Novel Separation Processes
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Lecture No. # 24
External Field Induced Membrane Separation Processes (Contd.)

Good morning everyone. We are discussing about the charged colloidal particles, electro filtration of the charge colloidal particles, but before doing the know quantification of how much what is the effect of electric field, we have to find out what is the what is the external electrophoresis induced friction of the movement of the charge particles. And we have seen that the electrophoresis will take~~s~~ place under the external electric field over the charged particles. The charged particles will move in the electrolytic solution in the particular direction based on the difference in the potential.

Now, in order to find out the electrophoretic velocity, and the electrophoretic mobility one need to find out what is the affect in electrolyte concentration in the on the charge sphere charge colloids, and ultimately what is the effect of the potential distribution that will be the around the sphere. So in the last class, what we have seen first we have to considered it charged flat surface is placed in an electrolytic solution, and how the Debye length will be defined and that will be the (( )) giving in the idea or estimate of the influence the region of influence are the external electric field we play the dominant role. Now, you are also saying what is the electric potential field that will be distribution around the charged sphere, so will just take a look at it and then continued to discussion our estimate the what is the electrophoretic velocity and then influence if external electric field.

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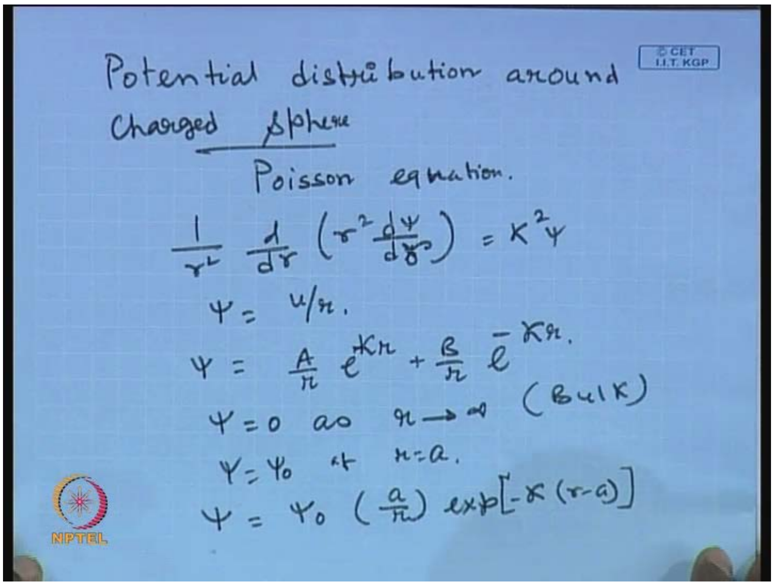

Potential distribution around Charged sphere

Poisson equation.

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = \kappa^2\psi$$

$$\psi = u/r.$$

$$\psi = \frac{A}{r}e^{\kappa r} + \frac{B}{r}e^{-\kappa r}.$$

$$\psi = 0 \text{ as } r \rightarrow \infty \quad (\text{Bulk})$$

$$\psi = \psi_0 \text{ at } r = a.$$

$$\psi = \psi_0\left(\frac{a}{r}\right)\exp[-\kappa(r-a)]$$

So the potential distribution around the charged sphere is very important as for as the actual application is concerned, so we have to take request to the Poisson equation, and you remember to the Poisson equation has to be written in the spherical co ordinates in r the Laplace will be written in r theta and phi there is symmetric is considered one dimensional the flow field only. And then in the concentration will be replaced by the Boltzmann distribution Poisson Boltzmann distribution has to be written in the spherical polar co ordinates, then will take the request to the Debye Huckel approximation that means psi will be the surface potential will be less than 25 milli volt.

So, you will be getting some kind of simplification in your calculation finally, you will getting the expression of potential distribution as 1 over r square d dr of r square d psi dx d psi dr is equal to kappa square psi, now if you put the change if do a substitution of change in value psi as u by r and the simplify the equation, and then the equation has to be solved the boundary condition the solution of the equation turns out to be A by r e to the power of kappa r plus B by r e to the power minus kappa, in fact told to you to I just one step in between I told you to include the this variation of transformation of variable and solve this equation.

And if you do not do that, then do yourself and these equation solved against to the boundary condition so can estimate the integration constant A and b this boundary condition are psi is equal to 0 as r turns to infinitive this will be condition out the bulk and psi is equal to surface potential as at r is equal to A. So, the final solution of the potential distribution so basically using this boundary condition estimate the e n e and the final solution transfer to be psi not A by r exponential minus kappa r minus A, where is the radius of the charged particles that we have already derived in the last case. Now in this class, we will define what is the net charge residing on the particle and how the electrophoretic mobility will be estimated.

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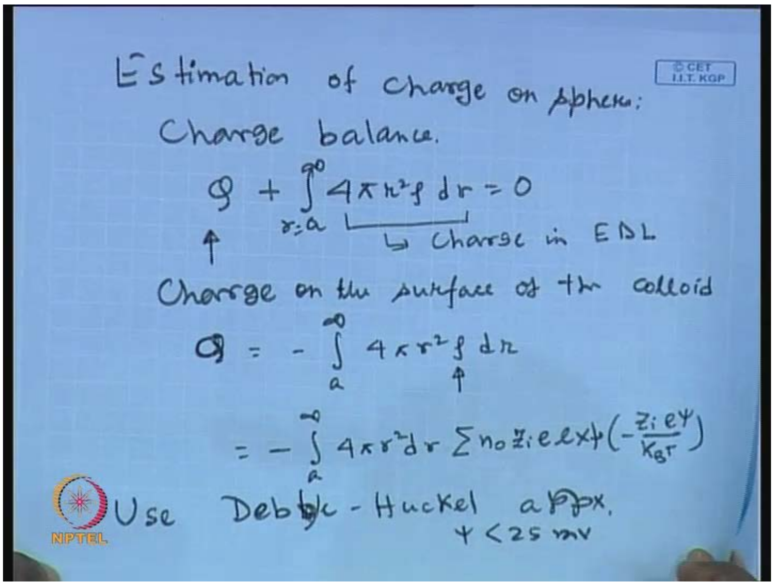

Estimation of charge on sphere:

Charge balance.

$$\underset{\uparrow}{Q} + \underbrace{\int_{r=a}^{\infty} 4\pi r^2 \rho \, dr}_{\text{Charge in EDL}} = 0$$

Charge on the surface of the colloid

$$Q = -\int_a^{\infty} 4\pi r^2 \underset{\uparrow}{\rho} \, dr$$

$$= -\int_a^{\infty} 4\pi r^2 dr \sum n_0 z_i e \exp\left(-\frac{z_i e \psi}{K_B T}\right)$$

Use Debye - Huckel appx.

$\psi < 25$ mV

Next, will see estimation of charge on the sphere, if you know the flow field in a potential distribution you can estimate the charge residing on the sphere charged sphere, so one can do a charge balance it carry out the charge balance that means the net charge, because the bulk of the solution will be electro neutral, there is the excess charge, so what is the excess charge excess charge is that charge residing on the solid particles plus charge that is that is within the electrical double layer in the solution. So summation of these two charges will be equal to 0, so the bulk of the solution it will be 0 in electro neutral so charge residing on the particles plus the charge in the double layer this 4 pi r square rho dr r is equal to A to infinitive r must be ~~is~~ equal to 0, so it is the charge on the surface outer surface of the colloid and these term will be the charge net charges in electrical double layer.

So what is the Q? Q can be estimated the from these equation 4 pi r square rho dr and then you put the expression of you know of charge density rho that will we derived in the last class in this equation so it turns to be minus of integral a to infinitive 4 pi r square dr and going to write down the expression of rho that is the net charge density and summation n 0 Z i e exponential minus z i e psi over K B T. Now again, you so that the this is the expression and you will getting the after carrying out integration will be getting the net charge residing on the solid colloid particle but the point is again one can it is difficult to carry out the integration because r square they are they are an the psi is there so in order to do that what you can do, you can again request to the Debye Huckel approximation and open up these exponential term use Debye Huckel approximation that means z e psi is less than 25 milli volt.

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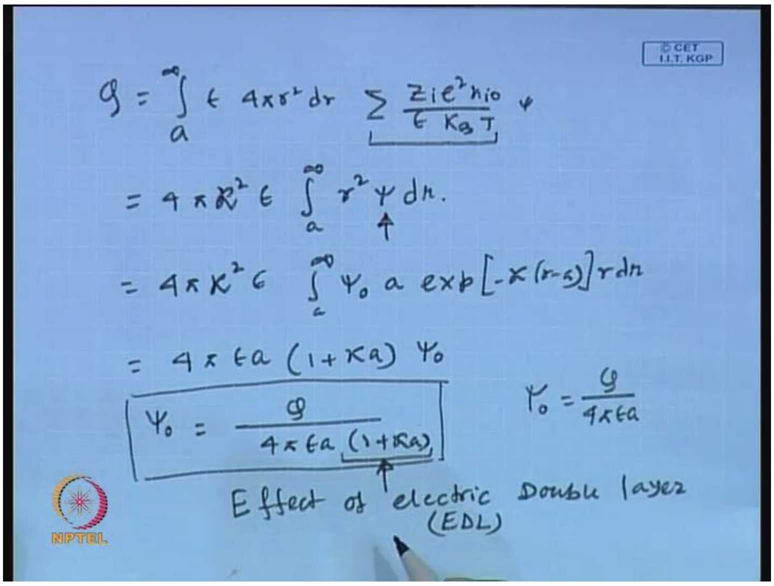

$$q = \int_a^\infty \epsilon \, 4\pi r^2 dr \underbrace{\sum \frac{z_i e^2 n_{i0}}{\epsilon K_B T}}\psi$$

$$= 4\pi \kappa^2 \epsilon \int_a^\infty r^2 \underset{\uparrow}{\psi} dr.$$

$$= 4\pi \kappa^2 \epsilon \int_a^\infty \psi_0 \, a \exp[-\kappa(r-a)] r dr$$

$$= 4\pi \epsilon a (1+\kappa a) \psi_0$$

$$\boxed{\psi_0 = \frac{q}{4\pi\epsilon a \underbrace{(1+\kappa a)}}} \qquad \psi_0 = \frac{q}{4\pi\epsilon a}$$

Effect of electric Double layer (EDL)

And under that condition the whole equation is simplified and this is becomes a to infinitive epsilon 4 X r square dr summation of z i e square n i naught epsilon K B T psi and so and what is this? This is nothing but the Debye inverse of Debye square kappa square by ~~the~~ definition so this is becomes 4 pi kappa square epsilon integration a to infinitive r square psi d r. Now already define derive the expression of potential distribution about the charge sphere as the function of r and will be it inserted there so these become 4 pi kappa square epsilon a to infinitive psi naught times a exponential minus kappa and r minus a r dr these become after carrying out integration carry out the integration this become 4 pi epsilon a 1 plus kappa a psi naught so you are psi naught are surface potential become Q divided by 4 pi epsilon a 1 plus kappa a.

Now if you know the surface potential psi naught you can estimate the value of Q from these expressions and kappa that is the inverse of the Debye length is the function of electrolyte concentration the nature of the electrolyte and the property of the solution dielectric constant so and so forth. So, if that you just see what is the different between the electrostatic and this in the system if you are if you complex that will explain by the electrostatic that means the stationary charged sphere with the with the surface charge is Q then what has the psi naught in the surface that has the Q divided by 4 pi epsilon a.

So, what is the correction these correction 1 plus kappa a and these becomes you know very important whenever the talking about the electrolytic solution. so this the effect of electric double layer so these the effect of electric double layer ideal in the in the solution and those who have come late could not attend the last class, we can we can clarified the significant electric double layer is basically the region around the around the charge sphere is a are a charge flat surface where the present of electric in the electrolytic solution will be realized or experienced beyond that the solution become electro neutral and the potential, because the electric potential because of the presence of the charge surface will be nill here earlier so that is the electric double layer and this is the affect, and how this is the way it will be influencing the surface potential and the charge residing of the particle.

Now, as they are mentioned earlier as they are mentioned earlier in an actual scenario when you are you are having a charged particle in an electrolyte solution how will estimate the charge you know surface potential is difficult to therefore, will be using depicted that is will be zeta meter zeta sized and the using the zeta meter one can estimate the zeta potential of the along the outer surface of the charged sphere, but at which location, because we are already seen that there will be always a stern layer associated with the charge sphere basically, the counter around will be the form a sort of layer that will be moving an it is moving along with the charge sphere as an envelope. Therefore, the outside surface to the stern layer becomes you know becomes exposed to the electrolytic solution and the potential that will be the measured by the zeta potential meter will be the potential that is residing, potential that is existing on the outer surface to stern layer and typically the stern layer is will be extremely small so if you can neglect the thickness of the stern layer.

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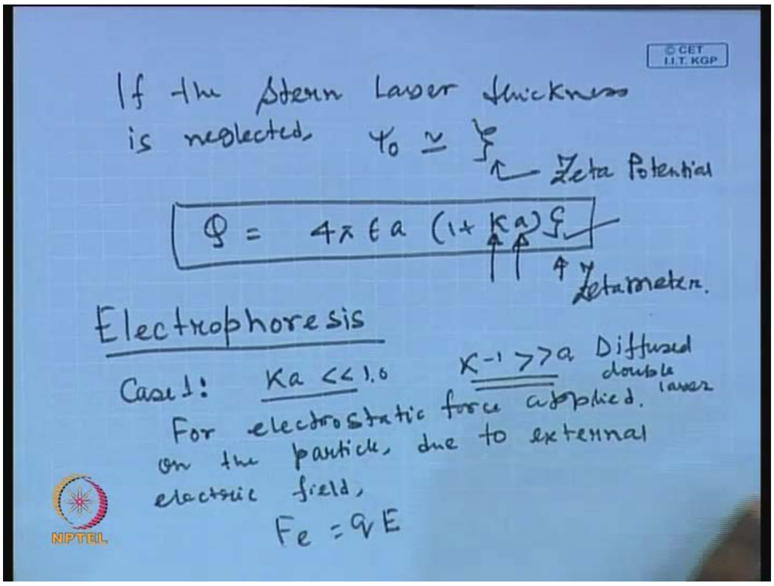

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If the Stern Layer thickness is neglected, $\psi_0 \simeq \zeta$ ← Zeta Potential

$$\boxed{Q = 4\pi \epsilon a (1+ \kappa a)\zeta}$$

Zetameter.

Electrophoresis

Case 1: $\underline{\kappa a << 1.0}$ $\underline{\underline{\kappa^{-1} >> a}}$ Diffused double layer

For electrostatic force applied on the particle, due to external electric field,

$F_e = qE$

NPTEL

So, if the stern layer thickness is neglected then is psi naught will be approximated as the zeta potential so this is nothing but the zeta potential so what I mean is that zeta potential is the measurable potential in the other hand psi naught that is residing the exact potential outer surface of the charge sphere it is difficult to measured because of the presence system here so one can measured the zeta potential and can estimate the value of Q that is 4 pi epsilon a 1 plus kappa a and psi naught will be replaced by the zeta potential, if we neglect the thickness of the stern layer what the potential drop across the stern layer these will be neglected are it will be in fact it will be the extremely small.

Therefore, what you have now using if you measure the zeta potential using the zeta meter and this kappa and radius of the particle will be known to you and the kappa that will be Debye length inverse of the Debye length in will be known to you because you hand the expression of Debye length and it is a function of the concentration on the electrolyte is now characteristic when it is 1 is to 1 electrolyte or 2 is to 1 electrolyte or 3 is to 1 electrolyte and the 400 property of the solution and epsilon will be known to you one can is the constant, so one can is estimate the net charged Q residing on the solid sphere of the charge colloids so next what will see now will be in position to get the electrophoretic velocity, so we go to the electrophoresis.

So basically whatever we have theoretical steady do that done the last class under the class before that basically two obtain the these expression, so I just wanted to show you that these expression does not come out of glue this is the solid solution will be finite. So, there are two cases of electrophoresis that will be the constant one is further case number 1 is kappa a is much much less than 1 so the electrolytic force that will be applied on that so there is the two case so case number 1 is the kappa much much less than 1 other case the kappa a much much greater than 1.

So, for electrostatic force applied on the particle due to external electric field it becomes F E is nothing but q multiplied by the few E multiplied by the E e is nothing but the fill straight (()) that all of you know now what is the implication of kappa a is much much less than 1 that means kappa inverse is much much greater than a right at kappa inverse much much greater than kappa is the nothing but the Debye length so the talking about a very high Debye length although the compare to one nanometre it may be less 7 times or 5 times or 8 times are like that. So these cases, we are talking of a diffused double layer. This condition kappa a is quite large compare to the particle gradient is known as the diffused double layer, so physically this condition kappa a much much less than 1 that means we are talking about a diffused double layer but the double layer thickness is quite large compare to the radius of the solid particles.

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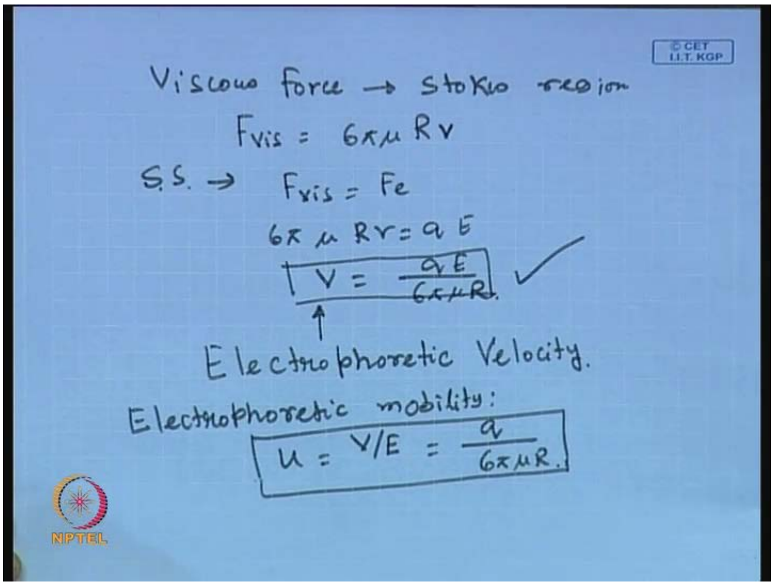

Viscous force → Stokes region

$F_{vis} = 6\pi\mu R v$

S.S. → $F_{vis} = F_e$

$6\pi\mu R v = q E$

$\boxed{V = \frac{qE}{6\pi\mu R}}$ ✓

↑ Electrophoretic Velocity.

Electrophoretic mobility:

$\boxed{u = V/E = \frac{q}{6\pi\mu R}}$

The viscous force typically it will be getting by the in the in the stroke’s region on a solid concentration on a sphere the viscous force becomes 6 pi mu R times g per V is basically, the velocity now under the steady state the viscous is force is the will be equal viscous force is will be equal to electric force so we can get a viscous is nothing but F E it will be getting 6 pi mu R V is nothing but q times E and electrophoretic velocity becomes q E divided by 6 pi mu R r is nothing but the nothing but a that is radius of the particle that we have to decided in the will discussed earlier so these so this is known as the electrophoretic velocity, this is the expression. Now, what is electrophoretic mobility? Electrophoretic mobility is defined as electrophoretic velocity per unit field strength so V divided by E; so this will becomes q divided by 6 pi mu R, so that is the expression of electrophoretic mobility. Therefore, depending on the situation, if you have kappa a much much less than 1 that means, if you are talking about the diffused double layer we are going to use this expression as electrophoretic velocity.

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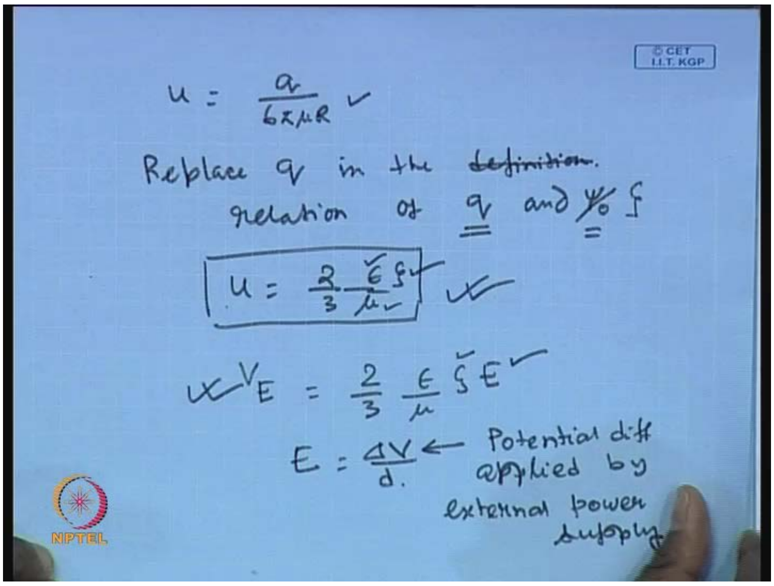

$u = \frac{q}{6\pi\mu R}$

Replace q in the ~~definition.~~
relation of $\underline{\underline{q}}$ and $\underline{\underline{\cancel{\psi_0}\ \zeta}}$

$$\boxed{u = \frac{2}{3}\frac{\epsilon\zeta}{\mu}}$$

$$\cancel{u}\ V_E = \frac{2}{3}\frac{\epsilon}{\mu}\zeta E$$

$E = \frac{\Delta V}{d}$ ← Potential diff applied by external power supply

So let us look into the other a stream other asymptote by using this electrophoretic mobility u is equal to q divided by 6 pi mu R and you can you can you can replace q in the definition in the in the definition means in the relation of relation of q and psi naught that is the charge residing on the particles psi naught that will be the just derived it if you if you put these expression q there then these becomes q becomes 2 by 3 epsilon zeta so these psi naught will be replaced by the zeta by 3 mu. Now next, go to and for a kappa a much much less than 1, 1 plus kappa a will be approximated one so this is the electrophoretic mobility that will be going to use what about the talk talking to the come coming to the analysis of the filtration of the charge particles, if kappa a is much much less than 1. Now, this expression will completely known to us because we can we can measured the zeta potential using a zeta meter psi is epsilon is known to as mu is the viscosity of the medium about a talking about and in ordered to get electrophoretic velocity this is will be nothing but V by E u becomes of the g v by so V by electrophoretic velocity is becomes 2 by 3 epsilon mu zeta times E.

And now E the external electric field steady state is become an operating condition what it how it becomes the operating condition becomes by using a an external power supply you are you are applying the voltage between the two points and between the top rate in the bottom rate you know the distance between the two points top to bottom rate so this d by d is nothing but the E so E is nothing but the V delta V by d, d is the total height of the channel and delta V is the potential difference that you are applying difference being applied by external power supply. So you can apply the 2 mill 2 volt the potential difference we can apply the 4 volt of potential apply you can 10 volt potential is difference; so it is entirely in operated charge so this is nothing but an operating condition. So in this expression operating condition will be known to you know the zeta potential of the particle because there is the measuring using by a zeta meter all the other quantities are known so you can estimate the electrophoretic velocity.

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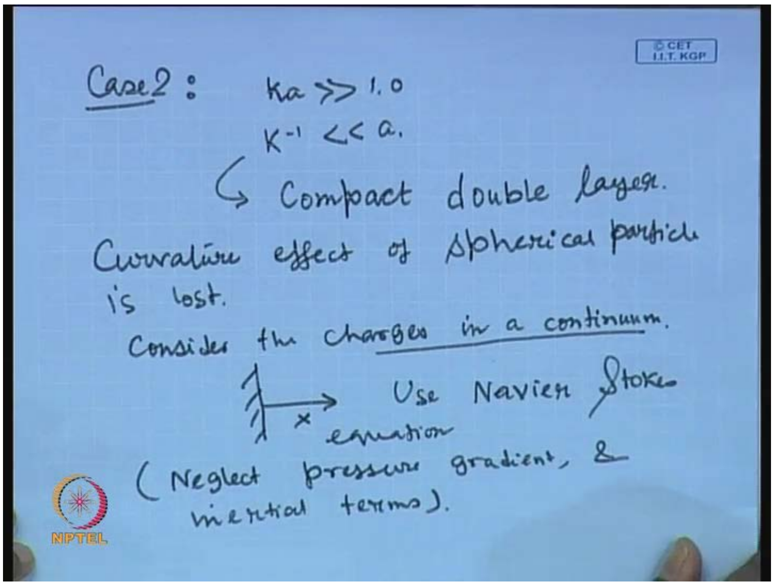

Case 2 : $\kappa a >> 1.0$

$\kappa^{-1} << a.$

↳ Compact double layer.

Curvature effect of spherical particle is lost.

Consider the charges in a continuum.

Use Navier Stokes equation

(Neglect pressure gradient, & inertial terms).

Now, let us look in to the other asymptote kappa a case number 2, where kappa is a much much greater than 1, now in this case we are talking about what is the kappa inverse, it is much much less than a. So, we are talking about the electric double layer which is very very thin so it is known as the compact double layer, so this condition is known as compact double layer and in these case the curvature affect of the particles will be lost spherical particles is lost how the curvature is will be lost? Since the talking about the compact double layer that means you are standing very close~~d~~ to the sphere so if you look into a I think we have discussed about this earlier, if you stand very close to a very big foot ball then it will be look in to the charge surface of the flat surface it will be lost and a and we can consider the charges in a continuum, so are analysis becomes a flat surface a distance X is away from it so, charge surface and distance surface and it is thin we can considered the charged in a small distance of a as a continuum we can use the Navier stroke equation to get to the electrophoretic velocity, so use Navier stroke equation, and in Navier stroke equation, we neglect the pressure gradient and internal terms completely.

So if you have to neglect the pressure gradient that means there because there is the pressure gradient only the electric external electric field pressure the presence of external electric field so pressure gradient is lost and the del p del X 1 will be there (()) an the inertial transmit that means the whole left hand side of Navier stroke equation will be neglect; so if you neglecting the inertial terms as well as so the right an side is viscous stern what is the left right hand side is the left the viscous stern of the system as well as the what is force time.

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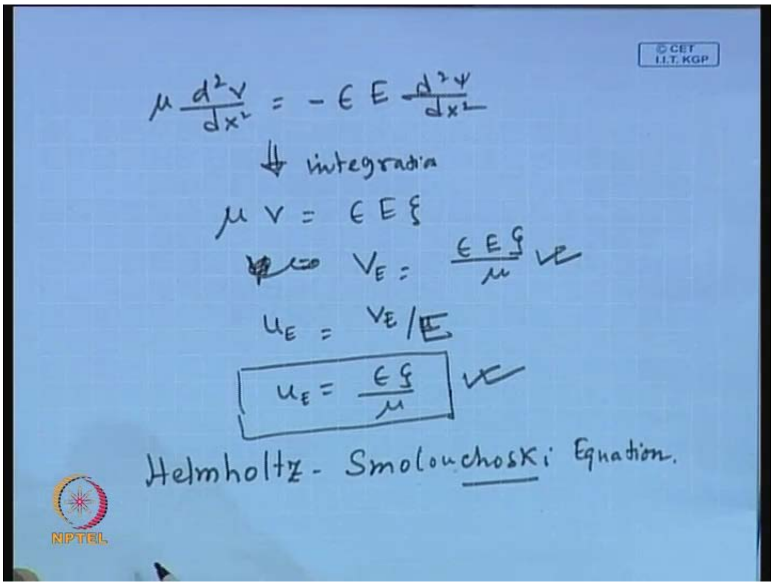

$$\mu \frac{d^2 v}{dx^2} = -\epsilon E \frac{d^2 \psi}{dx^2}$$

$\Downarrow$ integration

$$\mu v = \epsilon E \zeta$$

so $V_E = \frac{\epsilon E \zeta}{\mu}$ ✓

$$u_E = V_E / E$$

$$\boxed{u_E = \frac{\epsilon \zeta}{\mu}}$$ ✓

Helmholtz - Smolouchoski Equation.

So it becomes new d square V dx square equal to minus epsilon E d square psi by dx square now you can integrate these equation, upon integration will be getting u mu terms V is nothing but epsilon E zeta and u becomes so electrophoretic velocity under this condition will be epsilon E zeta by mu and electrophoretic mobility will becomes to V e by u by E and this is becomes epsilon zeta mu, so depending on the situation if kappa a is much much greater than kappa a much much greater than 1 for going to use this expression of electrophoretic velocity in our equation, now this equation is known as Helmholtz smolouchosk (()) equation you can you can you can measure~~d~~ the zeta potential are you know the speed property and one can get the electrophoretic mobility or electrophoretic velocity under these condition.

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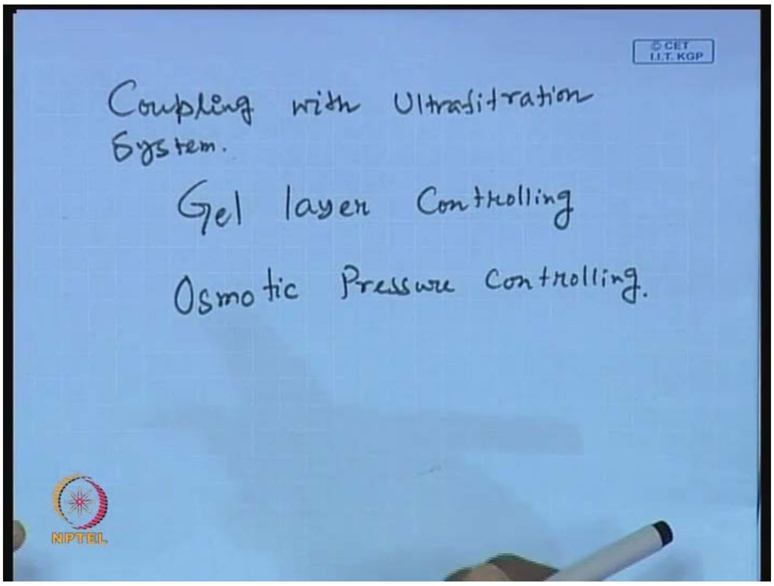

Coupling with Ultrafiltration System.

Gel layer Controlling

Osmotic Pressure Controlling.

Now next, what will you do a couple system with the actual ultra filtration process now the ultra filtration depending on the situation it will be there will be two types of ultra filtration we are talking about one is the osmotic pressure control and another one is the gel layer control, so gel layer controlling case another is the osmotic pressure control is case in a gel layer controlling in a it will be since we have discussed in the last about the two processes in the in the first case you are talking about the filtration of high molecules solute which will be forming a layer a gel or the thick gel of the membrane surface gel is nothing but a I will discussed over the membrane surface so there are two types of situation one can encounter the gel layer controlling filtration number.

One is that you can start with the osmotic pressure controlling filtration initially and then as a process along the channel length the membrane surface concentration keep on increasing beyond the particular point exit in the solid metric limit and the solid phase particle will be diffuse, and the membrane surface physically forming a porous network that will be the condition of gel. Second one is there are some solute which will be always forming the in a gel type of the membrane surface on the various beginning on the filtration for example, kept in polynomial alcohol things likes that there known as the gel in material these gel in material will be depositing on the membrane surface for forming a thick the thin layer of the highly viscous layer are it is a gel and the osmotic pressure controlling case there is basically, when you are talking about the filtration of low molecular solute the osmotic pressure of solute which will is pretty important that is osmotic pressure inversely proportional to the molecular way.

So, whenever talking about the low molecular solute osmotic pressure becomes very important and sometimes becomes comparable with the applied pressure difference so what happens are affective pressure difference will be decreased and it will be influence filtration performance.

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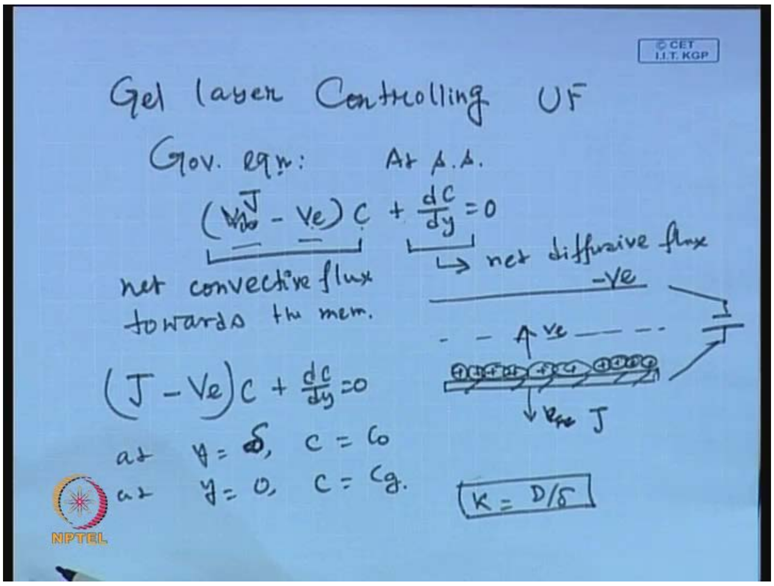

Gel layer Controlling UF

Gov. eqn: At s.s.

$$\underbrace{(\overset{J}{\cancel{V_w}} - V_e)\,C}_{\text{net convective flux towards the mem.}} + \underbrace{\frac{dc}{dy}}_{\text{net diffusive flux}} = 0$$

↳ net diffusive flux -ve

$$(J - V_e)\,C + \frac{dc}{dy} = 0$$

at $y = \delta$, $c = c_0$

at $y = 0$, $c = c_g$.

$$\boxed{k = D/\delta}$$

Now in case of... So, we will look into the both the cases is one after another of them and another gel layer controlling ultra filtration and it is basically the governing equation in this case becomes V w minus V e times C plus dc dy is equal to 0 at this steady state the one dimensional analysis, so these two velocity opposite in direction so you so you are going to have system custom something like that is a channel and the membrane is placed here in the bottom of feed topless let say all the particles they will be forming a gel type of layer over the membrane surface and they are positively charged.

Now, you use this two plates and connect them with external electric field with the negative potential on the topless so what it will be do it will be attract so there will be external electric field that will be influence in the system and it will be attract in the charge particle twice it by simple electrolytic attraction so electrophoretic velocity will be in the opposite direction right and what is the V w V w is the permeate velocity are will be called upon the J, which is denoted in the J the permeate velocity of the permeate flux in the opposite direction so J minus V e times C plus d V dy is equal to 0.

So net compact this will be giving you net convective flux towards the membrane and these will be the net diffusive flux from the membrane (()) at the steady state we discussed that is fluxes will be equal so you can you can integrate this equation becomes finally, J minus V terms V as dc dy is equal to 0 and we can use the boundary condition at y is equal to delta y is equal to delta there is the thickness of the mass transfer boundary layer C is equal to C naught and that y is equal to 0 by C is equal to C gel that is the gel layer concentration and we define the mass transfer coefficient that as d by delta the film mass transfer coefficient is d by delta and you can integrate this equation integrate the differential equation across the mass transfer boundary area thickness as from 0 to delta.

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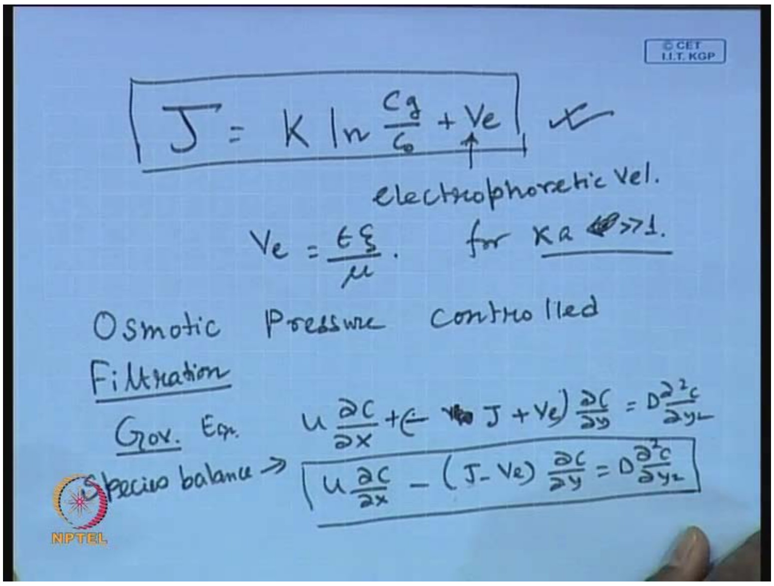

$$\boxed{J = K \ln \frac{C_g}{C_0} + V_e}$$

electrophoretic vel.

$V_e = \frac{\epsilon \xi}{\mu}$. for $\kappa a >> 1$.

Osmotic Pressure controlled Filtration

Gov. Eqn. $u\frac{\partial c}{\partial x} + (-J + V_e)\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2}$

Species balance → $\boxed{u\frac{\partial c}{\partial x} - (J - V_e)\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2}}$

So, if you really do that what will be getting is J will be nothing but equal to K l n C g by C naught plus V e so if you remember the case without external electric field this term was not present it was simply J is equal to K l n C g by C naught and it will be in case electric field associated ultra filtration gel layer control the electrophoretic velocity will be it will be added of (()) in second term so, and if for the small choose equation you can find out that the zeta times epsilon divided by the mu right so for kappa a much much less than 1 greater than 1 for this case you can you can estimate the zeta potential you know epsilon you know mu so you can estimate the value of V so the permeate flux for the productivity of the system will be enhanced by the same amount by this amount.

This is the very simplest scenario. In fact, in an actual case it will be the quit will be more complicated because the motion of the charge particles the electrophoretic velocity is velocity it coupled with the solution properties and thin viscosity of electro affect and things like that so roughly one can have a first case how much the permeate flux are productivity will be an enhanced by applying the external electric field by this situation under the gel layer control filtration, next we talk about the osmotic pressure control filtration.

Now in this case, this will be typically arise for the filtration of protein solution in you are solid solution are the calling the electrolytic solution the governing equation we be almost same, but it will be like the little bit modified because the pressure surface in an electric field so mu del C del X there is the diffuse balance equation special balance equation within the mass transfer boundary layer it will be u del C del X minus minus J plus V e del C del y is equal to D del square C del y square so u del C del X minus J minus V e del C del y equal to d del square C del y square, so these will be the governing equation of the of the concentration basis balance will be the mass transfer boundary layer. If you remember, when a whenever V derives this governing equation will be the without the external electric field this term will not true so as the present so they ~~ad~~just opposite. So V e was is equal to 0 in the case and if you put V is equal to 0 will get the governing equation without electric field. Now the boundary condition will be changed, which boundary conditional changed?

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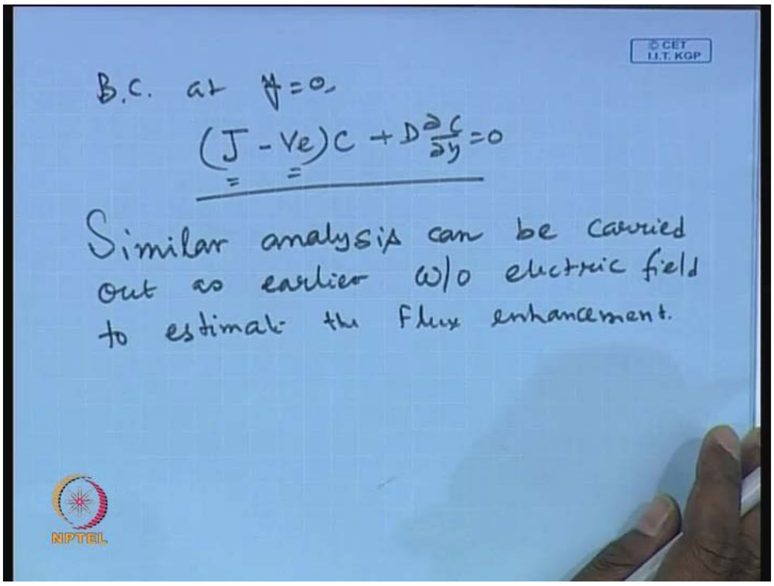

B.C. at y = 0

$$\underline{(\underset{=}{J} - \underset{=}{Ve})C + D\frac{\partial C}{\partial y} = 0}$$

Similar analysis can be carried out as earlier w/o electric field to estimate the flux enhancement.

The boundary condition at y is equal to 0 will be change and these case it will be the J minus V e times C plus d del C del y this is the change in the boundary condition that will that will occur in the case of osmotic pressure control filtration and if you if you remember these becomes the governing equation in case of gel layer controlling filtration there is the one dimensional model, so the governing equation one dimensional becomes the boundary condition of the two dimension module. Now, these set of equation in case of osmotic potential control filtration solving it here they it can be solved in the same way by using similarly, conformation what about than earlier so similar analysis can be carried out as earlier without electric field to estimate the flux enhancement. So probably in the tomorrow class, I will collect to gone in more detail in these aspect, but before that it is it is it is better to have a feel what is the what is the meaning of Debye length, what is the typical order of magnitude of Debye length for given a typical concentration of the electrolyte solid solution, etcetera.

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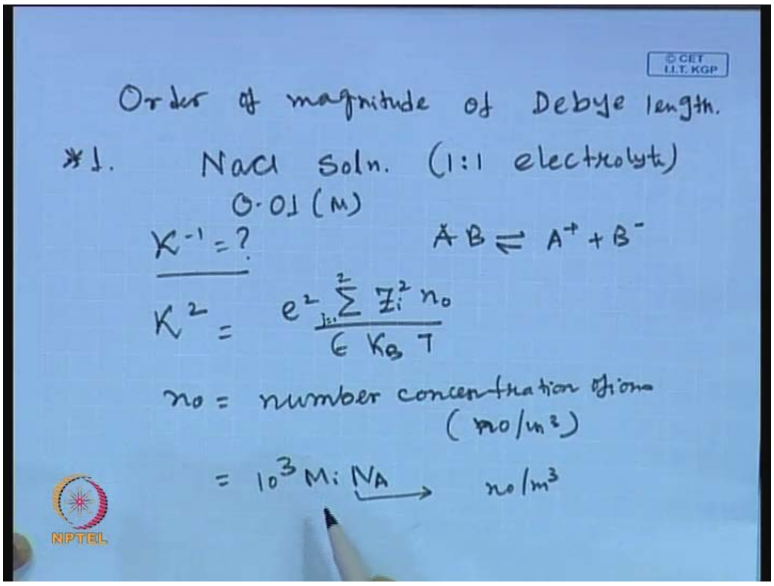

Order of magnitude of Debye length.

*1. NaCl Soln. (1:1 electrolyte)

0.01 (M)

$\kappa^{-1} = ?$

$AB \rightleftharpoons A^+ + B^-$

$$\kappa^2 = \frac{e^2 \sum_{i=1}^{2} z_i^2 n_0}{\epsilon K_B T}$$

n_0 = number concentration of ions (no/m^3)

$= 10^3 M_i \underset{\longrightarrow}{N_A}$ no/m^3

So, I will just do couple of calculation in order to estimate the order of magnitude of Debye length of Debye length given a concentration of the salt. Now the first example it is basically talking about sodium chloride solution it is 1 is to 1 electrolyte it is also called 1 is to 1 electrolyte and it is let say strong is electrolyte so it is the 0.01 molar sodium chloride solution. Now, we are going to find out the value of kappa inverse what is the Debye length particular salt concentration?, and will you have it is like A B these it is going to them disserted as A plus plus B minus where A is nothing but the sodium and B is nothing but the chloride spaces now if you look in to the expression of kappa square this will become E square summation z i square n naught divided by epsilon K B T.

These summation is over the ionic spaces there are 2 ionic spaces here one is the probably ca~~u~~tions and another one is anions is the chloride (()) and so what is n naught? n naught is basically number of ions it is the concentration number concentration of the ions right of ions is return as the unit or as number per unit meter cube so, if you have molar concentration these molar concentration should be converted into per litre sorry per meter cube that means you have to multiplied by the 1000 so becomes gram mole per meter cube now in 1 gram there will be Avogadro number of species present so it should be multiplied by the Avogadro number so these has the unit of number per unit meter cube. So this is the conversion, if you would like to calculate the number concentration you multiply the molar concentration by the Avogadro number multiplied by the 1000 so that the number concentration will be in the unit of number per unit meter cube, if you do not multiplied by the 1000 it becomes number per unit litre.

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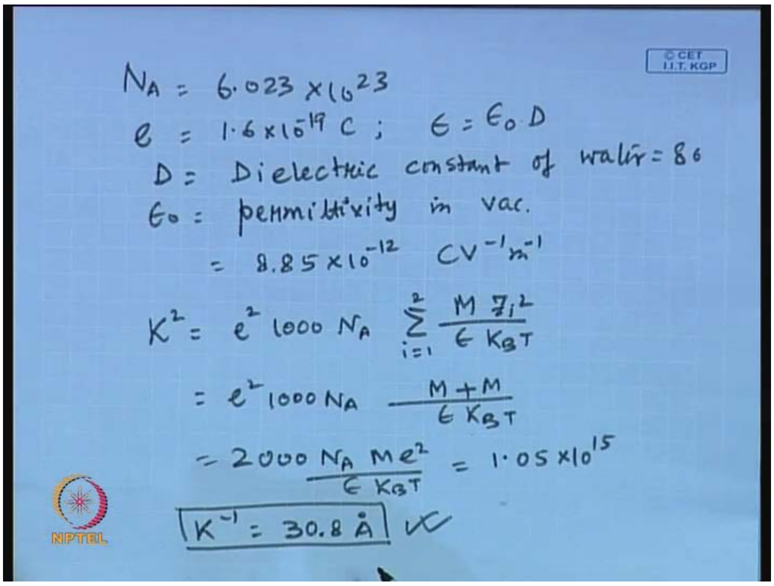

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$N_A = 6.023 \times 10^{23}$

$e = 1.6 \times 10^{-19}$ C ; $\epsilon = \epsilon_0 \cdot D$

D = Dielectric constant of water = 80

ϵ_0 = permittivity in vac.

$= 8.85 \times 10^{-12}$ $CV^{-1}m^{-1}$

$$K^2 = e^2\, 1000\, N_A \sum_{i=1}^{2} \frac{M z_i^2}{\epsilon K_B T}$$

$$= e^2\, 1000\, N_A \frac{M + M}{\epsilon K_B T}$$

$$= \frac{2000\, N_A M e^2}{\epsilon K_B T} = 1.05 \times 10^{15}$$

$$\boxed{K^{-1} = 30.8\ \text{Å}}$$

Now, the Avogadro number is 6.0 2 3 into 10 to the power of 23,~~20 3~~ the coulombic charge is 1.6 into 10 to the power of minus 19 coulomb, the epsilon is the permittivity of medium is nothing but epsilon 0 times D is the dielectric constant of water and these value is 80 and epsilon 0 is the permittivity in vacuum and this value is 8.85 into 10 to the power minus 12 coulomb or volt inverse meter inverse that is the unit of it and now you put everything in the definition of kappa square it becomes square 1000 N Avogadro number summation molar concentration z i square epsilon K B T and this i is over 1 to 2 that is the ionic species present in the since present in the solution so e square 1000 N A and what is what is now this i is over z i is basically and molar concentration of the both the spaces will be equal of the bulk; otherwise the electro is maintained this is 1 is to 1 electrolyte keeping there in mind now it is open upon the summation of it is epsilon K B T ~~exedra~~etcetera is the constant, so we can write it epsilon K B T.

So, N also it is the constant there is the matter basically z i square so it will be plus 1 square plus minus 1 square so it is the 1 plus 1 is 2 so this become N plus M so this becomes 2000 N A M E square over epsilon K B T. Now, if you put all the values these turns to be 1.0 5 in to 10 to the power of 15 and kappa inverse turns out to be 30.8 Armstrong. So, if you are talking about a 0.01 molar Nacl solution than kappa inverse will be 30.8 Armstrong, if you increase the concentration of the solute of the solid electrolyte 0.01 to 0.1 this is the will be kappa inverse will be further decreased so the so the idea is if you increase the electrolytic concentration the Debye length will become more and more compact.

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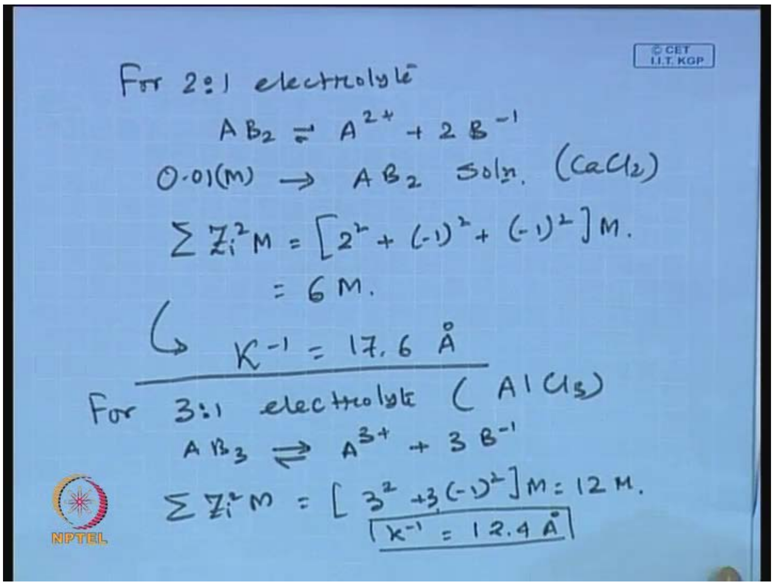

For 2:1 electrolyte

$$AB_2 \rightleftharpoons A^{2+} + 2B^{-1}$$

$0.01(M) \rightarrow AB_2$ Soln. $(CaCl_2)$

$$\sum Z_i^2 M = [2^2 + (-1)^2 + (-1)^2]M.$$
$$= 6M.$$
$$\hookrightarrow \kappa^{-1} = 17.6\ \text{Å}$$

For 3:1 electrolyte $(AlCl_3)$

$$AB_3 \rightleftharpoons A^{3+} + 3B^{-1}$$
$$\sum Z_i^2 M = [3^2 + 3(-1)^2]M = 12M.$$
$$\boxed{\kappa^{-1} = 12.4\ \text{Å}}$$

For let us look into the type of the electrolyte for 2 is to 1 electrolyte if you have some A B 2 that means A square plus plus to B minus right and let us talk about if 0.01 molar A B 2 solution and what is the typical A B t solution it is ~~N~~C a C l 2 solution is a calcium chloride let us say. Now, in this case the summation z i square M will be having 3 terms 3 parts 1 will be one for A another will be there are 2 B's this so one for A another for one A one B another for the second B, so will be having 2 square. So z i square will be 2 square plus minus 1 square for one B there are two B's this minus 1 square of times M so it becomes 1 plus 1 plus 4 so 6 M.

Now, if you use the same molar concentration let say 0.01 molar C l to solution the Debye length becomes 17.6 Armstrong so it is decreased so if you if you are talking about the 2 is to 1 electrolyte double layer become more compact, so for A 3 is to 1 electrolyte that means typically A l C l 3 so 3 is to 1 electrolyte, in A B 3 so this become

A 3 plus plus 3 B minus so you summation z one square M becomes 3 square plus minus 1 square plus minus 1 square, plus minus 1 square, 3 of them are there so 3 into that times M that mean 9 plus 3 so it is 12 M and in this case the electric double layer thickness will be becomes Debye length Debye length is become 12.4 Armstrong.

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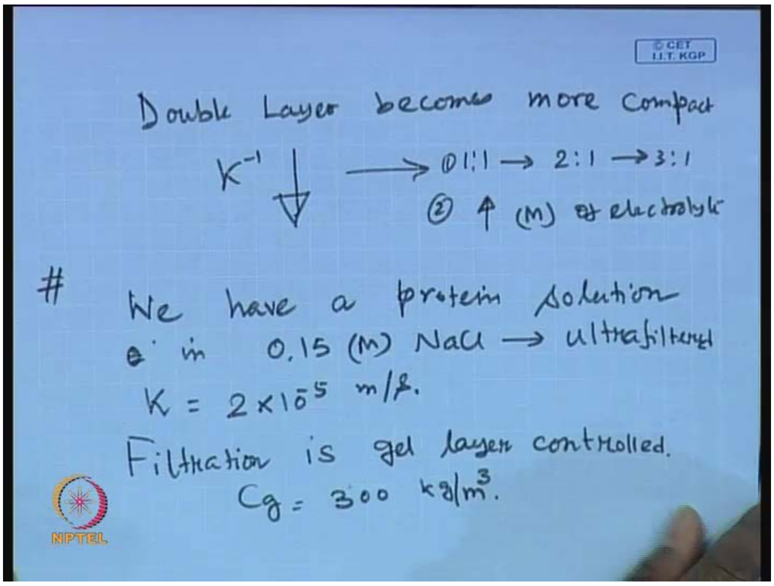

Double Layer becomes more Compact

$\kappa^{-1} \downarrow \longrightarrow$ ① 1:1 → 2:1 → 3:1

② ↑ (M) of electrolyte

#

We have a protein solution in 0.15 (M) NaCl → ultrafiltered

$K = 2 \times 10^{-5}$ m/s.

Filtration is gel layer controlled.

$C_g = 300$ kg/m^3.

So double layer becomes more and more compact becomes more and more compact that means kappa inverse decreases under these two conditions, if you go from 1 is to 1 to 2 is to 1 to 3 is 1 electrolyte number 1, number 2 if you increase the concentration that means molar concentration morality of electrolyte if you if you considered the sodium chloride only if you increase the molarities it becomes are compact if you increase the same molarities if you if you maintaining the same molarities and increase the nature of the nature of the electrolyte 1 is to 1, 2 is to 1, 3 is to 1, the double layer thickness, it becomes more and more compact becomes less. So this gives the typical calculation of what will be the order of the magnitude of Debye length you are talking about for a for a typical concentration for the electrolyte that will be appearing.

Why I have taken 0.0 molar ins reside solution? Because if whenever you are talking about the protein solution all the protein solution will be having if you talking about the filtration of the protein particles the protein solution will be having a 0.01 molar in NaCl that is typically given as the preservative so all the protein solution will be having the electrolyte present in it typically 0.01 molar 0.0, 0.15 molar like that so that is why I have taken those types of values. In the next example, I will do the small calculation on now the ultra filtration of protein solution in order which is assistant (()) by the external electric field and then we will see how the per how much the permeate will be enhanced, ~~if a~~ what is the typical order of magnitude of permeate flux one can one can expect to be enhance of the external electric field, let say we have a protein solution in 0.15 molar NaCl there is there is the preservative and it has to be ultra filtered will be will be the cross per velocity etcetera, (()) the mass transfer coefficient because 2 into 10 to the power minus 5 meter per second, the filtration is a gel layer control and J concentration is given as 300 kg per meter cube.

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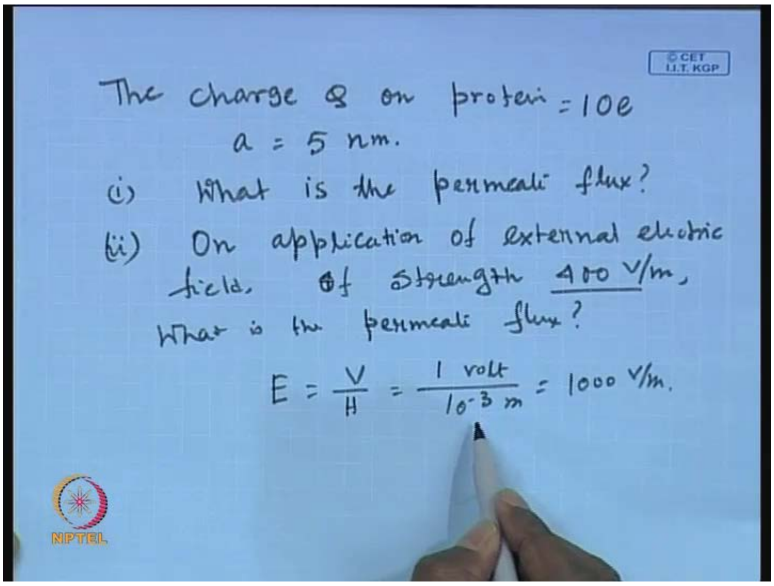

The charge Q on protein = 10e

a = 5 nm.

(i) What is the permeate flux?

(ii) On application of external electric field, of strength 400 V/m, What is the permeate flux?

$$E = \frac{V}{H} = \frac{1\ \text{volt}}{10^{-3}\ \text{m}} = 1000\ \text{V/m}.$$

The charge number of the protein is characterised, the charge that means Q on spherical protein it is known it is it is basically 10 times e that means 10 into 1.6 into 10 to the power of minus 1 meter coulomb an it has a radius a, which will be nothing but 5 nanometre. So the first question is what is the permeate flux, second is suppose you apply external electric field, on application of external electric field of strength 400 volt per meter what is the permeate flux? So this is the typical operating condition for 400 volt per meter.

Now, you should not think that you are applying a very (()) is really very high the value itself if very high, but it is not 400 volt the value is basically by using an external power supply you are using probably 1 volt or 2 volt and not more than that (()), but since the height is very small the whole thing~~s~~ become magnified for example, E becomes V by d b you know full height of the channel. Now suppose, we are applying you get 1 volt we apply an external electric field 1 volt and your typical the order of magnitude height is 1 millimetre. So 1 millimetre means 10 to the power of minus 3 meter so, this much of 1 volt will be result and electric field of 1000 volt per meter; so that means for a channel of thickness 1 millimetre if you apply a external electric if you if you use an external power supply to get a voltage of let say 0.4 volt that will give the free strength of 400 volt per meter. So, it is very small example that mean you are not incurring a huge power supply are you know in occurring the huge cross as well as the stern in electric field is concerned.

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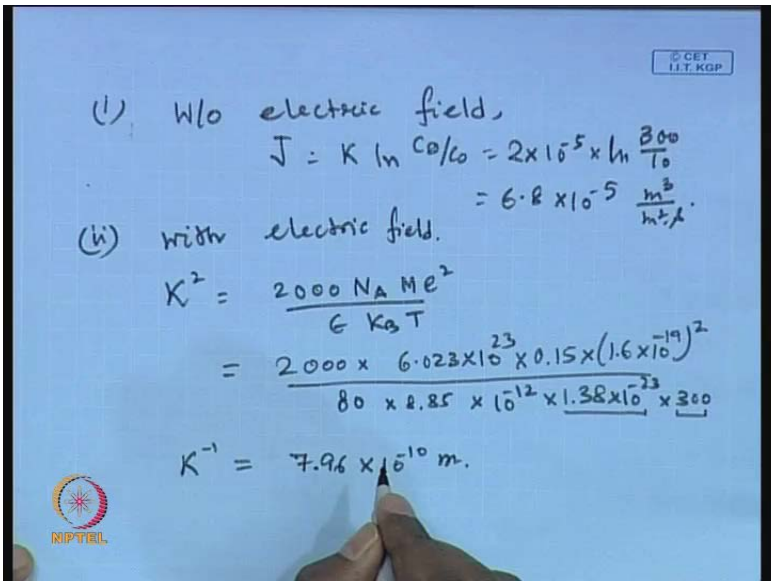

(i) W/o electric field,

$$J = K \ln C_0/C_0 = 2\times10^{-5}\times\ln\frac{300}{T_0}$$

$$= 6.8\times10^{-5}\ \frac{m^3}{m^2.s}.$$

(ii) with electric field.

$$K^2 = \frac{2000\, N_A\, M e^2}{\epsilon\, K_B T}$$

$$= \frac{2000\times 6.023\times10^{23}\times0.15\times(1.6\times10^{-19})^2}{80\times8.85\times10^{-12}\times\underbrace{1.38\times10^{-23}}\times\underbrace{300}}$$

$$K^{-1} = 7.96\times10^{-10}\ m.$$

Now, for first thing is what is the permeate flux the permeate flux without external electric field without electric field these becomes K l n C g by C naught the standard formula these gives you 2 into 10 to the power minus 5 into l N 300 by 10 that is gives you value of 6.8 into 10 to the power ~~of~~ minus 5 meter cube per meter square second and let us calculate with electric field the first thing is you have to calculate the Debye length value of the Debye length kappa square is 2000 N A times e times e square divided by epsilon K B T. I just the opened up the summation series and for in a N a C l zeta square will be summation of zeta square will be 1 plus 1 2 so 1000 was already multiplied to converted in to the number per litre to meter cube so it is become 2000. Now, if you put all the all the values here these becomes to 2000 into 6.0 2 3 into 10 to the power of 23 into 0.15 into 1.6 into 10 to the power of minus 19 square divided by 80 into 8.8 5 into 10 to the power ~~of~~ minus 12 in to 1.38 into 10 to the power ~~of~~ minus 23 into 300; 300 is the temperature that is 27 ~~7~~ centigrade per 300 Kelvin and this is nothing but the Boltzmann constant now ~~0~~1.38 into 10 to the power ~~of~~ minus 23 zeta Kelvin these turns out to be kappa inverse turns out to be 7.9 6 into 10 to the power of minus 10 meter.

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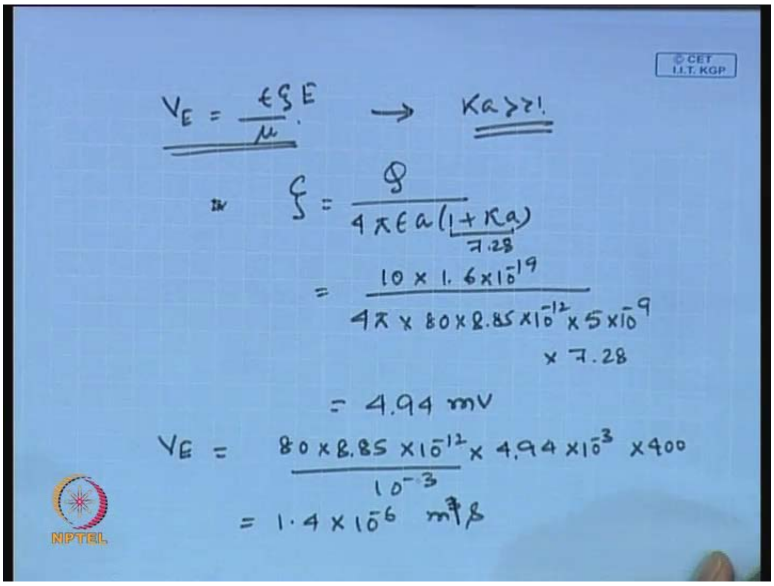

$$V_E = \frac{\epsilon \zeta E}{\mu} \rightarrow \kappa a \gg 1$$

$$\zeta = \frac{q}{4\pi\epsilon a(\underbrace{1+\kappa a}_{7.28})}$$

$$= \frac{10 \times 1.6\times10^{-19}}{4\pi\times 80\times 8.85\times10^{-12}\times 5\times10^{-9}\times 7.28}$$

$$= 4.94\ \mathrm{mV}$$

$$V_E = \frac{80\times 8.85\times10^{-12}}{10^{-3}}\times 4.94\times10^{-3}\times 400$$

$$= 1.4\times10^{-6}\ \mathrm{m/s}$$

Now, you know the electrophoretic velocity; the electrophoretic velocity will be epsilon zeta E by mu and this is basically for the case for kappa a much much greater than 1 so since the condition so you have going to use formula calculate the electrophoretic velocity so you know all the values, if do not know what is the value of zeta suppose if you have not measured zeta potential for the particular problem charge is given. So, you can calculate the value of zeta potential from the formula that we have defined, we obtained by the theoretical analysis basis earlier is whole by epsilon a 1 plus kappa a, so becomes surface charge is known if the surface potential you know when you go to use that now in this particular case is charge is known.

So, Q E will be 10 E so 10 times 1.6 into 10 to the power ~~of~~ minus 19 coulomb so this is 4 pi epsilon 80 into 8.5 into 10 to the power ~~of~~ minus 12 a radius 5 nanometre so 10 to the power of minus 19 multiplied by this whole term comes out to be 7 0.2 8 volt the calculate it this transfer 7.2 8 that mean the kappa a around is 6.28 so much much greater than 1 that is why, we are use this formula use to calculate in the electrophoretic velocity this turns out to be 4.9 milli volt, so you can now will be position in a calculate the electrophoretic velocity by using this formula so this becomes 80 into 8.85 into 10 to the power of minus 12 into zeta potential is 4.94 millivolt into 10 to the power minus ~~of~~ 3 electric field strength so that is 400 volt per meter that is already given to you divided by epsilon divided by mu; mu is the water viscosity 10 to the power of minus 3 and it turns out to be 1.4 into 10 to the power minus 6 meter per second.

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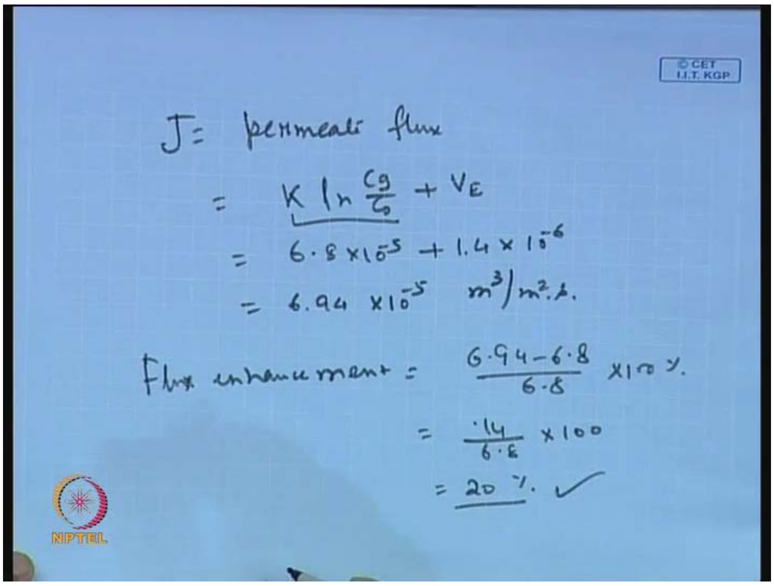

J = permeate flux

$$= \underbrace{K \ln \frac{C_g}{C_0}} + V_E$$

$$= 6.8 \times 10^{-5} + 1.4 \times 10^{-6}$$

$$= 6.94 \times 10^{-5} \; m^3/m^2.s.$$

Flux enhancement $= \frac{6.94 - 6.8}{6.8} \times 100\,\%$

$$= \frac{.14}{6.8} \times 100$$

$$= \underline{20\,\%} \; ✓$$

Now, just put the formula of permeate flux permeate, flux will be nothing but K l n C g by C naught plus V e and this was 6.8 so 6.8 into 10 to the power of minus 5 and this 1.4 into 10 to the power minus 6. So, this is 6.94 into 10 to the power minus 5 meter cube per meter square second, so the flux enhancement you get it be will nothing but 6.14 minus 6.8 divided by 6 point into 100 percent you just calculated how much phase 6.14 divide by 6.8 into 100 percent so it is around 20 percent. So, it is 14 14 divided by 7; so it is around 20 percent now if you increase the field strength 400 volt per meter so 600 volt per meter this will be further enhanced.

So, this is the typical order of magnitude of enhancement of the productivity of the flux are that is basically looking for 20 percent 30 percent of more than that, but is it possible to enhance it further for the that means you can increase the very high electric field will that called very high flux enhancement so the system will ~~be~~ fail, why the system will fail? Simply because it will it will it will basically at very high electric field strength the electrolysis will occur utilises of the solution of the two electrolyte will occur so in that case what will that what is the problem it will form the bubbles gasses will be form now this gas is will be going in to the membrane force it is block the membrane you know force and the permeate flux will be in fact (()) will be reduced so you cannot go and increasing the electric field strength you know unprofitably at the particular level when the before the on set of electrolysis this you have to keep on going the strength so the maximum operating per condition of the external electric field strength it will be decided by the concept for the electrolysis we have to keep the field strength as the (()) electrolysis will not occur at the particular field strength will continues this study tomorrow class thank you very much.