Novel Separation Processes

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Lecture No. # 20
Membrane Separation Processes (Contd.)

Good morning everyone. So, we are starting with this in this class some of the problems that whatever you have solved in the last class; you are just looking into some of the examples, in the last class, we have, we have looked into example osmotic pressure control ultra filtration; one gel layer control ultra filtration, and another batch gel layer control ultra filtration problem. Now, in today's class, we will we will do a problem will start with the problem which requires, not a constant boundary layer thickness or film theory, we will be utilizing the two-dimensional model for developing concentration boundary layer, and how to utilise those equations to get better accurate prediction of permeate flux of permeate concentration.

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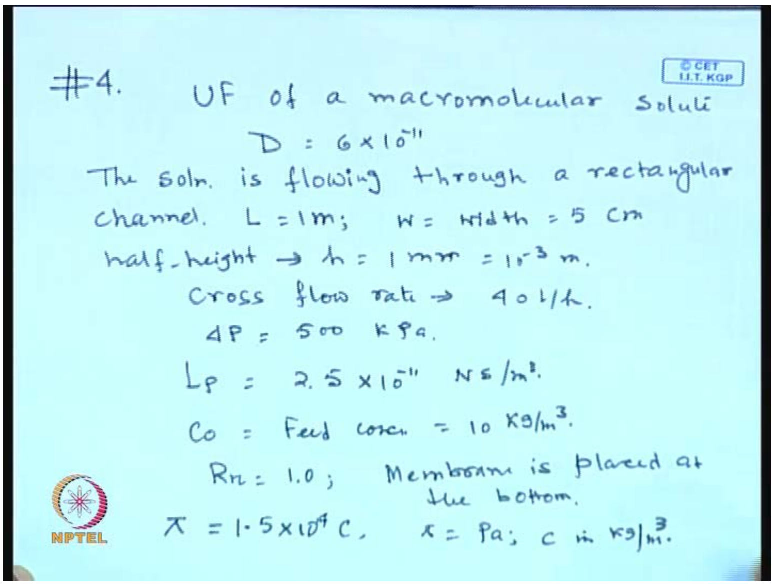

#4. UF of a macromolecular solute

$D = 6 \times 10^{-11}$

The soln. is flowing through a rectangular channel. $L = 1 m$; W = width = 5 cm

half-height → $h = 1 mm = 10^{-3} m$.

Cross flow rate → 40 l/h.

$\Delta P = 500$ kPa.

$L_p = 2.5 \times 10^{-11}$ Ns/m³.

C_0 = Feed concn = 10 kg/m³.

$R_r = 1.0$; Membrane is placed at the bottom.

$\pi = 1.5 \times 10^4 \, C$. π = Pa; C in kg/m³.

So, this problem, this problem number 4, it is it deals with the ultra filtration of a macro molecular solutes, solute. It has the diffusivity of 6 into 10 to the power minus 11, and this solution is flowing through a rectangular channel channel of length 1 meter width 5 centimetre and half height half hight of the channel that is h is 1 millimetre. So, that is 10 to the power minus 3 meter and the flow rate, cross flow rate is 40 litre per hour, and trans membrane pressure drop delta p is 500 kilo pascal, membrane permeability is given as 2.5 into 10 to the power minus 11 newton second per meter cube. The feed concentration C naught is 10 k g per meter cube, the real retention is given as 1 and membrane placed at the bottom only at the bottom of the channel that means only 1 single level and pi expression with concentration is given as 1.5 into 10 to the power 4 times c pi pi is pascal and C is in k g per meter cube.

So, this is the problem we have to find out what is the permeate flux. So, basically our aim is to find what is the permeate flux of this process this is a steady state process, but in this case we are not we are not saying that it is a it is a constant boundary layer thickness or film theory will be applicable, it will be utilizing the method that is that we have derived for the developing mass transfer boundary layer or two-dimensional analysis, but what will be doing here will not be solving any of the differential equations will be taking the expression of sherwood number relationship and the average root number relationship in terms of average permeate flux, and the shortcut method that whatever we have developed earlier discussed earlier.

So, it is say will be using that method, so that method will be more accurate than the film theory method, and will be basically in this problem, we are trying to separate filter the solution of a of a protein solution, which will be having a having a diffusivity of 6 into 10 to the power minus 11 flowing through rectangular channel with the cross flow rate for 48 per hour. The dimension of the of the various dimensions of the channel are given and the operating pressure trans membrane pressure drop is also given the membrane permeability is also given, the feed concentration is given, real retention is known and that is 1. And the relationship of osmotic pressure as a function of concentration unknown so you you would like to find out what is at the steady state what is the value of permeate flux.

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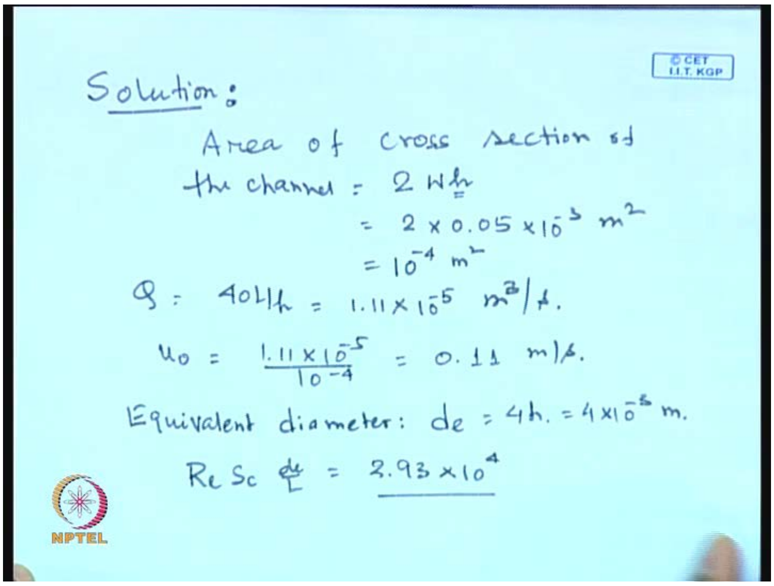

Solution:

Area of cross section of the channel $= 2wh$

$= 2 \times 0.05 \times 10^{-3}\ m^2$

$= 10^{-4}\ m^2$

$Q = 40\ L/h = 1.11 \times 10^{-5}\ m^3/s.$

$u_0 = \frac{1.11 \times 10^{-5}}{10^{-4}} = 0.11\ m/s.$

Equivalent diameter: $d_e = 4h. = 4 \times 10^{-3}\ m.$

$Re\, Sc\, \frac{d_e}{L} = \underline{2.93 \times 10^4}$

So, let us look into the solution to this problem. First you calculate the since the flow rate you would like to require to find out the cross flow velocity, right in order to calculate the reynolds number and in order to calculate the mass transfer coefficient. Therefore, you have and the flow rate is given us you should know the area of the cross section of the channel, the area of cross section is 2 w h. So, this h is the half height, so 2 h is the total height, w is the width and this turns out to be 2 into 0.05 into 10 to the power minus 3 meter square these turns out to be 10 to the minus 4 meter square. The Q is the flow rate that is 40 litre per hour, if you convert the flow rate into meter cube per second, it turns out to be 1.11 into 10 to the power minus 5 meter cube per second.

So, if you divide the flow rate by the cross sectional area, you will be getting the length average area the area average the cross flow velocity. This will be 1 0.11 into 10 to the power minus 5 divided by 10 to the power minus 4, so these will be 0.0 0. 11 meter per second is the cross flow velocity. Now the Reynolds is number will be defined at the equivalent diameter; so, you should calculate the equivalent diameter, the equivalent diameter is d equivalent to 4 into half height and these turns out to be 4 into 10 to the power minus 3 meter 4 millimetre. You should calculate the Reynolds smite d e by l and it turns out to be 2.93 into 10 to the power 4. I am not doing in these details so you can calculate the just puts the numbers and get the relationship get value.

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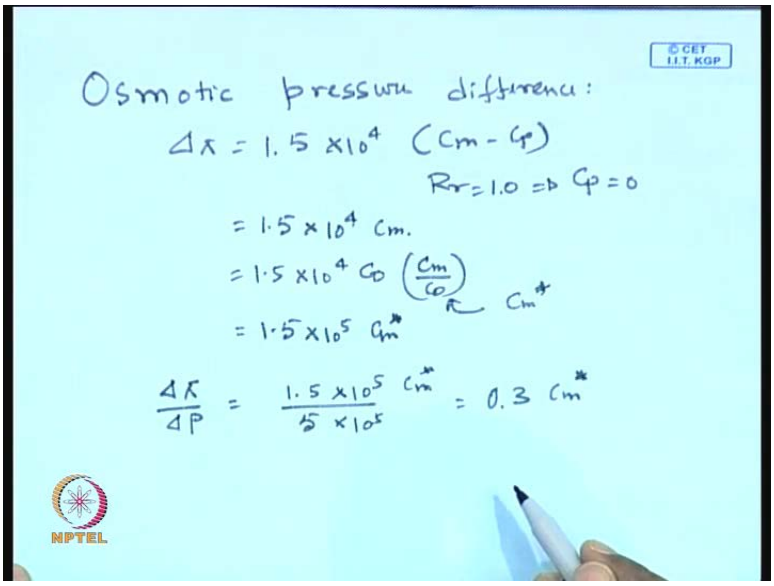

Osmotic pressure difference:

$$\Delta\pi = 1.5 \times 10^4 (C_m - C_p)$$

$R_r = 1.0 \Rightarrow C_p = 0$

$$= 1.5 \times 10^4 C_m.$$

$$= 1.5 \times 10^4 C_0 \left(\frac{C_m}{C_0}\right) \leftarrow C_m^*$$

$$= 1.5 \times 10^5 C_m^*$$

$$\frac{\Delta\pi}{\Delta P} = \frac{1.5 \times 10^5 C_m^*}{5 \times 10^5} = 0.3 C_m^*$$

So, this will be required to calculate the Sherwood number. The osmotic pressure difference delta pi is 1.5 is 10 to the power 4 c m minus c p, it is basically pi at the membrane surface minus pi at the permeate stream. So, since pi is given as a linear function of concentration so, this will be 1.5 into 10 to the power of 4 c m minus c p. Since your c p will be equal to 0 since R r is equal to1 that means, permeate concentration will be equal to 0. So, delta pi will be nothing but1.5 into 10 to the power 4 times c m right. So, you can make it non dimensional that means, you multiplied both multiply and divided by c naught. So, this will be c m by c naught, and this will be nothing but c m star. So, if you put the value of c naught I think that will be 10 to the power that will be 10 to the power k g per meter cube. So, delta pie will be 1.5 into the 10 to the power of 5 c m star at c m star is nothing but the non dimension concentration on the membrane surface.

So, let us look into the value of delta pi by delta p. The osmotic pressure difference will be 1 .5 into 10 to the power 5 so it will be c m star and what is delta p the trans membrane pressure drop that is the 5 into 10 to the power of 5. So, it will be 0.3 times c m star. So, depending on the value of c m star let say c m star will be about 5 times 4 times so, delta pi will be around 1.5 you know, if it is 5 times it will be 0.05. So, it will be 15 percent of the total Trans membrane pressure drop that will be that is constant to be more or less significant. Now will be you will be utilising the the method of shortcut method for the two dimension analysis that whatever you have discussed earlier that means, the analysis we are going to adapt in this particular problem is not the film theory, it is basically developing mass transfer boundary layer and will be utilizing the method of shortcut method that we have discussed earlier in terms of length average Sherwood number.

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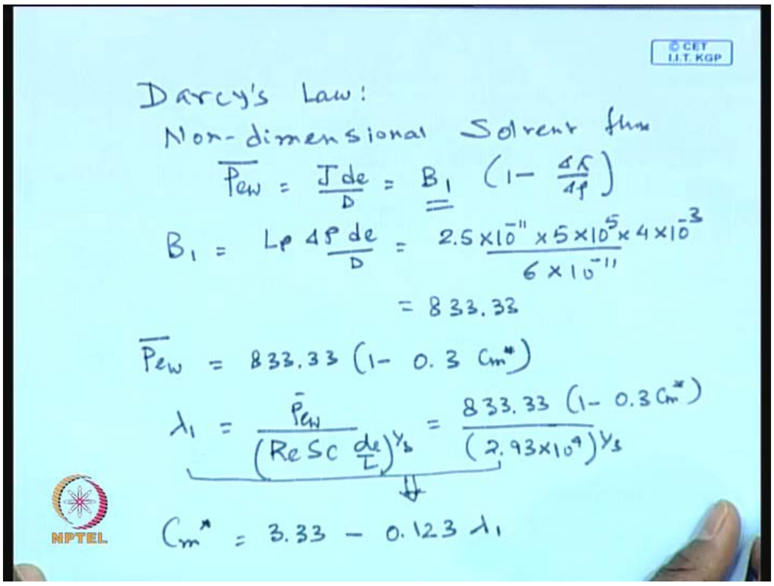

Darcy's Law:

Non-dimensional Solvent flux

$$\overline{Pe_w} = \frac{J d_e}{D} = \underline{\underline{B_1}} \left(1 - \frac{\Delta\pi}{\Delta P}\right)$$

$$B_1 = \frac{L_P \Delta P d_e}{D} = \frac{2.5\times10^{-11}\times5\times10^{5}\times4\times10^{-3}}{6\times10^{-11}}$$

$$= 833.33$$

$$\overline{Pe_w} = 833.33\,(1 - 0.3\, C_m^*)$$

$$\underbrace{\lambda_1 = \frac{\overline{Pe_w}}{\left(Re\, Sc\, \frac{d_e}{L}\right)^{1/3}} = \frac{833.33\,(1-0.3C_m^*)}{(2.93\times10^{4})^{1/3}}}_{\Downarrow}$$

$$C_m^* = 3.33 - 0.123\,\lambda_1$$

So, we can find out what is the Darcy’s law the solvent flux using Darcy’s law. The non dimensional solvent flux that is p e w is in nothing but J d e by d, this is equal to if you remember1 b1 minus into1 minus delta pi minus delta p write b1 is the parameter it is a non dimensional parameter if you remember it is L p times del p d equivalent that is divided by diffusivity if you put the values you can get an idea what is the what is this non dimensional number. The was 2.5 into 10 to the power minus 11 delta p is 5 into 10 to the power of 5 pascal, the equivalent diameter will be 4 into 10 to the power minus 3, and diffusivity will be 6 into 10 to the power minus 11 and it turns out to be in the order of 833.33. So, the the value the parameter non-dimensional parameter b1 for the case of ultra filtration will be always in the order of 1000.

So, you can have p w is equal to 833.33 into 1 minus 0.3 c m star. And if you remember if you look into the derivation of of the shortcut method earlier, we have seen that this the length average permeate flux right. If this c m star is the length average concentrate on the membrane surface. The lambda 1 we have to defined as p w divided by reynold smite d e by l rest to the power 1 upon 3 and, if you put the value expression of this p e w here in lambda 1 is turns out to be 833.33 into1 minus point 3 c m star divided by this we have already obtained 2.93 into 10 to the power 4 rest to the power 1 upon 3. Now you can rearrange this expression can and can express c m star in in in terms of lambda 1. If you really do that that means, you rearrange from these equation you can get c m star is equal to 3.33 minus 0.123 lambda 1.

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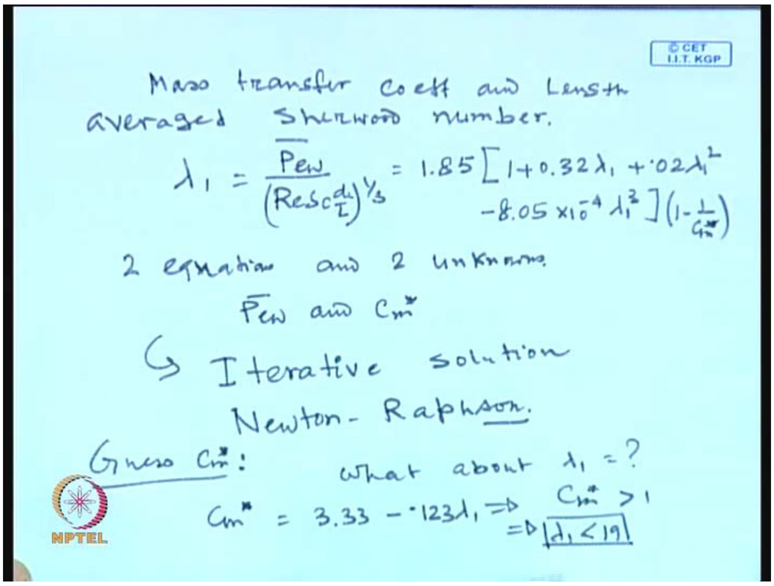

Mass transfer coeff and Length averaged Sherwood number.

$$\lambda_1 = \frac{\overline{Pe_w}}{\left(Re\,Sc\,\frac{d_e}{L}\right)^{1/3}} = 1.85\left[1+0.32\lambda_1+{\cdot}02\lambda_1^2 - 8.05\times10^{-4}\lambda_1^3\right]\left(1-\frac{1}{C_m^*}\right)$$

2 equation and 2 unknowns

$\overline{Pe_w}$ and C_m^*

↳ Iterative solution

Newton- Raphson.

Guess C_m^*: What about $\lambda_1 = ?$

$$C_m^* = 3.33 - {\cdot}123\lambda_1 \Rightarrow C_m^* > 1 \Rightarrow \boxed{\lambda_1 < 19}$$

Now, in terms of mass transfer coefficient and length averaged Sherwood number, you look into your notes and looking into the relevant expression during when we derive the relation here in this class. So, lambda 1 was p e w bar divided by Reynold smite d e by l rest to the power1 upon 3, will be is equal to 1.85 into 1 plus 0.32 lambda 1 plus 0.02 lambda 1 square minus 8.05 into 10 to the power minus 4, lambda1 cube1 minus1 over c m star. These comes p e w is equal to 1.85 Reynold smite d e by l rest to the1 upon 3 is k times1 minus1 by c m star (( )) these we have derived in the class earlier. So, these equation so will be will be getting 2 equation, those 2 equation up to the solved with with two unknowns. The unknowns are so the two equations and two unknowns that will be p w bar and c m star.

So, I am going to put p w bar, so you are you had one equation that relates p w bar and c m star that is this equation. The another equation is p w bar and lambda 1 we have already found out lambda one can be expressed in terms of c m star earlier, that we are going to substitute here that will constituted new equation. So, will be getting two equations two unknown system and these are algebraic equation. So, we are going to do that so you have to solve it by iterative technique solution may be Newton Raphson. Now we are c m star the so with the with the Newton Raphson or any iterative solution you should guess the first guess the value of c m star. So, guess c m star and what is the you know, traditional case of c m star, the minimum value of c m star is 1. So, the c m star has to be greater than 1 but you should know that, but what what about lambda what about lambda 1.

Now we can get in upper limit of lambda 1, because c m star is equal to 3.33 minus 0.12 3 lambda 1, and the condition c m star is always greater than1 that will give you a lambda1 which is always less than 19 18.94 right. So, this will give a bound on lambda1 what are the different values of lambda 1, that you are going to select. Now if you combined, so we we identify two equation two unknown.

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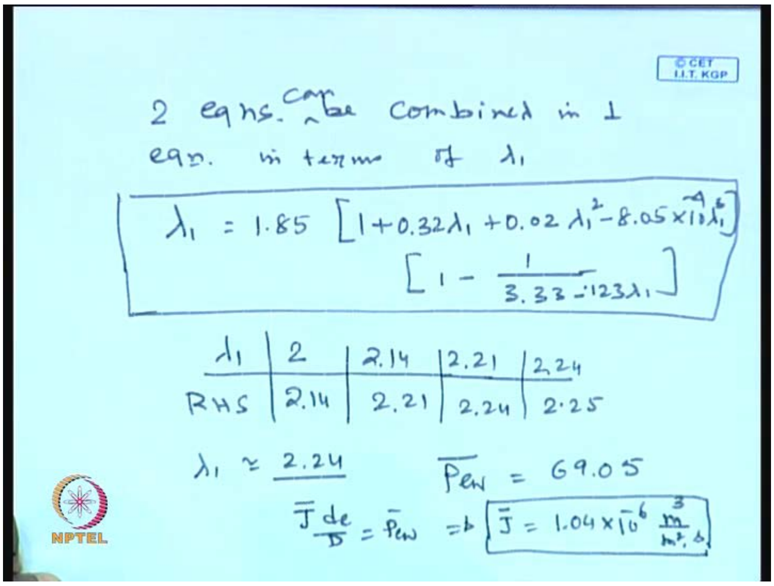

2 eqns. can be combined in 1 eqn. in terms of λ_1

$$\boxed{\lambda_1 = 1.85\left[1+0.32\lambda_1+0.02\lambda_1^2-8.05\times10^{-4}\lambda_1^3\right]\left[1-\frac{1}{3.33-.123\lambda_1}\right]}$$

λ_1	2	2.14	2.21	2.24
RHS	2.14	2.21	2.24	2.25

$\lambda_1 \simeq \underline{2.24}$ $\qquad \overline{Pe_w} = 69.05$

$$\frac{\bar{J}\,d_e}{D} = \overline{Pe_w} \Rightarrow \boxed{\bar{J} = 1.04\times10^{-6}\ \frac{m^3}{m^2 \cdot s}}$$

If you combine the two equations two equations can be combined in 1 equation, in terms of lambda 1. What is that equation lambda 1 is equal to 1.85 into1 plus 0.32, lambda 1 plus 0.02 lambda 1 square minus 8.05 in to 10 to the power minus 4, lambda 1 cube into1 minus 1 divided by 3.33 minus 1.123 lambda 1, it is 1 minus 1 by c m star. So, the c m star will be can be expressed in terms of lambda 1 that is 1 minus 1 by c m star, so c m star. So, the these equation can be expressed lambda 1 so, these equation has to be solve in order to get the value of lambda 1 once you get the value of lambda1 you can get the value of c m star once you get the value of c m star you can get the value of permeate flux.

So, this equation can be solved again using iterative technique either either you you cost the combined these 2 equation in terms of c m star or cost in these 2 equation in terms of lambda1; so in this particular case, have cost in terms of lambda 1. Now for different values of lambda1 so you selected value of lambda 1 evaluate the right hand side, and check whether the right hand side, come is coming close to the assume value one there is an idea. So, the lambda1 and right hand side of the equation if you select two these comes out to be 2.14 if you select 2.14, it is 2.21 this is if it is 2.21, it is 2.24 if you select 2. 24 then it will be 2.25. So, this is this is called a successive substitution technique. We guess a value of lambda1 and just put it there, and converged value will be the next case and see. So, the converged value will be around 2.24. So, if you select that then you can calculate, what is the length average permeate flux transfer to be 69.05 and if you put J bar d e by d is equal to equal to p e w bar, the length average permeate flux becomes 1 .04 into 10 to the power of minus 6 meter cube per meter square per second.

That is the value of length average permeate flux it turns out to be 69.5 and if you put J J bar d by d is equal to p w bar the length average permeate flux becomes 1.04 into 10 to the power minus 6 meter cube per meter square second that is length average permeate flux. So, you can calculate, what is the permeate flux, what is the membrane surface concentration?

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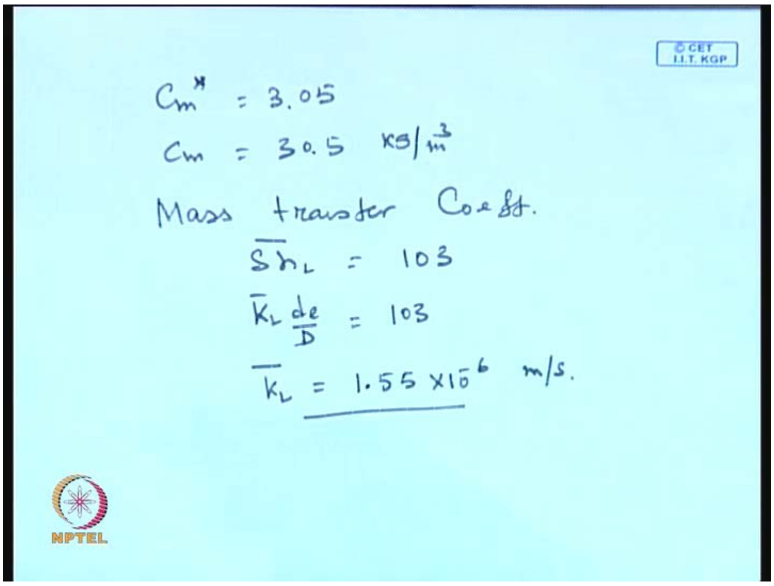

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$C_m^* = 3.05$

$C_m = 30.5 \text{ kg/m}^3$

Mass transfer Coeff.

$\overline{Sh}_L = 103$

$\overline{k}_L \frac{d_e}{D} = 103$

$\overline{k}_L = 1.55 \times 10^{-6}$ m/s.

So, membrane surface concentration turns out to be 3 .05 c m star. So, c m is 30.5 k g per meter cube, they have seen the for a for this particular problem the membrane surface concentration average length average membrane surface concentration is 3 times the feed concentration. And the mass transfer coefficient you can calculate, that is Sherwood number length average it turns out to be 103 for this particular problem, and I just put the definition of the mass transfer sherwood number in terms of mass transfer coefficient and diffusivity equivalent diameter etcetera, the average length average mass transfer coefficient turns out to be 1.55 into the 10 to the power minus 6 meter per second. So, these this demo this problem gives the demonstration, how we are going to use a more accurate method that we have discussed earlier that is a two-dimensional approach in terms of length average sherwood number and length average permeate flux and all the length average permeate membrane surface concentration all this all this quantity of the length average.

So, in this process also although we have use two-dimensional model two dimension model we are not solving any differential equations of any ordinary equation partial differential equations, avoid equation will be basically boiling down in the solution of two simultaneous algebraic equation, exactly like the film theory equation for the any formula formulation is change. Now, you can use you can use this data and use the film theory model, and get the prediction and see what is the value of permeate flux you are going to get, and it is a expected using film theory you are going to get a permeate flux which will be less than the flux that we have calculated by this method it is going to under predict. That I am I am not sure, it is it just it has an assignment and do it by yourself.

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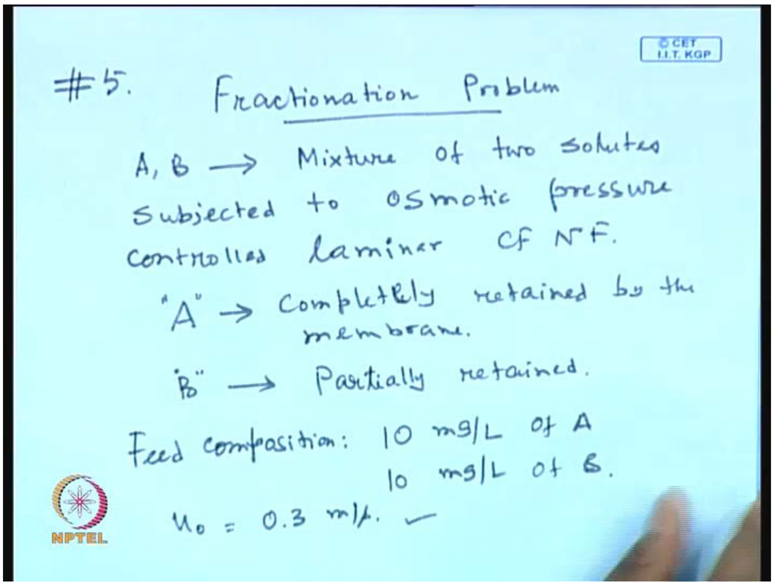

#5. Fractionation Problem

A, B → Mixture of two solutes subjected to osmotic pressure controlled laminar CF NF.

"A" → Completely retained by the membrane.

"B" → Partially retained.

Feed composition: 10 mg/L of A
10 mg/L of B.

$u_0 = 0.3$ m/s. ✓

The next problem that we are going to solve, is is a problem of a mixture of 2 solutes. Is basically a fractionation problem, till now we have we are talked about the separation and concentration problem. Now we are talking about a fractionation problem, initial of 2 solutes A and B their subjected to osmotic pressure controlled laminar cross flow nano filtration, Subjected to osmotic pressure of controlled laminar cross flow nano filtration. Solute A is completely retained and solute B is partially retained by the membrane. A completely retained by the membrane and the whole system is a steady state and B is partially retained that means, real retention for the component A return will be is equal 1, real retention of component for the component B will be something, which is not equal to1.

So, it is given the so it is it is the feed mixture, so let us look into the feed composition into 10 ppm of A and B each. So, 10 milligram per litter of a and 10 ppm means, milligram per litter 10 milligram per litter of B that is the feed composition. The average cross flow velocity in the channel, it is given that u naught is 0.3 meter per second, if the flow rate is given and the channel dimension is given is also find it out. Flow rate divided by the cross section area normal to the direction of the flow. So, in this particular problem it is given as 0.3 meter per second

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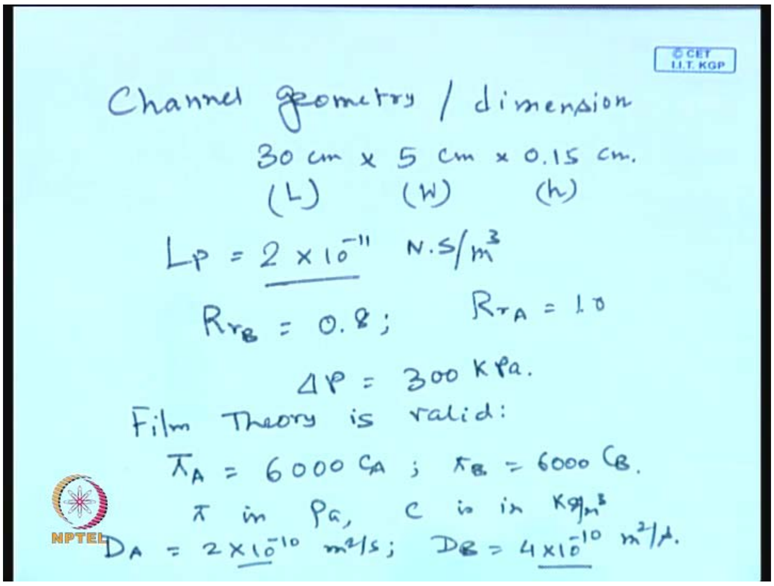

Channel geometry / dimension

30 cm × 5 cm × 0.15 cm.
(L) (W) (h)

$L_p = \underline{2 \times 10^{-11}}$ N.S/m^3

$R_{r_B} = 0.8$; $R_{r_A} = 1.0$

$\Delta P = 300$ KPa.

Film Theory is valid:

$\pi_A = 6000\, C_A$; $\pi_B = 6000\, C_B$.

π in Pa, c is in Kg/m^3

$D_A = 2\times\underline{10^{-10}}$ m^2/s; $D_B = 4\underline{\times10^{-10}}$ m^2/s.

So, you can calculate and find out what is the flow rate from here. The channel geometric is given channel geometry or dimension, the dimension is 30 centimetre 5 centimetre cross 0.15 centimetre, this is the length 0.3 metre, this is the width 0.05 meter and this is the height 0.15 meter. The membrane permeability is given that is 2 into 10 to the power minus 11 newton second per meter cube, which is the difficult order of magnitude any nano filtration membrane. The real retention of membrane for solute b is given as 0.8 and that for solute a is equal to1, there is completely retain by the membrane delta p is given as 300 kilo pascal that is the operating trans membrane pressure drop use film theory. Film theory is valid all though we have seen several shortcoming of film theory during film theory during or calculation, because it is easier to demonstrate how to calculate the problem.

And will just now see what examples, which use the 2 dimensional models in the shortcut method. Osmotic pressure of the solution of the of the for the complaint a of the solution is varying as 6000 times C A, and pi of B is and 6000 c b, where pi in pascal both pi in pascal and c is in k g per meter cube. That means, milligram per litre has to be convert into k g per meter cube, diffusivity of the 2 solutes are given diffusivity of A is given as 2 into 10 to the power minus 10 meter square per second, diffusivity of B is twice of that 4 into 10 to the power of minus 10 meter square per second, solution viscosity and density of taken to be same that of water. So, if you look in to the the diffusivity of these solutes, the diffusivity of the solutes are in the order of 10 to the power minus 10 meter square per second. Diffusivity protein in the order of 10 to the minus 11 meter square per second, and the solve will be having the maximum diffusivity in the order of 10 to the power minus 9 metre square per second the sodium chloride will be having diffusivity of 1.5 into 10 to the power of minus 10 meter square per second.

Therefore, we are talking about some solute which will be having diffusivity in the order of 10 to the power minus 10 meter square per second that means, this solute typically dies, those will be having molecular weight in the range of 400, 500, 600, 700 in that range and nano filtration is typically is used to separate this type of solutes.

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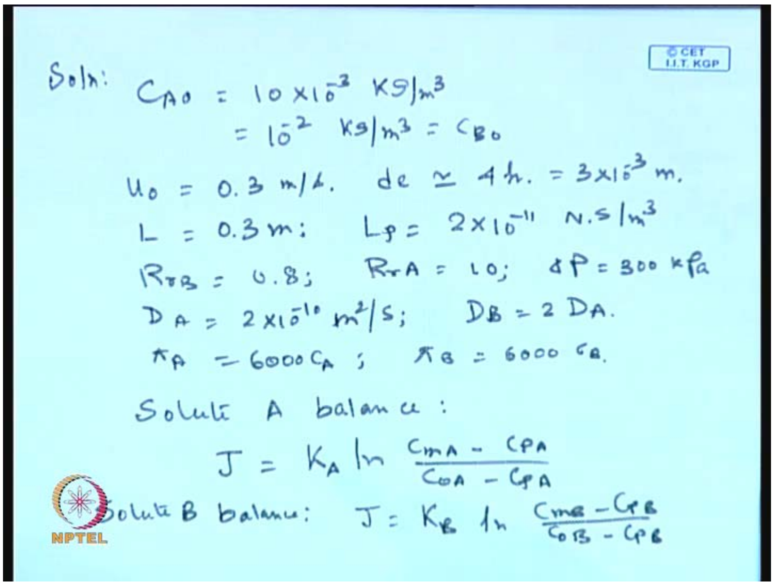

Soln:

$C_{A0} = 10 \times 10^{-3}$ Kg/m^3

$= 10^{-2}$ Kg/m^3 $= C_{B0}$

$u_0 = 0.3$ m/s. $d_e \simeq 4h. = 3 \times 10^{-3}$ m.

$L = 0.3$ m: $L_p = 2 \times 10^{-11}$ N.s/m^3

$R_{rB} = 0.8;$ $R_{rA} = 1.0;$ $\Delta P = 300$ kPa

$D_A = 2 \times 10^{-10}$ m^2/s; $D_B = 2 D_A.$

$\pi_A = 6000 C_A;$ $\pi_B = 6000 C_B.$

Solute A balance:

$$J = K_A \ln \frac{C_{mA} - C_{PA}}{C_{0A} - C_{PA}}$$

Solute B balance: $J = K_B \ln \frac{C_{mB} - C_{PB}}{C_{0B} - C_{PB}}$

So, solution let us look into C A naught is equal to 10 to the power minus 3 k g per meter cube. So, it will be 10 to the power if minus 2 k g per meter cube, and that is same as C B naught right get A both the 10 ppm each. So, so u naught given as 0.3 meter per second d equivalent diameter is 4 of 4 into half height; so the dimension that was given in the channel, dimension that was half height; so this 4 into half height it was 4 into half height. So, these turns out to be 3 into 10 to the power minus 3 meter. Length as given as thirty centimetre so 0.3 meter, l p was given as 2 into 10 to the power minus 11 newton second per meter cube, R r b 0.8 R r a1 delta p 300 kilo pascal diffusivity of a is given as

2 into 10 to the power minus 10 meter square per second, diffusivity of b is 2 times that of diffusivity of a and pi A is 6000 C A and pi B 6000 C B.

Now what we can do, we can do a solute mass balance over the membrane surface and exactly whatever the same way we have done the we have derive film theory equation. If you remember how you did the film theory equation, we derive the film theory equation once you will did the solute mass balance over differential element towards the membrane surface, and integrate across the boundary layer thickness, because the boundary thickness will be constant for a stagnant boundary layer film. Now, in this case will do the solute, balance solute a balance over the same element and solute b balance over the same element; so if you do that what will be getting we will be getting two different solution two different equation corresponding to solute number A and another1 cross flow to solute number. But the permeate flux will be remaining same that there that is independent solvent flux will be independent of the solutes. So, if you do a separate solute balance solute a balance over the mass transfer near the membrane surface, you will be getting J is equal to k time k a, L n C M A minus C p A divided by C naught a minus C p A.

This mass transfer coefficient will be now correspond will be having the property of solute a, because is the solute balance d a by delta and all this concentration will be corresponding to solute a. Solute b balance will ready j is equal to k B L n C M B minus A p B divided by C naught B minus C p b at the steady state. And1 our our thing let us given that C p A is equal to 0 because, real retention is equal to1. So, in the permeate there is no concentration of the solute A. If it is instead of real rentention1 if it is would have been something else that would have been put it here. So, for the safe of simplicity will assume that real retention of first component A is equal to so, the permeate will be basically you will be having an only one component that will be component B.

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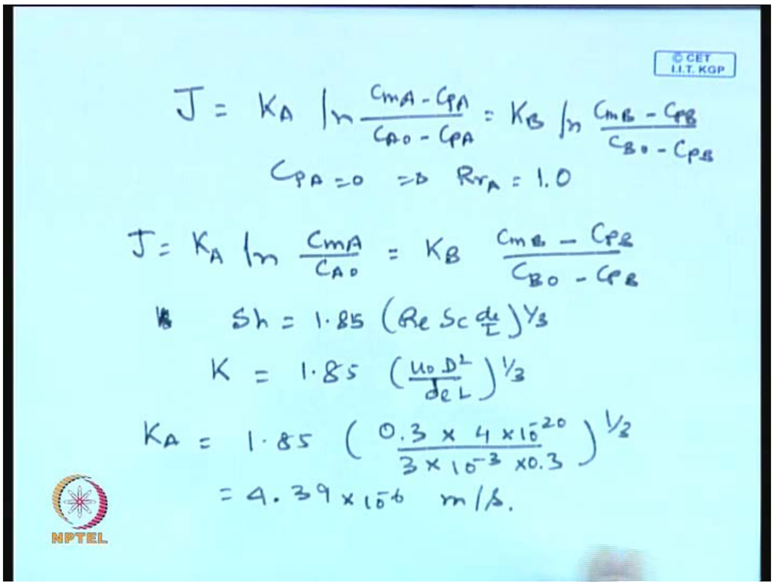

$$J = K_A \ln \frac{C_{mA} - C_{pA}}{C_{A0} - C_{pA}} = K_B \ln \frac{C_{mB} - C_{pB}}{C_{B0} - C_{pB}}$$

$$C_{pA} = 0 \Rightarrow Rr_A = 1.0$$

$$J = K_A \ln \frac{C_{mA}}{C_{A0}} = K_B \frac{C_{mB} - C_{pB}}{C_{B0} - C_{pB}}$$

$$Sh = 1.85 \left(Re\, Sc \frac{d_e}{L}\right)^{1/3}$$

$$K = 1.85 \left(\frac{u_0 D^2}{d_e L}\right)^{1/3}$$

$$K_A = 1.85 \left(\frac{0.3 \times 4 \times 10^{-20}}{3 \times 10^{-3} \times 0.3}\right)^{1/2}$$

$$= 4.39 \times 10^{-6} \ m/s.$$

So, you write1 equation J is equal to K A l n C M A minus C p A C A naught minus C p A is equal to K times B l n C M B minus C p B divided by C B naught minus C p B and we have C p A is equal to 0, because real retention of a is equal to1. So, you will be having J K A l n C M A divided by C A naught is equal to K B C M B minus C p B C B naught minus C p B and K will be laminar of flow the sherwood number will be 1.85 Reynolds smite d e by l rest to the power 1 upon 3 using that equation, and for K A and K B will just put the values of diffusivity or solute property corresponding to A or corresponding to B. Therefore, from this K becomes1.85 u 0 d square divided by d equivalent time’s length rest to the power of 1 upon 3. So, K A if you put the value of value of diffusivity here for the solute a will be getting the mass transfer coefficient corresponding to a, if you put the value of diffusivity of the solute b will be getting the mass transfer coefficient corresponding to solute b.

So, K A will be 1.85 0.3 into 4 into 10 to the power of minus 10 20, divided by 3 into 10 to the power of minus 3 into 0.3 rest to the power1 upon 3, and these turns out to be 4.39 into 10 to the power minus 6 meter per second. Similarly, we put the value of diffusivity corresponding to the solute B there and we can able to calculate the mass transfer coefficient of the solute B.

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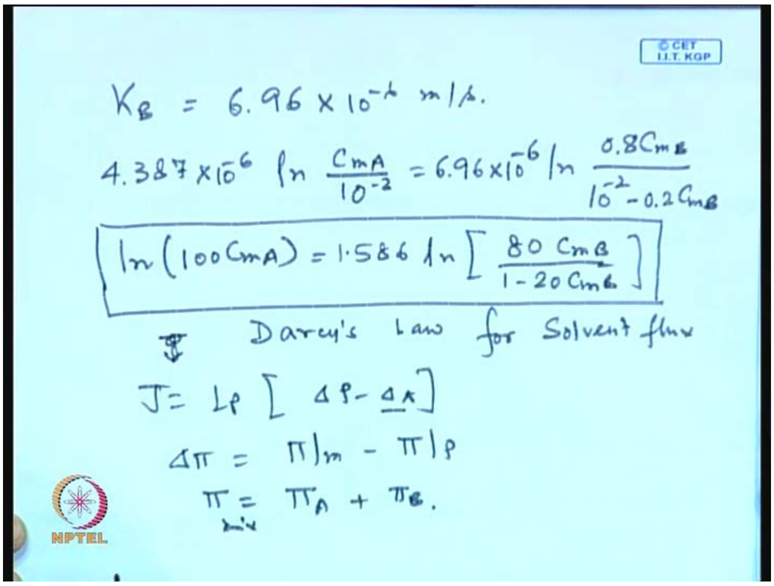

$$K_B = 6.96 \times 10^{-6} \ m/s.$$

$$4.387 \times 10^{-6} \ln \frac{C_{mA}}{10^{-2}} = 6.96 \times 10^{-6} \ln \frac{0.8 C_{mB}}{10^{-2} - 0.2 C_{mB}}$$

$$\boxed{\ln (100 C_{mA}) = 1.586 \ln \left[\frac{80 C_{mB}}{1 - 20 C_{mB}} \right]}$$

$\Downarrow$ Darcy's Law for Solvent flux

$$J = L_p [\Delta P - \underline{\Delta \pi}]$$

$$\Delta \pi = \pi|_m - \pi|_p$$

$$\pi_{mix} = \pi_A + \pi_B.$$

So, just put in the value of there and it turns out to be K B becomes 6.96 into 10 to the power minus 6 meter per second. Now we put this values in the in the equation so, you are you are getting an equation. So, just equate this equation this equal to this. So, if you do that what you will be getting is 4.387 into 10 to the power minus 6, l n of c m a divided by 10 to the power minus 2 to that is the concentration of a in the feed equal to 6 .9 6 into 10 to the power minus 6, log of 0.8 c m b you are you are replacing in terms of real are10tion c p in terms of real re10tion of B and C M of b 10 to the power of minus 2 minus 0.2 times c m b. So, in in short, these will a equation something this log 1000 times c m a is equal to 1.586, log of 80 C M B divided by 1 minus 20 same.

So, you have 3 equations basically 3 unknowns here, C M A and C M B and J their 3 unknowns in this particular problem. So, 1 equation is that the second equation is will be getting is there is the Darcy's law for the solvent flux, Osmotic pressure model what is that J is equal to l p times delta p minus delta pi. Now this is the mixture mixture of the dilute solution so delta pi should be delta pi should be pi at the membrane surface osmotic pressure, on the membrane surface minus osmotic pressure on the permeate stream, but this osmotic pressure will be the osmotic pressure of the mixture. And since it is a dilute solution dilute solution in the order of 20 milligram per litter p p m level. So, osmotic pressure of the mixture can be considered as a simple super position of the osmotic pressure of individual component pi of a plus pi of b.

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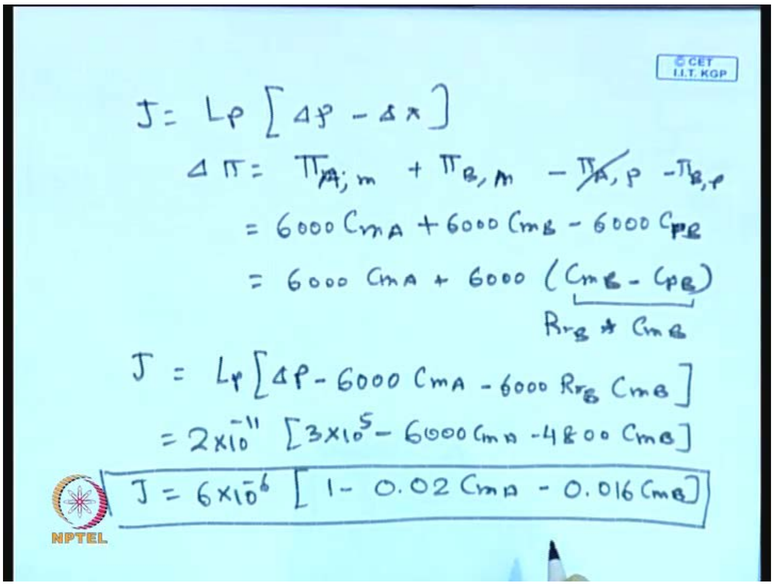

$$J = L_p [\Delta P - \Delta \pi]$$

$$\Delta \pi = \pi_{A,m} + \pi_{B,m} - \cancel{\pi_{A,p}} - \pi_{B,p}$$

$$= 6000 C_{mA} + 6000 C_{mB} - 6000 C_{pB}$$

$$= 6000 C_{mA} + 6000 \underbrace{(C_{mB} - C_{pB})}_{R_{rB} * C_{mB}}$$

$$J = L_p [\Delta P - 6000 C_{mA} - 6000 R_{rB} C_{mB}]$$

$$= 2 \times 10^{-11} [3 \times 10^5 - 6000 C_{mA} - 4800 C_{mB}]$$

$$\boxed{J = 6 \times 10^{-6} [1 - 0.02 C_{mA} - 0.016 C_{mB}]}$$

If you really do that then what will be getting is, J is equal to L p times delta p minus delta pi, where delta pi will be pi of a on the membrane surface plus pi of b on the membrane surface, minus pi of a and the permeate stream, minus pi of b on a permeate stream. Now this will be a 6000 times C m A this will be 6000 into C m B C A p C A p will be equal to 0. So, that will not be there is becoming 6000 C M C e p B this will be 0 because, it is real retention 1 basically is equal to1. So, these become 6000 C M A plus 6000 times c m b minus minus C p B write And this will be nothing but real retention of B multiplied by the C m B. Now it is point is multiplied by C M B so, you will be getting J is equal to l p times del p minus 6000 C M A minus 6000 R r B C m B. So, let us put the values 2 into 10 to the power minus 11, 3 into 10 to the power 5 minus 6000 C M A minus, 4800 C m B just take 3 into 10 to the power 5 outside. So, this is becomes 6 into 10 to the power minus 61 minus 0.02 C m A minus 0.01 6 C m B. so this the second equations.

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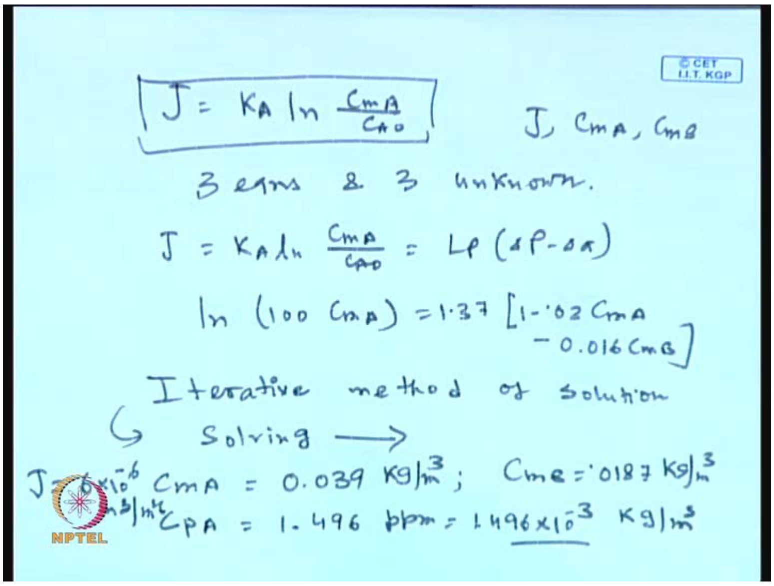

$$\boxed{J = K_A \ln \frac{C_{mA}}{C_{A0}}}$$

J, C_{mA}, C_{mB}

3 eqns & 3 unknown.

$$J = K_A \ln \frac{C_{mA}}{C_{A0}} = L_p(\Delta P - \Delta \pi)$$

$$\ln(100\, C_{mA}) = 1.37\,[1 - .02\, C_{mA} - 0.016\, C_{mB}]$$

Iterative method of solution

$\hookrightarrow$ Solving $\longrightarrow$

$J = 6 \times 10^{-6}$ $m^3/m^2 s$

$C_{mA} = 0.039\ kg/m^3$; $C_{mB} = .0187\ kg/m^3$

$C_{pA} = 1.496$ ppm $= \underline{1.496 \times 10^{-3}}\ kg/m^3$

And the third equation will be getting J is equal to K A l n C m A divided by C A naught. So, this is the third equation. So, 3 unknowns J C m A C m B and will be getting 3 equation so 3 equation 3 unknown system. Therefore, it is a completely definable for the defined problem and 1 can get the solution. So, you just use this relationship so there are several 1 can do. So, you can use J is equal to K A l n C m A times C A naught to l p into del p minus delta pi, if you do that will be getting this expression l n 100 times C m A is equal to 1.37 into 1 minus 0.02 c m a minus 0.016 c m b. These gives the relationship between the C m A and C m B, and know you can you can so basically there are 3 equations and 3 unknown and one can do an iteration iterative method, because they are non-linear of solution 3 algebraic equation non-linear and three unknowns.

So, this can be solved and upon solution solved in a particular way you can solved in another so there are several ways right so, the 3 equation 3 unknown that that can be solved and c m a turns out to be 0.039 k g per meter cube; and this 39 ppm right c m a will be 0.0187 18 0.7 ppm k g per meter cube. If we multiplied k g per meter cube by thousand it will be getting milligram per p p m. So, in permeate C p A turns out to be 1.96 p p m or you know1 .4 9 6 into the 10 to the power minus 3 k g per meter cube. And the value of permeate flux, J turns out to be J transfer to be around 6 into 10 to the power minus 6 meter cube per meter square second. So, this a typical problems we have solved were with there are 2 components are present, if this is the case of fractionation previously we have solved in the case of separation and concentration and another case we are used, we have solve for case of concentration in case of batch ultra filtration mode. Now this is the particular problem that we have solve for the case of fractionation and 1 can really get the system performance once by looking in to the solution.

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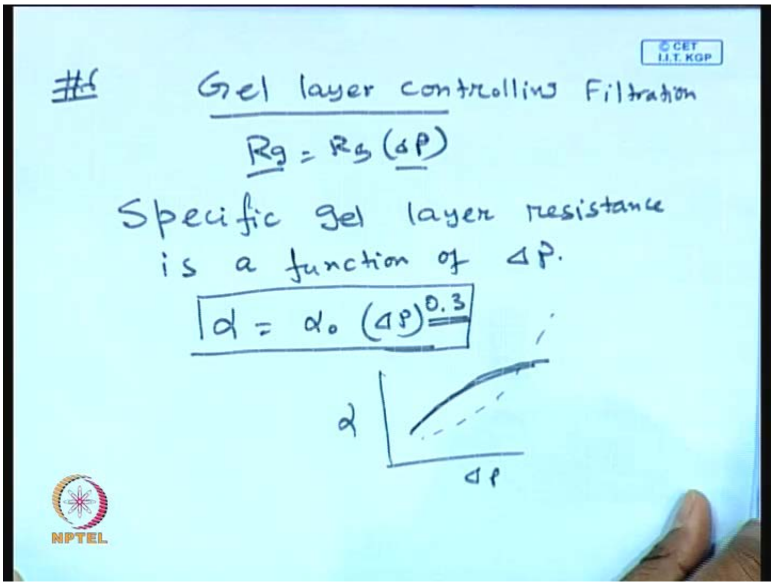

#6 Gel layer controlling Filtration

$R_g = R_g(\Delta P)$

Specific gel layer resistance is a function of ΔP.

$$\boxed{\alpha = \alpha_0 (\Delta P)^{0.3}}$$

Now next problem that we have going in to solve, is again a gel layer controlling filtration, but in this case the the typical what is the difference between these problem and previous problem is that, in this case the gel layer is a function of delta p, gel layer thickness or gel layer resistance is a function of delta p. What what does mean that mean, if you increased the offer Trans membrane pressure drop the gel becomes compact. So, if the gel layer resistance is independent of delta p that means, is incompressible incompressible the gel layer is in compressible, If it is a slow function of concentrate delta p becomes compressible. So, you will be getting a slide modification of variation of the Trans membrane of the permeate flux with operating pressure. So, 1 can find what is the an optimum operating condition or optimum trans membrane pressure drop 1 has to be applied in case the gel controlling filtration.

So, in this case we we assume that specific gel layer resistance is a function of delta p. R g is if you remember R g is nothing but alpha into 1 minus excellence g rho g times l alpha is the specific gel layer thickness gel layer resistance the specific gel layer resistance is a function of trans membrane pressure drop and that functional form is given us alpha is equal to alpha naught delta p rest to the power 0.3. How will determine these relationship, if you remember when we discussed about the gel controlling filtration in a in a on stead batch cell, you have to plot1 by1 by J square versus time something like that, right then they will a straight line from the slope of the straight line will be put calculate the specific gel layer resistance. If you conduct the experiment with 3 different of 4 different pressure will be getting the corresponding gel layer specific gel layer resistance corresponding to 4 delta p or 4 pressures.

Then if you plot alpha versus delta p, that will be typically these will be typically result a curve something like this, with a diminution slope diminution slope means, your exponent should be less than 1. If the exponent is greater than1 then the nature of the curve would have been something like this, but difficulty will be getting these that mean, at higher pressure, what what this this curves signifier and higher pressure it is independent of delta p that means if the gel becomes totally in compressible so at lower pressure becomes compressible. So, that is how you will be getting this relationship or establishing this relationship by conducting batch experiment under various operating pressure drop.

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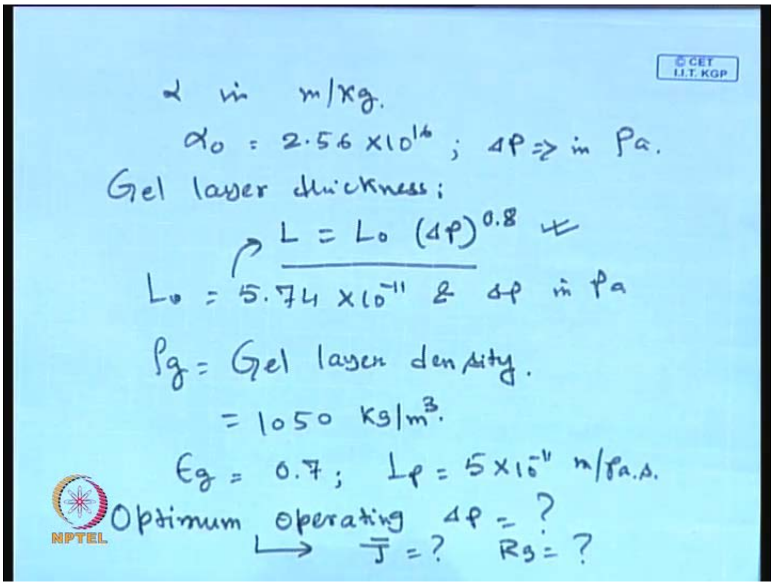

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α in m/kg.

$\alpha_0 = 2.56 \times 10^{16}$; $\Delta P \Rightarrow$ in Pa.

Gel layer thickness;

$\underline{L = L_0 (\Delta P)^{0.8}}$ ✓

$L_0 = 5.74 \times 10^{-11}$ & ΔP in Pa

ρ_g = Gel layer density.

= 1050 kg/m^3.

$\epsilon_g = 0.7$; $L_P = 5 \times 10^{-11}$ m/Pa.s.

Optimum operating ΔP = ?

$\longrightarrow$ $\bar{J}$ = ? R_g = ?

Now here alpha is in meter per k g that is the unit alpha alpha naught is given as 2.56 into 10 to the power 16 appropriate unit, and delta p will be in pascal. And gel layer thickness is founded to be varying as function of different function of delta p at it is it is establishes l equal to l naught delta p in to 0.8. How to obtain this you can obtain these thing by macroscopic microscopy observation that means, after the experiment dry out the solution whatever is depositing over the membrane surface, put under a microscope and measure the thickness then do a calculation and check how much volume of the previously, because it was in the solution it was in the in the gel state. So, 1 can back calculate really find out what is the actual length thickness of the gel layer under the solution under the weight condition.

Suppose you conduct the experiment different delta p and found out the relationship between l and delta p is something like this, while l0 is given as 5.74 into 10 to the power minus 11 and delta p again in pascal in this correlation. The gel density gel layer density is given as 1050 k g per meter cube, and gel layer porosity is given as 0.7, and membrane permeability is given as 5 into 10 to the power minus 11 meter per pascal second. Now what is avoiding in the particular problem we have to find out the limiting operation pressure optimum operating pressure. Operating pressure delta p is how much and what are the value corresponding value of the permeate flux at this at this delta p, what is the corresponding value of the permeate flux and gel layer resistance. So, basically now what we have to do.

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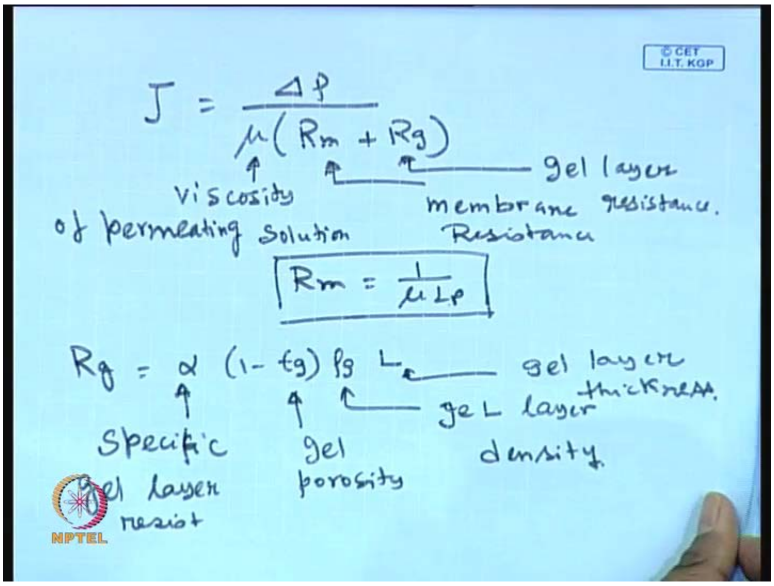

$$J = \frac{\Delta p}{\mu(R_m + R_g)}$$

$$\boxed{R_m = \frac{1}{\mu L_p}}$$

$$R_g = \alpha\,(1-\epsilon_g)\,\rho_g\,L$$

We have to write down the phenomenological equation. What is the phenomenological equation that is J was driving force divided by the resistance? So, delta p divided by mu times R M plus R g. This is the driving force for delta p mu is the viscosity of the permeating solution; this is the membrane resistance, this gel layer resistance. And membrane resistance know it is 1 over mu times L p, permeability multiplied by the viscosity of the permeating solution inverse of that will give you the resistance of the hydraulic resistance of the membrane. Now, what is R g? If you remember the definition of the R g alpha into1 minus excellence g rho g times L, alpha is the specific gel layer resistance excellence g is gel density gel porosity, this is gel layer density L is the gel layer thickness. Now you put different values of expression of alpha has the function of delta P and L as the function delta p in this equation.

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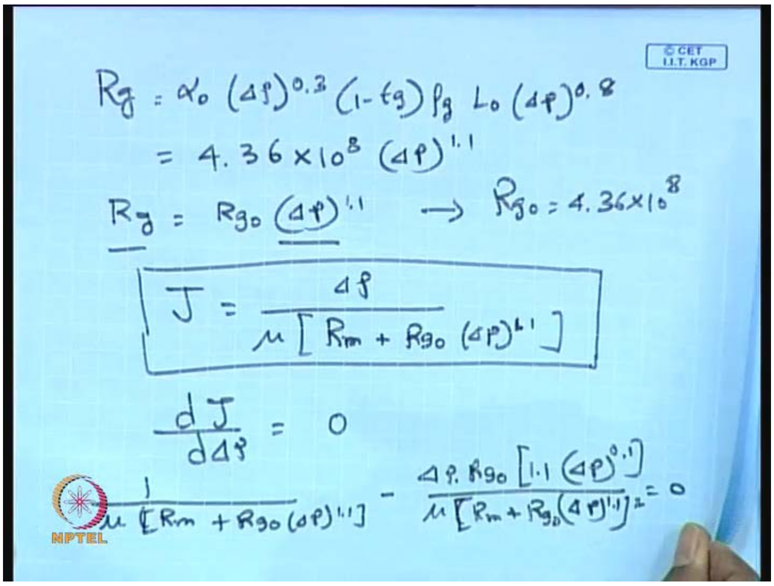

$$R_g = \alpha_0 (\Delta P)^{0.3} (1-\epsilon_g) \rho_g L_0 (\Delta P)^{0.8}$$

$$= 4.36 \times 10^8 (\Delta P)^{1.1}$$

$$\underline{R_g} = R_{g0} \underline{(\Delta P)}^{1.1} \longrightarrow R_{g0} = 4.36 \times 10^8$$

$$\boxed{J = \frac{\Delta P}{\mu [R_m + R_{g0} (\Delta P)^{1.1}]}}$$

$$\frac{dJ}{d\Delta P} = 0$$

$$\frac{1}{\mu [R_m + R_{g0} (\Delta P)^{1.1}]} - \frac{\Delta P . R_{g0} [1.1 (\Delta P)^{0.1}]}{\mu [R_m + R_{g0} (\Delta P)^{1.1}]^2} = 0$$

So, what will be getting is that R g is equal to alpha naught, delta p rest to the power point 3 in to1 minus excellent g rho g times l naught delta p rest to the power 0.8. And just put different values whatever different values are given excreta this turns out to be 4 .3 6 into 10 to the power 8 times delta p rest to the power 1.1. Put all the values together and therefore, 1 can get in the form R g is equal to R g naught into delta p rest to the power 1.1 where R g naught is nothing but 4.36 into 10 to the power 8. Now you put this expression of R g which is the function of nothing but delta p in the governing equation of permeates flux, which is delta p is divided by mu, R m plus R g naught delta p rest to the power 1.1. So, in this in this expression the permeate flux is a sole function of delta p all the other parameter known and we are going to find out what is the optimum delta p are the limiting delta p.

So, I take the differential equation with respect to delta p and put into equal to 0. So, d J d delta p if you do that what will be getting is you put d J evaluate d J d delta p and put into the equal to 0. And these turns out to be 1 over mu R m plus R g naught delta p rest to the power of 1.1 minus delta p R g naught1.1 delta p rest to the power 0.1 divided by mu R m plus R g naught delta p rest to the power1 point1 square of that will be equal to 0. So, just take the derivative of the with respect to delta p and put into the equal to 0 you will be getting some expression.

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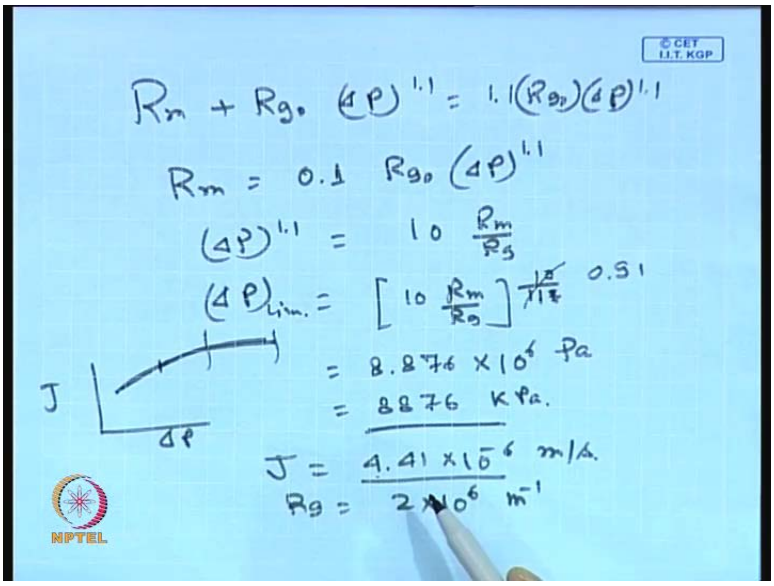

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$$R_m + R_{g_0}(\Delta P)^{1.1} = 1.1(R_{g_0})(\Delta P)^{1.1}$$

$$R_m = 0.1\, R_{g_0}(\Delta P)^{1.1}$$

$$(\Delta P)^{1.1} = 10\frac{R_m}{R_g}$$

$$(\Delta P)_{\lim.} = \left[10\frac{R_m}{R_g}\right]^{\cancel{\frac{1}{1.1}}\frac{10}{11}} \quad 0.91$$

$$= 8.876\times10^6 \text{ Pa}$$

$$= 8876 \text{ kPa.}$$

$$J = 4.41\times10^{-6} \text{ m/s.}$$

$$R_g = 2\times10^6 \text{ m}^{-1}$$

And that expression will be R m after simplification plus R g naught delta p rest to the power 1.1 will be equal to 1.1 R g naught delta p rest to the power of 1.1; and it turns out to be R m is equal to 0.1 R g naught delta p rest to the power 1.1 and delta p rest to the power 1.1, becomes 10 times R m by R g. And delta p should be 10 R m by R g rest to the power 1 by 1.1. So, this is becomes 10 by 11 evaluate the it terms turns out to be 0. 91 these these exponent turns out to be 0.91 and delta p will be around 8.876 into 10 to the power 6 pascal, and in kilo pascal it turns out to be 8876 kilo pascal.

So, it is it is extremely high that means, if you plot J versus delta p if you really plot J versus delta p the plot will go something like this, and the pressure independent join where the gel layer almost incompressible that will occurred at a very high delta p. That will be that will be around divided by 101 this turns out to be around an 88 atmosphere. So, about a about a pressure of about 88 atmosphere, these particular gel layer which will you function this given function of delta p gel layer thickness and the gel layer resistance, that will be incompressible will be independent of delta p, when you are operating pressure will be extremely high.

At that limiting pressure, this is the limiting pressure and at that limiting pressure permeate if you put the value of you know this pressure the permeate flux turns out to be 4.41 into 10 to the power minus 6 meter per second. And r g turns out to be 2 into 10 to the power 16 in meter inverse. So, naturally one can 1 can ask why I should do this exercise, because at that high delta p at that the high delta p, which will be which is almost you know difficult to realize in a in a experimental condition, if you like to have a and operating pressure are 90 atmosphere, that will be extremely difficult know there is there is every possibility having an accident. So, why should I do this calculation do this calculation, because the permeate flux beyond this particular expresser the permeate flux becomes constant. So, you plot j verses delta p we conduct the experimental lower pressure and say that and this this value becomes 4, this is actually this plot becomes very white function of delta p.

So, if will say that the flux will be that that getting around 4 into 10 to the power of minus 6 meter per second at a pressure delta p thin atmosphere that means, by increased the delta p know I will not getting watch improvement in permeate flux, because an limiting flux will be 90 atmosphere at the atmosphere, I will be getting of a 4 .41 into 10 to the power of minus 6, there is not game increasing in the pressure; so I should not increase the energy cost in order to act in higher productivity of the system. So, I should operate detective operated more operating pressure at 8 at 10 atmosphere in order to the at most same as the permeate flux or productivity of the system. So, that why this is the limiting calculation, one should do before identify what will be the operating the delta p that we are going to the have in your system. It is stop here; in the next class will be doing some over problems and module design and dialysis then will moment to the next topic thank you.