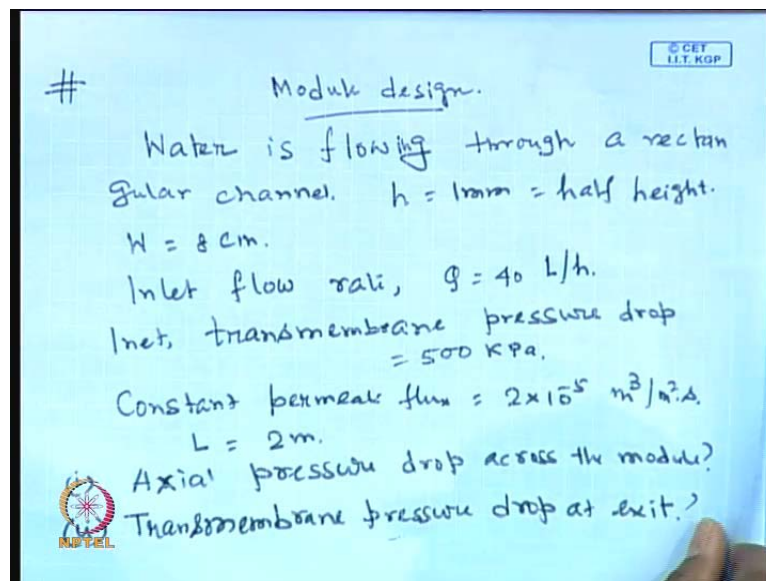


Novel Separation Processes
Prof. Dr. Sirshendu De
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture No. # 21
Membrane Separation Processes

Good morning every one. So, we are looking into the some of the example problem on whatever we have done in this course still now. And in today's class, we have already seen some problem how to solve problems in case of ultra filtration gel layer controlled, and osmotic pressure controlled under you know various conditions batch steady state so and so forth. Into today's class will be looking in to couple of example from design of the membrane modules and the dialysis.

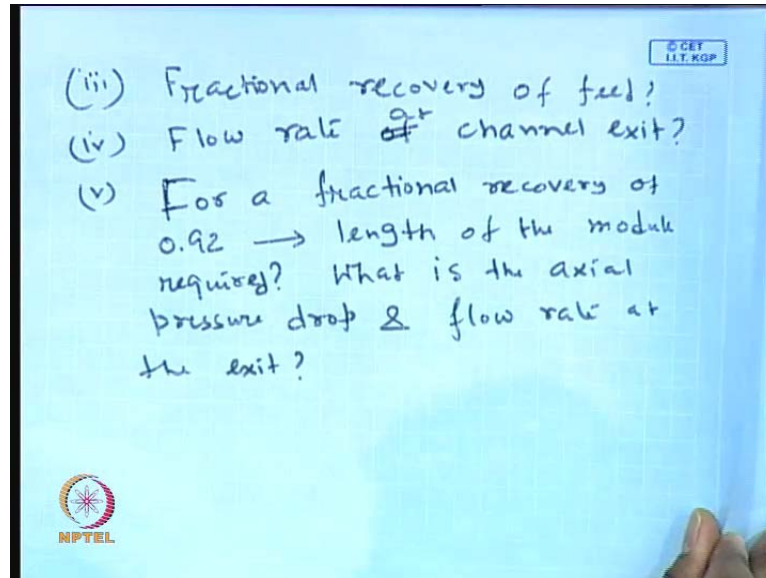
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The first problem that have design today that is the module design, membrane module design. So, in this problem we will just write it at the statement is that water is flowing through rectangular channel is plain water, through a rectangular channel of half height is 1 millimetre this is the half height, and width of the channel is 8 centimetre, the inlet flow rate Q is 40 litter per hour. The inlet Trans membrane pressure drop is 500 kilo pascal, 500 kilo pascal, and it results a constant permeate flux of constant permeate flux of 2 in to 10 to the power minus 5 meter cube per meter square second, and the length of the channel is 2 meter. So, these are the specification and based on these you are going to

calculate some of the things, the first think will calculate what is the actual pressure drop in the module? Actual pressure drop across the module. The Trans membrane pressure drop at the module exit, **trans membrane pressure drop at the exit**.

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Then will calculate the fractional recovery of the feed, because that is very important from the amount, that is going in to the system the maximum you will recover the permeate that will be the **that will be** better fractional recovery of feed and flow rate at the channel exit, **flow rate at the channel exit** and if we like to recover more feed then what is the length of the module that you that is required, if for a recovery fractional recovery of 0.92 that means, very high what is the length required? Length of the module required and in that case what is the actual pressure drop at the exit actual pressure drop and flow rate at the exit. Now in this problem basically up to question number 4 the same channel we would like to calculate various quantity in the last problem, we would like to increase the fractional recovery for that what you have to design of the module? You have to find out what is the length required for the particular purpose?

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Solution: $h = 10^{-3} \text{ m}; W = 0.08 \text{ m};$
 $\Delta P_i = 5 \times 10^5 \text{ N/m}^2; J = 2 \times 10^{-5} \text{ m}^3/\text{m}^2 \cdot \text{s}$
 $L = 2 \text{ m}; Q_i = 40 \text{ L/h} = 1.11 \times 10^{-5} \text{ m}^3/\text{s}.$

(a) $\Delta P_{\text{axial}} = \frac{3}{2} \frac{\mu}{h^3 W} Q_i x \left(1 - \frac{v_w W x}{Q_i}\right)$
 $= \frac{3}{2} \frac{10^{-3} \times 1.11 \times 10^{-5} \times 2}{(10^{-3})^3 \times 0.08} \left(1 - \frac{2 \times 10^{-5} \times 0.08 \times 2}{1.11 \times 10^{-5}}\right)$
 $= 416.25 (1 - 0.29)$
 $= \underline{296 \text{ Pa}}$

Now, let us look into the solution and it is in this case, this is a pure water and there is no effect of osmotic pressure and thinks like that. So, the first formulation simplest formulation that what you have done to a particular design that formulation will whole good. So, let us write down various quantities half height is the 10 to the power minus 3 meter that is 1 millimetre w is 0.08 meter ΔP at the inlet point is 500 kilo pascal 5×10^5 into 10 to the power 5 Newton per meter square J is the constant flux 2×10^{-5} meter cube per meter square second. L is the 2 meter flow rate at the inlet point is 40 liters per hour and if you changed into meter cube per second this Trans out to the 1.11×10^{-5} meter cube per second.

Now, let us solve the first part, if you look into the expression of ΔP actual, the actual pressure drop will be $\frac{3}{2} \frac{\mu}{h^3 W} Q_i x \left(1 - \frac{v_w W x}{Q_i}\right)$ that is the that is the expression of the actual pressure drop just put different values. So, this becomes $\frac{3}{2} \frac{10^{-3} \times 1.11 \times 10^{-5} \times 2}{(10^{-3})^3 \times 0.08} \left(1 - \frac{2 \times 10^{-5} \times 0.08 \times 2}{1.11 \times 10^{-5}}\right)$ that is the flow rate, x is the length of the module that is 2 meter and this is h^3 10^{-3} cube of that point width is 8 centimetre $1 - \frac{2 \times 10^{-5} \times 0.08 \times 2}{1.11 \times 10^{-5}}$ that is the flux and width is point 0.08 multiplied by 2 that is the length 1.11×10^{-5} that is the flow rate. So, this Trans out to be $416.25 (1 - 0.29)$. So, this is 296 pascal, so the actual pressure drop for this particular system is only at is about 300 pascal.

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(i) Transmembrane pressure drop at the exit
 $\Delta P_{\text{exit}} = \Delta P_i - \Delta P_{\text{axial}}$
 $= 499.7 \text{ kPa.}$

(ii) Fractional recovery of feed in the permeate
 $f = \frac{2JWL}{Q_i}$
 $= \frac{2 \times 2 \times 10^{-5} \times 8 \times 10^{-2} \times 2}{1.11 \times 10^{-5}}$
 $= 0.576$

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And now you will be able to calculate the quantities. The second part is the trans membrane pressure drop, **trans membrane pressure drop** at the exit point it becomes delta P exit is nothing but delta P i minus delta P axial that means, the initial pressure minus the drop. So, that will be giving you the final 1. So, this trans out to be 499.7 kilo pascal that is for a particular module the actual pressure drop will be really very low. Now will see how and what condition these will be more and thinks like that all those interpretation will come. Now, the third part is the fractional recovery of the feed, of feed in the permeate. Now if you look in to the expression of their these drift trans out to be 2 J w L divided by Q i and this trans out to be 2 into 2 into 2 into 10 to the power minus 5 into 8 into 10 to the power minus 2 that is the width times length divided by Q i is 1.11 into 10 to the power 5. So, it is Trans out to the 0.576. So, you will be recovering about 58 percent of the feed inform of the in the form of permeate, rest will be the retained to the rest will go to the retained stream.

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(iv) Flow rate at the module exit

$$Q = Q_i \left(1 - \frac{2JwX}{2Q_i} \right)$$

At the channel exit, $X=L$

$$Q = 1.11 \times 10^{-5} \left(1 - \frac{2 \times 10^{-5} \times 0.08 \times 2}{1.11 \times 10^{-5}} \right)$$

$$\approx 29 \text{ l/h.}$$

(v) $f = 0.92 = \frac{2JwL}{Q_i}$

$$L = 0.92 \times \frac{1.11 \times 10^{-5}}{2 \times 2 \times 10^{-5} \times 0.08} = 3.2 \text{ m.}$$

Now, you can calculate the flow rate at the channel exit, flow rate at the let say module exit, this becomes the expression of cross flow rate becomes Q is equal to Q_i 1 minus $2JwX$ divided by $2Q_i$. So, this 22 will be cancelling out. So, that will be the amount of permeate at amount of feed at your re drawing. Now at the channel exit x is equal to l the length of the channel 2 meter. So, Q becomes 1.11 into 10 to the power minus 5 , 1 minus 2 into 10 to the power minus 5 into 0.08 into 2 divided by 1.11 into 10 to the power minus 5 , and the Trans out to be around 29 liter per hour. That was the permeate flow rate the there is the channel the returned at the channel exit. Now let us come down to the design problem, the design problem was that fractional recovery was 0.92 what will be the length of the module you required? Fractional recovery of the feed will be point 0.92 .

So, $2Vw$ is nothing but the J , $2JwL$ divided by Q_i for the same flow rate, same channel width section a etcetera section rate the length can be calculated now the length will be 0.92 into 1.11 into 10 to the power minus 5 divided by 2 into 2 into 10 to the power minus 5 into 0.08 , and it trans out to be about 3.2 meter. So, for 2 meter long modules you are you are recovering about 57 percent of the feed in the form of permeate. In case of a fractional recovery of 92 percent you recovery a larger length of the module and that the Trans out to be 3.2 meter for this particular case.

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$$\begin{aligned}\Delta P_{\text{axial}} &= \text{Axial pressure drop.} \\ &= \frac{3}{2} \frac{\mu}{h^3 w} Q_i X \left(1 - \frac{2 J W X}{2 Q_i}\right) \\ \underline{X=L} \\ &= \underline{360 \text{ Pa.}} \\ Q_2 &= \text{Flow rate at the channel} \\ &\quad \text{exit} \\ &= \underline{21.6 \text{ L/h.}}\end{aligned}$$

And let us look into the axial pressure drop, ΔP_{axial} . So, it is the axial pressure drop this become just put the values $\frac{3}{2} \frac{\mu}{h^3 w} Q_i X$ minus $\frac{2 J W X}{2 Q_i}$. You put x is equal to 1 for the throughout the whole module and if you put the number this is Trans out to be about 360 pascal. And you see this time the axial pressure drop will be higher compare to the earlier case, and the flow rate will be, flow rate at the channel exit or the module exit will be around 21.6 litter per hour. It will be further lower, because you are extracting more permeate out of the module so flow will be less. So this is the more simpler case for doing a for designing a module, and in the and in this particular problem seen that if you like to recover more permeate more feed if the form of permeate then **the length has to be** length of the module has to be increased. In the next problem will see what is the effect of if you reduce the channel height.

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#2 For #1, $h = 0.5 \text{ mm}$. ✓

(a) Axial pressure drop.

(b) Transmembrane pressure drop at the module outlet



(c) Fractional recovery of feed.

(d) Flow rate at the module outlet?

(a) $\Delta P_{\text{axial}} = \frac{3}{2} \times \frac{10^{-3} \times 1.11 \times 10^{-5} \times 2(1-0.29)}{(0.5 \times 10^{-3})^3 \times 0.08}$

$= 2364.3 \text{ Pa}$

$= \underline{2.36 \text{ kPa}}$

Now, in the next problem the problem number 2 for today's problem, in the above problem that means, problem 1, for problem number 1 if the channel height is now reduce to 0.5 millimetre is reduce by half, then you have to find out the axial pressure drop, you have to find out the trans membrane pressure drop at the module outlet, fractional recovery of feed, and flow rate at the channel outlet at the module outlet. Now the first case the axial pressure drop you just calculate it delta P axial just putting the numbers in the formula 10 to the power minus 3 divided by 0.5 into 10 to the power minus 3 raise to the power 3 into 0.08 1.11 into 10 to the power minus 5 , into 2 1 minus 0.29 just put the values in the formula it trans out to be case 2364.3 pascal that means, it is about 2.36 kilo paschal,. So, you see that now you have reduced the channel height, the height of the module and the axial pressure drop has increased tremendously **ok**.

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(b) Pressure drop at the module outlet:
 $\Delta P_L = 5 \times 10^5 - 2364.3$
 $\approx 497 \text{ KPa.}$

(c) $f = 0.576$ (as 'h' does not appear in the exp.)

(d) $Q_L = 28.5 \text{ L/h.}$

So, the pressure drop at the outlet can be calculated, module outlet is becomes delta P L is nothing but 500 kilo pascal 5 into 10 to the power 5 minus 2364.3 this trans out to be 497 kilo pascal. So, that is pressure drop that will you are going to get at the channel outlet. Now the fractional recovery, now if you look into the definition of fractional recovery there is no the channel height does not appear so it remains same. So, channel height the fractional recovery remains it is remains has to be 5.576 as h does not appear in the expression similarly, the flow rate will remain unchanged it the flow rate at the module outlet, it remains unchanged and it remains same value 28.5 litter per hour.

So, if you decrease or increase the height of the channel and maintain the other quantities of almost invariant, then what it will be it will be manifest, it will be manifest that effect will be manifest state in to the pressure drop. So, you can you can design your module accordingly whether you required higher pressure drop, axial pressure drop, low axial pressure drop how to control it? How easier how difficult to controlled this parameters.

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#3. Module design for an UF Process.

A protein solution; $C_0 = 0.2 \text{ kg/m}^3$ has to be concentrated by UF.

$\Delta P_i = 5 \times 10^5 \text{ Pa.}$
 $W = 0.08 \text{ m.}$
 $h = 1 \text{ mm} = 10^{-3} \text{ m.}$
 $L_p = 10^{-11} \text{ N.s/m}^2$
 $Q_i = 40 \text{ L/h.}$ $L = 2 \text{ m.}$

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Now, in the next problem will see that this is not a, this is a little bit complicated one, but in this case will do the assumption that the osmotic pressure is negligible compared to the trans membrane pressure drop, then only you can do the analytical solution otherwise you have to do numerically. So, next problem will be doing a module design for an ultra filtration membrane process. This case a consider a protein solution, the concentration feed concentration is given as 0.2 kg per meter cube. It has to be concentrate by ultra filtration, this protein solution has to be concentrated by U F the operation condition like this at the channel entrance ΔP_i is 500 kilo pascal 5×10^5 pascal, that is the trans membrane pressure drop at the channel entrance W is the width of the channel and this is same as 8 centimetre or 0.08 meter and the channel half height is let say 1 millimetre. So, this becomes 10^{-3} meter the membrane permeability is given as 10^{-11} Newton second per meter square.

So, this basically is in the order of 10^{-11} may be 2×10^{-11} for this particular problem, let us have it is in the order of 7×10^{-11} the feed flow rate that is the channel entrance that is Q_i is 40 litter per meter hour and the channel length is 2 meter. And we neglect the osmotic pressure of the protein solution, it is high molecular protein, so osmotic pressure of protein solution compare to the trans membrane pressure drop.

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Negligible osmotic pr. compared to transmembrane pressure drop.

(a) Pressure drop across the module?
(b) Fractional Recovery of feed?
(c) Velocity & concentration at the channel exit?

$Q_i = 40 \text{ L/h} = 1.11 \times 10^{-5} \text{ m}^3/\text{s}$

$u_{ol_i} = \frac{Q}{2hw} = 0.07 \text{ m/s}$

So, negligible osmotic pressure compare to Trans membrane pressure drop. Let us say what we have to calculate, you have to find out the pressure drop across the channel, across the module it will find out the fractional recovery of the feed, velocity and concentration at the channel exit and channel exit. So, we have different quantities ΔP_i is 5×10^5 pascal width is 0.08 meter height is half height is 10^{-3} meter, membrane permeability is given Q_e at the inlet point is given 40 litre per hour, it trans out to be 1.11×10^{-5} meter cube per second. So, you can find out what is the velocity at the cross section average velocity at the entrance of the channel at the modules, this is becomes Q by $2hw$ that will be the area cross section of the flow, and it trans out to be 0.7 meter per second. Now in this case we are assuming you that osmotic pressure is negligible is small. So, therefore, permeate flux is proportional to trans membrane pressure drop at any point of time because $\Delta p - \Delta \pi$ the π is negligibly small. So, we will go to the second formulation for the membrane module design and we calculate accordingly.

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$$\lambda = \sqrt{\frac{3\mu L P}{h^3}} = \sqrt{\frac{3 \times 10^3 \times 10^{-11}}{10^{-9}}} = 5.48 \times 10^{-3}$$
$$\beta = \frac{3}{2} \frac{\mu Q_i}{h^3 W \lambda \Delta P_i}$$
$$= \frac{3}{2} \times \frac{10^3 \times 1.11 \times 10^{-5}}{10^{-9} \times 0.08 \times 5.48 \times 10^{-3} \times 6 \times 10^{-5}}$$
$$= 0.076$$
$$\lambda L = 0.011 ;$$
$$\sinh(\lambda L) = 0.011$$
$$\cosh(\lambda L) = 1.0$$

So therefore, we have to calculate the value of lambda there if you remember that was $\frac{3\mu L P}{h^3}$, μ is the viscosity of **viscosity of** water protein solution let us say it is same as water viscosity 3×10^{-3} pascal second L is given as 10^{-11} in the order of 10^{-11} this will be in the order of this h^3 10^{-9} to the power of minus 9. So, it Trans out to be 5.48×10^{-3} . Then there is another parameter beta you have to calculate that that is $\frac{3}{2} \frac{\mu Q_i}{h^3 W \lambda \Delta P_i}$ that is the inlet flow rate $h^3 W \lambda \Delta P_i$ at the inlet point, and this trans out to be these $3.2 \times 10^{-3} \times 1.11 \times 10^{-5}$ divided by $10^{-9} \times 0.08$, the value of lambda is 5.48×10^{-3} and ΔP_i is 6×10^{-5} and it trans out to be 0.076.

So, lambda L you have to find out lambda L trans out to be 0.011 and $\sinh(\lambda L)$ and cosine $\cosh(\lambda L)$ or you will that do calculate utilise. So, this Trans out to the 0.011 and $\cosh(\lambda L)$ trans out to be 1. So, by computing all the quantities you know all the parameters you have to put them in the formula.

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(a) Pressure drop

$$\frac{\Delta P(L)}{\Delta P_i} = \cosh(\lambda L) - \beta \sinh(\lambda L)$$
$$= 0.999$$
$$\Delta P(L) = 499.5 \text{ kPa.}$$
$$\Delta P_{\text{axial}} = \underline{500 \text{ Pa}}$$

(b) Fractional Feed recovery

$$f = \frac{2wLp\Delta P_i}{Q_i \lambda} [\sinh(\lambda L) - \beta \cosh(\lambda L) - 1]$$
$$= \underline{0.147}$$

The image shows handwritten mathematical derivations on a light blue background. Part (a) calculates the pressure drop ratio and axial pressure drop. Part (b) calculates the fractional feed recovery. Logos for I.I.T. KGP and NPTEL are visible in the corners of the slide.

First calculation is pressure drop, axial pressure drop if you look into the definition of that ΔP at the length L divided by ΔP at the entrance point is nothing but $\cosh \lambda L$ minus $\beta \sinh \lambda L$ and it turns out to be 0.999. So, it is negligible small and the Trans membrane pressure drop at the outlet will be 499.50 kilo pascal. And ΔP_{axial} , the axial special drop will turn out to be around 500 pascal only really this is really very small. For a channel of length 2 meter and for the particular half height.

Next we calculate the fractional feed recovery in the form of permeate in the formula was $2w$ that is the width of the channel, multiplied by $L p \Delta P$ divided by $Q_i \lambda \sinh \lambda L$ minus $\beta \cosh \lambda L$ minus 1. So, these is ΔP at the entrance point is ΔP_i and if put all these values it turns out to be 0.147 that means, in this particular problem you have recovering only 14 percent or about 15 percent of the feed in the form of permeate, that may not be feasible. So, you have to change the parameters for the example channel half height or width or other parameters, design parameters to increase the fractional feed recovery, even you can set the fractional feed recovery about 90 percent or 85 percent and that calculate what will be the dimension of the channel and design appropriately.

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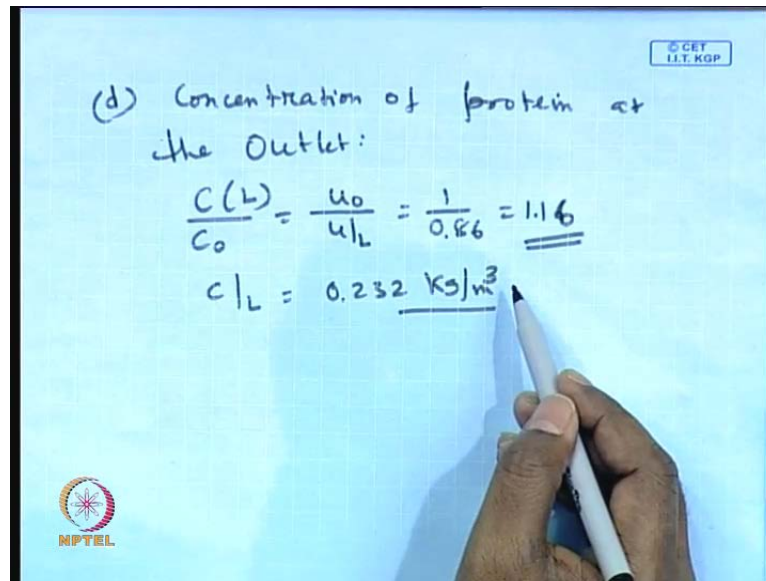
The image shows a handwritten derivation on a grid background. At the top right, there is a small logo for '© CET I.I.T. KGP'. The text '(c) Velocity at the outlet:' is written in blue. Below it, the following equations are written:

$$\frac{\bar{u}|_L}{u_0} = 1 - \frac{L \Delta p_i}{h \lambda u_0} \left[\frac{\sinh(\lambda L) - \beta x}{\cosh(\lambda L) - 1} \right]$$
$$= 1 - \frac{10^{-11} \times 5 \times 10^5}{10^{-3} \times 5.85 \times 10^3 \times 0.07} [0.011]$$
$$= \underline{\underline{0.86}}$$

Below the final result, the velocity is given as $\bar{u}|_L = \underline{\underline{0.06 \text{ m/s}}}$. In the bottom left corner, there is a logo for 'NPTEL'.

Now, 3rd problem was velocity at the outlet, it was \bar{u} at length L divided by cross sectional average velocity that the inlet $1 - \frac{L \Delta p_i}{h \lambda u_0} \left[\frac{\sinh(\lambda L) - \beta x}{\cosh(\lambda L) - 1} \right]$. So, if you put all the values it is trans out to be that is $10^{-11} \times 5 \times 10^5$ that is the trans membrane pressure drop at the entrance point, h is $10^{-3} \times 5.85 \times 10^3$ u_0 0.07 and this turns out to be 0.011 and it is 0.86 . So, \bar{u} at length L at the module exit it will be 0.06 meter per second. So, there is 14 percent loss in the cross flow velocity, because the rest amount of feed is, because the feed you are extracting out of feed. So, the velocity decreased so about 14 percent loss in the velocity compare to the inlet of the module.

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(d) Concentration of protein at the Outlet:

$$\frac{C(L)}{C_0} = \frac{u_0}{u_L} = \frac{1}{0.86} = \underline{\underline{1.16}}$$
$$C|_L = \underline{0.232 \text{ Kg/m}^3}$$

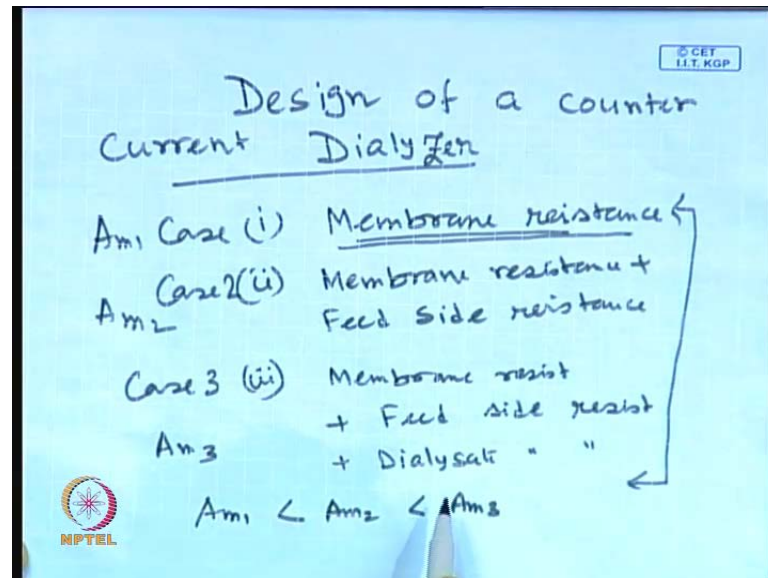
The image shows a hand holding a white marker pointing to the final result of the calculation. The whiteboard has a grid pattern and logos for I.I.T. KGP and NPTEL.

Next is concentration at the outlet of protein at the outlet. So, C_L divided by C_0 is inverse of the previous problem u_0 divided by u_L that is 1 over 0.86 these Trans out to be 1.16 . So, C_L Trans out to be 0.232 kg per meter cube. So, in this particular problem for this module, for these particular modules in dimension in the operating conditions you are only concentrated it only by 16 percent from C_0 by C_L is about 1.16 . Now if you would like to increase you would like to concentrate it more. So, let us say 2 times, 3 times 4 times it is the 1.16 times you can design your module appropriately, you can find what will be the length, what will be the width, what will be the height and what will be the operating, you can thinker with the operating condition as well that you can say that will the same inlet trans membrane pressure drop will be such.

So, that I can increase the concentration at the outlet by 2 fold or 3 fold. So, I am just, I have just selected some of the typical numbers to give you a feel and idea how to do that calculation and what will be the for some typical values, what will be the final results you are going to obtained. So, if you would like to recover to like that you would like to concentrate the protein by 2 folds **right** in that case you put C_L by C_0 at the outlet by C_0 is equal 2 or 3 is 3 folds concentrate, concentration then, according to that you can you can design your module, you can select your design parameters at the operating condition they are two thing only design parameter in the operating conditions and of course, the full properties, but that is fixed once you have selected your feed that you are

what is the protein solution, or you are walking with the effluent. So, that is fixed based on the requirement one can find in or design in the geometric parameters of the module as well as the operating conditions to achieve the goal that you are setting.

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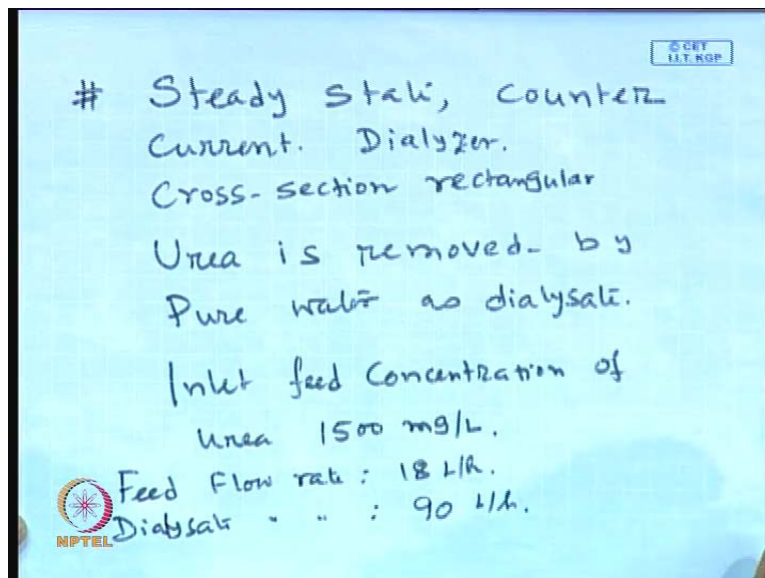
So, next set of problem, I am just select or selected for the dialysis 1 and this is a design of a counter current dialyzer. And this problem is a this problem has a very carefully selected and here, we are we have shown you that, will be doing the one dimension module calculation for example, using the long mean, concentration difference idea will be doing the design. Now this problem is selected such that in one case we have selected that the. So, there are 3 resistances as we are discussed earlier one is the resistance in this feed side, another is the resistance within the membrane, and resistance in the dialyzer it side. Typically, if the flow rate in the feed side in the dialyzer side or quite high we can neglect those resistances. So, the main resistance will be the membrane resistance. So, first will do the calculation and find out the area required for getting a particular, you know separation I am coming to the problem in elaborately and if you consider membrane resistance only then what will be area that is that will be required?

Now, typically the feed flow rate is 2 times or 3 times lower than the dialyzer flow rate to have the better efficiency. Now if you calculate, if you consider of resistance of membrane as well as the feed flow rate, feed resistance then what will be the area membrane area that you are going to get particular you know separation of the dialysis

process. The third case this case 1, this is case 2, this case 3, in the third case we have considered that including membrane resistance, to includes feed side resistance as well as the dialysate side resistance and see what membrane area going to get. The compensate that the if you include these resistance only the area that will be the minimum here, this is $A_m 1$, in this case this $A_m 2$, and in this case this is the $A_m 3$ that the membrane area in this third case. And you are you are supposed to get $A_m 1$ is less than $A_m 2$ is less than $A_m 3$ in that order.

Now, what will be the area and order of magnitude area that will see later on? Now so basically for an actual dialysis case you should considered all the 3 resistances in that, but what happens if you neglect one of them, if you neglect two of them, if you include all of them so let us looks for.

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Now, let us come down to the problem, it is a steady state dialyzer, steady state counter current, counter current dialyzer of cross section rectangular that means, you are having a small channel rectangular channel. Then you said put the membrane and then another rectangular channel two rectangular channels are separated by a dialysis membrane in one channel you are you are passing the feed, and in the other channel you are passing the dialysate in the opposite direction. So, that forms the counter current dialyzer and the membrane area will be nothing but the height of the channel multiplied by the length of the module that will be the membrane area and the cross section area of the flow will be width of the channel multiplied by the height of the membrane or height of the module.

So, that will proper formulate that will give you the cross sectional area normal to the flow, and length of the membrane module multiplied by the height of the module, the membrane or module will give you the area of the membrane. Now in this case urea is separated is removed, by pure water at dialysate. An inlet feed concentration of urea is 1500 milligram per litre. The feed flow rate given as 18 liters per hour, dialysate flow rate will be 5 times of that let say 19 litre per hour.

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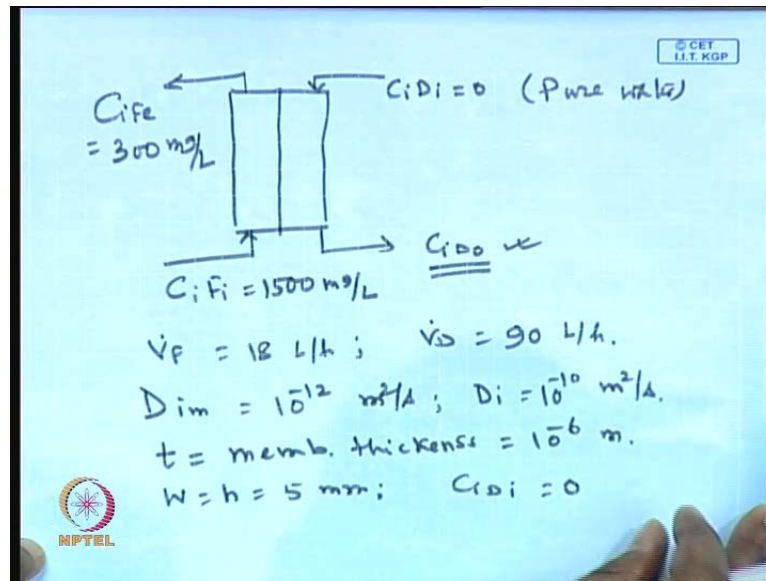
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D_{urea} in bulk soln = $10^{-10} \text{ m}^2/\text{s}$
 D_{urea} in the memb. = $10^{-12} \text{ m}^2/\text{s}$
Memb. thickness: 10^{-6} m
Feed and dialysate chambers
are identical.
Width: 5 mm
height: 5 mm.
Feed sid: urea conc. from 1500 (inlet)
to 300 mg/L (output)
Find membrane area required.

NPTEL

Diffusivity of urea, **diffusivity of urea** in bulk solution is given as 10^{-10} meter square per second, and diffusivity of urea in the membrane is two orders of magnitude less that will be in the order of 10^{-12} meter square per second. Membrane thickness 1 micron 10^{-6} meter, feed and the dialysate chambers are identical, they are the channel width is 5 millimetre and full height is 5 millimetre, **5 millimetre height**, 5 millimetre width both the channels. In the feed side the urea concentration has to be reduced from 500 to 300 milligram per litre from 1500 at the inlet to 300 milligram per litre at the outlet. Now we have to find out the membrane area required for this particular problem.

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Now, we just draw the semantic diagram of the counter current dialyzer. C_{iF_i} is the feed of urea that is going in to system chamber is 1500 milligram per litre. C_{iF_e} that is the urea that is going out should be 300 milligram per litre, we are going to reduce from 1500 into 300 per litter. It is the C_{iD_i} will be equal to 0, because it is pure distilled water C_{iD_o} outlet will be something that we are going to find out. So, v_f dot 18 liter per hour that is the flow rate of the feed flow rate in the dialysate side is 90 liter per hour, D_{im} is 10 to the power minus 12 meter square per second, that is the urea diffusivity in the membrane surface D_i will be 10 to the power minus 10 meter square per second.

Thickness of membrane, membrane thickness we called it as l probably does not matter is thickness is the 10 to the power minus 6 meter 1 micron and w and height both are 5 millimetre C_{iD_i} will be equal to 0. So, what will the first step what will be doing you have to calculate the log mean, concentration difference. In order to calculate the log mean concentration different you required to know what is the value of $C_{iD_{naught}}$. So, that can be calculated using overall an urea balance in the system that amount that is urea going in to this. So, the amount of urea that is being lost in the feed chamber should be gain by the dialysate chamber.

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$$\begin{aligned} \dot{m} &= \text{urea transport rate} \\ &= \dot{V}_F (C_{iF_i} - C_{iF_e}) \\ &= \dot{V}_D (C_{iD_o} - C_{iD_i}) \end{aligned}$$
$$18(1500 - 300) = 90 (C_{iD_o} - 0)$$
$$\underline{C_{iD_o} = 240 \text{ mg/L}}$$

Case 1: Neglect mass transfer resistance on both sides. Membrane resistance prevails.

So, \dot{m} is the urea, rate of urea, transport urea transport rate and this will be nothing but urea transported means, from the feed chamber to the dialyzer chamber through the membrane. So, this will be nothing but $\dot{V}_F (C_{iF_i} - C_{iF_e})$ is equal to $\dot{V}_D (C_{iD_o} - C_{iD_i})$. This is the amount of urea lost in the feed chamber, is equal to amount of urea that is gain in the dialysate chamber. So, if you do that you will be getting $18(1500 - 300)$ is equal to $90(C_{iD_o} - 0)$ and C_{iD_o} will be 240 milligram per litre. So, the outlet concentration in the dialysate will be will be containing 240 milligram per litre urea at the steady state. And now will solve the case 1.

So, C_{iD_o} in this particular case will be 240 milligram per litre that have calculated. Now you will be in the, because all the 4 you know concentration are unknown at the 4 is 2 entry point and two 2 point you will be in the position to calculate the log mean concentration difference. So, ΔC_1 will be will be $300 - 0$ and ΔC_2 will be $1500 - 240$. Based on that you can calculate the log mean, concentration difference **right** it is exactly 7 log mean, temperature in this case of heat transfer. So, case number 1 we neglect the mass transfer resistance on both sides the only resistance membrane resistance prevails. So, you can calculate that.

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The image shows handwritten notes on a blue background. At the top right, there is a small box containing the text '© CBT I.I.T. KGP'. The main text defines K_0 as the overall mass transfer coefficient and calculates its dimensions as $\frac{\text{Dim}}{t} = \frac{10^{12}}{10^6} = 10^6 \text{ m/s}$. Below this, the Log Mean Temperature Difference (LMTD) is defined as $(\Delta C)_{\text{LMTD}} = \frac{(\Delta C)_1 - (\Delta C)_2}{1 + \frac{(\Delta C)_1}{(\Delta C)_2}}$. The calculations for $(\Delta C)_1$ and $(\Delta C)_2$ are shown as $(\Delta C)_1 = 1500 - 240 = 1260 \text{ mg/L}$ and $(\Delta C)_2 = 300 - 0 = 300 \text{ mg/L}$. In the bottom left corner, there is a logo for NPTEL.

$$K_0 = \text{Overall mass transfer Coefficient}$$
$$= \frac{\text{Dim}}{t} = \frac{10^{12}}{10^6} = 10^6 \text{ m/s}$$
$$(\Delta C)_{\text{LMTD}} = \frac{(\Delta C)_1 - (\Delta C)_2}{1 + \frac{(\Delta C)_1}{(\Delta C)_2}}$$
$$(\Delta C)_1 = 1500 - 240 = 1260 \text{ mg/L}$$
$$(\Delta C)_2 = 300 - 0 = 300 \text{ mg/L}$$

Now, K_0 the overall mass transfer resistance, mass transfer coefficient, the inverse of the overall mass transfer resistance coefficient it trans out to be Dim / t divided by the thickness of the membrane and this will be 10^{12} , divided by 10^6 to the power minus 6, that will be 10^6 meter per second. Now you will be you calculate the ΔC_{LMTD} , we write it LMTD should be LMCD is basically the idea is same this is ΔC_1 , minus ΔC_2 divided by $1 + \frac{\Delta C_1}{\Delta C_2}$. So, it will be ΔC_1 will be 1500 minus 240. So, this trans out to be 1260 milligram per litre, and the ΔC_2 will be 300 minus 0 that is 300 milligram per litre. So, it is at the inlet point of the feed, these at the outlet point of the feed, the temperature concentration difference at the inlet point of the feed, and this is the concentration difference at the exit point.

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$$\begin{aligned}(\Delta C)_{\text{LMCD}} &= \frac{1260 - 300}{\ln \frac{1260}{300}} \\ &= 668.95 \text{ mg/L.}\end{aligned}$$

\dot{m} = mass transport rate of urea across the membrane

$$\begin{aligned}&= \dot{V}_F (C_{iF_i} - C_{iF_e}) \\ &= 18 \text{ (L/h)} \times 1200 \frac{\text{mg}}{\text{L}} \\ &= \underline{21.6 \text{ g/hr.}}\end{aligned}$$

So, you will be in position to calculate delta C L M you should called LMCD, but anyway since it you know the meaning of it 1260 minus 300 divided by ln 1260 divided by 300 it trans out to be 668.95 milligram per litre. So, the log means concentration difference for the full dialyzer module it Trans out to be 668.95 milligram per litre, and you can calculate the mass transport rate of urea, rate of urea across the membrane. How much urea has been transparent across the membrane? The amount of urea that has been lost in the feed chamber **right**. So, it will be nothing but V F dot times C i F i minus C i F e the amount of urea that is lost in the feed chamber must have been transported to the other side at the steady state. So, it will be 18 litres per hour multiplied by 1200 milligram per litre, so you can convert it into you know. So, it so it Trans out to be 21.6 gram per hour. So, you can you can calculate it find it out you can convert the unit and it is 21.6 gram per hour. Now these mass transport rate of urea can be can be retained terms of overall mass transfer coefficient.

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$$\begin{aligned} m &= K_o A_m (\Delta C)_{LMTD} \\ &= \underbrace{10^{-6}}_{(m/s)} \underbrace{A_m}_{(m^2)} \underbrace{668.5}_{\frac{mg}{l}} \\ &= 10^{-6} \times 668.5 A_m \times 3600 \frac{g}{h} \\ &= \underline{2.41 A_m} \quad (g/h) \\ 2.41 A_m &= 21.6 \\ A_m &= 8.96 m^2 \\ &\approx 9 m^2 \end{aligned}$$

This is nothing but, $K_o A_m \Delta C LMTD$ if you remember. So, these to will be equated so what K_o naught? K_o naught will be 10^{-6} this is A_m and $\Delta C LMTD$ is 668.95 milligram per liter. Now, there is the unit problem here cannot is meter per second this is meter square, and this will be in milligram per liter. You convert into appropriate unit and these Trans out to be in fact milligram per liter will be nothing but so if you multiplied it, I think you multiplied by 3600 and this is trans out to be gram per hour that is unit conversion. So, meter cube **meter cube** and liter will be cancelled out if you convert liter into meter cube and milligram converted into gram and this if you multiply both side multiplied by divided by 3600 that second will be converted into hour. So, it will be in the form of gram per hour. So, it Trans out to it 2.41 times A_m the unit will be gram per hour.

So, you should very careful about our inlet, now will be equating the amount urea transported by depletion amount depleted in the feed chamber that is 21.61 per gram feed chamber with this. So, this will be 2.41 A_m times 20 is equal 21.6, and area of the membrane will be nothing but 8.96 meter square, so it will be around 9 meter square. So, given the dimension of the given the dimension of the channel and the operating condition the specification the amount of urea that has to be you know that is going in to system amount has to removed, if you would like to designed the membrane dialyzer, you have to supply a membrane area of above 9 meter square, then you will be realizing those you know the target that we are going to be set.

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Case 2
Neglect Dialysate side mass transfer resistance & include feed side resistance.
 $\dot{V}_F = 1.8 \text{ l/h}; W = H = 5 \text{ mm.}$
 $\text{Area of c.s.} = 25 \times 10^{-6} \text{ m}^2$
 $u_f = \frac{5 \times 10^{-6}}{25 \times 10^{-6}} = 0.2 \text{ m/s.}$
 $d_e = 2H = 10 \text{ mm} = 10^{-2} \text{ m}$
 $Re_f = \frac{\rho u_f d_e}{\mu} = \frac{2000}{\text{Laminar flow}}$

In the case number 2 will be considering, the dialysate side will be neglect the dialysate mass transfer coefficient resistance, neglect dialysate side mass transfer resistance and include feed side resistance, and see what you get and calculate the membrane area for this particular performance of the dialyzer. So, \dot{V}_F will be 18 litre per hour, and you are W and height will be 5 millimetre each. So, area of cross section normal to the flow will be nothing but the W times full height that is 5 into 5. So, 25 into 10 to the power minus 6 meter square. So, that u_f that velocity the cross sectional average velocity in the feed chamber will be 5 into 10 to the power of minus 6 divided by 25 into 10 to the power minus 6. This 5 into 10 to the power minus 6 is basically, if you convert 18 litres per hour into meter cube per second, these Trans out to be 18 litre per hour is equal to 5 into 10 to the power minus 6 meter cube per second.

So, meter cube per second divided by meter square. So, the velocity in the feed chamber will be getting it will be you will be getting as point 0. 2 meter per second. Now equivalent diameter you calculate the Reynolds number once, you calculate the Reynolds number based on the value of the Reynolds number you have to select the appropriate mass transfer coefficient or by selecting the appropriate for relationship for Sherwood number. So, equivalent diameter will be 4 into half height. So, 2 times full height **right**. So, it will be 10 millimetre to be 10 to the power minus 2 meter. So, Reynolds number you can calculate in the feed chamber as $u_f d_e$ by $\rho u_f d_e$ by μ is you take density of water 10 cube density of the viscosity of water as 10 to the power minus 3 u_f is point

2 and d_e equivalent is 10 to the power minus 2 it trans out to be 2000. So, this is less than 2200 so it is a laminar flow.

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The image shows handwritten mathematical derivations on a light blue background. In the top right corner, there is a small logo for 'CET I.I.T. KGP'. In the bottom left corner, there is a logo for 'NPTEL'. The derivations are as follows:

$$Sh = \frac{k d_e}{D} = 1.85 \left(Re Sc \frac{d_e}{L} \right)^{1/3}$$

$$k_f = 1.85 \cdot \left(\frac{u_F D^2}{d_e L} \right)^{1/3}$$

$$= \frac{1.08 \times 10^6}{L^{1/3}}$$

$$k_o = \frac{1}{\frac{1}{k_f} + \frac{t}{D_i}} = \frac{1}{\frac{L^{1/3}}{1.08 \times 10^6} + \frac{1}{10^6}}$$

$$= \frac{10^6}{1 + 0.93 L^{1/3}}$$

Now, if it is a laminar flow you can calculate the Sherwood number from this relationship $K d_e$ by D is equal to 1.85 Reynolds smite d_e by L rest to the power 1 upon 3. And k we put the velocity. So, k_f will be 1.85 $u_F D$ square divided by d_e times L rest to the power 1 upon 3, and if you put all the values, you do not know the length of the module. So, it Trans out to be 1.08 into 10 to the power minus 6 divided by L to the power 1 upon 3 really is not know. You are in position to calculate the overall mass transfer coefficient and in this case it will be 1 over K_f plus 1 over l means, thickness over D_i thickness of the membrane by D_i . So, if you put all the values it Trans out to be 1 to be power 1 upon 3 is 1.08 into 10 to the power minus 6 plus 1 over 10 to the power minus 6. And this will be 10 to the power minus 6 divided by 1 plus 0.93 1 to the power of 1 upon 3. So, that is the value of overall mass transfer coefficient.

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Handwritten derivation on a light blue background:

$$\dot{m} = 21.6 \text{ g/h. } \checkmark$$

$$\dot{m} = K_o A_m (\Delta C)_{LMTD}$$

$$= \frac{18.6}{1+0.93 L^{1/3}} A_m 668.95 \times 3600 \text{ (g/h)}$$

$$A_m = H \neq L$$

$$\Rightarrow L = 200 A_m$$

$$= \frac{2.41 A_m}{1+5.44 A_m^{1/3}} \checkmark$$

$$\frac{2.41 A_m}{1+5.44 A_m^{1/3}} = 21.6 \Rightarrow \boxed{341 \text{ m}^2}$$

Logos: NPTEL (bottom left), IIT KGP (top right)

In this particular case so you have already calculated that \dot{m} is 21.6 gram per hour. And now you can calculate \dot{m} from the definition of overall mass transfer coefficient as a $\dot{m} = K_o A_m (\Delta C)_{LMTD}$. So, it trans out to be 18.6 to the power minus 6, divided by 1 plus 0.93 L to the power 1 upon 3, $A_m 668.95$ into 3600 and this unit will be gram per hour. Now A_m what is the A_m will be the full H multiplied by the length of the membrane **right**. So, L if you L can be replaced in form of in terms of total membrane area. So, L will be nothing but 200 times A_m so if you put that so these trans out to be 2.41 times A_m divided by 1 plus 5.44 A_m to the power 1 upon 3.

Now we can equate this **this** it becomes 2.41 A_m 1 plus 5.44 A_m to the power of 1 upon 3 is equal to 21.6. And from these basically you can you can these **these** quantity will be pretty large you can neglect 1 with respect to that and you can get direct solution of A_m . And A_m Trans out to be 341 meter square in this particular problem. So, you we if you neglect the mass transfer resistance of the feed side the membrane area required is around 9 metre square, but if you include the mass transfer resistance of the feed side it will be shooting up to 340 meter square. So, inclusion of all the resistance very important, because the feed side feed side is in the laminar flow. So, mass transfer resistance will be significant. So, if you neglect that then you are be you will be under estimating the whole problem.

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(iii) Consider resistances on both sides.

$$\dot{V}_D = 90 \text{ L/h} = 2.5 \times 10^{-5} \text{ m}^3/\text{s}$$
$$u_D = 1 \text{ m/s}$$
$$Re_D = 10,000 \text{ turbulent.}$$
$$k_c Sh = \frac{K_d L}{D} = 0.023 (Re)^{0.8} (Sc)^{0.33}$$
$$K_D = 7.616 \times 10^{-6} \text{ m/s}$$
$$K_D = \frac{10^{-6}}{1.131 + 0.93 L^{1/3}}$$

Now, in the second case will be considering both side resistances that 3rd cases consider resistances on both sides. So, you can you can calculate the velocity in the dialysate side these trans out to be 2.5 into 10 to the power minus 5 meter cube per second, and the dialysate side velocity will be 1 meter per second and the reynodes number in this particular problem trans out to be 10,000. So, it is in the turbulent. So, we are selected in the problem solves that in 1 case it will be laminar flow and another side will be turbulent flow. So, you will be you will be leading requiring the Sherwood number relationship as the 10.023 reynodes to the power 0.8 and smite to the power 0.33 the calculation was straight forward and the K D Trans out to be 7.616 into 10 to the power minus 6 meter per second for this particular case. So, there is no 1 there so you can calculate the reynodes number smites number exedra and it is Trans out to be this. Now you overall mass transfer coefficient will be 10 to the power minus 6 divided by 1.131 plus 0.93 to the power into 1 to the power 1 upon 3 in this particular problem.

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$$21.6 = K_0 A_m (\Delta C)_{LMTD}$$
$$\downarrow$$
$$\frac{A_m}{1.131 + 5.44 A_m^{1/3}} = 8.96$$

→ Trial & error soln.:

$$A_m = 355 \text{ m}^2$$
$$9 \text{ m}^2 < 340 \text{ m}^2 < 355 \text{ m}^2$$

Now, if you equate that 21.6 gram per hour to $K_0 a_m \Delta C_{LMTD}$ and you know the simplify these things final expression that you will be getting is a_m divided by 1.131 plus 5.44 a_m to the power 1 upon 3 is equal to 8.96. So, by you can do a trial and error solution trial and error solution gives you a_m as 355 meter square. So, you are getting first 9 meter square, then you are getting 340 meters square, then now we are getting 355 meter square. So, if you go on increasing the resistance the area required for the dialyzer module will be will keep on increasing. So, although there is not much difference between these two areas, because you know dialyzer sides flow rate is real in the turbulent condition and the resistance will be less and mass transfer coefficient will pretty high. On the other hand if you include if you do not include the any of the feed side dialyzer resistance will be really under predicting the design. So, you have to considered all the resistances at least the feed side resistance to get a realistic picture or estimate of the membrane design. **thank you** very much now in the next class will be moving on another topic.