Novel Separation Processes
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Lecture No. # 18
Membrane Separation Processes (Contd.)

Well, we are discussing about the two dimensional model and more complicated realistic model of dialysis process, continuous dialysis process.

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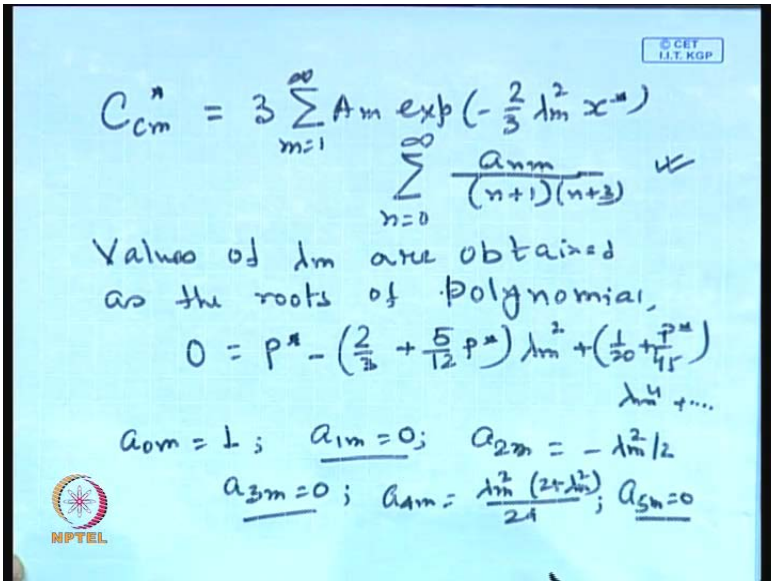

And what we have discussed that the removal of toxic material, the rate of removal of toxic material would be obtained. If you if you if you compare that depletion of concentration from the fixed f side to at any location, in terms of cup mixing concentration. Why it cup mixing concentration, because it should be the area averaged. Because, that is the cup mixing concentration is the only measurable quantity.

Now we, what we already had earlier, we already by solving the equation of motion and species balancing equation, we couple them and we got the concentration profile in terms of the x and y. And once you get the averaging in the y direction that concentration profile will give you the cup mixing concentration profile and we have looked into the solution; the solution of cup mixing concentration profile gives by this expression. The first term comes from the x varying part; the second term comes from the series solution of the y varying part. Now the lambda in the eigen values of this problem will be obtained from the roots of this characteristic equation that will be obtained from the boundary condition at y star equal to one.

Now, let us compute some of the values of a n m, these a zero m is equal to one. If you really look in to the series solution, these are the coefficients that will come out, a one m will be equal to zero; a two m is equal to minus lambda squared by two; a three m is equal to zero; a four m will be lambda m squared 2 plus lambda m squared divided by 24; a five m is equal to zero. Similarly, so that means, it comes from the series solution and use of the boundary condition, all the odd coefficients will turn out to be zero and the even coefficient will be having some values.

Similarly, you will be getting a seven m to be equal to zero and things like that.

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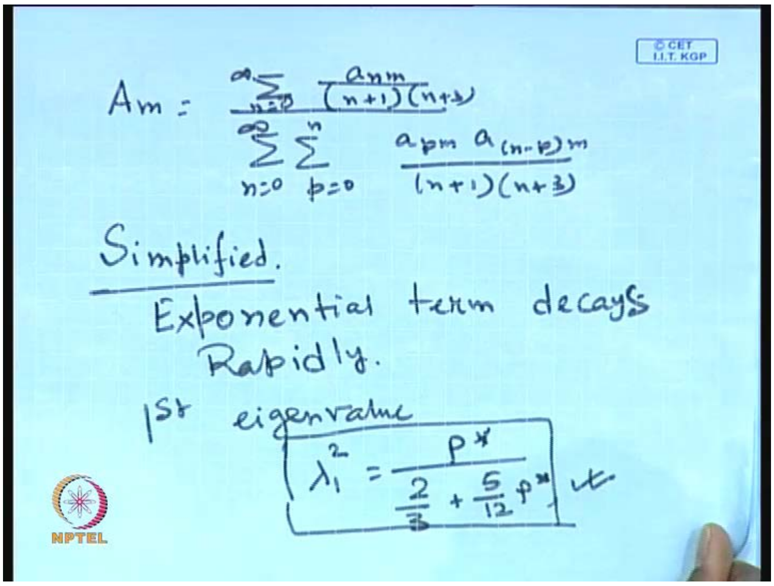

And the capital a is given as n is equal to zero to infinity, a n m divide by n plus 1 into n plus 3, n is equal to zero to infinity, p is equal to zero to n, a p m, a n minus p times m divided by n plus 1 into n plus 3. Now, I am doing all these, do not try to memorize any of these equations, do not try to you know go into mathematical complications. I am just writing these things for the sake of completeness. Now, i will just go through it and look into the application of these equations.

Now, these equation can be simplified, the threatening form of these equation, do not get threatened by looking into it, just they will be simply simplified, if you look into the exponential term. Now all the solution, if you look into the solution of cup mixing concentration; the cup mixing concentration, the exponential terms becomes very important. Because, if you will look in to the values of these lambda; the eigen values, these eigen values will be in in this the first eigen value will be around 0.3 or 0.4, second eigen value will be around 3.3 or 3.4. They will be typically in the order of arithmetic progression of three, next will be let us say six; next will be, fourth be around nine like that.

But, it will be also multiplied with the square of that. So, as you go on increasing the number of terms of the summation series, these terms, these exponential of minus becomes extremely small. So, if you go up to the second term, third term like that, the contribution of these will be so small. When it is multiplied with the other quantities, those will be negligibly small. So, you can very well work with the first eigen value itself.

So the first eigen value, thus because exponential terms decay rapidly and we can we can very well work with the first eigen value itself. The first eigen value will be lambda 1 squared is equal to p star divided by 2 by 3 plus 5 by 12 p star. So, you can very well get with the first eigen value. And consider, the first term of the summation series and if you do that then the whole equation becomes very simplified. You need not to bother about the second term, third term onwards and lambda m squared will be replaced by the lambda 1 squared and lambda 1 squared expression is given by this. Now, you are in a position to plot log of c m C c m star versus x star.

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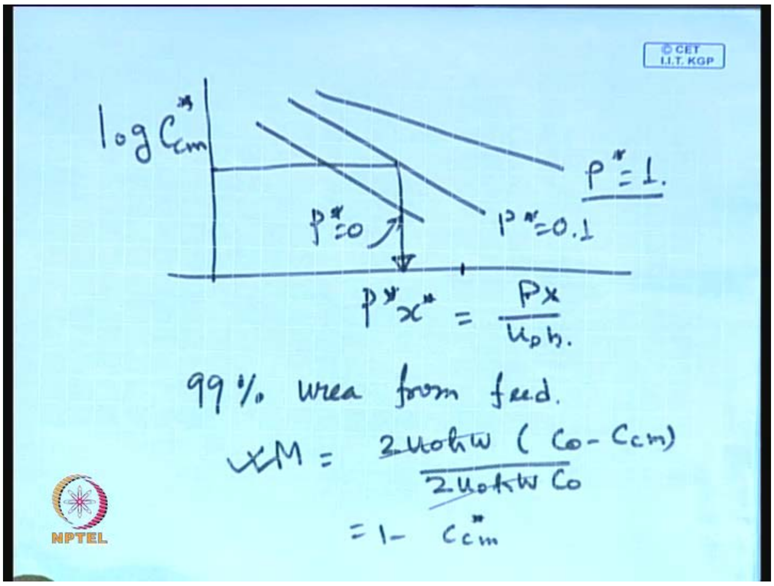

Now, if you really do, you will be getting an expression something like this, log of C c m star versus p star x star. Because that becomes a non dimensional, this becomes p x over u zero h and you will be getting the curves something like this. This is p star equal to zero, this is p star is equal to 0.1, this is p star is equal to 1.

So, what is p star? p star is nothing but, p times h divided by diffusivity of the solute in the bulk. So, we know that diffusivity of the solute in the bulk, we know the channel have fight and p is a constant, which is a characteristic for membrane solute system. So, you know the p star, so various other membrane, so p star becomes an operating condition and that is selection of the membrane and selection of the solute, same membranes and different solutes, the value of p star will be different. Now, one can get the, so if you would like to remove, let us say ninety percent of phenol in the feed side or ninety percent of urea from the feed side. You know the feed concentration; you know the amount that is entering in to the feed. If the removal rate is known to you; ninety percent let us say, you can calculate, what is the C c m star? What is the cup mixing concentration?

If you know the cup mixing concentration, you can consult this plot and let us say, p star is equal to 0.1, let us say C c m star; log of C c m star, may be this. Then you can go there and can get the value of p star x star. In this quantity, p is known to you, u zero is known to you, h is known to you. You can find out, what is the value of x or what is the length of the dialyzer. Therefore, if your removal rate is specified that i would like to remove 99 percent of urea, let us say from the feed. So, that is you know the removal rate, if you remember, it was 2 u zero h w multiplied by c zero by minus C c m divided by 2 u zero h w divided by c naught. So, that is the percentage efficiency sort of thing.

So, the whole thing becomes, it will be canceled out. So, it becomes 1 minus C c m star. So, you can so this will be specified; m will be specified, so you find out the value of what is C c m star. You can go to this plot and you know, what is the characteristic value of this parameter p star, because you are selecting the membrane and the solute. And therefore, you can go to the appropriate plot and can get the value of p x star and from the p x star, you get the value of x or the length that is required of the dialyzer. That way, the two dimensional analysis of a of a of a of a continuous dialyzer can be done and it will be more accurate than the one dimensional analysis that we have talked earlier.

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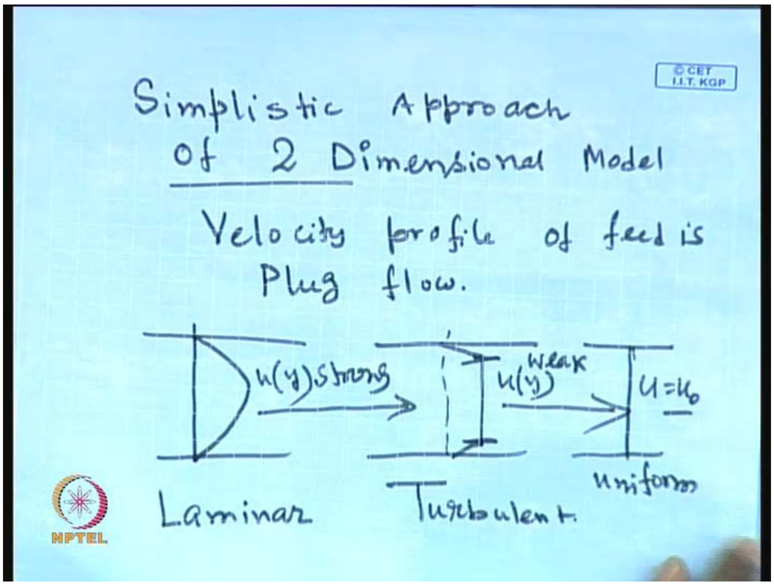

Now, in order to get a closer view to a system. Now, we will be looking in to a simplistic approach. So and this simplistic approach is basically as per as the mathematics of the analysis is concerned, simplistic approach of 2 dimensional model. So here we will consider that instead of having a parabolic velocity profile, the feed will be going for by a plug flow, the feed velocity the velocity profile of the feed; feed is plug flow. That means, the it is it is called uniform velocity profile. If you look into the, you know velocity field for three different cases, this is for a laminar one; this is for a turbulent one, turbulent velocity profile; this is for a uniform velocity profile.

For the laminar profile, it is parabolic in nature with the maximum at the middle of the channel. In the turbulent one, again there will be the turbulent core will be prevailing, the turbulent core is basically the velocity almost constant, the turbulent core will be prevailing in the ninety five percent; above ninety five percent of the full channel length. And then you will be having only small portions, where the viscous sub layer and the transitional region will lie and in the uniform plug flow, the velocity is considered to be uniform throughout the whole channel. That means, if you increase the velocity of the feed, the transition will be from the laminar to turbulent and from the turbulent to the uniform velocity profile. The, what is the, you know simplicity as far as the mathematics is concerned, u is a strong function of y here, u is a weak function of y here, because ninety five percent of the channel length, the velocity will be almost equal to turbulent core and u is independent of y here.

So, these are strong variation of y, this is weak variation of y and this is independent of y. So, u is equal to u naught and the whole analysis becomes simplified, you need not to do a series solution to compute the y varying part as we have discussed in the earlier sub problem.

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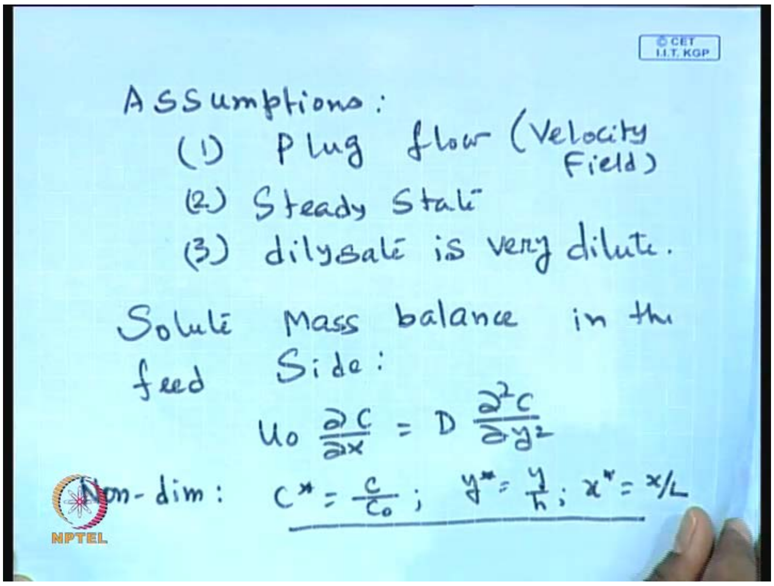

So, the the assumptions those are required for this problem. To solve this problem, one is it is a plug flow; second is, plug flow means only the, as far as the velocity field is concerned; second is, it is a steady state of course; third one is, (( )) is extremely dilute, dilysate is extremely dilute, that means the solute that is coming to dilysate side instantaneously. It will be washed away, so there is no concentration in the dialyzer sub state.

So therefore, the solute mass balance equation in the feed side becomes u zero del c del x is equal to D del squared c del y squared. These are same equation as we have done in the earlier case, but in this case, u is equal to u naught in state of the parabolic velocity profile. And we define the, you know the non dimensional quantities as c star is equal to c by c naught, y star is equal to y by h, x star is equal to x by l, if you know the length of the dialyzer. So, or you can, you could have defined the x star that whatever we have defined in the earlier case as well. So, what I have done? I have solved this problem using x by x by l, it does not matter really.

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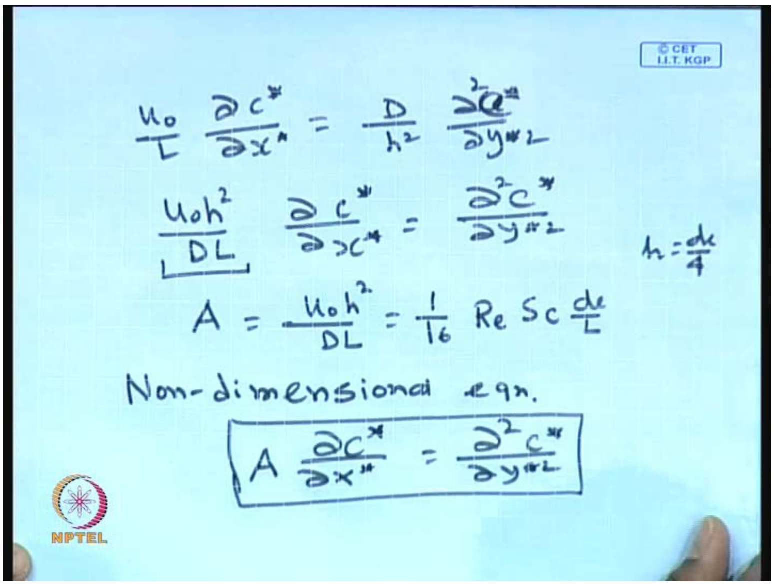

So, if you substitute all these quantities there, this becomes u naught del c star over l del x star is equal to D over h squared del c del c star over del squared c star over del y star squared. So, you will be getting u zero h squared by D l is equal to multiplied by del c star del x star is equal to del squared c star del y star squared. And this is nothing but the operating conditions and this can be written as, A equal to u zero h squared by D l and this is nothing but, an h can be replaced by equivalent diameter divided by 4, as you have done earlier. So it will become 1 by 16 Reynolds Smith d e by L. So, A becomes really a non dimensional parameter and it is defined as 1 by 16 Reynolds Smith d e by L.

So therefore, the non dimensional, let us let us write down the non dimensional equation and non dimensional boundary conditions. Non dimensional equation becomes A del c star del x star is equal to del squared c star del y star squared and the boundary conditions, let us write down to for this governing equation and let us make them non dimensional as well.

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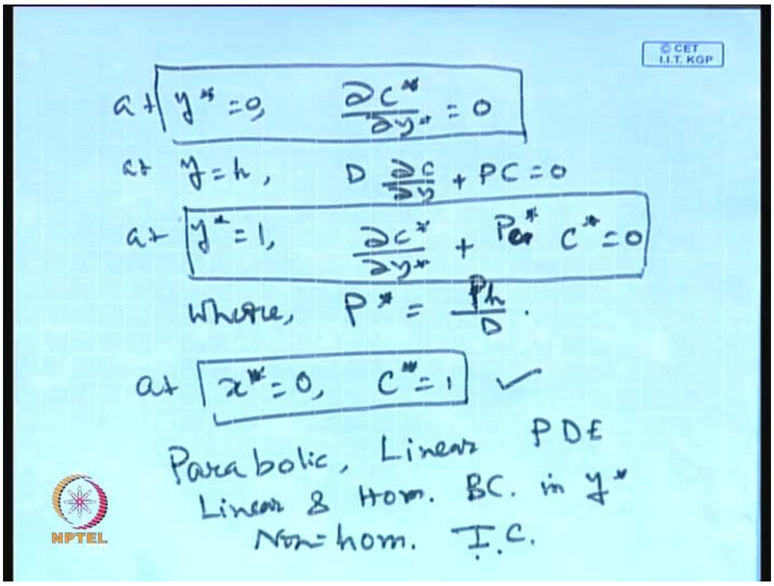

If you do that at y star is equal to zero, you had del c star del y star, so initially you had del c del y will be equal to zeros, it will del c star del y star will be equal to zero. At y is equal to h, you will be having D del c del y plus p c will be equal to zero. So that means, at y star is equal to one, you have del c star del y star plus p e m plus p star c star will be equal to zero, where p star becomes p h over D.

The non dimensional solute permeability through the membrane and of course, at x star x is equal to zero, you had c is equal to c naught, so x star is equal to zero; c star is equal to one. So, this is one boundary condition at y star is equal to zero; this is another boundary condition at y star is equal to zero; this is another boundary condition, these another, these basically nothing but, the initial condition at x star is equal to zero. Now, in this system, there is no terms like one minus y star squared and things will come. So, this is a straight forward case of parabolic partial differential equation, parabolic and linear PDE and linear and homogeneous boundary conditions in y star and non homogeneous initial condition. Non homogeneous initial condition, that is at x star is equal to zero, c star is equal to one.

So obviously, this problem will be definitely a candidate for having a separation of variable type of solution and we are going to do a complete solution by using separation of variable.

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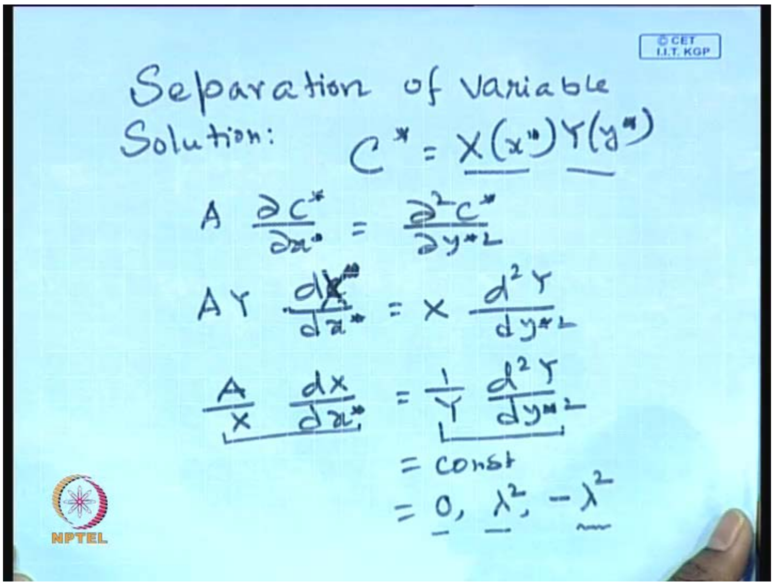

Solution, we assume C star is equal to X x star Y y star, so that means we can completely disassociate the x varying part and y varying part and the overall solution will be constituted by the product of x varying part and y varying part, if you if you if you can evaluate them separately. Now, you just substitute this in your governing equation, your governing equation evolves A del c star del x star is equal to del squared c star del y star squared. Let us put c star is equal to x into y, so what you will be getting is as d c star d X star is equal to X d squared Y d y star squared, just divide both side by X Y, what you'll be getting is A by X d X d x star is equal to one over Y d squared Y d y star squared.

Now, the left hand side is completely a function of x; the right hand side is completely a function of y and they are equal and that means, this equality will be some constant. So, this constant can be zero, can be positive, can be negative. If it is zero or positive, we can show that we will landing up with a trivial solution which you are not looking for. So, for a nontrivial solution, this constant must be negative and this constant will be minus lambda squared..

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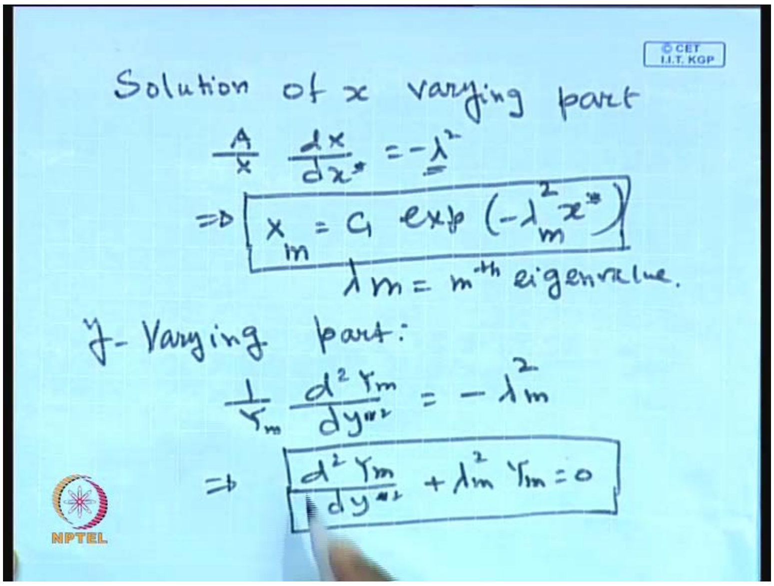

So, if you really do that then we can solve the x varying part and y varying part. The solution of the x varying part becomes A by X d x by d x star is equal to minus lambda squared. So, X becomes some constant exponential minus lambda squared times x star and this constant lambda is called the eigen value of the problem. So, we write a lambda m and the corresponding solution as X m where lambda m indicates the m-th eigen value.

And we really do not know, what is the value of lambda m till now. So, we have the complete solution of the x varying part in terms of eigen value and the y varying part, if you look in to the governing equation and the boundary condition, y varying part becomes 1 over Y d squared Y d y star squared is equal to minus lambda m squared. We could substitute subscript m in y as well, because that corresponds to m-th eigen value. So, this becomes d squared Y m d y star squared plus lambda m squared Y m will be equal to zero. And we know the solution of this equation, the solution is constructed by the sine and cosine functions.

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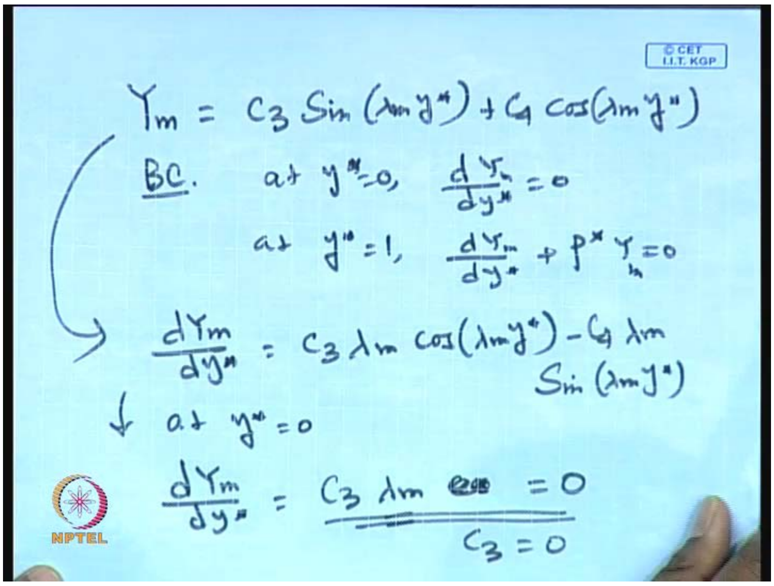

And let us look into the boundary conditions, the solution of this equation is constituted by the sine and cosine functions. And Y m becomes let us say some constant C 3 sine lambda m y star plus another constant C 4 cosine lambda m y star. And where the boundary conditions; boundary conditions will be the boundary conditions in the y direction of the original problem must be satisfy by the boundary conditions of this y varying part that means at y star is equal to zero. your d Y d y star will be equal to zero and at y star is equal to one, d Y d y star plus p star Y will be equal to zero, let us put the subscript m to, may be all the definitions consistent. So, if you do that, let us put the first boundary condition, at y star is equal to zero, d Y m d y star will be equal to zero, let us take the derivative of this d Y m d y star will be nothing but, C 3 lambda m cosine lambda m y star minus C 4 lambda m sine lambda m y star.

Now, in order to impose this boundary condition, you have to evaluate this derivative at y star is equal to zero. So, if you do that d Y m d y star becomes at y star equal to zero means sine varying part will be equal to zero, so you will be having C 3 lambda m cosine zero will become one, so that will equal to zero. And lambda m cannot be equal to zero, because lambda m is equal to zero then whole thing boils down to a trivial solution. Therefore, in order to satisfy this boundary condition, your C 3 must be equal to zero.

So, if your C 3 is equal to zero, the whole solution of y varying part becomes Y m is equal to some cosine variation.

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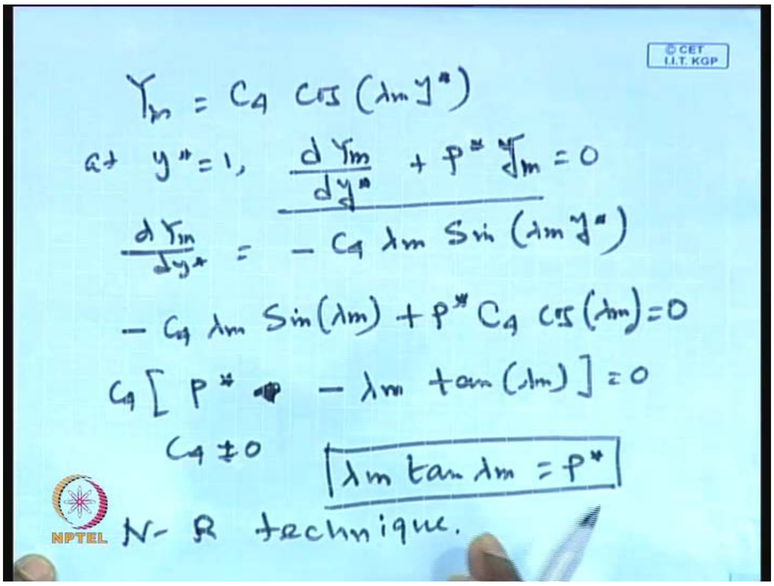

So, Y m is nothing but C 4 cosine lambda m y star. Now, let us use the other boundary condition at y star is equal to 1, you have d Y m d y star plus p star y m will be equal to zero. That means, let us get d Y m d y star means, it is nothing but minus C 4 lambda m sine lambda m y star. So, if you put this boundary condition here, what you will be getting is? Minus C 4 lambda m sine lambda m, that is the is this part evaluated at y star is equal to one, plus p star y m evaluated at y star is equal to one, that means this will be nothing but C 4 cosine lambda m will be equal to zero.

So, what you get? Your C 4 multiplied by p star plus minus lambda m tan lambda m will be equal to zero, I divide both sides by cosine lambda m. So you, now there are two probabilities, if either C 4 is equal to zero or whole term in the bracket will be equal to zero, if C 4 is equal to zero, you will be getting y m equal to zero that means you are going to get a trivial solution, but you are not looking for it. So, C 4 cannot be equal to zero that is ruled out. That means, what is equal to zero? You will be getting lambda m tan lambda m will be is equal to p star. Now, what is this equation? This equation called a characteristic equation or it is also known as the transcedental equation and depending on the value of p star, if whether it is 0.1 or 1 or 0.5 or whatever, you will be getting the solution of lambda m tan lambda m.

So, you can use by doing an iterative technique by like a Newton Raphson technique. And if you really plot this equation, you will find that this will give you the infinite number of solution. So, this will be first eigen value, lambda 1; this will be the second eigen value, lambda 2; lambda 3, lambda 4 likewise you will be getting infinite number of solutions.

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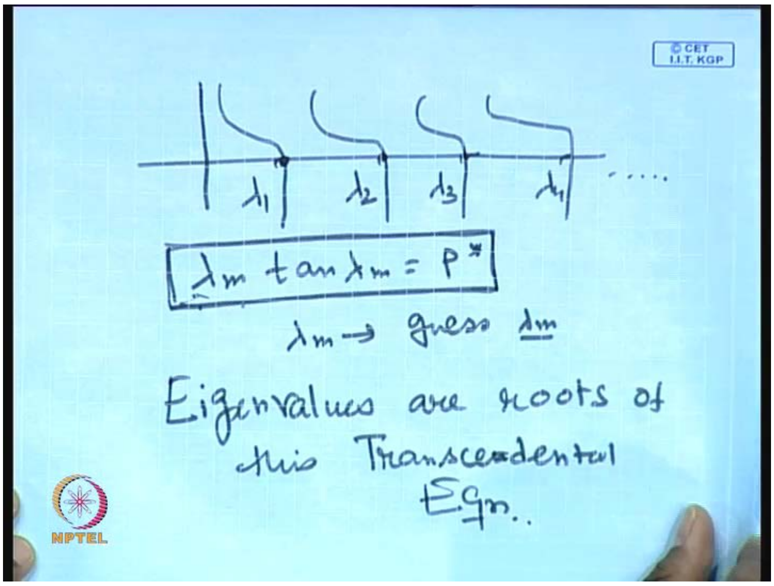

Now, how to the point is, how to compute these the solutions of the roots of this equation which are nothing but the eigen values to the problem, in fact that is a whether it is a very interesting problem. I just taught this thing in the last course, but you can also do it by yourself, what do you do? You depend, so p star will be known to you, let us say p star is equal to some value, given that value, you use a Newton Raphson technique with a guess value. So, guess a value of lambda m, let us say 0.5 or 0.1 or 0.1 whatever.

Now put in a Newton Raphson loop, you will be getting the first conversed value of lambda m that will be giving you the first value of first eigen value. Now, the typical characteristic of this equation is that the roots will be appearing in the arithmetic progression of around 3 or 4, it will be around 3. Therefore, you put another loop over the Newton Raphson, where the guess value, the second guess value will be, let us say previous one plus 3. So and using that guess value, if you solve this equation by using Newton Raphson, you will be landing up with the second root. So, the number of times you are iterating the outer loop, you will be getting number of number of eigen values. If you compute the outer loop, if you evaluate the outer loop four types that means, if you use the guess value four times with an arithmetic prorogation of 3, you will be landing up with first four eigen values.

Similarly, since it is in the computer is doing the work and you are not basically, you are just instructing it, you are instructed to compute the first ten values, so you will, it will compute the first ten eigen values and these things like that. So that is why, so eigen values are the roots of this transcedental equations,

Let us stop for the mathematics and let us come back to the actual problem, let us look, what is the expression of the final solution?

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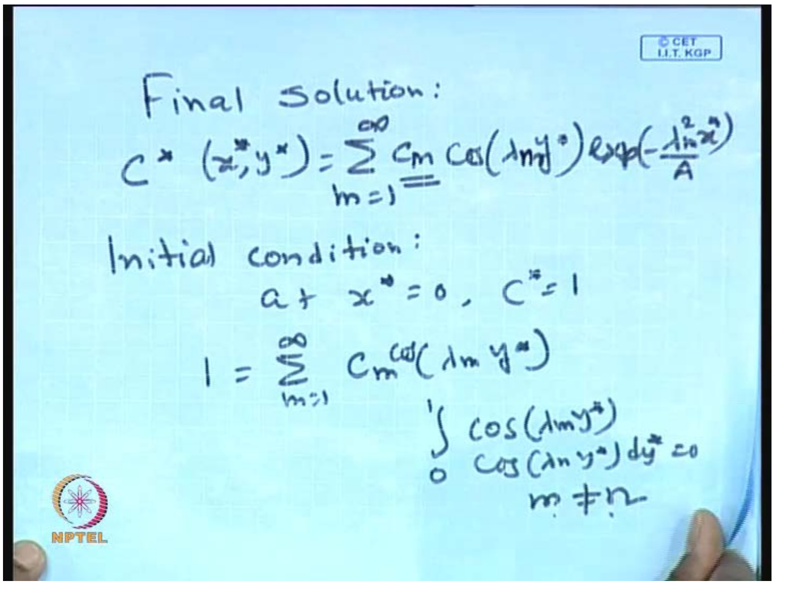

Final solution of concentration profile becomes C star x star y star is nothing but summation of c m, when the run c m cosine lambda m y star exponential minus lambda m squared x star divided by A, let us say, let us put it at the m. So, that everything becomes consistent, the index m runs from 1 to infinity.

So, it is a infinite number of series, there are infinite series with this form. Now, you have the initial condition, now the whole solution is now formulated, except the constant C m that will be obtained from the nonzero initial condition. That is the one condition, still we are leaving to utilize. The initial condition becomes at x star is equal to zero, your c star is equal to 1, so therefore you will be getting 1 is equal to m is equal to 1 to infinity C m cosine lambda m y star. Now, again I am not going into mathematical detail, in fact from this equation, this equation can be solved and you can determine the value of C m, if you utilize the orthogonal property of the cosine functions and sine function, Cosine functions are orthogonal to each other that means, integral of cosine, there is a cosine; cosine lambda m y star, cosine lambda n y star, d y star will be is equal to zero, from zero to 1. This is known as the orthogonal property of the cosine function that means you multiply both sides by cosine lambda m y star d y star and integrate across the domain of y star that is from zero to 1.

If you do that on the right hand side, if you open up the summation series, these orthogonal properties valid for lambda for m not is equal to n. That means, are you getting my point? So, you are multiplying both side by cosine lambda n y star d y star and integrate across the domain of y star that is from zero to 1 and then you open up the summation series on the right hand side. So, on the left hand side, you will be having only one term that is zero to 1 cosine lambda n y star d y star and the right hand, you will be having a summation series, you open up the summation series. Now, for all the terms when m is not is equal to n, this term will vanish, only one term will survive when m is equal to n. So, from that you will be getting you can you can evaluate the value of this constant C m.

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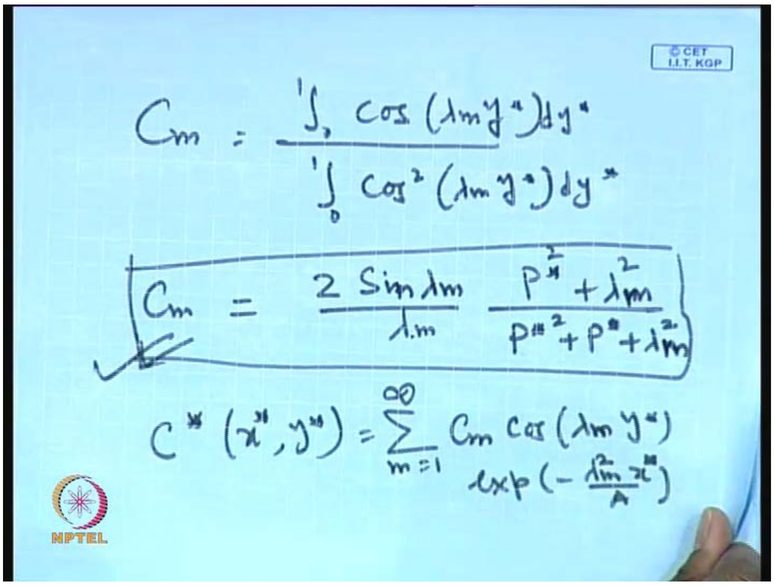

And this C m then becomes zero to 1 cosine lambda m y star d y star zero to one cosine squared lambda m y star d y star. Why it is cosine squared, because in that case n becomes equal to m, only one term will survive. So, cosine lambda m and cosine lambda n, so both becomes cos squared, when n is equal to n, all the other terms will vanish, because of the orthogonal property of the cosine function. Now, if you really carry out this integration, I am not; I am just leaving the couple steps to you, these becomes 2 sine lambda m divide by lambda m p star squared plus lambda n squared divided by p star squared plus p star plus lambda n squared lambda m.

Thes are straight forward integration, you can carry out this integration by yourself and you can check that the value of the constant C m turns out to be this, if you and since lambda m are known, because lambda m are the roots of the characteristic equation that we are going to evaluate numerically. So, sine lambda m will be known; lambda m will be known; p star is an is nothing but an operating condition depending on the characteristic of the membrane, so you can evaluate the constant C m. Once you will be evaluating the constant C m, the full concentration profile is now known to you. Because, by the function of x star and y star m is equal to 1 to infinity C m cosine lambda m y star exponential minus lambda m squared x star divided by A and A is the thing that we already know, it is 1 by 16 Reynolds Smith d e by l.

So, once you know the full concentration profile, because all the quantities are not now known to you. You can now in a position to evaluate the cup mixing concentration that means you are going to do a y averaging, what is concentration profile and can get the cup mixing concentration.

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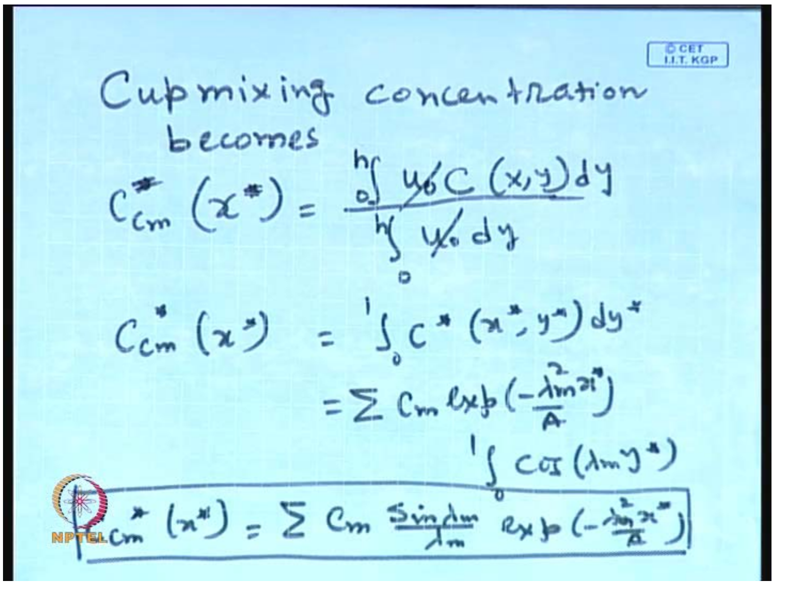

So therefore, the cup mixing concentration becomes C c m, star which will be a function of x star is nothing but u naught; that is constant in this case, C x y multiplied by d y divided by u naught d y zero to h; zero to h, these these no star, let us say these in dimensional form. You can just divide by c naught everything becomes star, in fact u naughty; u naught will be canceled out, because it is a plug flow velocity, so it will becomes constant.

So, you can you can really do that and find out the cup mixing concentration and the cup mixing concentration as a function of x star, it becomes C c m star, it becomes you know zero to 1 c star x star y star d y star and this is nothing but summation of c m exponential minus lambda m squared x star over A integration zero to 1 cosine lambda m y star and if you really evaluate this integral, the whole expression of C c m star becomes summation of c m sine lambda m over lambda m exponential minus lambda m squared x star over A.

After scanning out this integration, you will be in a position to get the value of cup mixing concentration. So, once you get the cup mixing concentration then the rate of removal of pollutants will also be known at a function of x star.

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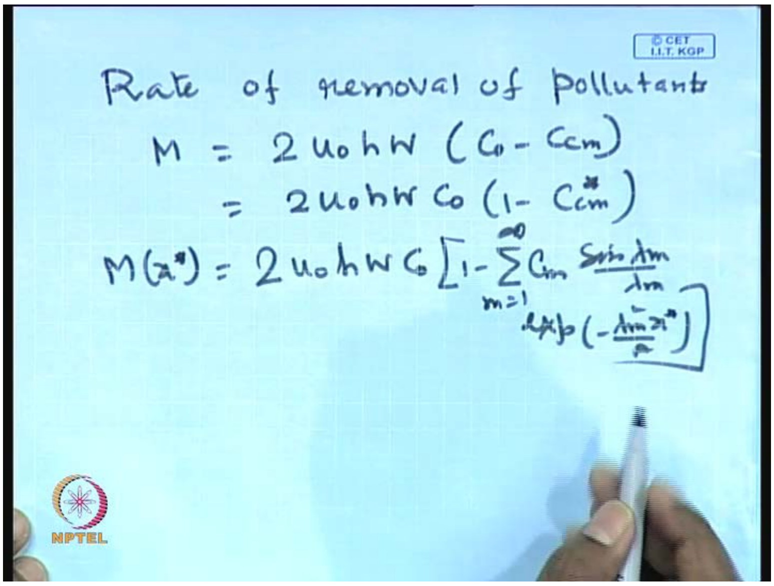

If you remember, the definition that we have put for the rate of removal of pollutants that is m is 2 u zero h w times c zero minus C c m. So, you can just divide by c naught and this becomes 2 u zero h w times c naught one minus C c m star and you can substitute the expression of C c m star as 2 u zero h w c naught one minus is summation c m sine lambda m by lambda m exponential minus lambda m squared x star divided by A.

So, you will be in a position to get how the removal rate is varying as a function of x star and you can define a particular value of removal less than ninety percent or ninety five percent and you can estimate, what is the value of x star? You will be requiring to get that to to to to find out the length of the dialyzer. Now, the mass transfer coefficient becomes very important and if you for the design of this and if you look into the; let us let us try to establish the expression of mass transfer coefficient, since this problem it has been simplified to almost an analytical solution, the expression of mass transfer coefficient will also becomes very very very simple.

And one more thing is that if you, in order to get; in order to evaluate the summation series as we have discussed earlier, the values of lambda m becomes very high, as you go for the second eigen value, third eigen value or second term, third term in the series, square becomes further higher, so these exponential terms becomes lower and lower. So, you can compute the first term or first couple of terms that is good enough, you need not to go to compute, let us say first ten terms or first twenty terms; first two terms will be good enough, because the exponential term becomes smaller and smaller, they will be contributing less and less towards the summation series as you number of term as the number terms are getting increased.

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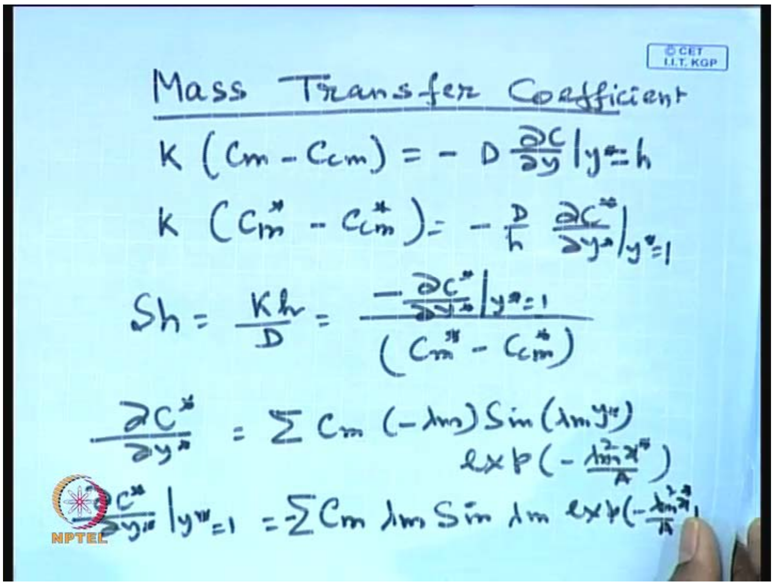

Now, in the next, we evaluate the mass transfer coefficient and we start with the definition k is nothing but c m minus C c m, in this case the bulk concentration is replaced by the cup mixing concentration minus D del c del y at y star is equal at at at y is equal to h, because our coordinate system now is at the middle. So, in terms; in terms of you just make it non dimensional, why you defined y is y is y star, you define as y by h, so this becomes k c m star minus C c m star will be nothing but minus D by h del c star del y star at y star is equal to 1.

So, Sherwood number is defined as k h over D, for this particular problem. And in the non dimensional quantity, it will be del c star del y star at y star is equal to 1 divided by c m star minus C c m star. So, that is the definition of Sherwood number and you can put the values and if you if you write down the first, for the first eigen value then see what you get, del c star let us let us first evaluate del c star del y star; del c star del y star will be nothing but summation of c m minus lambda m sine lambda m y star exponential minus lambda m squared x star over A. And del c star del y star evaluated at y star is equal to 1 becomes summation of c m lambda m with a negative sign, sine lambda m exponential minus lambda m squared x star divided by A. So, that is del c star del y star evaluated at y star is equal to 1 that will come to the numerator.

Now let us write down, let us just compute this expression or this you know, the Sherwood number for the first eigen value.

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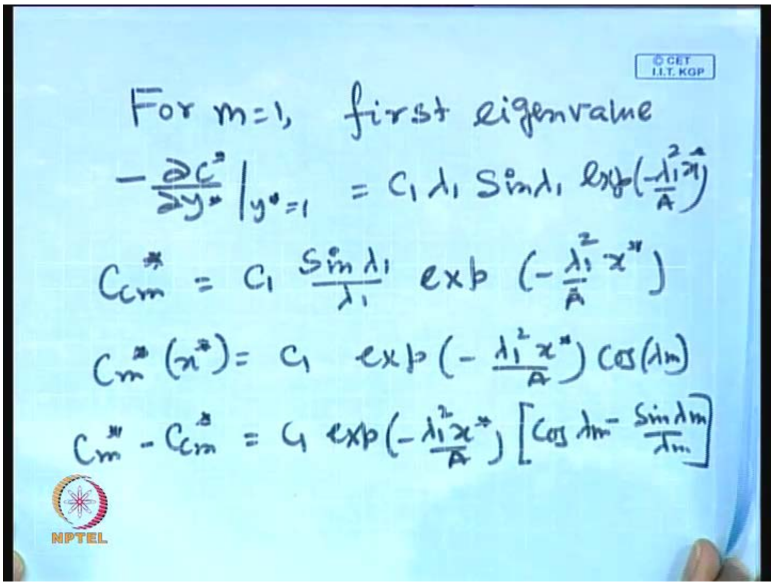

You will be you will be getting a neat solution for n is equal to 1. That means, for first eigen value, you will be getting minus del c star del y star at y star is equal to 1 as c 1 lambda 1 sine lambda 1 exponential minus lambda 1 squared x star over A, C c m star that is means the cup mixing concentration, it becomes c 1 sine lambda 1 divided by lambda 1 exponential minus lambda 1 squared x star by A. And what is c m star x star? That is at, when it is at y star is equal to 1, so c m star becomes c 1 exponential minus lambda 1 squared x star over A cosine lambda m, when y star equal to one.

Now, c m star minus C c m star, this becomes c 1 exponential minus lambda 1 squared x star over A times cosine lambda m minus sine lambda m over lambda m. Now, you put all the values in the definition of Sherwood number then you will be landing up with the Sherwood number, when m is equal to 1, for m is equal to 1, that is a first eigen value.

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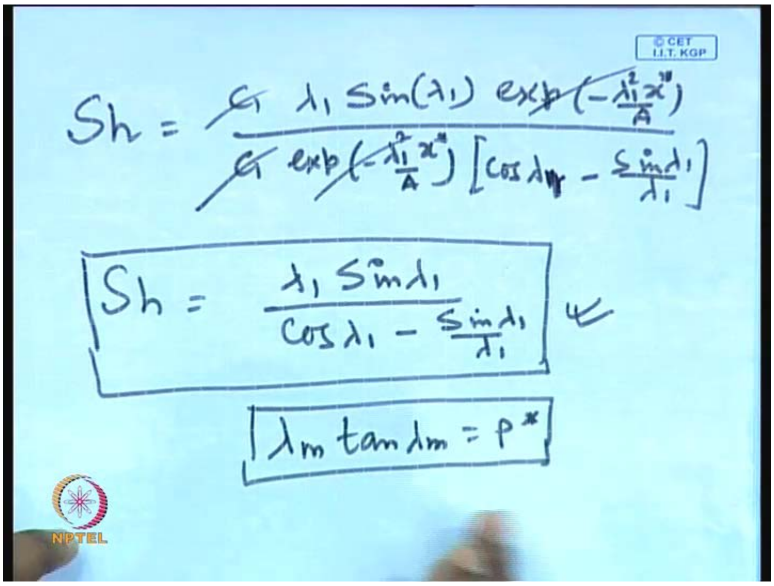

So, if you do that the expression of Sherwood number becomes c 1 lambda 1 sine lambda 1 exponential minus lambda 1 squared x star over A divided by c 1 exponential minus lambda 1 squared x star over A times cosine lambda m lambda cosine lambda 1 minus sine lambda 1 divided by lambda 1.

Now, c 1 c 1 will be canceled out and this exponential term will also be canceling out. So, the expression of Sherwood number becomes lambda 1 sine lambda 1 divided by cosine lambda 1 minus sine lambda 1 divided by lambda 1. So, this is the value, this expression of Sherwood number will be obtaining and if you remember, the lambda 1 will be a function of p star. If you remember, it was lambda m tan lambda m is equal to p star, I think that is the expression. So, the first eigen value, so it will be depending on the value of p star, so depending on the value of p star, the first eigen value from this expression that you are going to get take you using that one can get the value of Sherwood number computing the first eigen value. You can compute, let us say first two or three terms and can get the overall expression of Sherwood number and what is the mass transfer coefficient that will that will be appearing in this particular system. I have given this example for the purpose that, all though this is this is an unrealistic case, because most of the dialysis operations are undertaken, you know under laminar flow conditions. But, this is not only a turbulent, this is a very high turbulent. So that, the turbulent velocity profile can be constituted as a uniform velocity profile plug flow.

I have given this example simply, because if you really look into the actual case, where that where the laminar profile is existing, the mathematics become very complicated. And in order to show you that under the certain simplification, how the mathematics comes out. Now probably with these example, it will be very clear to you, the various mathematical entry cases involved in this calculations. So, you can go back and now really confidentially solve the actual problem that whatever we have done in the previous example.

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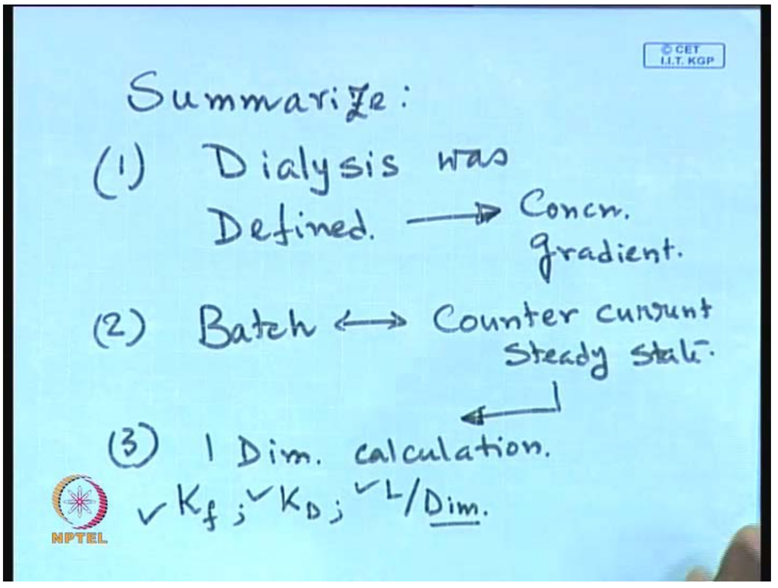

So that covers the whole dialysis operation. So, let us summarize, whatever we have done till now. The first we defined the… define the dialysis process was defined. And here, we said that it is a process, which is given by the concentration gradient only. Now, we talked about two dialysis processes, one is the batch; another is the counter current steady state. So, our aim is to design or you to evolve the, to get the design equations for this counter current steady state dialyzer. Because, that is most common and most important and most useful one.

So, what we did? We identified the, whatever the resistances that will be occurring for during your calculations. So we, first we take request to a one dimensional calculation. In this one dimensional calculation, we assume that concentration is a function of x only and concentration is not a function of y that means we are assuming that same concentration is prevailing over the cross section of the channel.

Second one is, the velocity is also considered to be uniform and we are basically computed several parameters in terms of cross sectional average velocity. Now, we identified various resistances or mass transfer coefficient; mass transfer coefficient is nothing but the inverse of the film resistance. So, we identified the film mass transfer coefficient in the feed side, we identified the mass transfer coefficient in the dialysate side and the membrane resistance, a lower D i m.

Now, these three resistances are basically put in series and you will getting the overall mass transfer coefficient or overall resistance. Then, we carried out an analysis, these exactly like counter current heat transfer problem. And we ultimately obtain a design equation, which is nothing but a log delta c l m t d sort of thing, log mean concentration difference, delta c l m t d sort of thing. So, it is instead of log mean temperature difference becomes log mean concentration difference.

So, by looking into those equation and if you carefully, you know calculate these three parameters, one will be able to obtained or design the the the dialyzer. But, among these three quantities, it will be easier to calculate the k f and k d, because the Sherwood number correlations are available to you. So, one can use the operating conditions in the feed side, I mean the dialysate side, one can use the geometries of the feed chamber and dialysate chamber can easily find out, the mass transfer coefficient in the feed side as well as in the dialysate side.

On the other hand, although the it is not a problem, it is not a much of a problem, which now a days, the analysis analytical facility that we are having in the laboratories, it is very easy to calculate the or estimate the thickness of the membrane. But, it will be extremely difficult to calculate the solute diffusivity within the membrane phase.

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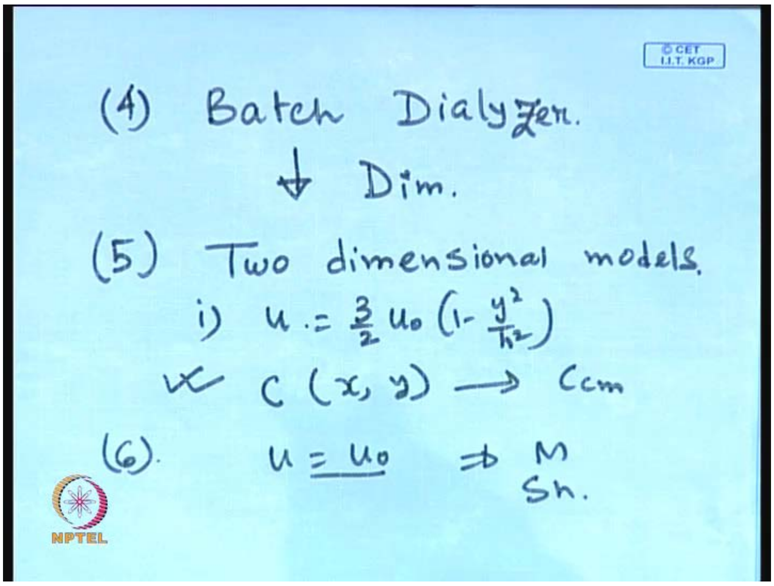

So, in that case, what we can do, we can take request to the batch dialyzer. And carry out a simple experiment in the batch dialysis cell and keep on monitoring the concentration of the solute in the dialysate side. By the suitable plot of this dialysate concentration is a function of time, one can estimate the value of D i m more accurately. In that, we have we have elaborate one another method to estimate the value of D i m, but that contains some parameters which are difficult to estimate.

Then we, so our one dimensional counter current buzz dialyzer was all over. Then, we identified the you know short coming of that and we went for a two dimensional models or more accurate models. In two dimensional model, we assume that, first we assume that velocity is laminar and there exist a velocity profile in the in the channel and which is nothing but a parabolic velocity profile. So, using this we obtain the concentration is a function of x and y. We define the percentage efficiency or the amount of solute that has been you know removed. And based on that, we we define the cup mixing concentration and obtain how the removal efficiency is varying as a function of x star. By defining a particular removal efficiency, let us say fixed at ninety eight percent or ninety five percent. we will be we will be in a position to calculate, what will be the length of the dialyzer.

Then, in order to explain the mathematical intricacies involved in this process, we simplified this, we have step number 5, assuming a uniform velocity profile u naught and completely solve the problem, that is a class room problem can be solved using the analytical tools. And we have obtained the the value of you know, the removal efficiency and the Sherwood number in this case. So, after whoever will be doing these courses or these or these lectures on dialysis etc., after doing this class, I will be, i am just expecting that each of you will be able to at least design a dialyzer, given the conditions geometry and the of the of the and the removal efficiency either one dimensional case and you can go for the two dimensional analysis. And you can compare, what is gaining? What is what accuracy you are gaining? If you go for higher dimensional modeling or design a questions.

So, I will stop here in this class, the next class onwards I will be solving some of the example problems, which will be very crucial as for your examinations concerned. So, most probably in the next week, I will be getting couple of classes, we are solving some example problems only.

Next week.

Thank you.