Novel Separation Processes
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Lecture No. # 19
Membrane Separation Processes

Well, Good morning every one. So, we will just in this class will look into some of the problems, numerical problems and see how to solve these problems. And so, you know we have now a fairly detailed idea of various theories those will be a associated for calculation of performing parameters of any membrane surface process for example, permeate flux, permeate concentration so and so forth. Now, in about two, three classes we will be looking into some of the practical examples, and how to a really solves some of the problems, how to utilize those theories that we have already gone through, develop and gone through.

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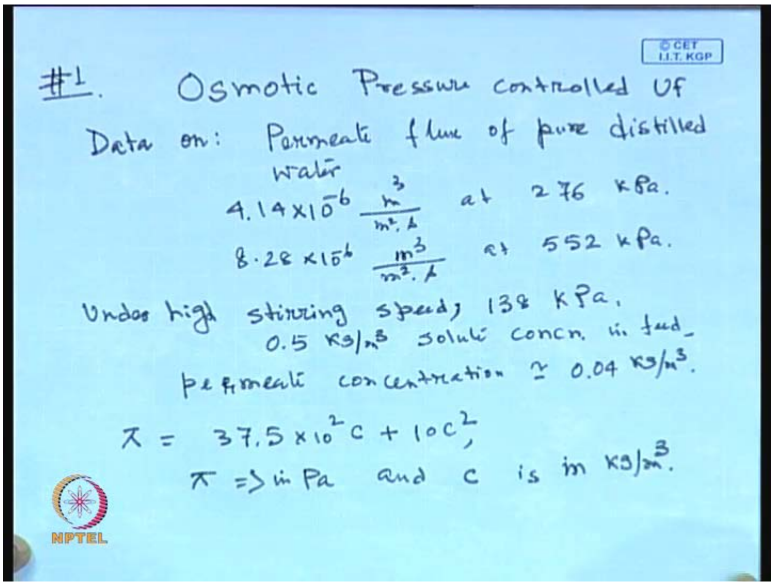

So, the first problem that we are talking about it is basically an ultra filtration process, and we are talking about the Osmotic pressure controlled filtration, controlled ultra filtration. The first data that is given, they have given some data on permeate flux using pure distilled water for example, you have got the flux of 4.14 into the 10 to the power of minus 6 meter cube per meter square second at 276 kilo Pascal, and 8.28 into 10 to the power minus 6 meter cube per meter square second at 552 kilo Pascal, these data are given.

And another data is given that under high stirring speed under high stirring speed at 138 kilo Pascal pressure, and 0.5 kg per meter cube of solute concentration in feed, one conducts experiment and gets the permeate concentration about 0.04 kg per meter cube, so these data are given. The osmotic pressure of the solution is also given as pi is equal to 37.5 into 10 square C plus 10 C square, this is the osmotic pressure as the function of concentration is given where the unit is the very important, because is an empirical relation pi is in Pascal, and C is in kg per meter cube. So these are the data given.

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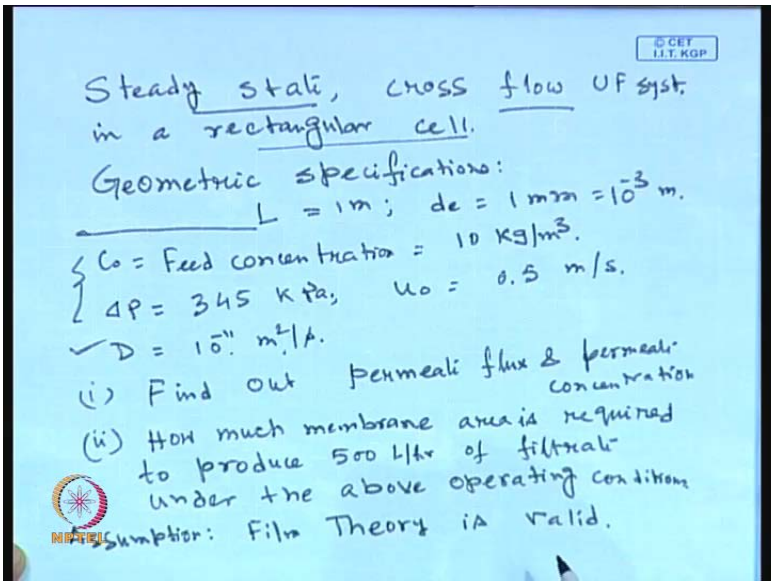

And what is the actual problem? We are actually talking about it steady state, cross flow ultra filtration system in a rectangular cell. So, rectangular cell diameter and a geometric specification should be given to you, so geometrics specification and given as the cell length is 1 meter, and equivalent diameter is 1 millimeter, that means around 10 to the power minus 3 meter.

Now, you conduct an experiment with feed concentration of solute 10 kg per meter cube, delta p trans membrane pressure drop of 345 kilo Pascal, cross flow velocity is given as 0.5 meter per second, solute diffusivity in that bulk is given as 10 to the power minus 11 meter square per second. So, these are the geometric specifications two data already given, these are the operating conditions, and these are property of the solute given in the feed. Now, what you have to find out is that? You have to find out permeate flux, and permeate concentration that we have to find out.

You have to also find out how much membrane area is required to produce 500 liter per hour of filtrate under the above operating conditions and the assumption is Film theory is valid. the film theory is valid So, let us… So, this is the actual problem, and let us sees what we can get from the first information that those are given.

From these data what you can calculate, you can calculate… this is the basically permeate flux that you were optioning, using distilled water at various pressure. So, from these data one can calculate what is the membrane permeability, so we can obtain the permeability from this data. Now, under high stirring speed 138 kilo Pascal pressure, that means that is a very low pressure, and 0.5 kg per meter cube solute concentration in the feed, that means this is very dilute solution, that means we are talking about these conditions means, this is a low polarization conditions, this conditions favor low concentration polarizations.

Now, if you remember we defined the definition of real retention, that under low polarization conditions the observed retention is almost equal to real retention. So therefore, under these conditions whatever the permeate concentration you are getting, using that data you can estimate the value of real retention, because from these if you calculate the what is the observed retention, that will be get almost same as the real retention.

And this is the osmotic pressure versus concentration relation and diffusivity of the solute is given as 10 to the power minus 11 meter square per second, these the steady state cross flow system in a rectangular cell, so you should required the specification of the rectangular cell length is given as 1 meter, equitant diameter is given by 1 millimeter, the feed concentration is 10 kg per meter cube, delta p is given, cross flow velocity is given, you have to find out the permeate flux and permeate concentration.

And also, if you would like to and have the productivity of 500 liter per hour, what will be the membrane area is required at the particular operating conditions? We assume that the film theory is valid, that means there is a constant film thickness that is the present over the membrane surface that is valid. And we are going to calculate what the various quantities that have been asked for are.

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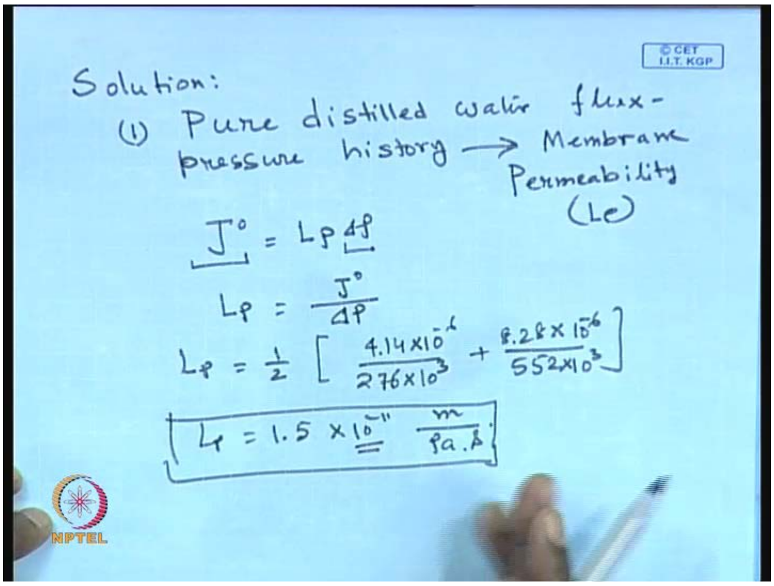

So, the first data let as look into the solution now. The first from the first data per pure distilled water data, we will be able to calculate the membrane permeability ability. Pure distilled water flux pressure history; we can calculate the membrane permeability L p. So, what is L p? L p will be nothing but J naught L p del p, if you write down the expression J naught is equal to L p del p that means the pure water flux will be proportional to the trans membrane pressure drop. Since it is pure water, it is distilled water, there is the no question of osmotic pressure, so osmotic pressure will be 0 and you can calculate what is the value of L p, L p will be nothing but J naught divided by delta p. Now, J naught value, the pure water flux value at two defined pressure drops are given in the problem.

So therefore, we just take the arithmetic average of the two data, so L p will be given as, L p will be half of 4.14 into 10 to the power minus 6 divided by 276 into 10 cube plus 8.28 into 10to the power minus 6 divided by 552 into 10 cube. So, the it was given that 276 kilo Pascal pressure you will be getting a permeate flux of 4.14 into the 10 to the power minus 6, and 552 kilo Pascal you will be getting around 8.28 into 10 to power minus 6. So, using both the condition, both the values you can take the, you can calculate average permeability and it Trans out to be 1.5 into 10 to the power minus 11 meter by Pascal second.

So, from the first set of data we will be able to calculate the permeate, the membrane permeability or how (( )) the membranes is. Now, as I told earlier, that if you look in to the order of magnitude of the L p value, we will be getting an idea what kind of membrane you are talking about, since the L p in the order of 10 to the power minus 11, we are talking about an ultra filtration membrane, if it is in the order of 10 to the power minus 12 then will be talking about a reverse osmosis membrane, if it is in the order of 10 to the power minus 10 then will be talking about a micro filtration membrane.

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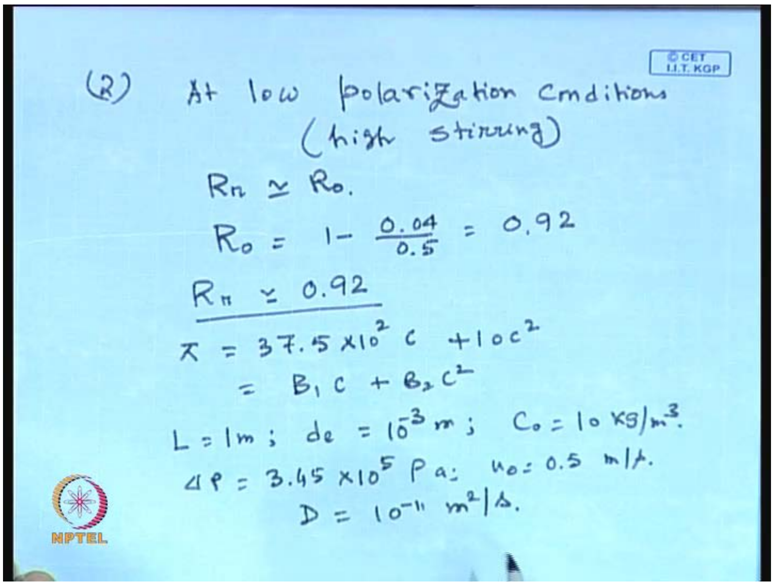

Now, the second information that will be getting from the data given, that is at low polarisable condition low polarization conditions. Low polarization condition means, you are going to have a very high stirring in the system of very high turbulence, so in that case real retention will be approaching to observed retention, the value of the permeate concentration is given, and the value of the feed concentration is given, so you can calculate what is the value of observed retention 1 minus 0.04 divided by 0.5, so 0.5 was the, 0.5 is kg per meter cube was the feed concentration, and 0.04 is the kg per meter cube was the permeate concentration, and this trans out to be 0.92, so real retention for this particular membrane is about 0.92.

So, these are the data, so now your all the information are complete, you know the value of L p, you know the value of real retention, the variation of osmotic pressure as a concentration of the solute in a function form is given, that is 37.5 into 10 square C plus

10 C square. So, this becomes nothing but B 1 C plus B 2 C square l is equal to 1 meter, d e equivalent is given as 10 to minus cube meter 1 millimeter, C naught is given as 10 kg per meter cube, delta p is given 345 kilo Pascal, so this will be 3.45 into 10 to the power 5 Pascal, u 0 is given as 0.5 meter per second, diffusivity is absolute is given as 10 to the minus 11 meter square per second.

So now, will be using the film theory equation and calculate the stuff, but before doing that one has to estimate the mass transfer coefficient.

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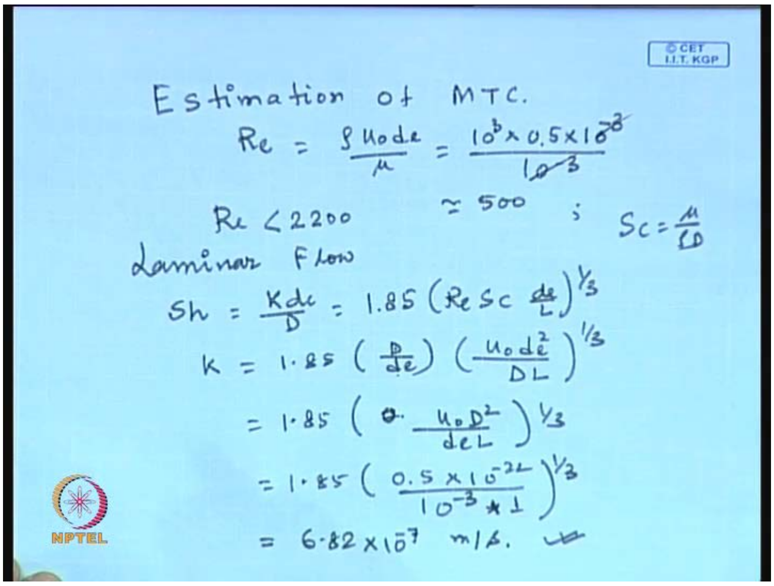

The first estimating mass transfer coefficient is calculate the estimation of mass transfer coefficient, you calculate the Reynolds number, and depending on the Reynolds number rho u 0 B by mu, you will be deciding whether the flow will be laminar or the flow will be turbulent, you take the almost water viscosity or density, so it will be 10 cube, mu naught is given as point 5 meter per second, d equivalent is 10 to the power minus 3 meter, and mu will be 10 to the power minus 3 Pascal second, so d will be how much? 10 to the power of minus 3 will be cancelling out these will be around 500, so Reynolds number will be much less than 2200, so will be having a laminar flow.

So, for the laminar flow you have to use the scheduled number relationship which is valid for the laminar flow region, that is k d e by d is nothing but 1.85 Reynolds smite d e by l rest to the power 1 upon 3, if the Reynolds number would have turned out to be around more than 4000, it would have taken the (( )) number relationship for the turbulent flow is the detours bolder relationship. So, you can calculates, so these k becomes 1.85 d by d e u 0 d e square by d l rest to the power 1 upon 3, you put the expression of Reynolds number, you put expression of smite number mu by rho d mu and rho will be cancelled out in this bracket, and what is remaining will be u 0 d e square by d l.

Now, put the values, if you put the values it trans out to be, so this become u 0 d square d e l rest to the power 1 upon 3, if you take if you observe d by d e within this bracket. So, this will be turn out to be 1.85 into 0.5 in to 10 to the power minus 22, 0.5 is velocity value, d is 10 to the power minus 11, so d square will be 10 to the power minus 22, d e will be 10 to the minus 3, length will be 1 rest to the power 1 upon 3, and this will return your value of 6.82 in to 10 to the power minus 7 meter per second. So, you can the estimate the mass transfer coefficient by looking into the flow region, and selecting the appropriate shroud number relationship for that particular flow region.

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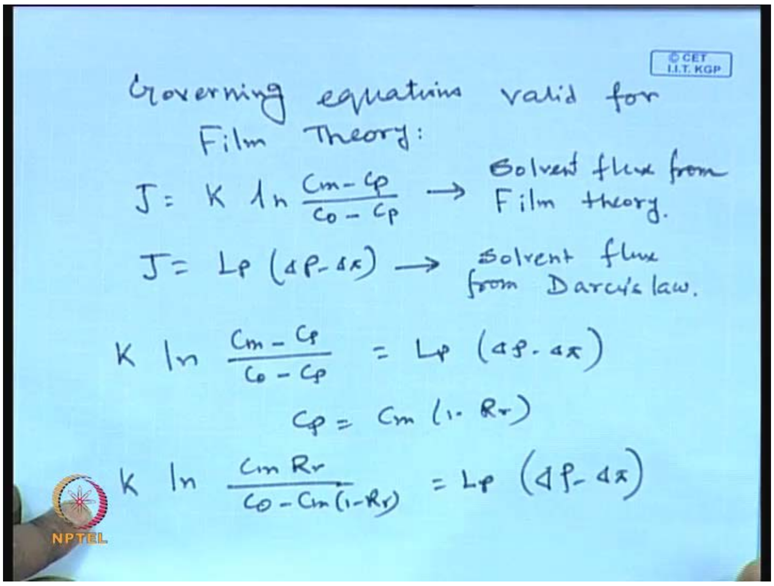

So, once you do that now you write down the governing equations for the film theory, equations valid for film theory J is equal to k L n C m minus C p if you remember, this you obtain by using the solute mass balance over the membrane surface, and another relation was J L p times del p minus del pi, that is the solvent flux you obtained from boundary value analysis, from film theory, and this you obtain from the solvent flux, from darcys law that is valid for the transport through the porous medium. So now, we will be equating them up, so it becomes k L n C m minus C p divided by C naught minus C p is equal to L p delta p minus delta pi right, now will replace C p in favor of C m in terms of, in the definition of real retention.

So, C p is nothing but C m into 1 minus R r, so this becomes k L n C m R r the numerator, and the denominator becomes C naught minus C m into 1 minus R r is equal to Lp times del p minus del pi. Now, in this expression we have already estimated real retention and it was around 0.92.

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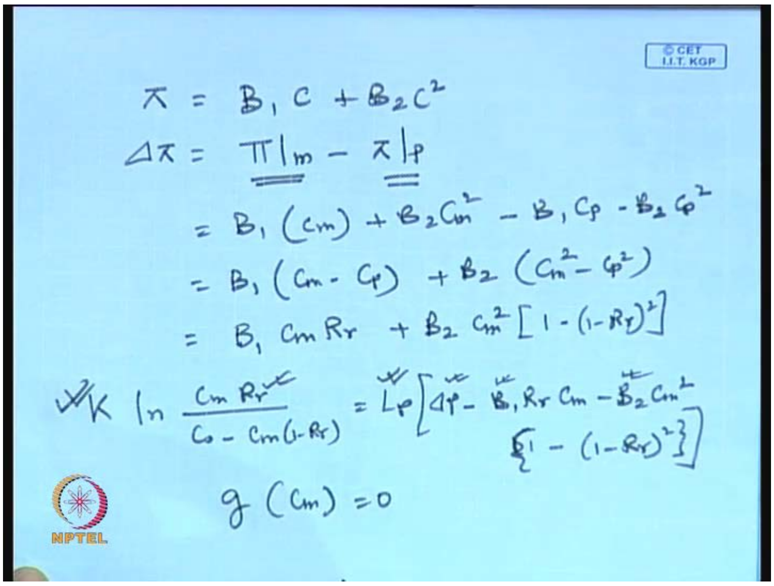

Now, we will just simplify the expression of delta pi, the expression of pie was given as B 1 C plus B 2 C square what the coefficients B 1 and B 2 is given, so what is delta pi delta? Delta pi will be nothing but pi at the membrane surface minus pi at the permeate stream, pi at the membrane surface will be function of C m, pi at the permeate stream will be the function of C p. So therefore, these will B only 1 cm plus B 2 C m square minus B 1 C p minus B 2 C p square , so this becomes B 1 times C m minus C p plus B 2 times C m square minus C p square.

Now, again we will replace C p in terms of C m through the definition of real retention if you do that, this become B 1 C m R r plus B 2 C m square 1 minus 1 minus R r square. So, what you get? So, these expression k l n C m R r divided by C naught into C m 1 minus R r is equal to l p del p minus B 1 R r C m minus B 2 C m square 1 minus 1 minus R r square.

Now, in this equation we have already estimated the value of k, that will be in the order of 10 to the power minus 7 meter per second that will have already done it, real retention we have already estimated that is the value of 0.92, so left hand side is completely known except the value of C m, on the right hand side we have already estimated the value of l p that is the membrane permeability, we know what is the operating pressure, trans membrane pressure drop delta p, B 1 and B 2 are the coefficient of osmotic pressure those are given to you, and value of real retention are known. So, this will be a non-linear algebraic equation in terms of C m only, so you can you can take this the equation on the left hand side, or take this equation on the right hand side and write it in the form of g times C m is equal to 0.

So, it becomes a non-linear algebraic equation in one variable C m only. Now, if you really do that, if you really put the values of different quantities in this equation.

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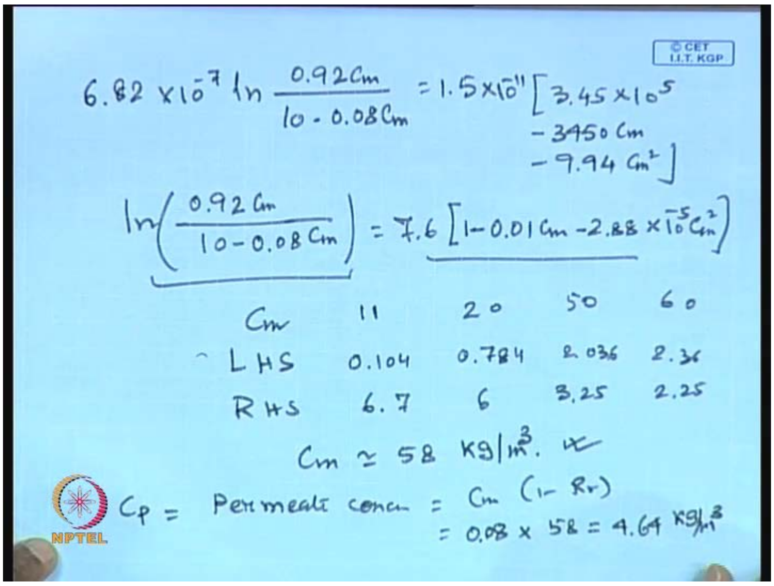

These trans out to be 6.82 into 10 to the power minus 7 l n 0.92 C m divided by 10 minus 0.08 C m is equal to 1.5 in to 10 to the power minus 11 that is the value of l p, 3.45 into 10 to the power 5 that will be the value of del p minus 3450 C m minus 9.94 C m square. So, in a neat form this become 0.92 times C m 10 minus 0.08 times C m bracket is equal to 7.6 1 minus 01 C m minus 2 .88 in to 10 to the power minus 5 C m square.

Now, we have to really do a trial and error solution in ordered to get this you know to get left hand side is equal to right hand side, so you take different values of, you guess various values of C m, so I just give a table C m left hand side, right hand side, this left hand side, this is the right hand side, if you guess a value of C m of 11 kg per meter cube, and the idea is C m should be always greater than C naught , so you should, your guess value should never be in below the value of C m C naught, so if you guess the value of C m equal to 11, this becomes 0.104, right hand side 6.7, if it is 20 the left hand side is 0.784, the right hand side becomes 6, if it is 50 and becomes 2 .036 , 3.25, and so it is now less if it is around 60, so it will be 2.36 and 2.25, so this becomes higher so it will be between 50 and 60 right, so the difference changes and C m will be roughly around 58 kg per meter cube, it will be around, the convergence will be around 58 kg per meter cube.

In fact, this can be done even in a programmable calculator so you can do it in the exam, but in an actual problem where it becomes a bit complicated, we can use neutral absent routine to do it numerically. In fact, in an actual problem this will be coupled with the set of o d is in the actual module design we have already seen, so there you cannot use the programmable calculator, we have to really do the computer coding numerically. So, we can… so for this particular problem the value of same will be 58 kg per meter cube, and the permeate concentration C p becomes C m into 1 minus real retention, and the value of real retention is 0.98, so it will be 1 minus R r is nothing but 0.02, 0.92 so it becomes 0.08. So, it becomes 0.08 into 58 and that turns out to be 4.64 kg per meter cube.

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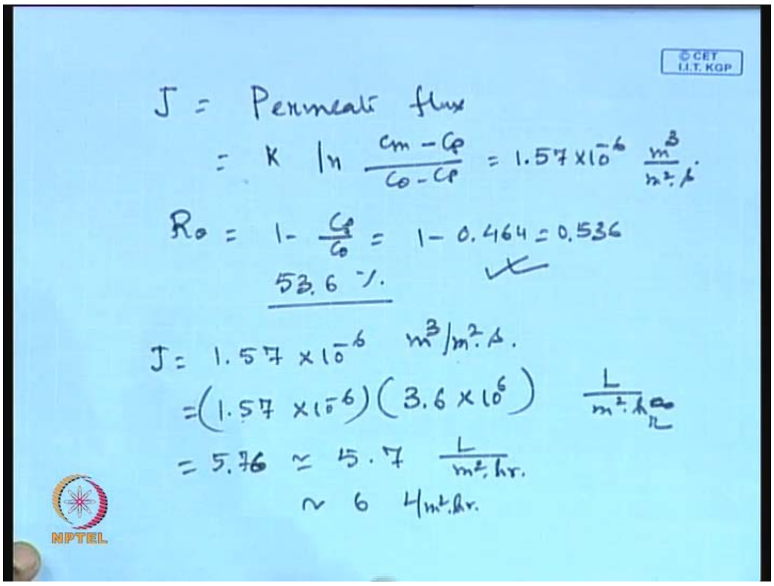

So, J and the value of permeate flux, J is equal to k L n C m minus C p divided by C naught minus C p, if you really put the values then it trans out to be 1.57 into 10 to the power of minus 6 meter cube per meter square second. So, the value of permeate concentration will be getting around 4.64 kg per meter cube, your feed concentration was 10 kg per meter cube, so the observed retention that will be getting is C p by C naught, so that will be 1 minus 0.464, so you are going to get around 53.6 percent rejection of this particular solute, using this particular membrane, and the value of permeate flux of the…

So, this gives the quality of the permeate, and the quantity of the permeate will be giving the filtrate rate, or the permeate flux that will be 1.57 into 10 to the power minus 6 meter cube per meter square second, if you like to express it in liter per meter square per hour you can do that, so this become 1.57 into 10 to the power minus 6 meter cube per meter square second, and you have to multiplied by a factor of 3.6 into 10 to the power of 6 to convert it into liter per meter square hour, so 1.57 into 10 to the power of minus 6, it should be multiplied by 3.6 into 10 to the power 6 to get this unit conversion liter per meter square hour, so 5.76, so it will be around 5.76= 5.7 liter per meter square hour, I just taken it to 1.6, if you multiply….

So, roughly you will be getting a permeate flux of the productivity of the process around 5.7 liter per meter square hour, so about 6 liter per meter square hour, that means if you will be able to give provide the membrane area of 1 meter square, 1 meter square means 3 feet by 3 feet right, if you put… you will be able to provide a 1 meter square membrane area, then in 1 hour under the… For this particular solute with these concentrations, and operating conditions, and the composition, you will be… You are going to get around 6 liter per hour permeate flux of the productivity.

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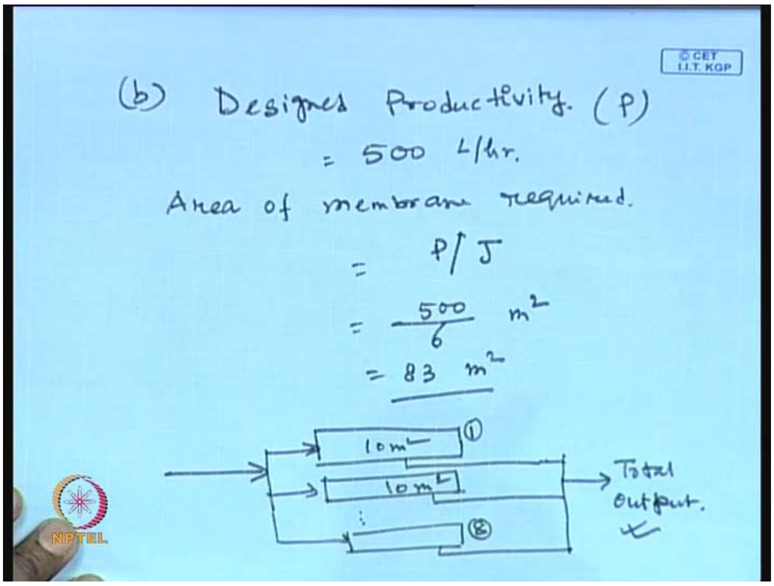

Now, in the second part of the problem the productivity it was designed as 500 liter per hour, designed productivity is 500 liter per hour, so the area required area of membrane that we are going to required is this productivity p divided by the permeate flux right, that will give you the area, so 500 liter per hour and you are going to have 6 liter per meter square per hour divided by 6 meter square around 83 meter square of membrane area will be requiring to achieve a productivity of the process of 500 liter per hour. Now, how to achieve 83 meter square? That is as pretty high number, so you can either take a spiral module or you can take a hello spiral module, so that the membrane area will be higher, or you can take number of modules, small modules in series in parallel right, number of modules in parallel so the area will be more.

So, same feed will be going to, let say if you one module will be having, you will able to give let say around 10 meter square, then same feed from a common header must be going to 8 such units. This is the 1st unit, this is the 8th unit, and permeate that we are going to collect, they will be collected in a common header and from there we will be getting the total output right. So, if you put all these modules in parallel then you are going to get that much of area, and if you have 10 meter square of a each module then we have to put 8 modules in parallel, and we are going to get the total output that is the 500 liter in 1 hour. So that goes the 1st problem.

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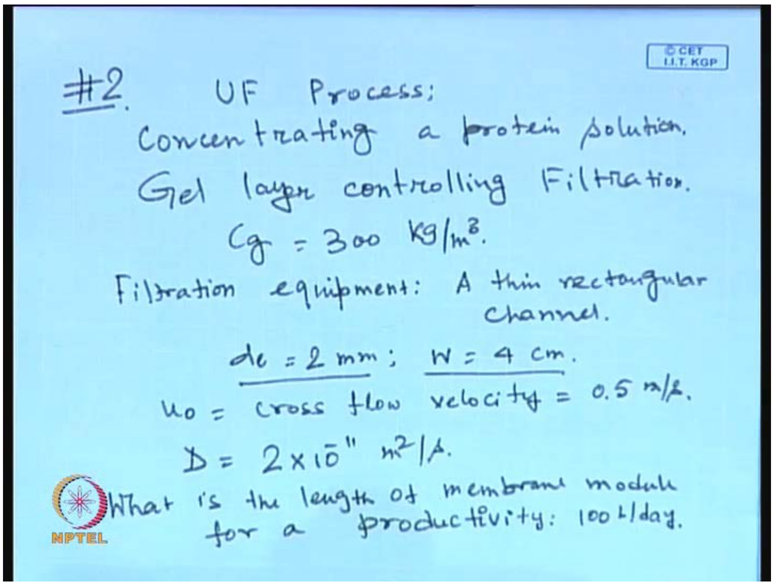

Now, let us look into the problem number two. The problem number two is again a concept of ultra filtration process, but in this case we are concentrating a protein, we are concentrating a protein solution. Now, in this case since the protein solution, protein will be having a high molecular weight, it will be having a larger size. So therefore, the filtration is gel layer control in this particular case, the previous problem we have done for the osmotic special control case.

Now, it is a Gel layer controlling filtration, the Gel concentration is given as 300 kg per meter cube, and filtration takes place in a thin rectangular channel, the filtration, the equipment is a thin rectangular channel with a equivalent diameter is given, equivalent diameter is 2 millimeter and width is given as 4 centimeter, the flow velocity is u naught, the cross flow velocity is given as 0.5 meter per second, and the protein diffusivity in the aqua solution is given as 2 in to10 to power minus 11 meter square per second. In this case, we are going to design the equipment, what is the other design parameter we required to get this equipment? You want to have the length of the channel.

So, what is the length of the channel membrane module if you are targeting for a filtration rate for productivity 100 liter per day? So, if you would like to produce a concentrated protein solution at the rate of productivity 100 liter per day, what is the requirement of the length of the channel that we are going to design?

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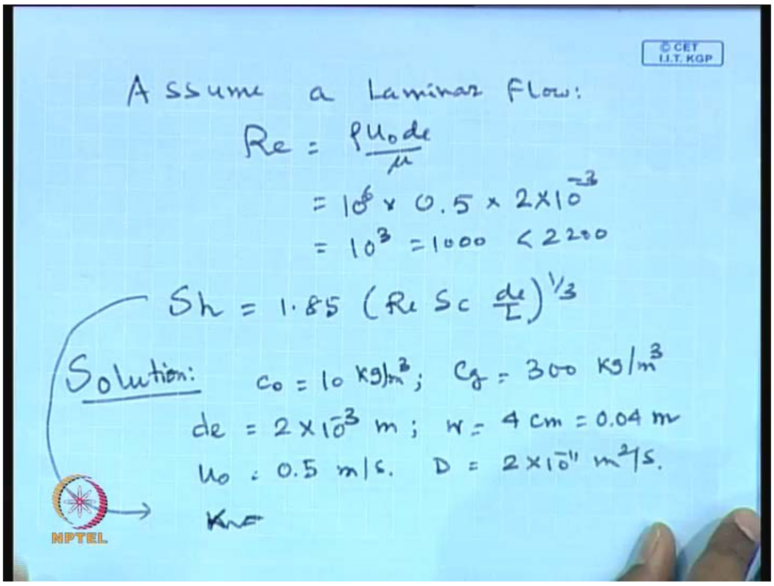

Again we are…And here will assume the laminar flow and assume the laminar flow in that the flow becomes laminar right, the u naught will be 0.5 meter per second, and the equivalent diameter is 2 millimeter, you can calculate the Reynolds number, it becomes rho u 0 d e by mu, and if the density and viscosity are almost like a water, so this becomes 10 to the power 6 rho by mu, rho is 10 to the power 3, mu is 10 to the power minus 3, so rho by mu will be 10 to the power minus 6, u 0 is 0.5, and d equivalent will be into 2 millimeter, so 2 into 10 to the power minus 3, so 2 into 0.5 is 1 is 10 to the power 6 minus will be 10 cube that is 1000.

That is the obviously less than 2200, so you are going to use the Sherwood number as 1.85, Reynolds smite d e by l rest to the 1 upon 3 for the laminar flow region. Now, let us look into the solution to this problem, this problem is straight forward, you have given all the necessary quantities that is C naught is 10 kg per meter cube, C gel concentration is given as 300 kg per meter cube, equivalent diameter is given as 2 millimeter 2 in to 10 to the power minus 3 meter, width of the module is given as 4 centimeter that is 0.04 meter, u naught is given as 0.5 meter per second, and diffusivity will be 2 into 10 to the power minus 11 meter square per seconds. So, if you can calculate the mass transfer coefficient from this relationship, the mass transfer coefficient will turns out to be, I am not doing that here.

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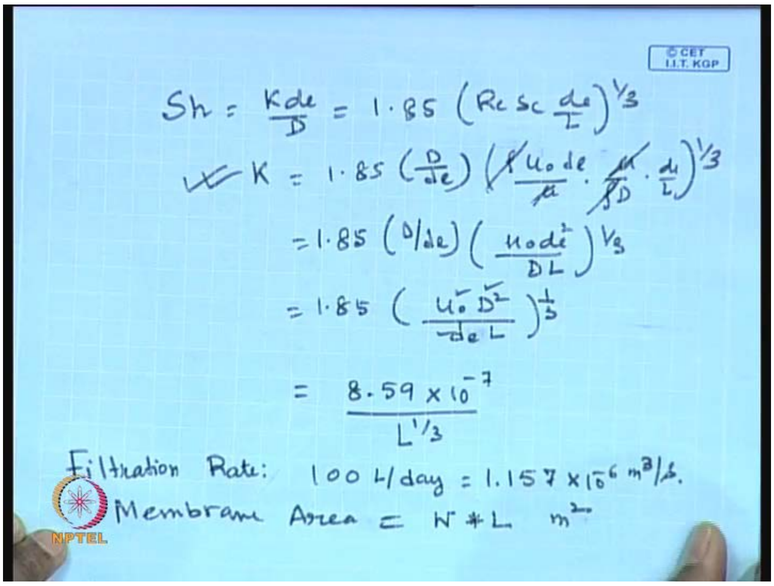

So, I have to do it probably then things will be clear to you. So, 1st step will be estimation of the mass transfer coefficient from this relationship, k d e by d because length is not known for the particular problem, 1.85 Reynolds smite d e by l rest to the 1 upon 3, so it will be k is nothing but 1.85 d by d e, Reynolds is rho u 0 d e by mu, smite is mu by rho d e by l and raise to the power 1 upon 3. So, this becomes 1.85 d over d e, rho rho will be cancelling out, mu mu will be gone, so it will be mu 0 d square by d e l rest to the 1 upon 3, so it becomes u 0 d square by d e l rest to the power of 1upon 3, that is the mass transfer coefficient. Now, you know all the values of u 0, you know the value of d and d e, but you do not know the value of l that we are going to find out.

So, if you do that I think it turns out to be, if you really put the values it turns out to be 8.59 into 10 to the power of minus 7 divided by l to the power of 1 upon 3, and fine that is the form the that is the the information given, you have estimated the mass transfer coefficient, then it calculate what is the filtration rate? Filtration rate and we have to transfer into the meter cube per second, it is given as 100 liter per day if you transfer into meter cube per second it turns out to be 1.157 into 10 to the power minus 6 meter cube per second, and what is the membrane area? Membrane area the width is given and the length you are going to find out, the membrane area will be w times l right, that much meter square.

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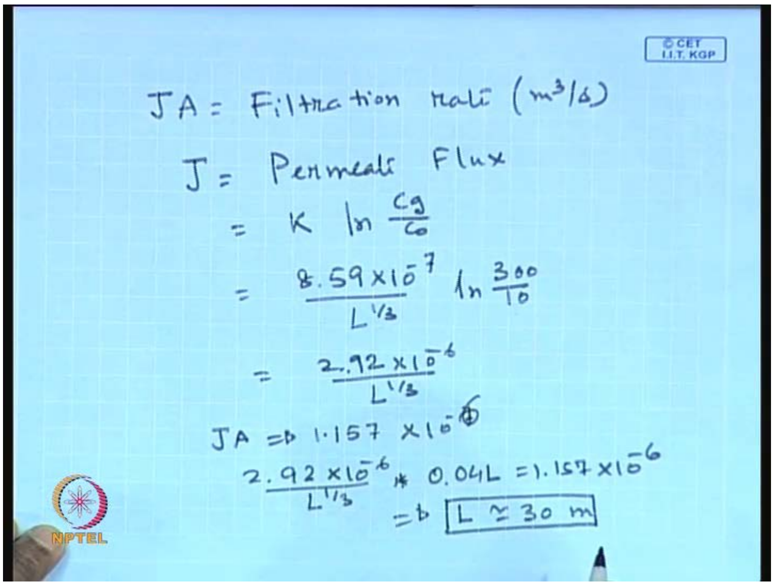

Now, you use the so J times a will be nothing but the filtration area sorry filtration rate, that is meter cube per second, and what is J? J is the permeate flux that for a gel controlled ultra filtration system you know the permeate flux k is equal to, J is equal to k l n C g by C naught simply.

So, these becomes 8.59 into 10 to the power minus 7 divided by l to the power 1 upon 3 l n 300 by 10, so j turns out to be 2.92 into 10 to the power minus 6 divided by l to the power of 1 upon 3, so you can calculate it j times a is equal to filtration rate that is 1.157 into 10 to the power minus 6, so this becomes 2.92 into 10 to the power of minus 6 divided by l to the power1 upon 3 multiplied by area, area we have already found out that it is 0.04 times l, width is 0.04 centimeter 0.04 meter. No, in this particular problem it is given only at the bottom, if it is… it will be specified in the problem, for this particular problem I have solved the problem keeping the membrane only in the bottom of the channel, if it is put in the top of the channel you will be, it will be twice of area.

That will be given in the problem otherwise, even if it is not stated, is that actually something we are assuming membrane is kept on both the walls and do it accordingly. So, in this particular problem we have use that area is membrane kept only at the bottom of the channel. So, 0.04 l is equal to 1.157 into 10 to the power minus 6, this will be 10 to the power minus 6.

So, these turns out to be l is equal to around 30 meter, so you required about 30 meter of membrane module to get this much of filtration area, if you put membrane only at the bottom of the channel. Now, you can… It is difficult to keep a, to get a 30 meter that means around 90 feed of the module, so we can even break down this module into three small module, and get them let say 10 meter each, or 4 or 5 of them and put them in parallel, so that will be provide that much of membrane area.

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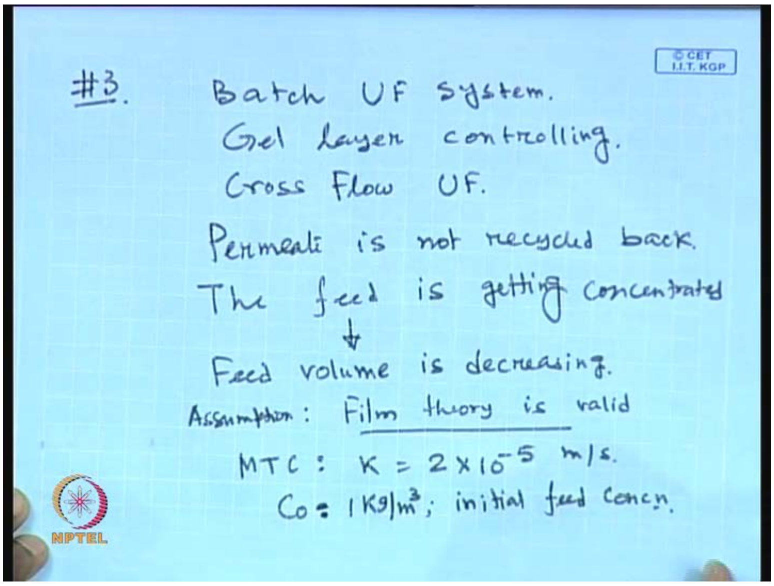

So, now let us go to the 3rd kind of problem, this problem is for a batch ultra filtration system where it is gel layer controlling. In this case, a is the cross flow ultra filtration, but in this particular case the permeate is not recycled back, so permeate in a steady state process what we you used do, the permeate is recycled back, the (( )) is also recycled back, and you take a small sample from the permeate to analysis, for analysis of the permeate concentration. The amount that you are taking from the permeate stream for analysis will be very small, it does not matter, it does not really matter.

So, India in that case what happens? The feed concentration will always be constant, so you are basically pumping, filtering the same feed concentration, so that will be the steady state. Now, in this case what will happen? Permeate is not recycled back means, your extracting water, mostly water out of gel controlled polarization process, the permeate concentration will be almost 0, it will be almost pure water, so you are extracting water out of the solution, the feed sample, so therefore the feed is going to get concentrate all the time.

So, permeate is not recycled back in this process, so therefore the feed is concentrating, feed is getting concentrated, and since you are extracting water out of feed, the feed volume getting decreased, right? And we will assume that film theory is valid, that is an assumption. And the mass transfer coefficient in this case, it is directly given otherwise, you can.., if the recycled cross flow velocity is given you can evaluate the Reynolds number, and can split an approximate mass transfer coefficient or Sherwood number relationship, and can evaluate the mass transfer coefficient. But in this particular problem value mass transfer coefficient is given as k is equal to 2 into 10 to the power minus 5 meter per second.

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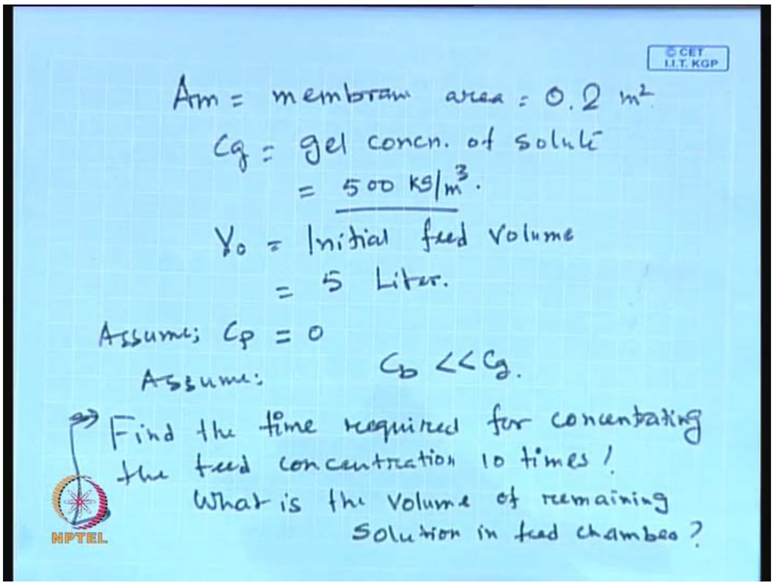

Initial feed concentration is 1 kg per meter cube, the initial feed concentration is given there, effective filtration area of the membrane is given as 0.2 meter square, point to meter square and gel layer concentration of the solute gel concentration of solute is given as 500 kg per meter cube, initial feed volume is given as that is given as 5 liters. Assuming there is no solute present in the permeate stream because it is gel concentration, gel layer control layer filtration C p is equal to 0, and (( )) concentration is much less than gel concentration, that means gel concentration is pretty high, and we are going to use a feed concentration of 1 kg per meter cube, and at any point of time we assume the bulk concentration is much less than C g, that is a valid assumption.

Because, if you are starting feed concentration is 1 kg per meter cube, and the gel concentration is 300, 500 kg per meter cube, that it is 500 times of that, so at any point of time during the operation, although the bulk concentration will be increasing, it will be increasing let say 1 kg per meter cube to 2 kg per meter cube, 3 kg per meter cube, like that. It will be always less than, much less than the gel concentration. And what you have going to find out? Find the time required for concentrating the feed concentration 10 times, what is the volume remaining in the chamber, volume of remaining liquid, remaining solution? If you started with 5 liter of solution, how much will, what is the volume of the feed liquid that is remaining in the feed chamber? So, that is the questions.

So basically, this will be very important problem for anyone who will be doing the bio chemical, bio technology experiments. You would like to concentrate a particular protein solution from a particular concentration (( )), 5 times of that, 10 times of that, so the volume will be much less, and if the membrane or ultra filtration, all the specifications like the channel, dimension, geometric area, etcetera all known, that means because you are having a system at your hand, at your disposal, and how long we will continue this operation, what is the time required? That we are going to find out in this particular problem.

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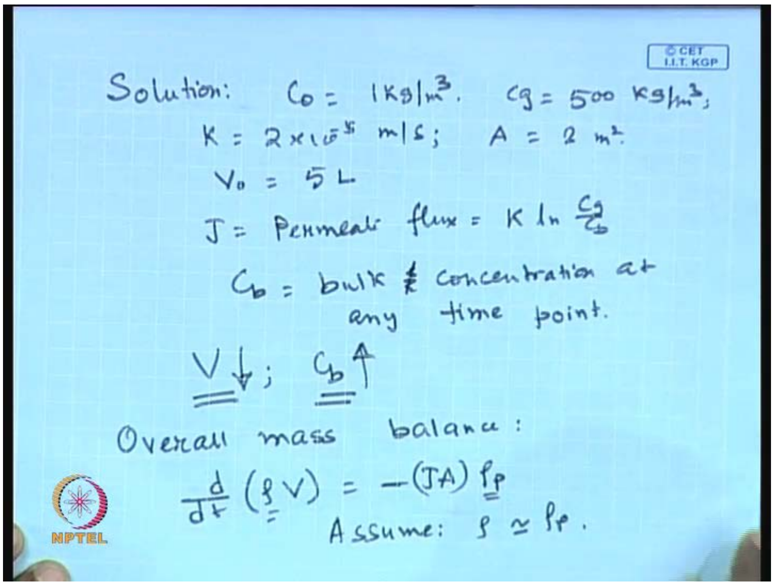

So, let us look into the solution, C naught is 1 kg per meter cube, that is the feed concentration, that is c B bulk concentration at time t is equal 0, that is the initial concentration of the feed, C g is given as 500 kg per meter cube, so these are the information let us write them down, mass transfer coefficient is given as 2 into 10 to the power minus 5 meter per second, if you know the geometric of the flow domain, if you know the properties you can estimate the mass transfer coefficient independently, as well by identified the appropriate Sherwood number relationship, and area of the membrane is given as 2 meter square, v naught is equal to initial feed volume that is 5 liter.

Now, what is the permeate flux? k L n C g by c B right? k is given and C g is also known, and what is c B? c B is the bulk concentration or feed concentration at any point of time. Initially, this c B was equal to C naught at time t is equal to 0, c B is basically bulk concentration, now there is nothing like feed, feed you have just given in initially and after that it is going on, so it is basically bulk concentration at any time point. So, what is the difference of this problem compare to the earlier problem? In the earlier problem the bulk concentration was constant, in this case the bulk concentration is not constant, and it is a function of time, why it is function of time? Because you are extracting water out of it therefore, the there is a reduction of feed volume, and because of the reduction of the feed volume the concentration getting increased.

So, in order to…. So therefore, volume is decreasing and concentration is increasing, so you must be writing overall mass balance and material balance in order to trap the variation of the volume, variation of thee feed concentration. So, first you do overall material balance, what is overall material balance? Rate of accumulation is equal to rate of material in minus rate of material out. Since, it is that process you have dumped everything out, so there is nothing called rate of material in, it will be equal to zero. But since you are extracting water out of it, so there will be some material that is going out of the system. So, d dt is the rate of accumulations rho times v, rho times v is basically total mass that is remaining into the feed. So, we are neglecting the volume of the pipeline etcetera that will be very small.

So, rho times v is the total material at any point of time is equal to minus, there is nothing called material in but there is material out, so rate of in minus rate of out, so minus J times A, that is the total material that is going out in meter cube per unit time, you should multiply by the density of the permeate, so that will be converted into the total material that is going out in kg per unit time, right? It should multiplied by rho permeate. Now, we have already discussed earlier that viscous, the density is a… So what is the difference between the rho in the bulk and rho in the permeate? In the permeate stream there is the no solute, but the retention stream, in the bulk stream it is loaded with the solution, solutes.

So therefore, concentration of solute is more in the bulk, but since density is a weak function of concentration we can assume that rho in the bulk, and rho of the permeate will be almost equal, we assume so that is an assumption, and it is valid assumption because density is the very weak function of concentration.

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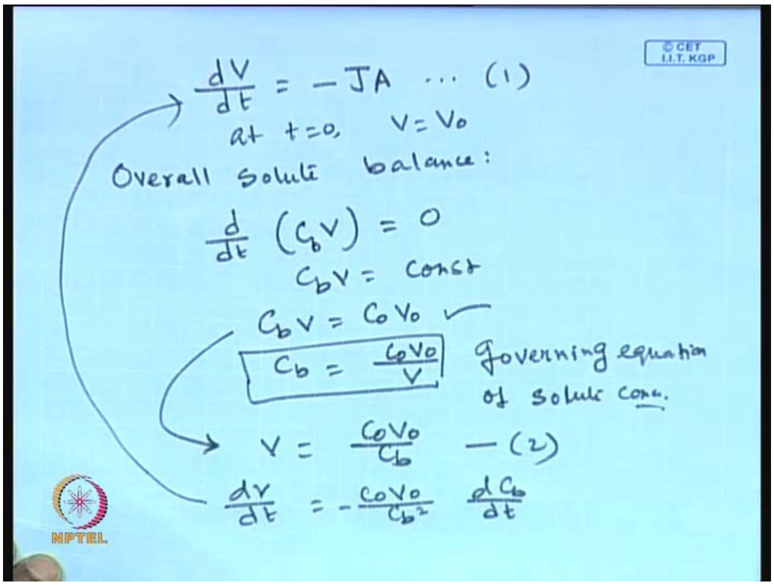

If that is the case, we get the governing equation of volume change in the feed as minus J a, so this is equation number one, at time t is equal to 0 we know, v is equal to v naught, v naught was given to you.

Now, next we get the governing so this is the governing equation of volume, next get the governing equation of bulk concentration, for that what you have to after do? We do an overall material solute balance, rate of accumulation is equal to rate of material that is solute that is going in to this minus rate of going out of system, there is nothing going in. So, you will be having d d t of C times v, this is the concentration C bulk let say C bulk times v, where v is the volume at any point of time, C B is the concentration at any point of time, that is the amount of material amount of the particular solute present in the system is equal to, there is nothing going into the system minus nothing going out of the system.

Because, since it is a gel layer control filtration the permeate concentration will be equal to 0, so there is no solute present that is going in the permeate stream, so c B will be equal to 0 right, so this will be equal to 0 that means c B times v will be equal to constant, that means c B times v that is equal to nothing but C naught v naught, because this values are known at time t is equal to 0, so at any point of time you can evaluate c B that is nothing but C naught by v naught, C naught times v naught divided by v, since the volume is decreasing the function of time that will be give in corresponding increase in bulk concentration, so this is the governing equation of solute concentration.

Now, you can write it in terms of v C naught v naught divided by c B, and we know the equation one d v by d t is nothing but j times a, so you just take the time derivative of this expression and put it there, so equation one and this let say this is equation two, combine equation one and two ,combine equation 1 2 means take the time derivatives of v so what is this d v d t will be nothing but C naught v naught divided by c B square with the minus sign d c B by d t, right? That is the derivatives of d v d t and put in there.

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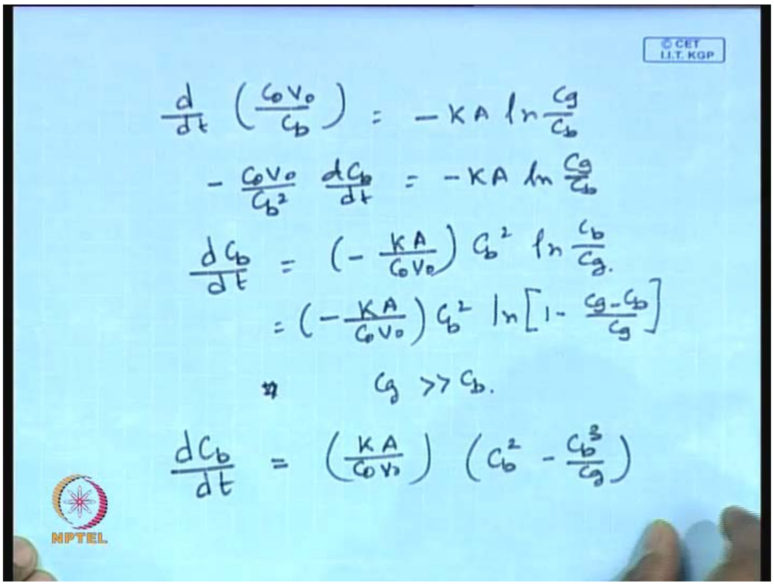

If you do that you will be getting d d t of C naught v naught divided by c B is equal to minus k a l n C g by c B, and just get the derivatives of that that we have already done minus C naught v naught by c B square d c B d t will be minus k a l n C g by c B, and we are writing the final expression, so d c B d t will be minus k times a divided by C naught v naught c B square l n c B by C g right?

Now, let us see how we can simplify it, because these integration you can take c B square in the denominator and take d t on the other side and you can integrate, but that will give you numerical, that you required numerically integration, you do not have an analytical expression for this, but this can be simplified further by the assumption that c B at any point of time will be much less than C g So, c B square these l n c B by C g can be written as 1 minus C g minus c B divided by C g. So, this will be giving you, and since the approximation that C g is much greater than c B, so using that approximation I am just omitting number of steps you would please try to derive them, the expression of d C p is trans out to be k a divide by C naught v naught c B square minus c B cube divided by C g, that means under the this condition you can expand these 1 minus C g by c B c g minus c B by C g assuming these will be very small, so you do an expression try to get this expression.

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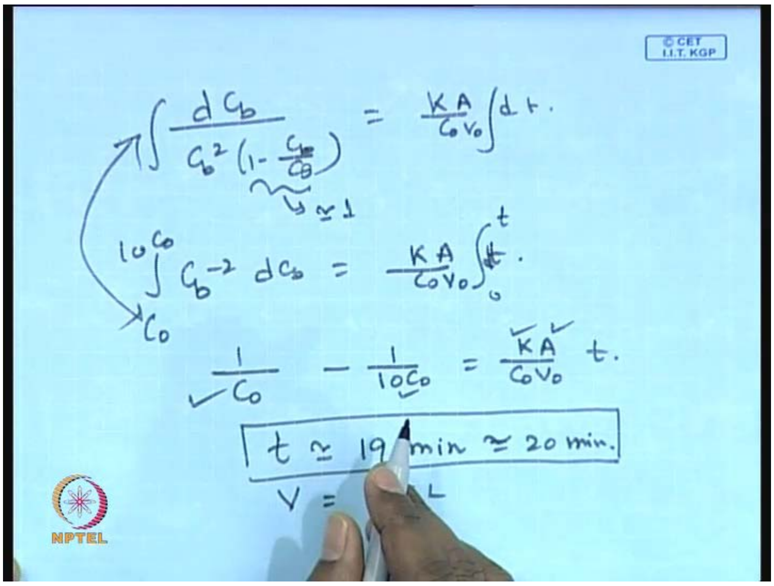

And now, you will be able to get an analytical integration, so this becomes d c B c B square 1 minus c B by C g is equal to k a C naught v naught d t, and you can really integrate it out in fact, c B by C g will be very small right? So, 1 minus c B by C g will be roughly approximated as 1, so it will be roughly approximately as 1 so this will be nothing but c B minus 2 d c B k a C naught v naught times t, so it will be formed 0 to t and this will be from C naught to 10 times of C naught right? So, it gives an expression something like this 1 by divide by C naught minus 1 by 10 C naught is equal to k a C naught v naught times t, everything will be known, C naught is known, k is known, a is known, v naught is known, and we can estimate the values of t and this t it trans out to be around 19 minute let say, 20 minute and final volume trans out to be point 5 liter.

The most important thing is that, we have to carry out the concentration experiment for 20 minutes under these condition, given conditions to concentrated feed concentration, concentration for a protein solution by in 10 times. In fact, these simplification are useless, you can directly do a numerical integration after getting the formulation, because I would like to have you, give you a feeling of what is the (( )) order of the magnitude of the time that you require for this operation, I did this assumption and got this simplification. Otherwise, one can directly do numerically integration by using a trapezoidal rule, can estimate the time required for this to achieve this particular concentration of the protein. So, in a next class, in tomorrow's class will be solving some more problem those will be relevant for, as far as our examination is concerned. Thank you.