Novel Separation Processes
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Lecture No. # 17
Membrane Separation processes (Contd.)

Good morning every one. So, we are discussing about the dialysis and in the last class whatever you have seen you have seen the how the design equation of a continuous dialyzer or derived and how they going to utilize and what are the various internal parameters that will be require to design this dialysis to to to design the dialyzer. And we looked in to that from the flow geometry and the flow region, one can get the mass transfer coefficient in the feed side and in the dialyzed side, and there are the resistor it is offered by the membrane phase they only one parameter that requires separate set of experiment to be conducted that is the diffusivity of the solute to the membrane phase. For that we have one can one can undertake and experiment which is a in a batch mode, which is pretty is ready to conduct and using the batch mode experiment and the and the data that is obtain form the let the concentration profile that is obtained from the dilysate side one can rearrange the equation and can plot the concentration profile suitably and one can get the value of diffusivity through the membrane phase.

Now, in today class first will look in to one aspect of the batch dialyzer in batch dialyzer. What are happens the when the console solute concentration from the feed gets transported to the dialyzer side, then what happens the concentration in the dialyzer side those are increasing. So, as time of operation progressive the concentration difference are concentration gradient decreases. So, in order to prevent that one can use some solutes in the dialyzer side which will react to the solute the will be getting transported to the dialyzer side. So, therefore, the concentration of the solute in the dialyzer side will always the maintain that 0 level. So, that will maintain the maximum concentration different.

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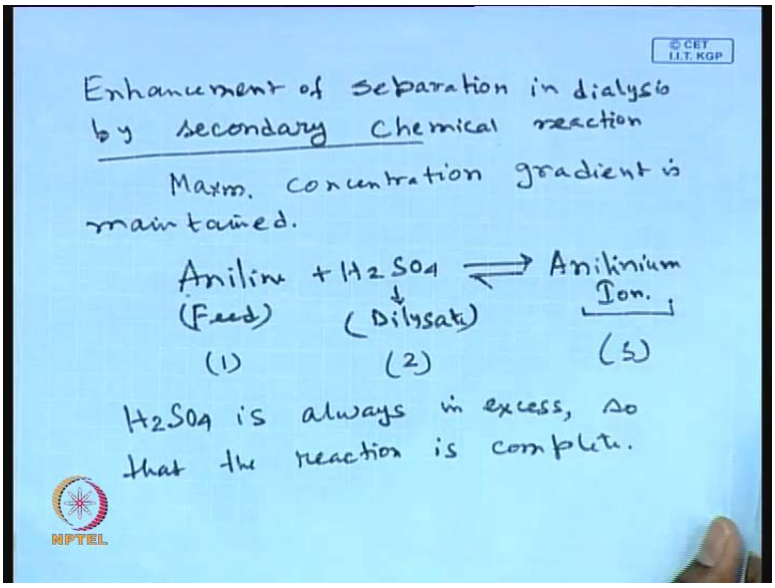

So, the transport of the solute to the dialyzer will be enhanced. So therefore, today will be first talk about the enhancement, of separation in dialysis by secondary chemical reaction, will allow a secondary chemical reaction to occur in the dialyzer side in the solute that is coming from the feed chamber. So, that the concentration prevents you the 0. So, in the dialyzer chamber you have to select the dialyzer such that the diffused components react and product ions are not cannot diffused, but therefore, a maximum concentration difference maximum concentration gradient is maintained. I just take up one example, aniline plus H 2 S o 4 gives you anilinium Ion it forms a complex. So, if you like to remove aniline from the feed side and in the dilate. So, aniline is there in the feed side and in the dialysate side will be having sulphuric acid solution. So, aniline causes over the membrane and comes to the dialyzer side it reacts with the sulphuric acid present in the dialyzer side forming the anilinium ion.

This anilinium ion is bigger in size it cannot diffused back to the feed chamber. So, therefore, in the dialyzer side the concentration of aniline is will be always the aniline the minimum level. So, that that provides a maximum concentration different at towards the membrane called is as component 1, this is the component 2, this is the component 3 and component two; that means, H 2 S o 4 always in excess size of the reaction always complete is always in excess. So, that reaction is complete. So, that is the cells of the of the idea is the aniline diffuses through the dialysis membrane comes to dialyzer side immediately react to sulphuric acid forming the anilinium ion. So, therefore, in the

dialyzer side you are maintaining a 0 concentration of aniline. So, therefore, the concentration gradient across the membrane is maintaining a maximum.

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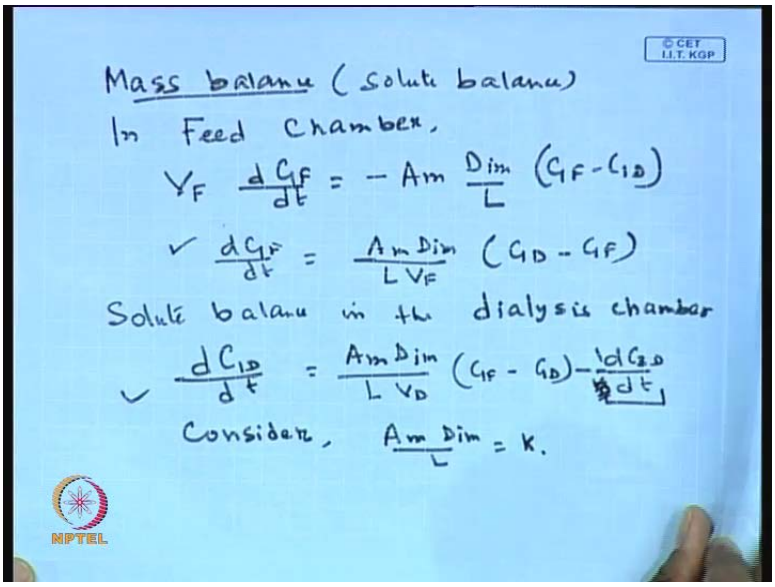

Now what will do will right a mass balance equation in feed chamber mass balance use in terms of solute balance basically, in feed chamber v F d C 1 F d t is equal to minus A m d I m over L C 1 F minus C 1 D 1 represent the aniline concentration. So, therefore, this is the aniline mass balance in the feed chamber the accumulation that is there it is basically negative the accumulation, because in the feed chamber the amount of aniline is decreasing C 1 F and C 1 D concentration of the aniline in the feed and the dialyzer chamber respectively at any point of time.

This equation can be rearranged as d C 1 F d t is equal to A m d I m over L times v F C 1 D minus C 1 F this minus term this absorbed within the bracket and aniline balance of solute balance in the dialysis chamber becomes d C 1 D d t is equal to A m d I m and L times v d C 1 F minus C 1 D d C 3 d over d t. So, in the dialyzer chamber for what you have we have we have the you are the came in the aniline concentration that is coming from the feed size as well as there is the zinc term present in the dialyzer side which will indicates will be v d this which will be which indicates that there will be decrease in the this basically consumption in the aniline, because the presence of formation of the anilinium ion it reacts with the sulphuric.

So, there is the see term present there. Now it considered it defined A m by d I m over L as the constant k. So, these 2 equations and they should not be allowed d t there. So, this 2 equation can be retail in a much more compact fashion and in terms of k.

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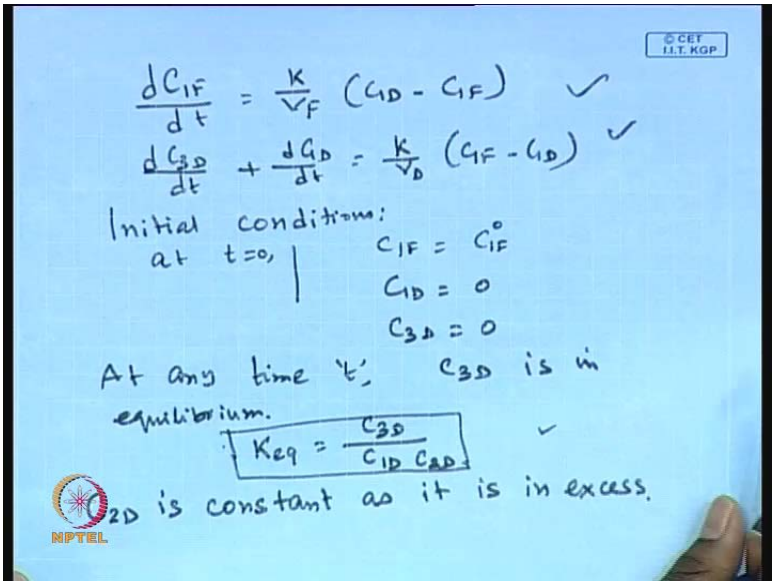

So, if you do that this 2 equation becomes d C 1 F d t is equal to k divided by v F C 1 D minus C 1 F d C 3 d d t plus d C 1 D d t is equal to k by v d C 1 F minus C 1 D and subscribe F stands for the feed side and subscribe D stands for the dialyzer side. Now you have the initial condition. So, this is an ordinary differential equation and the initial conditions as specified by the initial condition are at time t is equal to 0. You have C 1 F is equal to C 1 F naught; that means, initial concentration as C 1 F naught at time t is equal to 0 you have C 1 D is equal to 0; that means, there is at the end at the start of the process there is no solute crossing over to the dialyzer side and C 3 d is equal to 0, because there are no solute present in the dialysis size that is no chance of having a reaction. So, the component 3 that is the aniline ion the concentration of that will be equal to 0 at time t is equal to 0.

Now, at any time in t at any time t the concentration of anilinium ion in the dialyzer side is in equilibrium, because the reaction is equilibrium. And you have the equilibrium in terms of equilibrium constant you can express the concentration as C 3 d divided by C 1 D times C 2 d. So, this concentration of anilinium ion this is the concentration of anilinium dialysis side, and 2 is the concentration of sulphuric acid in the dialysis side

this equilibrium relation gives you the concentration. So, that will be the constant for particular temperature. So, this gives a relationship between the anilinium ion and concentration of the aniline in the dialyzer chamber where this equation. Where C 2 d is the concentration of sulphuric acid since it is in excess this concentration will be prevail as constant. So, C 2 d is constant as it is in excess. So, so the therefore, we can we can solve this 2 equation and expression C 3 in terms of C 1 b and equilibrium constant in this equation. So, let us see how will do that.

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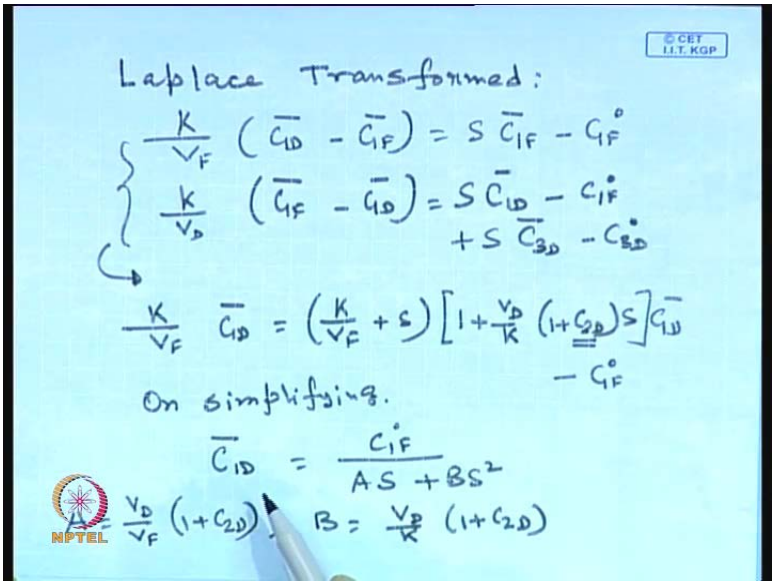

So, what will take again we take the Laplace transform this is the coupled worries. So, take the Laplace transform equation and the transformed and we can work with the transform equation if you take the Laplace transform the following to result following 2 equation are resulted k over v F times C 1 d minus C 1 F bar this bars transfer the transform quantities s C 1 F bar minus C 1 F naught k by v d is C 1 F bar minus C 1 D bar is equal to s C 1 D bar minus C 1 F naught plus s C 3 d bar minus C 3 d 0. So, this will be the Laplace transform of the 2 equations will result this equations and 1 can simplify this 2 equation and can get a expression of C 1 D bar.

So, because basically you are going to monitor the concentration of the aniline and the dialyzer side will just check up a sample and analysis. So, if you do that will be get in k by v F C 1 D bar is equal to k bar v F plus s into 1 plus v d over k 1 plus C 2 d times s C 1 D bar minus C 1 F naught and remember that the thing with in the 3rd bracket becomes

completely constant, because C 2 b is an excess. So therefore, can be detailed as constant.

You can you can rewrite this equation on simplification and simplifying finally, you can get and expression something like this C 1 D bar is equal to C 1 F naught is equal to s divided by a s plus b s square where the term as is defined as v d by v F 1 plus C 2 d since v d and v F of the curve values of the feed on the dialysis chamber and C 2 this constant the value of a is also constant v is equal to v d over k 1 plus C 2 d. Now this equation can be the terms in the denominator can be represent in the form of 1 plus 1 over s minus 1 over the 1 plus k plus something like that. So, what I mean, this fraction the denominator can be represented as the summation or subtraction of of the real fraction. Then 1 can take the inverse Laplace transform and can get the expression of how the concentration of sulphuric acid is varying as the function of time in the dialyzer chamber.

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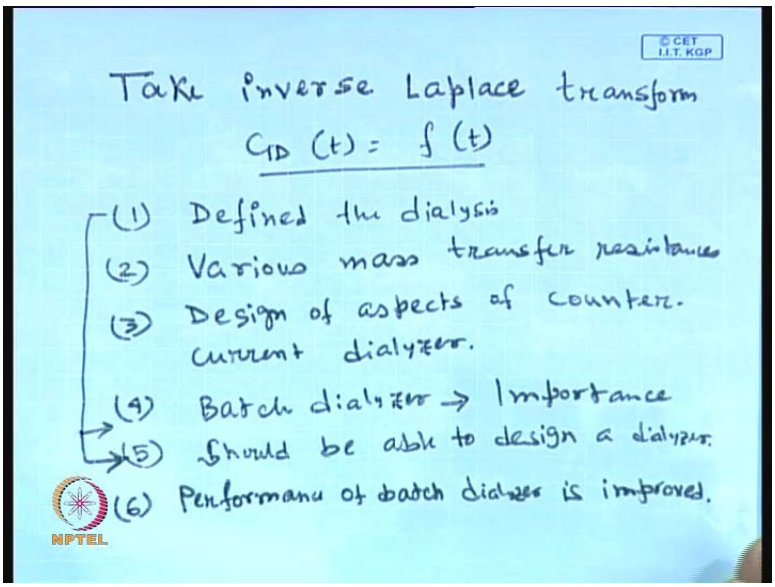

So, one can take the inverse Laplace transform and one can get the C 1 D as the function of time and the doing that replace carryout this exercise the by the yourself and find the concerned profile I the of of the solute in the dialysis side. So, let us now summarize whatever we obtain till now we have got the definition we have we have defined the dialysis process and we have analyzed the various mass transfer resistance that will be coming across the dialysis process. This mass transfer resistances then we are looked in

to the design aspects of design aspects of counter current dialyzer dialyzer in the counter current dialyzer you have found out there are 3 resistance is the that have to be determined 1 is the mass transfer coefficient that feed side mass transfer coefficient in the dialysis side and membrane resistance out of this 3 to can be easily obtain the looking in to the flow region and expression of Sherwood number relation.

Now, on the other hand the parameter the diffusivity through the membrane phase from the estimated by a separate set of experiment in a batch dialysis. Batch dialyzer and how important this batch dialyzer is, because the importance is that it gives you the concentration it gives you important parameter that you the diffusivity of the solute through the membrane phase and which will be utilize in the design equation of the continuous counter current dialyzer.

So, using this information step number 1 into 4 will be able to really design should be able to design a dialyzer. And today you have just large how the performance of batch dialyzer is improved. So, what we have to do that case in that case in the dialysis chamber you gives some some material some chemical which will be reactive with the solute that is given in transport in to the dialysis side. So, that the concentration of solute will be always maintenance 0 and a maximum driving force will be maintained all the time.

Now, the design of continuous dialyzer in the counter current mode that whatever the than till now that is 1 dimensional model, but that is good enough to get a first information from the design of the dialyzer. Now for the for the sake of complete completeness and for the scientific interest one has to go in to the two-dimensional model more complicated model of the steady state dialysis process. Why it will be required it is required, because the channel the dialyzer channel and the feed channel that whatever have actually in 1 dimensional model you considered in the velocity profile is flat; that means, is something something like a plug flow. So, talking about the cross sectional average velocity are we are evaluating the mass transfer coefficient exedra from the cross sectional average velocity also we are assuming that concentration profile that is existing in the system in the channel is always in in in this 1 dimensional it it is called as ten.

In fact, the bulk concentration constant there is no profile existing their which is not the actual case if you talk about more 75 terms both velocity feed and concentration feed even the fully develop region do you function of you know why are in the truancies direction in the channel. So, in the one dimensional case we are neglecting the variation of the concentration and the velocity profile, when the two dimensional case you will be including the actual velocity profile and concentration profile in the in the channel. In fact, you have you have is the fully develop velocity profile and will be deriving the concentration profile and from that expression will be doing length, you know cross sectional averaging is called cross section averaging that is called cup mixing concentration or cup mixing temperature. You remember the concept of the average temperature or concentration in case of heat transfer or mass transfer.

In an actual case is if you would like to take put the sample of if you like to monitor the temperature particular location, or if you like to measure a concentration you cannot get the instantaneous you know point point value of the temperature or concentration. So, what we have to do you you get you you can able to measure that will be the cross sectional average value. So, once you get a profile you have to average it out over the cross section in the y direction, and that particular value after the averaging is known as the cup mixing temperature in case of retransfer or cup mixing concentration in case for mass transfer. So, these are the memorable quantities. So, so will be getting the expression of that and form that will from the first principle will look in to the dialysis and performance and it is efficiency and from that we can design a dialyzer more accurately.

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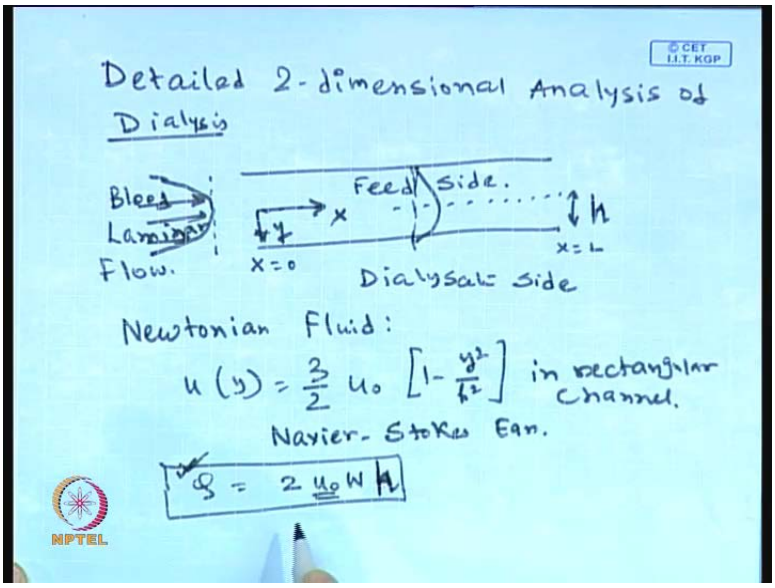

So, let us look in to the detailed 2 dimensional analysis of dialysis. So, the x starts from here and is the 12 this is the dialysate side and this is the feed side. The blood enter sphere with a profile from the like this, it is a fully developed parabolic velocity profile blood going in to the system under the laminar flow condition, this is the half channel height let say is h and the will put hour is set of coordinate system in this case in the middle of the channel that will that will simplify our life as for as the mathematics concerned.

So, that is in the centre and the concentration profile is also parabolic 1 if you solve you will be getting a more or less parabolic concentration profile. So, in the earlier case we have consider the velocity profile is plot on the concentration profile also plot. So, this is a typical dialysis the concentration term velocity profile in a rectangular geometric dialysis dialysis system, for a Newtonian fluid the velocity profile becomes u times y is equal to 3 by 2 u naught 1 minus y square by h square, that is the profile of the velocity in rectangular channel.

So, let say h small h is the half height of the channel, and y is the middle plane. So, this curves from the Navier strokes equation solution of Navier strokes equation for equation of continuity. And we also assume that the channel width is very large compare to the channel height, and mass transfer is therefore, 2 dimensional. Now q is 2 u naught w times h, what is this q. Thing q is the flow rate of the plug and u not is the cross section

average velocity and should be multiplied by the area cross section w times 2 h w times 2 h we need to the small h half height w time P h is the full height. So, that give you 2 w h is the full cross sectional area which is normal to the direction of the flow of the material. So, you can put the Rota meter in 1 of the lies and can measure the velocity over the flow rate and divide the flow rate by the cross sectional area, because the channel geometric exedra known to you. So, you can estimate the value of u naught.

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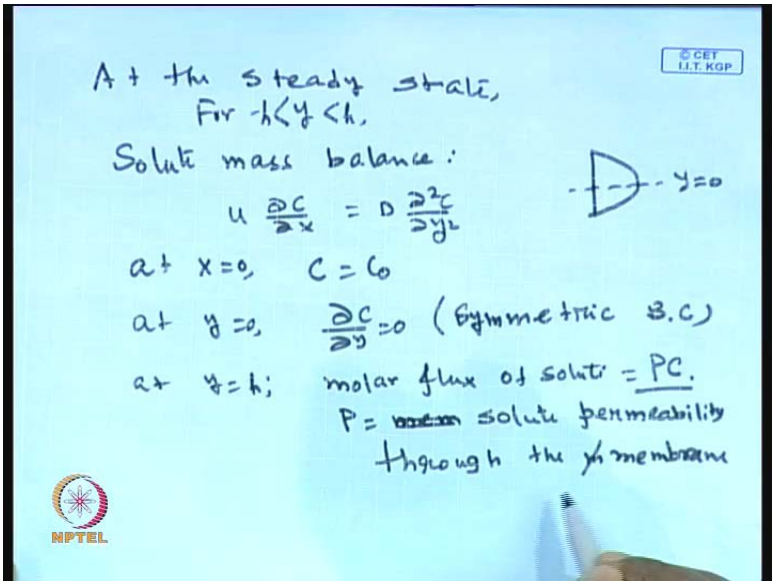

Now, at the steady state at the steady state you can write down the solute solute mass balance, for y lying between minus h to plus h. You write down the solute mass balance that be the simply u Del C del x is equal to d Del square C del y square. If you remember why a talking about the pressure driven membrane the separation process is will note down the concentration balance equation within the mass transfer boundary area there are an extra term layer that was plot v del C del y. Why that term they was appearing the term was appearing, because there is a trans membrane pressure drop, because of the trans membrane pressure drop there was convective flow of the solute towards the membrane surface, but in the case of dialysis since there is known trans membrane pressure drop it is the driving force is only concentration gradient the velocity component in the y direction does not exist.

So, it will be u del C del x is equal to d del square C del y square and if the term on the left hand side in the second term which appeared earlier that will be turning out to be 0.

So, we required to have 1 boundary condition at x is equal to 0 and 2 boundary condition at on y. So, at x is equal to 0 we had C is equal to C naught that is the feed concentration that is going in to the system and at y is equal to 0 you have del C del y will be equal to 0; that means, the concentration profile is undergoing a maximum at the center line this is also known as the symmetric boundary condition. So, we have the concentration profile something like this. So, the maximum will be going the at the middle of the channel. So, at the middle of the channel since it is maximum Del C del y equal to 0 and at y is equal to h. Let us say the molar flux of the solute solute is given as P times C what is P is nothing but the membrane permeate solute permeability solute permeability through the membrane.

So, and and that has to be this solute permeable through the membrane; that means, P times C and we have assuming that concentration in the dilaysate side will be equal to 0, because the dialysate is maintained the efficiency high speed of the whatever the solute that is travelling to the dialysate side immediately it will be wash it away. So, therefore, through through the molar flux of solute through the membrane should be P time C mi C at the membrane feed interface minus membrane dialyzer interface, but we are assuming that concentration will be equal to 0 in the dialysate side, because the dialysate flow rate is maintain sufficiently high velocity. So, that it becomes will be get it it it becomes 0 and an it will be it will be watched away that simplifies the whole analysis otherwise one can put that and whole thing have to you solved numerically.

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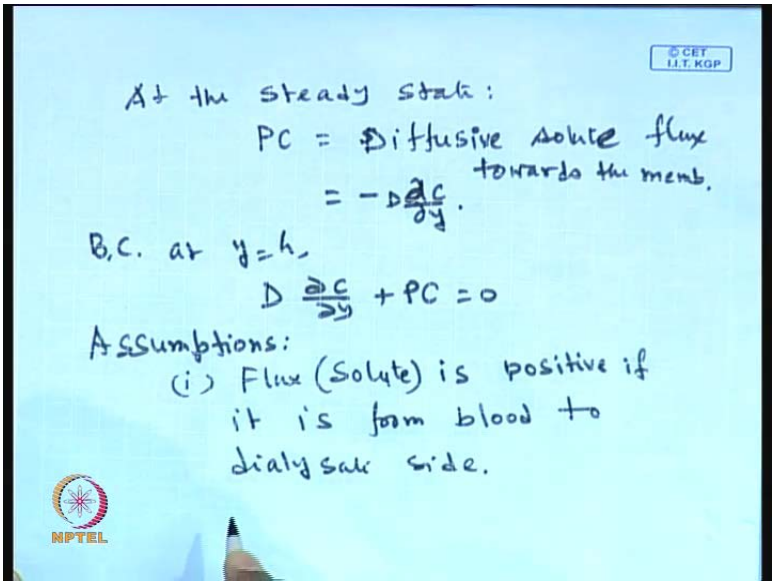

So, let us look in to and that at the steady state at the steady state that has to be P C there is the it has to be matched by the solute diffusive solute flux solute flux towards the membrane. What is diffusive solute solute flux that will be minus d d C del C del y minus d del C del y there is the fix law so; that means, amount of amount of solute per unit time per unit cross section area that will arriving of the membrane surface it will be immediately getting transported through the to the other side by the permeate the solute permeability, because of the solute permeability present there. So, solute permeability P is a known quantity for u.

So, now let us write down the so therefore, the condition the boundary condition at y is equal to h becomes d Del C del y plus P C will be equal to 0. And if you remember the this become almost the same boundary condition of the pressure driven membranes special process is reverse osmosis ultra filtration exedra everything now this missing is they are we had C minus C minus C minus C p, but here we will will will not be having C minus C b, because we are assuming C b will be instantly will be equal to 0. So, let us write down the assumption that we have made to be analysis one is solute flux is positive, if it is from blood to the dialysate side and blood side is nothing,

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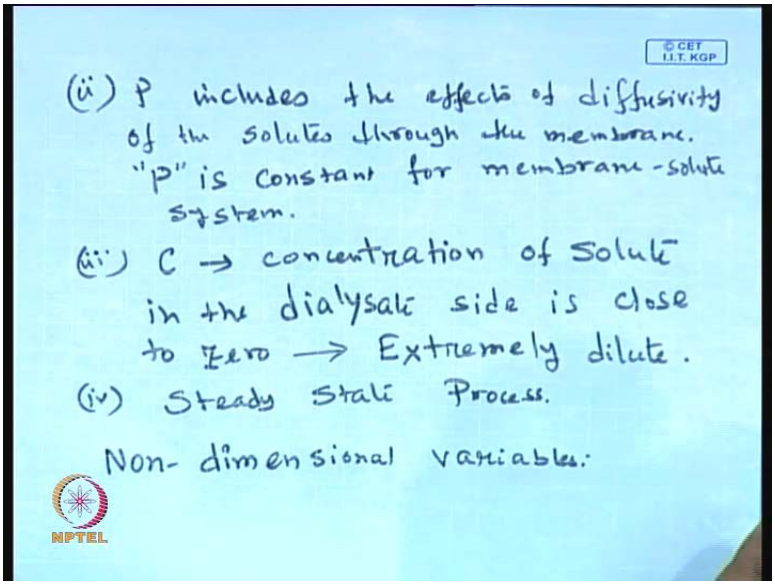

but the feed side. The second assumption is the parameter P includes the effects of diffusivity of the solutes through the membrane; that means, the term d I m by L we are couple we are we are just coupling the term d I m by L in to the parameter P which will

be constant for a membrane solute system; that means, P is inherent P is constant for membrane solute system. 3rd is the C where concentration of solute solute in the dialysate side is close to 0; that means, it is extremely dilute. So, this are the various and of course, the steady it is a steady state process. So, now we are in a position under this set of assumption we are you can constant the governing equation of the boundary conditions once you do that now, let us right down in terms of non dimensional parameters and make the equation non dimensional. So, that the mathematical complication will be much much simpler.

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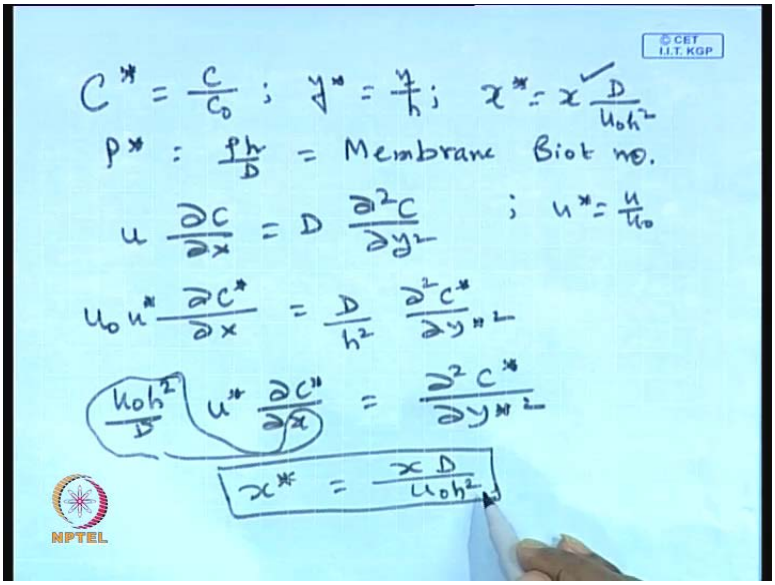

So, it defined a non dimensional terms variables as C star equal to C by C naught y star equal to y over h and x star is equal to x times d u 0 h square and P star is equal to P h over d that is known as the membrane biot number. How you obtain this x star if you remember the governing equation as Del C del x is equal to d del square C del y square and this talk about obvious the definition of C star, because you know the concentration C naught at the entry point and the height of the channel is known. So, therefore, the dimension in the y direction is known to you. So, therefore, you can put this non dimensional variable there. So, this becomes Del C star del x is equal to d divided by h square del square C star del y star square this is a u there. So, you can you can defined as u star is equal to u by u naught. So, therefore, it will be u naught u star. So, multiplied by both side h square by d.

So, becomes u 0 h square by d u star del C star del x is equal to del square C star del y star square. So, on the right hand side is fully 1 dimensional the left hand side u star is non dimensional C star is non dimensional; that means, this has to be a non dimensional quantity. So, it defined x star as x multiplied by the d divided by u 0 h square, that some the non dimensional term of you know x star will appear in the particular form. So, there is go for any non dimension of any governing equation, because if a if there are some non dimensional variable will be code obvious to, because in the some of the variables are coming from the operating condition some of them are known from the geometric you can defined those variables, but if some of the non dimension variable is not apparent through, because substitute the known non dimensional very variable there and try to neck the whole equation non dimensional and the form of non dimension variable which is not apparent to that will be automatically emergent. So, you can do that as well.

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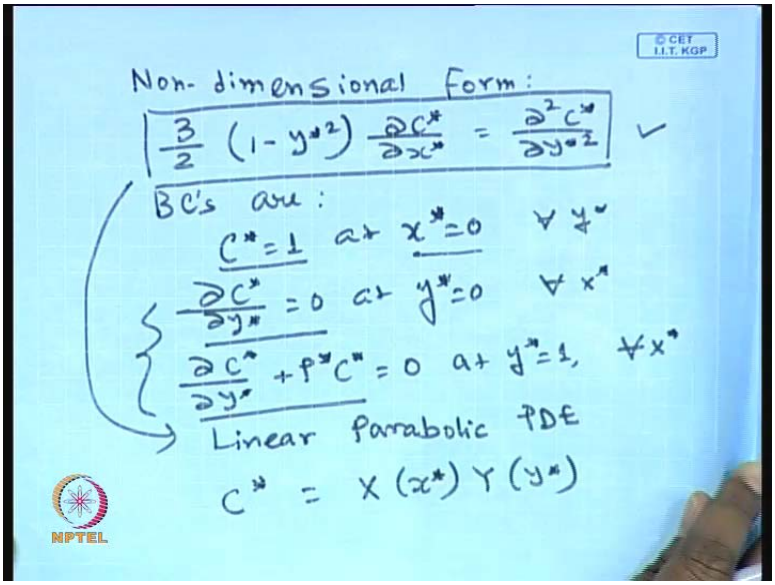

Now, let us write down the non dimensional form of our governing equation and it is boundary conditions 3 by 2 1 minus y star square, del C star del x star is equal to del square C star del y star square. This is the governing equation and the boundary conditions are C star equal to 1 at x star equal to 0, for all y star del C star del y star will be equal to 0 at y star equal to 0 from all x star and del C star del y star plus P star C star is equal to 0 at y star equal to 1 for all x star. Now, this equation so what what what what is the form of the equation is linear or non-linear, this equation this this is a parabolic partial differential equation and the equation is linear. So, if you remember the definition

of linearity or non-linearity of a partially differential equation if the if you have some terms you governing equation which is nothing but the multiplication of dependent variable and it steady very well for example, if you would of obtain C star into del C star del x star or the higher order derivatives del square del C star del x star rest to the power 2 order may be it may be more, than those equation becomes non-linear, but since y star is the independent variable it is multiplied.

So, this equation is a linear parabolic parabolic partial differential equation those you are done mathematics scores in the earlier semester it I think there is pretty you know apparent obvious to them. Now the boundary condition this of course, a linear and it is the non 0 it must the non 0 homogeneous condition there at the entry point at the at x star equal to 0 and will be having of homogeneous boundary condition at y star equal to 0 and homogeneous linear boundary condition at y star equal to 1. So, both the boundary conditions are homogeneous and linear of the governing equation becomes linear. So, must be having a separation of variable type of solution. So, the particular set of equation. Now that analytical solution is not obvious. So, what is done 1 can have a series solution in order to get the you know. So, it is basically it has to be solved by C star is equal to x star multiplied by y star.

So, what you can do you can separate you can you can express the concentration as the product of 2 variable x and y and is varying part is assume to be solve the function of x star and to be. So, linear function of y stars. So, if you put their in governing equation. So, this called a separation of variable take an idea of separation variable technique is that you must be having a governing equation a linear one the boundary conditions in y direction where it will be order two then it must be having the homogeneous boundary condition at the direction. So, if you really do that.

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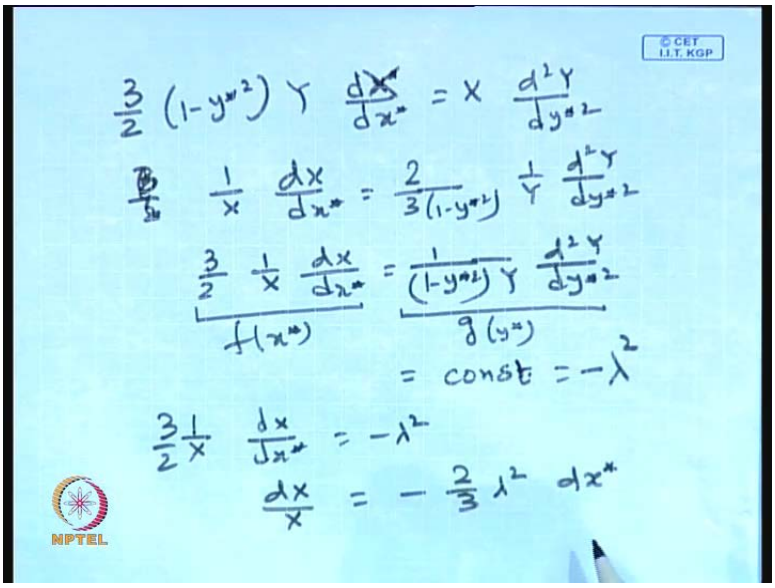

So, this becomes 3 by 2 1 minus y star square y times d x star d x star is equal to x time d square this x I this d square y d y star square. Now you can separate the variables 3 by 2 1 over x d x by d x star is equal to 2 by 3 1 over y star square 1 over y d square y d y square. In fact, it will be better if you take the 3 by 2 term of the left hand side. So, it become 3 by 2 1 over x d x d x star is equal to 1 over 1 minus y star square 1 over y d square y d y star square. Now the left hand side is completely a function of x the right hand side is completely a function of y and they are equal the; that means, their equal to some constant. Now this constant is can be positive can be negative or can be can be 0 now those you are done the mathematic scores in the last semester does it is known to them that if the constant 0 or positive will be getting a prevail solution and we are not looking for them.

So, therefore, this constant has to be a negative constant let say minus lambda square per lambda are called the Eigen values to be particular problem. Now I am not going in to retail of the solution. So therefore, the you you can just get an idea of the solution. So, the x varying part will be 1 over x 3 by 2 1 over x d x d x star is equal to minus lambda square and d x by x is nothing but minus 2 by 3 lambda square d x star. So, 1 can integrate this equation and can get the x varying part as. So, it will be it will be log of x integral of will be log of x and after integration this x can be expression at the exponential term.

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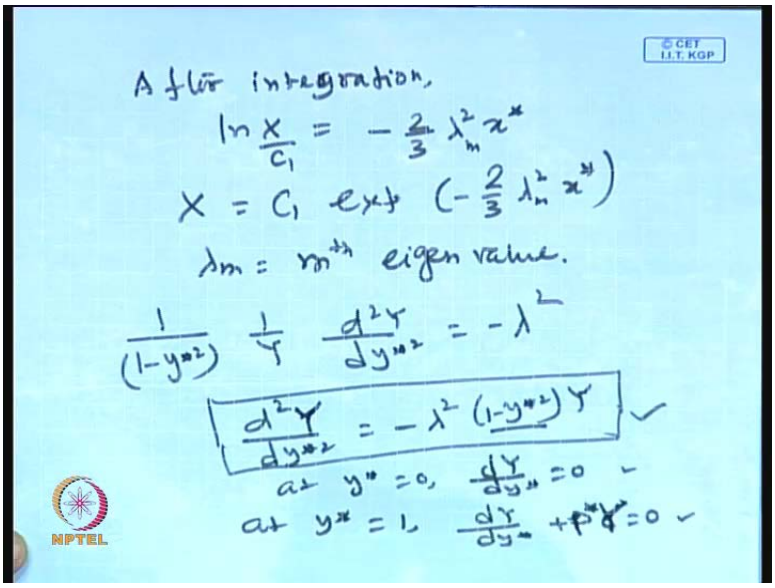

So, after integration after integration will be getting log of x is nothing but minus 2 by 3 lambda square x star and there will be a some constant C 1 and x becomes some constant C 1 exponential minus 2 by 3 lambda square. Let say this is lambda m m transfer for n th Eigen value lambda m is nothing but n th Eigen value of the system. Now once you do that now you will be will be able to now now will be you can you can solve the y varying part the y varying part will be 1 over 1 minus y star square 1 over y d square y d y star square is equal to minus lambda square. So, this becomes d square y d y star square is equal to minus lambda square 1 minus y star square times y. And what are the boundary condition the boundary conditions of on y varying part must satisfied the boundary condition on y in the original problem; that means, at y star is equal to 0 d y divided by d y star will be equal to 0 and at y star is equal to 1 you have d y d y star plus P star C is P star time y equal to 0.

So, you have or ordinary differential equations in terms of y and you have to homogeneous boundary conditions. So, again this can be solved almost you know with this term would not having their then you could of solubility quit easily, but because of the presence of this term the solution becomes is not that straight forward what what, but still 1 can get an analytical solution that analytical solution can be obtained by using a series solution. So, if you have you must be familiar, is the series solution of the ordinary differential equation that has to been term in the whole in the fast of second linear.

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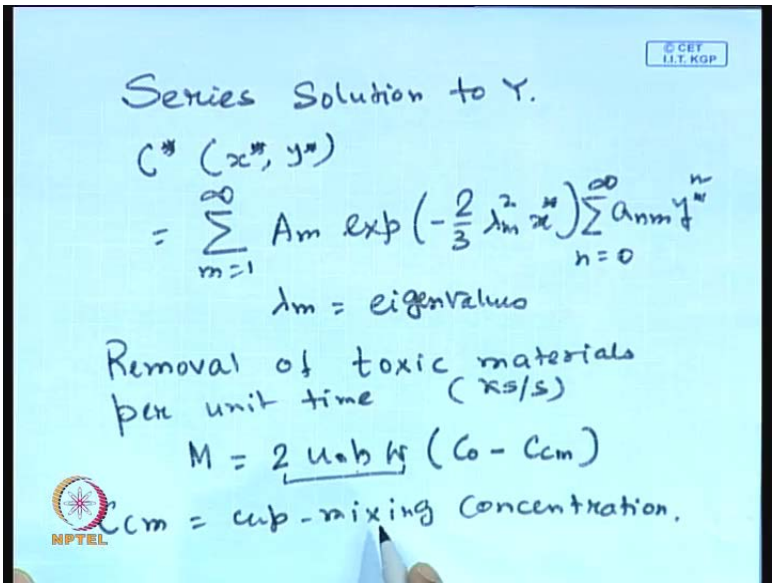

So, 1 can get a series solution to y varying part and I am not doing that exercise here, because I do not 1 to getting to that involved mathematic, because our purpose is something else. So therefore, I am just like in the overall solution in concentration. So, the final expression of concentration as the function of x star and y star becomes summation of A m where m is equal to 1 to infinitive exponential minus 2 by 3 lambda m square x star summation of a n m y star rest to the power n where n goes from 0 to infinitive.

Actually, this is the series solution this summation basically coming from the free solution of the y varying part that I just discussed earlier. Now lambda m are the Eigen values to the system and let us look in to the removal of toxic materials how that will be obtained per unit time; that means, rate of removal let say since k g per second; that means, concentration. So, expressed in k g per meter cube if that is the. So, how it will be defined it will be defined as the amount of concentration the amount of material of the solute, that is coming at the encamp entry entry point; that means, an x is equal to 0 minus the amount of solute that is the present at any particular location.

Let say if it is obtain the length L there it will be the after the channel. So, it will be 2 u naught h w multiplied by C naught minus C c m I will defined C c m let us try to understand what it is, 2 time h time w will be basically the cross section area multiplied by the u naught that will be give you giving you the flow rate, meter cube per second and

it should be multiplied by the amount of solute lets k g per meter cube meter cube meter cube will be cancelled. So, this volumetric flow rate q multiplied by the C naught will be the amount of solute that is entering in to the system. And what is C c m is the cup mixing concentration if you remember the solution solution is a in terms of x star and y star, but we cannot take any point value at any sample and determine the value of the concentration at any point in the at any point in the cross sectional cross sectional area of the channel.

So, if you take out a sample and measure it what it will give to give you a why averaged and average value of the concentration that is known as the cup mixing concentration. So, cup mixing concentration will be a function of x, but it will not be function of y, because you are averaging out the y dimension. So, cup mixing concentration at any location x star it is it is the cross sectional average concentration at that particular location x star.

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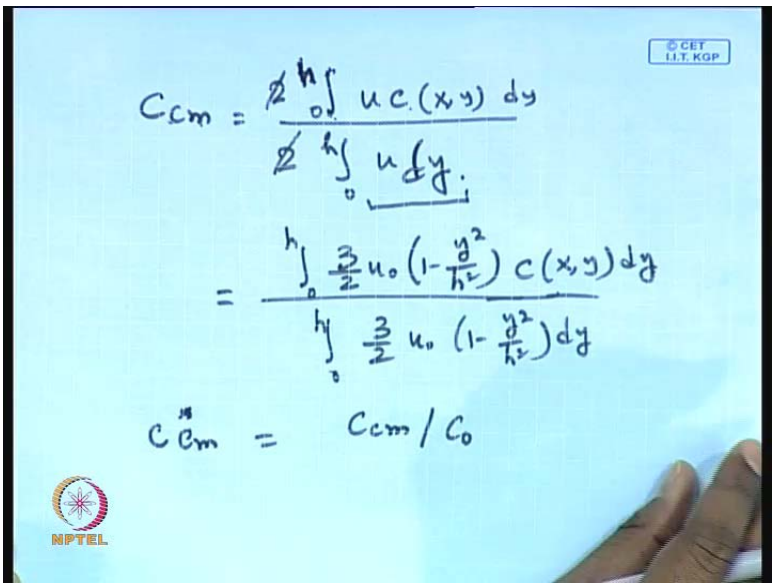

So, what is the definition of cup mixing concentration? The definition is integration 0 to h half height it is u naught u C, this is the function of x and y d y multiplied by 0 to h u d y. It is basically the amount of the solute that is coming and it should multiplied by 2 and should be multiplied by 2 and it w will be cancelled out all this thing cancelled out 2 will be cancelled out. So, it is basically twice of that, because because is symmetrical bidirectional. So, this gives this definition of cup mixing concentration gives you; that

means, have u is the function of void only, yes since it is fully developed in the y direction. So, it will be in the denominator will be getting a value. On the on the other hand in the numerator you are going to put the profile of u there that C will be function of x and y. So, therefore, after integration in the y direction all the y variation will be gone. So, therefore, C c m, but the x variation will be impact, because you are not integrating over the x.

So, C c m the concentration the cup mixing concentration will be a function of x star that will not be function of y star, because it is hour is to over the y. So, what we are going to do put in to us 0 to h 3 by 2 u naught 1 minus y square h square C then going to put the concentration profile we just obtained earlier the last slight 0 to h 3 by 2 u naught 1 minus y square by h square times d y. That is the definition of cup mixing concentration. You can you can defined a non dimensional cup mixing concentration divided by by concentration at the at the entry point. So, this C c m star will be nothing but C c m divided by C naught. Now if you now really put the concentration profile will go are and and integrate and multiply multiply with this and carry out this integration what will be getting is that I am just.

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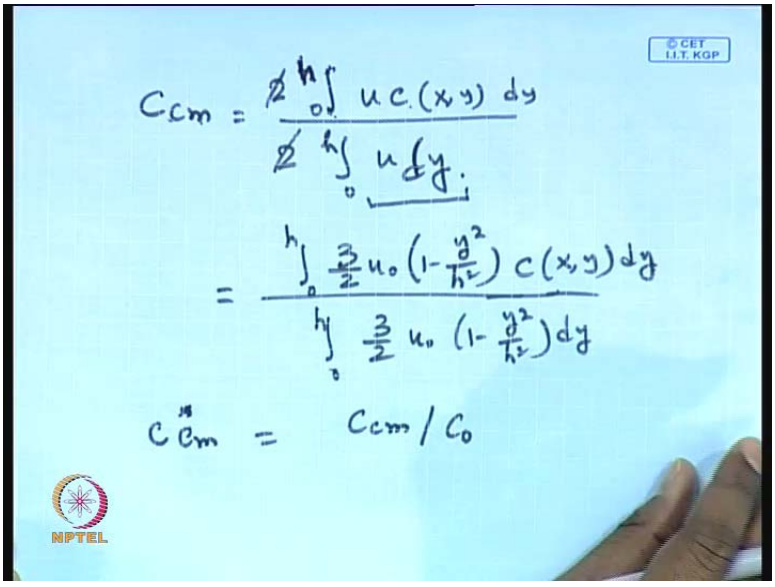

So, I just leaving at the exercise to you and just writing the final expression of C c m star the value of C c m star will be nothing but after carrying out the integration in the y direction will be nothing but 3 multiplied by A m where m is equal to 1 to infinitive

exponential minus 2 by 3 lambda m square x star another summation a n m divided by n plus 1 in to n plus 3. Where the n goes from 0 to infinitive this series is coming from the solution of the y dimensional that is coming from the series solution. And what are the values of lambda. Values of lambda m there is the Eigen values of this problem are obtained as the root of the polynomial.

What is the polynomial is just writing a polynomial in this form 0 is equal to P star minus 2 by 3 plus 5 by 12 P star times lambda m square plus 1 over 20 plus 1 over 45 P star lambda m to the power 4 plus. How obtain this equation this are we can obtain this equation writ writing the solution the boundary condition at y star is equal to 1 from that 1 can obtain then Eigen the about characteristic equation of Eigen values and the roots of this equation will be giving you the lambda m and and you just is. So, will stop here. So, will go to the lets some break and go to the next class and see hour the implication of the equation as for as the you know actual design is concerned when you are talking about a 2 dimensional flow fluid in the dialysis chamber, Thank you.