^{*2} = \left(\frac{36}{A C g^{*2}} \right)^{1/3} x^{*2/3}}$$

Mass Transfer coefficient

Definition $K (C_g - C_0) = -D \left(\frac{\partial C}{\partial y} \right)_{y=0}$

or $K = \frac{-D \left(\frac{\partial C}{\partial y} \right)_{y=0}}{C_g - C_0}$

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So, next we separate the variables and see what you get? Delta star square A delta star square by 12 d delta square star dx star is equal to 1 over C g star. So, delta star square d delta star is equal to 12 by A C g star and just type dx star, form 0 to x star and will be from 0 to del star. So, will be getting an expression of delta star and will be del star by 3 delta star cube by 3 so, delta star will be 3 into 12 30 6 A C g star raise to the power 1 upon 3 x star raise to the power 1 upon 3. So, that is how? The delta star is the thickness of mass transfer bounded area varies as the function of x star on the length in the module.

Now, let us look into the mass transfer coefficient and rest of the calculation mass transfer coefficient in the, if you? With the lets look into the definition K C g minus C naught is equal to minus D del C del y at y is equal to 0. This is the mass transfer coefficient C is the concentration at the at the lower surface minus C naught is the bulk concentration so, there is by the definition, this is by the definition. Now, make it non dimensional so, you can defined the Sherwood number you can you can K as minus D del C del y at equal to 0 divided by C g minus C naught define make it non dimensional that means, C star by C by C g star and y star by r and things likes that.

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$$K = - \frac{D}{R} \frac{\left(\frac{\partial C^*}{\partial y^*}\right)_{y^*=0}}{Cg^*-1}$$

$$\frac{KR}{D} = \frac{-\left(\frac{\partial C^*}{\partial y^*}\right)_{y^*=0}}{Cg^*-1}$$

$$Sh = \frac{Kd}{D} = -2 \frac{\left(\frac{\partial C^*}{\partial y^*}\right)_{y^*=0}}{Cg^*-1}$$

$$Sh = \frac{4}{\delta^*}$$

If you do that K is equal to minus D del C star will be the small R here, del y star at y star equal to and these will be C g star minus 1. This will be KR over D minus is equal to minus del C star del y star at y star is equal to 0 divided by C g star minus 1. And as Sherwood number is defined as Kd by D that means, twice of R so, it will be it will be minus it will be minus 2 del C star del y star at y star is equal to 0 divided by C g star minus 1. So, if you put the expression of del C star del y star y star equal to 0 I think we are already done that earlier.

del C star del y star at y star is equal to will be getting 2 by delta star multiplied by C g star minus 1 we can this term so, the definition of Sherwood number is negative negative will be positive and the 2 will be 4. So, it will be substitute that these derivatives here the C g star minus 1 will be cancelling from the numerical term numerator and will be getting Sherwood number is equal to 4 by delta star. And we are just derive, how delta star is varying the function of delta star? If you substitute there then will be obtain that how Sherwood number is varying the function of the x star? And then you can do an averaging length averaging Sherwood number.

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$$Sh = 4 \left(\frac{Acg^*}{36} \right)^{1/3} x^{*-1/3}$$

$$\bar{Sh} = \int_{x^*=0}^1 Sh dx^* = 4 \left(\frac{Acg^*}{36} \right)^{1/3} \int_0^1 x^{*-1/3} dx^*$$

$$\bar{Sh} = 6 \left(\frac{Acg^*}{36} \right)^{1/3}$$

Length averaged permeate flux:

$$K(Cg - C_0) = -D \left(\frac{\partial C}{\partial y} \right)_{y=0} = (J - ve)Cg$$

Definition of mass transfer coeff. Boundary Condition at $y=0$

So, if you do that the Sherwood number transfer to be 4 in to A C g star by 36 raise to the power 1 upon 3 x star is become minus 1 upon 3. You can do a length averaging that mean length average Sherwood number will be nothing but Sherwood dx star x star for 0 to 1. It is put the value so, becomes 4 all the things will be constant 4 A C g star divided by 36 raise to the power 1 upon 3 becomes X to the 1 upon minus 1 upon 3 dx star 0 to 1 so, it integration will be minus X to the power X to the power minus 1 upon 3 plus 1 divided by 1 minus 1 upon that so, 2 by 3 so, will be getting a value 3 by 2 or 1.5 here and after integrate that will be 1 so, the it will be 1.5 after integration the whole integration will becomes 1.5, 1.5 into 4 it becomes 6.

So, Sherwood number average becomes 6 A C g star divided by 36 raise to the power 1 upon 3. Now, you can put the value of a you will do that, next will find what is the length averaged permeate flux? The length averaged permeate flux can be obtain by the definition of mass transfer coefficient and the boundary condition at y is equal to 0. K C g minus C naught is equal to minus D del C del y at y is equal to 0 and this is equal to J minus V e times C g.

Now, what is this? The first equation is nothing but definition of the mass transfer coefficient if you remember you just read it right away definition of mass transfer coefficient and what is the last 2 terms? If you remember J minus V e times C g is equal to minus D del C del y is equal to 0 that is the boundary condition at y is equal to 0. The

boundary condition at y is equal to 0 so, if you equate K times C g minus C naught is equal to J minus V e times C g then will be getting an estimate of the permeate flux.

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Handwritten mathematical derivation on a blue background:

$$K(C_g - C_0) = (J - v_e)C_g$$

$$K(C_g^* - 1) = (J - v_e)C_g^*$$

$$\left(\frac{Kd}{D}\right)(C_g^* - 1) = \left(\frac{Jd}{D} - \frac{v_e d}{D}\right)C_g^*$$

$$Sh(C_g^* - 1) = (P_{ew} - P_{ee})C_g^*$$

$$P_{ew} = P_{ee} + Sh \left(\frac{C_g^* - 1}{C_g^*}\right)$$

→ length averaging.

$$\int_0^1 P_{ew} dx^* = \int_0^1 P_{ee} dx^* + \left(\frac{C_g^* - 1}{C_g^*}\right) \int_0^1 Sh dx^*$$

$$\overline{P_{ew}} = P_{ee} + \left(\frac{C_g^* - 1}{C_g^*}\right) Sh$$

So, will be getting K C g minus C naught is equal to J times V e C g make it non dimensional that means K C g star minus 1 is equal to J minus V e C g star that means divided C naught and then multiplied by Kd by D multiplied by diameter divided by the diffusivity both side will be getting C g star minus 1 is equal to J D by D minus V e D by D times C g star. So, what is this? This is nothing but the Sherwood number. Sherwood C g star minus 1 is equal to what is this? This is nothing but the P e w minus this will be P e e times C g star. So, P e e is nothing but P e e plus Sherwood average Sherwood multiplied by the C g star minus 1 divided by C g star.

Now, if you do a length averaging; how do a length averaging? Length averaging that means we multiplied by the both side by dx star and integrate over the dx I is basically integral P e w dx star is equal to P e e dx star integration X C g star minus 1 divided by the C g star that is constant so, this is to becomes Sherwood dx star integral now, 0 to 1 0 to 1 0 to 1 so, P e w dx star will be nothing but length average permeate flux. Now, P e e is constant it does not depend on the x star it will be outer the integral so, integral of dx star equal to 1 so, this is become P e e plus C g star minus 1 divided by C g star and this integral 0 to 1 S h is nothing but the length of average Sherwood number. This is the relationship of the permeate flux length average Sherwood number.

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Exp. of \bar{Sh}

$$\bar{P}_{ew} = Pe_2 + 6 \left(\frac{A G^*}{36} \right)^{1/3} \left(\frac{C_{g^*} - 1}{C_{g^*}} \right)$$

↓ insert "A"

$$\bar{P}_{ew} = Pe_2 + 0.32 (Re)^{0.58} (Sc \frac{d}{L})^{1/3}$$
$$\bar{Sh} = 0.32 Re^{0.58} (Sc \frac{d}{L})^{1/3} \left(\frac{C_{g^*} - 1}{C_{g^*}^{2/3}} \right)$$

The image shows a handwritten derivation on a blue grid background. It starts with the title 'Exp. of \bar{Sh} '. The first equation is $\bar{P}_{ew} = Pe_2 + 6 \left(\frac{A G^*}{36} \right)^{1/3} \left(\frac{C_{g^*} - 1}{C_{g^*}} \right)$. Below this, an arrow points down with the text 'insert "A"'. The second equation is $\bar{P}_{ew} = Pe_2 + 0.32 (Re)^{0.58} (Sc \frac{d}{L})^{1/3}$. The final equation, which is boxed, is $\bar{Sh} = 0.32 Re^{0.58} (Sc \frac{d}{L})^{1/3} \left(\frac{C_{g^*} - 1}{C_{g^*}^{2/3}} \right)$. There are logos for '© CET I.I.T. KGP' in the top right and 'NPTEL' in the bottom left of the slide.

Now, we put the value of expression of Sherwood number this length average permeate flux becomes $Pe_2 + 6 A C_{g^*} / 36$ raise to the power $1/3$ $C_{g^*} - 1$ divided by C_{g^*} . Now, insert the value of A insert the expression of A if you really do that finally, will be getting the direct I just forming to similar the similar step P becomes $Pe_2 + 0.32 Reynolds$ raise to the power 0.58 $smit D$ by L raise to the power $1/3$ $C_{g^*} - 1$ divided by C_{g^*} raise to the power of $2/3$ and length average, Sherwood number transfer to be $0.32 Reynolds$ to the power of 0.58 $smit d$ by L raise to the power of $1/3$. So, these things as expression of length of the Sherwood number in the turbo turbulent flow phase.

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$$\bar{P}_w = P_e + 0.32 Re^{0.58} \left(Sc \frac{d}{L} \right)^{1/3} \left(C_g^{*1/3} - C_g^{*2/3} \right)$$

if $C_g^* \ll e^3 = 20$

$$\bar{P}_w = P_e + 0.32 Re^{0.58} \left(Sc \frac{d}{L} \right)^{1/3} \ln C_g^*$$

And then we can go for the further simplification the \bar{P}_w length average becomes $P_e + 0.32 Re^{0.58} \left(Sc \frac{d}{L} \right)^{1/3} \ln C_g^*$. Now, this is the expression that will be really wanted if you know the Reynolds number? You know the electric field, this is the filter part velocity down link in term other properties on them you can find the permeate flux. For this can further simplify for the limiting case we have discussed this thing if C_g^* is much much less than e^3 that is 20 this terms down Poisson to the expression $P_e + 0.32 Re^{0.58} \left(Sc \frac{d}{L} \right)^{1/3} \ln C_g^*$.

These quantities become $\ln C_g^*$ by C_g^* , if C_g^* is less than 20 that we have already discussed in our earlier case to talk about the laminar flow gel layer control ultra filtration and the integral method of solution. So, these exercise this surely good idea how to apply the external electric field in gel controlling filtration in a tube so, in this exercise you know how to evaluate? The Sherwood number also in case of the tubular geometric under the laminar flow under the under the turbulent flow conditions so, we stop here and we go to the next topic in the next class thank you very much.