

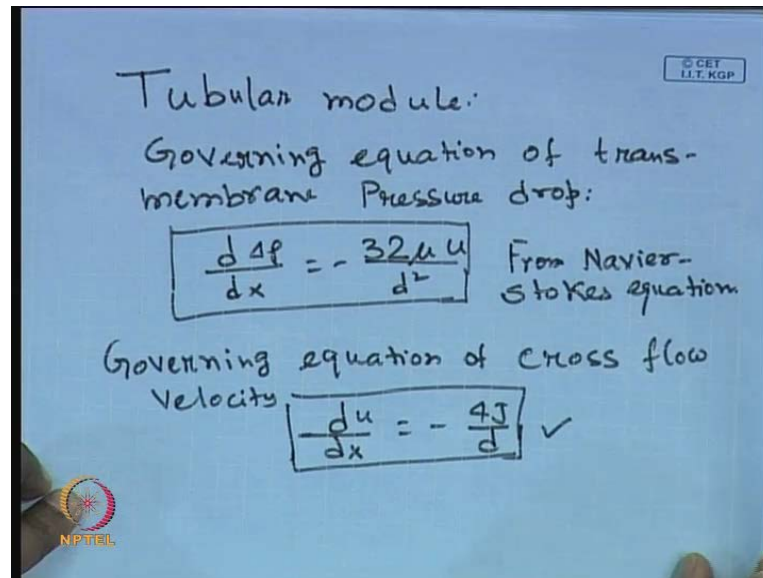
**Novel Separation Processes**  
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**Lecture No # 15**  
**Membrane Separation Processes**

Good morning everyone. So, we are discussing the about the modelling of membrane modules, and first we to took the laminar (( )) and nutritian flow through spiral wound model. Now, the spiral wound module and the flat platen frame modules, both can be analysed by taking request to rectangular copolar, rectangular coordinates system. On the other hand the tubular module will be, and hollow fiber module can be moduled using the polar coordinates. So, in the last class we have seen in detail how the you know various complexities can be added to the analysis, and first we considered the that the permeation, the permeation velocity the permeate flux will be constant.

In second case, we have taken considered it is proportional to delta p, but it is trans membrane pressure drop are it is the osmotic pressure affects are negligible. In the third in the more realistic analysis that we have considered in the last class, that is case number three is that the osmotic pressure is not really negligible. And it is substantial under this conditions how the membrane module design can be carried out. So, we have considered in the last class we have just finished how to do the modelling or analysis in case of the rectangular coordinates system that is applicable for the plate and frame and spiral wound module. In today class we start with the tubular module.

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So, here let us write down the governing equation, the governing equation for the tubular module, governing equation of trans membrane pressure drop is  $\frac{d \Delta p}{dx}$  is equal to minus  $32 \mu u$  by  $d$  square. In fact, this comes from the Navier Stokes equation or equation of motion for a flow through a tube. Now, this will be the governing equation from  $\Delta p$ , and the governing equation of cross flow velocity becomes  $\frac{du}{dx}$  is equal to minus  $4 J$  over  $d$ ,  $d$  is the diameter.

Now, how to obtain the equation? In the last class we have really did all the derivation in detail. What we did? We to get any infinite phase small element. And did an overall material balance that will lead to this expression that is a governing equation for cross flow velocity, and

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Analysis over a differential element in terms of solute balance:

$$u \frac{dc}{dx} = \frac{4J}{d} (c - c_p)$$

$$\frac{dc}{dx} = \frac{4J}{du} (c - c_p)$$

 Gov. eqn. of concn. in the bulk.

Bulk conc. & membrane surface conc.  $c_m$   
Related by an algebraic eqn. via definition of MTC.

If you really do carryout a differential analysis over a differential element, in terms of solute concentration expression or in terms of solute balance you will be getting the concentration balance equation. And that will be become  $u \frac{dc}{dx} = \frac{4J}{d} (c - c_p)$  so,  $\frac{dc}{dx}$  will be  $\frac{4J}{du} (c - c_p)$ . So, this will be giving the concentration balance equation governing equation of concentration of the concentration in the bulk. Now, we know the mass transfer coefficient and this the bulk concentration  $c$ .

And membrane surface concentration  $c_m$  they are related by an algebraic equation via the definition of mass transfer coefficient. In the last class we have derived that expression in detail and we have retain the  $(\frac{4J}{du})$  and we have understood that how to get the expression? So, you will be getting a relationship bulk between the bulk concentration and the membrane surface concentration  $c_m$   $c_n$   $c_m$  via the definition of mass transfer coefficient it will be an algebraic equation that we have already said earlier.

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$$K = \frac{1}{I} \left( \frac{u d^2}{dx} \right)^{1/3}$$
$$I = \int_0^{\infty} \exp\left(-\frac{8\phi^3}{9} - \frac{2}{3}\lambda\phi\right) d\phi$$
$$\lambda = \frac{Pew}{(Re Sc d/L)^{1/3}} = \frac{J d}{D}$$

"De & Bhattacharya, JMS, 109, 1997."

Now, in the expression you will be having the expression of mass transfer coefficient in K. And mass transfer coefficient in K for a turbulent flow region that becomes 1 over I u d square by d x rest to the power 1 upon 3 where the interior I is given as it is. The definite integral in this case becomes exponential minus 8 phi cube over 9 minus 2 by 3 lambda phi d phi where lambda is nothing but p e w divided by Reynolds smite d by L rest to the power of 1 upon 3. Now, where from you get the equation and you have read out the similarity solution for the rectangular case and we have derive the expression of mass transfer coefficient.

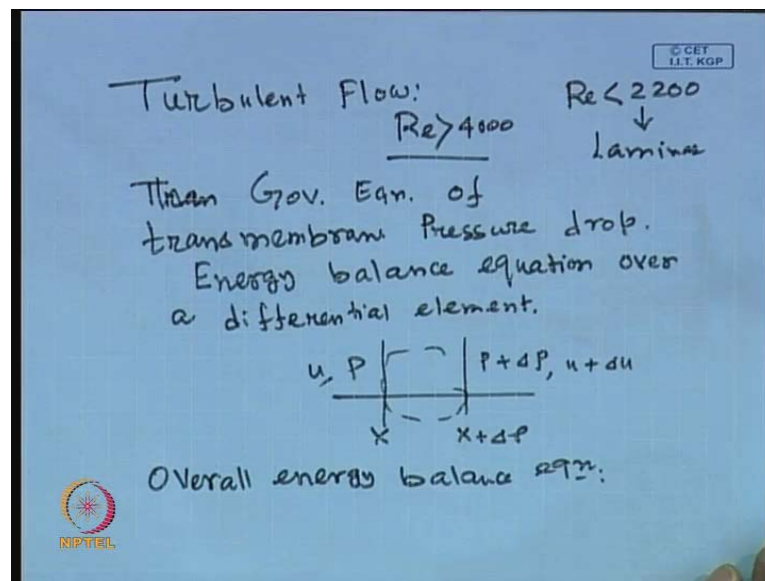
And you can carry out the same analysis in a tubular module and under the laminar flow condition then you will be getting the this expression. In fact, if you really interested you can look into the reference of De and Bhattacharya, journal membrane science volume 100 and 9 year 1997. I think the page number around the 100 9 to 15 or 16 whatever it is. So, just look into the reference in the reference the 3 geometric has been solved 1 is for the rectangular geometric, another is the tubular geometric, another is the radial cell radial geometric. In the class we have derive the fool expression analysis of rectangular geometric just look into the reference the see how the radial distribute (O) tubular polar coordinates has been solved.

And you will be getting this expression of mass transfer coefficient as the function of x. There and the definite integral form of the definite integral will be taken this form and

this  $p_e w$  is nothing but it is the length average per in term of permeate flux that mean this  $p_e w$  is nothing but  $J \bar{d}$  over  $d$  this non dimensional permeate flux. Now, again the algorithm that we have you know discussed in the last class that will still whole good in the class as well will be assuming a value of length average permeate flux.

And then calculate the I know will solve 3 ordinal; differential equation coupled to 1 algebraic equation there has to be solved. A at a every step of there has to be solved iteratively and the every step of are in the put of 4 algorithm for the solution of ordinal differential equations. So, in that case an you will be getting the profile of permeate flux in the function of  $x$  do a length averaging and check whether the calculated length average permeate flux is equal to the assume value or not. If not f to iterate number of times within 3 4 iterative equation will be getting the solution.

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So, that goes for the you know the laminar flow condition under the tubular flow module now we just look into some of the aspect of if the flow is turbulent. Till now done the analysis in laminar flow conditions that mean Reynolds number is less than 2200 there is the laminar and Reynolds number greater than 4000 that will be the really turbulent flow. Now, in this case what we have to do and overall the trans membrane pressure drop governing equation of trans membrane pressure drop. If you remember how will be obtain the last time this is the turbulent flow governing equation of trans membrane pressure drop will be no longer the way of obtained earlier.

That will be obtained by being an energy balance equation over a differential element. So, that means, you are again your considering the element is located at a  $x$  plus delta  $x$  where the pressure becomes  $p$  here this becomes  $p$  plus delta  $p$  here. And you know  $u$  becomes  $u$  and  $u$  plus delta  $u$   $c$  becomes  $c$  plus delta  $c$  something like that. Now, we have to write down the overall energy balance equation to on this differential element of the module. If you do that I am just writing the final expression it that going to the text book and see why are you get derive the it will be able to deriving this equation on the fundamental equation of the pressure balance.

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$$P - (P + dp) + \frac{\rho u^2}{2} - \frac{\rho (u + du)^2}{2} + \frac{A_m J \rho (n^2 - J^2)}{2} = \frac{4 \rho f dx u^2}{2 de}$$

Pressure Energy term  
 change in K.E. of permeating stream  
 Friction Loss Term

$$\frac{d \Delta P}{dx} = -\rho u \frac{du}{dx} + \frac{W J}{Q} \rho u^2 - \frac{W J}{Q} \rho J^2 - \rho \frac{f u^2}{2h}$$

$Q = 2Whu$  ;  $f = \text{friction factor}$

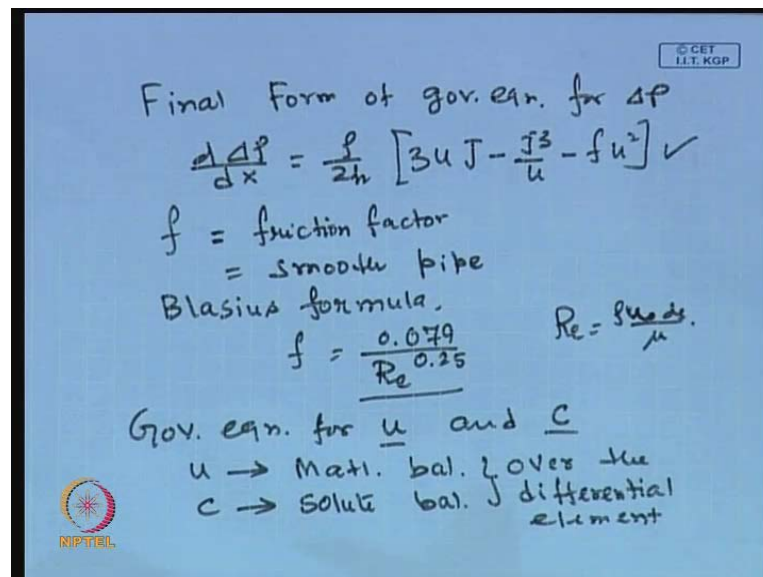
That is  $P$  minus  $P$  plus  $dp$ ,  $dp$  is nothing but delta  $p$  that is the  $p$  plus  $dp$  is the pressure drop of outlet of the element differential element plus  $\rho u$  square by 2 minus  $\rho u$  plus  $du$  whole square divided by 2 plus  $A_m$  by  $Q$  times  $J$  rho by 2  $u$  square minus  $J$  square is equal to  $4 \rho f dx u$  square by  $2 de$ . If you remember this becomes you know this is the pressure variation term delta  $p$  by rho pressure term this 2 becomes the kinetic energy term, this pressure energy, this is the kinetic energy. Again this is the this term associated with the this kinetic energy of the changing time is basically change in kinetic energy.

This is changing in pressure energy between in the inlet or outlet this changing kinetic energy of or of permeating stream. And this becomes a you know pressure loss term friction loss term this becomes the friction loss term. So, this equation finally, will take this form on the writing the final form. What we can do? You can divide both side by

delta x and really making to the differential form this becomes d delta p d x look delta p can various can we written as p minus p atmosphere.

So, d p d x can be written as d delta p d x for delta p is the trans membrane pressure drop d delta p d x minus rho u d u d x plus W J. W is the width of the module of the R the channel Q is the flow rate rho u square minus W J over Q rho J square minus rho f u square by 2 h. Where Q is nothing but 2 W h times u 2 h is basically the full height, W is the width, 2 w is nothing but the normal cross section of the radius of the flow, u is the velocity, but this will becomes is the flow rate and f is the friction factor.

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Now, once you do this then you will be getting the final form of pressure balance equation. The final form of governing equation for delta p becomes d delta p d x is equal to rho e by 2 h 3 u J minus J cube by u minus f of u square. Where f can be it is basically friction factor for a smooth pipe it is you can use Blasius formula. It can use Blasius formula as f is equal to 0.0 7 9 divided by Reynolds rest to the power 0.2 5 for Reynolds number is based on the equivalent diameter rho u 0 d e by mu. Now, this not u 0 will be u so, bulk velocity so, this is the finding friction Blasius formula now for the if the pipe surface this is rough. So, you can have various expression of friction factor in terms of roughness surface of that if you remember the (O) chart on Balsius chart.

Now, in this case the mass transfer coefficient is and you are governing equation. So, they that gives the governing equation of delta p, the governing equation for u and c.



That means, bulk velocity and bulk concentration can be obtained by the overall material balance in oral species balance over the differential element that we have already analysed earlier. Governing equation of  $u$  can be obtained by material balance and governing equation of  $c$  will be obtained as solute balance or species balance equation, over the differential element of the module that we have already discussed earlier.

Now, only thing is in this case what is new the newness of this the novel thing in the turbulent flow is the thought he expression of trans membrane pressure drop. Governing equation of that will be changed from the laminar flow and it will be obtained for the overall energy balance within the over the differential element. Now, again what you have to you have here the if you really would like to get the value of the permeate flux and permeate concentration.

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Minnikhan ti et al., JMS, 1999. An expression relating  $c$  &  $C_m$ . An Algebraic eqn. connecting  $c$  &  $C_m$  via definition of MTC.

For turbulent flow:

$$K(x) = 0.236 \frac{D}{I} \left[ \frac{Re^{1.75} Sc}{d_e^2 x} \right]^{1/3}$$

$$I = \int_0^{\infty} \exp\left(-\frac{n^2}{3} - 2.82 \lambda n\right) dn$$

$$\lambda = \frac{P_{ew}}{(Re Sc_T)^{1/3}} \frac{\bar{J} de}{D}$$

You much be having an expression of that that relation that an expression relating bulk concentration  $c$ , the membrane surface concentration  $c_m$ . That we can already obtained by the already seen that an algebraic equation connect this 2 algebraic equation connecting  $c$  and  $c_m$  via the definition of mass transfer coefficient. This mass transfer coefficient for the turbulent flow this is the mass transfer coefficient can be expressed in function of  $x$  and in this case becomes  $0.236 d$  divided by  $I$  Reynolds rest to the power  $1.75$  smit rest the smit divided by  $d_e$  square times  $x$  whole think rest to the power  $1$  upon  $3$ .



And  $I$  is given as  $0$  to infinitive exponential minus  $\eta$  cube by  $3$  minus  $2$  point  $8$   $2$   $\lambda \eta \theta$ . For  $\lambda$  is the suction parameter is the defined as  $p e w$  divided by  $\text{renal smit } d e$  by  $L$  rest to the power of  $1$  upon  $3$  and this  $p w$  average will be defined in the non dimensional length average velocity permeate flux, it becomes  $J \text{ bar } d e$  over  $d$ . Now, the everything is the now following in the how obtain this expression again this expression we have derived in the class. If you really interested you can look into the reference of this wok Minnikanti etal those the journal membrane signs in the year 1999. Just give as such to suns direct in journal membrane signs Minnikanti etal and 1999 you will be getting the article.

In article you can look into how the derived the turbulent? How you are similar the turbulent flow? And I will give you the rough idea how we really did the turbulent flow modelling. In fact, what we are assumed a fully developed turbulent velocity profile. And we you know that the topic of turbulent velocity profile for will be having  $3$  components  $1$  is viscous sub layer another is the traditional region, another is the fully developed turbulent core. And we have assume that our mass tran so, there is the fully developed velocity profile.

Now, a assume our mass transfer boundary layer lies within the viscous sub layer that means, that means you are viscous sub layer. If you remember that it lies between non dimensional distance from the from the wall up to a height of up to a height of a around  $4$  or so, I did not remember some value the after that is becomes a tangiest region unit becomes the turbulent core. Now, with in that region if you are  $I$  in most of the membrane based separation pressure you are talking about in the filtration of the proteins and higher molecular solute which will be having very low diffusivity.

If you have a very low diffusivity you are thickness of the mass transfer boundary layer is since it is reversely proportional to the proportional to the diffusivity. So, it is directly proportional reverse of mass transfer boundary layer put a thickness will be extremely small. So therefore, in this particular case for this solute be considering is in the membrane based separation process is the thickness of mass transfer boundary layer will be extremely small. We are assuming the thickness of the mass transfer boundary layer concentration boundary layer lies will in the viscous of the sub layer of the fully developed turbulent velocity profile.

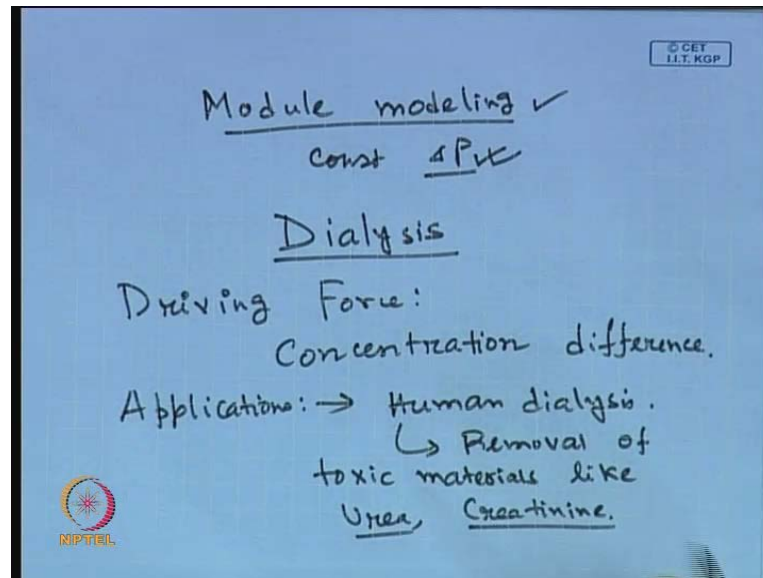
So, in that under this assumption the whole things becomes very simplified and it can be recast on the form of similarity solution that we have already derived earlier. That means, will be getting a parabolic partial differential equation with a boundary condition exacting an infinitive. So, again you can have the some various similarity solution that will be getting this expression. So, please if you are interested look into the paper and getting to the derivation and you can really do the derivation by reversal becomes that is almost same in analytical in nature. So, again in this case will be assuming the so, the basically again will be assuming will be getting 3 governing ordinal differential equation.

So, three governing equation for 3 quantities 1 is the trans membrane pressure drop, there is the bound velocity cost for velocity another is the bulk concentration. This an bulk concentration and membrane surface concentration will be connected via and algebraic equation to the definition of mass transfer coefficient. So, this three differential equation an 1 can coupled in algebraic equation constitute the d a e system d I p system that means, differential algebraic equation system and that is initial value problem.

And this can be solved by using in you know  $r K$  for at a fixed for  $r K$  for have to evaluate the nonlinear algebraic equation where using interruption and update the value of  $c_m$ . So, will be getting a profile of membrane surface concentration you will be getting a profile of permeate flux, will be getting a profile a permeate concentration as well because permeate concentration can be expressed as  $c_m$  into  $1 - r r$  real retention. So, if you know the value of  $c_m$  you will be able to calculate the value of  $c_p$  e.

So, once you get a value of you know a profile of permeate flux you can do a length average by using  $\left(\left(\right)\right)$  one-third of 3 8 whatever or  $\left(\left(\right)\right)$  whatever. And then you will be getting in the length average permeate flux and check whether that value calc calculated value coming close to the guest value or not otherwise we up to reiterate calculated number if times. So, thus now 1 have to do the module modelling and under the turbulent flow conditions.

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So, those who are not present in the last class again I will emphasize why the module modelling becomes very important. If you remember in the earlier class whenever we talk about the module modelling we talked about the constant  $\Delta p$  constant trans membrane pressure drop. But this constant trans membrane pressure drop can be considered as constant, if we are talking about a length of let say few centimetres the 10 centimetre, 30 centimetre like that, but actual membrane module is of larger in length. For example, it can be anything between 1 meter or higher, 1 meter means 3 feet already.

So therefore, in that over the length of the module there will be the drop in  $\Delta p$ . So, must be technical the how the pressure drop profiles varies as a function of length of the module. Because based on that you must be selecting a pump that will be pumping the whole flow you know liquid into the system maintaining the  $\Delta p$  which will be quite large. Because under the  $\Delta p$  there will be permeation of cost because, the filtration will be under the  $\Delta p$ . But also the same time it should be very careful about that the whether the loss in pressure at the out layer of the module will sustain the pressure so, that will be getting the some significant amount the permeation.

So, the pressure drop becomes very important in all the chemical engineering unit operation that I information earlier starting from fluidized bed reactor, back plate reactor, distillation column, absorption column including the membrane module. So, therefore, that is the need to calculate the you know governing equation if trans membrane pressure

drop. In the same way you can calculate the velocity you can formulate the governing equation of the bulk velocity if you formulate the governing equation of bulk concentration. And they are the requirements necessary for doing module modelling of the actual module.

And for a plate and frame and spiral wound module you use the coordinate system that we are going to adapt it rectangular coordinate system  $x, y, z$ . For a tubular or the hollow fibre system we are going to adapt the radial polar coordinate system. But the point is under these circumstances radial polar coordinate system will be boiled on 2 rectangular coordinate system. Simply because the thickness of the boundary layer that we are constant in this case will be in the order of micron and the actual dimension of the module becomes higher.

So, in that case if you neglect the curvature effect that means, you are standing inside a big sphere. And if you stand very close to the surface if you cannot experience the effect of curvature, the surface in front of you can appear to be a flat surface. So, they think it becomes similar to the thickness of mass transfer boundary layer so small that when you are talking about the mass transfer boundary layer looking at the wall so, it will be a flat surface.

So, under the situation that whatever we are doing here instead of all until we are talking about hollow fibre module where the thickness will be pretty small. In most of the other cases tubular module etcetera, the analysis will basically be boiled on to the flow to rate flat plate because of the loss of curvature effect. And why the curvature effect is a loss? Because the mass transfer boundary layer thickness will be extremely small. Why the mass transfer boundary layer thickness will be extremely small? Because we are talking about the system where the Péclet number will be pretty high. Why is the number pretty high? Because the diffusivity is pretty low. Why is diffusivity pretty low? Because we are talking about a solute which will be having very high molecular weight and low diffusivity.

So, that is the rationale of being in the module design in a comprehensive manner then will go to the next membrane based separation process that is dialysis. Now, in dialysis the driving force is the concentration difference till now we have looked [\look](#) into the pressure driven membranous separation process. For example, reverse osmosis it request

is very high the  $\Delta p$  the operating pressure it is generally in the order of 25 in the atmosphere to 40 atmosphere or even higher. Then we talked about the nano filtration how the pressure the del; pressure equipment becomes lower and it will be in the order of let say 10 to 12 10 to 15 atmosphere.

For ultra filtrations since the force size is larger the pressure requirement will be further lower and it will be around 6 to 7 atmosphere for micro filtration almost around 2 to 4 atmosphere will be sufficient. For in case of dialysis we required the no pressure gradient no pressure different and it only the driving force is the concentration difference. And dialysis as very tremendous useful and important application is basically the dialysis human the human dialysis here were basically removing removal of toxic material from the blood like urea, creatinine. Now, they will be having low molecular weight. So, you have to select the membrane which will be around in the order of let say 3000 to 4000 to 4000 cut off are even let say order of 10 to 12000 cut of molecular weight.

Here we just send the dialysis in the feed chamber and around the other can basically the dialysis membrane will be kept in to 2 chambers. 1 will be the feed side another will be dialysis side, dialysis side the fresh solution at distilled water or saline water will be circulated in the feed side the blood will be circulated. Now, (( )) etcetera will be will not be permeate to the membrane because membrane portion is pretty small. The only the urea creatinine in this mol molecular elements will be eliminated from the feed side and they will be transferred to the dialysis side. So, therefore, dialysis the concentrate the concentration of urea creatinine will be increasing from the in the dialysis side.

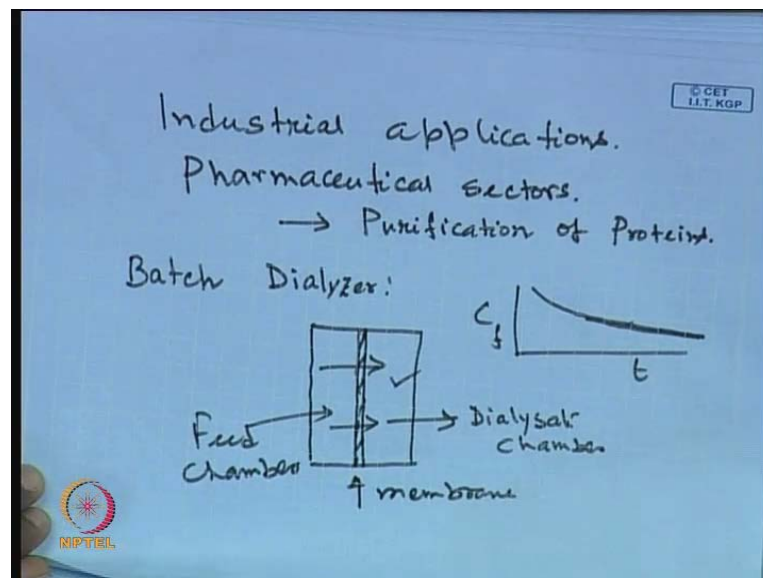
So, where so, basically generally in the hospital what they do they over a from an over a tank they send they circulate the fresh dialyzer fresh. What is the fresh dialyzer? Is basically distilled water solution it is not pure distilled water solution, it is basically a water with the composition of the blood that means, salinity etcetera maintained without the blood cells. So, in that case you are maintain the maximum concentration different right if a if you are sending the fresh dialyzer in the dialyzer side then the concentration of the urea creatinine a tiny point of time or a tiny location will be equal to 0 there. So, there are maintaining the maximum concentration different.

So, that the transport rate of urea etcetera toxic material from the feed side on the blood side with dialyzer become fast. And typically dialyzer what is generally done? Is

basically dialyzer the whole estimate is called it is composed of hollow fibre the small hollow fibre and the housing is called is basically hollow fibre membrane cottage or module. And this module we are not really a fabricating in India we are basically getting this module from outside all the corium modules curia Japanese, French and Jerman. Now, based on the fabrication you know the companying the manufacturer the cost vary from between 1500 to 2500 rupees.

And a kidney failure patient is required dialysis 3 to 4 times how with, but this Cartesian generally used a used for 1 time basically use and through time. But in India since become costly the doctor use them number of times for the same patient it will 4 times but every time is a efficiency is lower and lower because the blockage. Now, the point is Varun is the 1 of the student under be who are try to develop the dialysis Cortagges how on our own. So, if you can really do that then you can supply that dialysis carrier very cheap rate. We got from success, but this will be tested and they will be basically tested finally, tested by the doctors who will be prescribing them. But this testing cannot be done human directly there be several stages to do it anyway.

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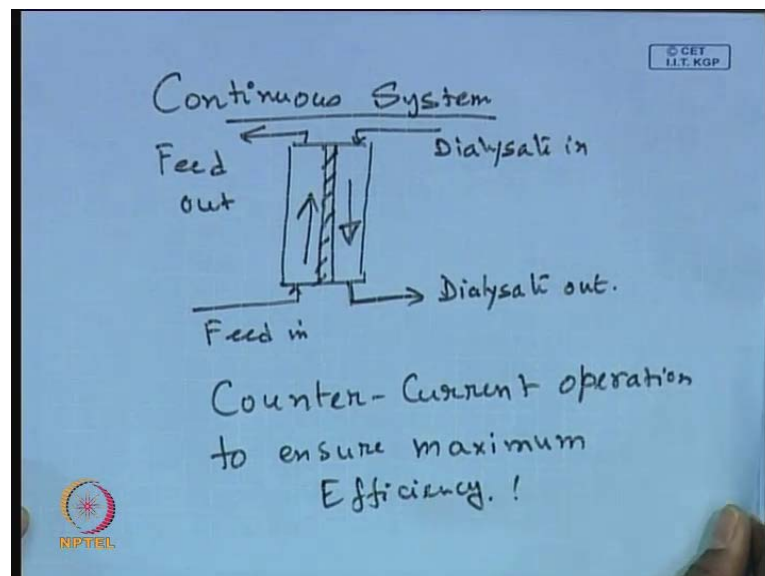
So, there they are several application apart from the medical application there are several other application of the dialysis as well industrial application. In this industrial application most of the application are in pharmaceutical sectors

purification of proteins. Now, in a stream a particular suppose you are targeting a particular protein it will not be pure protein they it will be establish them other proteins are small molecular material solute they can be separated out of the separated by the dialysis. There are several other the industrial application as well.

Now, let us look into the principles of dialyzer let us first look into the semantic of batch dialyzer this is the batch dialyzer, this is the feed chamber and this is the dialyzer chamber and here we have to put in the dialysis membrane. So, because of that in the dialyzer chamber initially there are no concentrations of the solute concentration is equal to 0 and the solute concentration is maximum in the feed. So, solute gets transported across the membrane and goes to the dialyzer side so, the dialyzer so, the concentration gradient gradually decreases.

So, if you look in to the concentration verses time in the feed side concentration are solute verses time in the feed side it will go on decreasing. And it will go on decreasing asymptotically on because you are maintaining the dialyser concentration the maximum concentration gradient here because of the dialyser you are not doing a recirculation or recycling.

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On the other hand in case of continuous system



In continuous system is the feed in, this is the feed out dialysate in, dialysate out. Now, there is the there is another problem in dialysis is that if you would like to remove small molecular material. For example, urea creatinine the etcetera in q you have to selected force side of membrane such that they will be permeable. Now, any membrane cannot be having a very sharp force side distribution sharp marginal cut off. So, this will be having the distributed forces there will be the distribution not only along with the urea creatinine some amount of serum will also be will be transported from the feed side to the dialyzer side.

So, there will be loss of vital body fluid so, there so, if you select a higher forces membrane the transport of urea creatinine will be much more much easier much Firstar, but at the same time loss of vital plate. But if you select a very small forces of the membrane they are the transport the loss of vital fluid is minimized, but at the same time the transported urea creatinine etcetera will be constructed. So, there will be there will be hinder diffusion, hinder transport. There should be a judicious selection of this of the 2 aspects the force side and selection of the membrane. Now, the thing is now in engine what is the inguinal in order to have a if you put a patient under dialysis it is basically taking the blood out of the body and again recirculation it.

So, that the patient cannot be having you know that much patient or that much in your stability if you carry out the dialysis over a long period of time. Generally dialysis is typically carry out for about 45 minutes to 45 to 90 minutes anything between that. So therefore, people what people use to do people use to select a large of forces membrane. So, that the transport of the urea creatinine will be maximized, but at the same time will be a will be facing a loss of body fluids. So, there will be there inject makeup body fluids in to the human system that how it is maintained.

In a continuous cell in a continuous system the direction of circulation of feed and the direction of circulate dialyzer circuit it will be in the opposite direction in order to have the maximum efficiency in your system. That means, this will be counter current operation; that means, the direction of the feed will be in the direction, the dialysate circuit will be in the opposite direction across the interface of the membrane. So, this is this will be having a counter current operation to ensure maximum efficiency.

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Transport mechanism:  
Diffusion of solute  
 $N_i =$  Mass flux of  $i^{\text{th}}$  species  
 $= D_{im} \frac{dC_{im}}{dx} = D_{im} \frac{\Delta C_{im}}{L}$   
 $D_{im} =$  Diffusivity of  $i^{\text{th}}$  solute through the membrane.  
 $L =$  Membrane thickness  
 $C_{im} =$  Concentration of  $i^{\text{th}}$  species in membrane phase.

Now let us look into less go into the details of the dialysis system let us look into the transport mechanism exedra. Main transport mechanism in the dialysis is concentration in different in concentration gradient the concentration gradient. What is the concentration gradient, what it will be cause? Basically triggering the diffusion of the solute. So, main transport mechanism is diffusion  $N_i$  we write it as mass flux or molar flux depends on the unit of concentration that you are selecting of  $i^{\text{th}}$  species this nothing but  $D_{im} \Delta C_{im} / L$ .

What is  $d_{im}$ ?  $D_{im}$  is diffusivity of  $i^{\text{th}}$  solute through the membrane  $L$  is the membrane thickness and  $C_{im}$  is concentration of  $i^{\text{th}}$  species in membrane phase. So, with this we can. So, basically the there is the concentration different across the membrane the triggers the diffusion of the solute suite.

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$$N_i = \frac{D_{im}}{L} [C_{im}^F - C_{im}^D]$$

Concentration between bulk & on the membrane surface  $\rightarrow$  related by a partition Factor.

$m_i = i^{\text{th}}$  species

$$\frac{C_{im}^F}{C_i^F} = m_i = \frac{C_{im}^D}{C_i^D}$$

Bulk concentrations in feed & dialysate

So, we can write down the  $N_i$  as  $\frac{D_{im}}{L} [C_{im}^F - C_{im}^D]$  there is in feed side minus  $C_{im}^D$  in the dialysate side. Now, there is a defence in again whenever there is the transport across the membrane some solute will be transport here. So, there will be a film of solute that will be form both in feed side as well as the dialysate side. What is this film is? Film is nothing but the mass transfer boundary layer beyond this  $C_{im}^F$  is basically the concentration of  $i^{\text{th}}$  species on the membrane surface in the feed side. And  $C_{im}^D$  is nothing but the concentration of  $i^{\text{th}}$  species on the membrane surface in the dialysate side.

And this is the bulk of the feed this will be bulk of the dialysate, between the bulk on the membrane surface there will be a formation of thin boundary layer mass transfer boundary layer. So, we assume the ratio the value of solute concentration and in bulk between the this point there is the concentration gradient right. This flat between this concentration between this point and this point it is defined by a patrician factor. So, it nothing but some kind of mass transfer coefficient that will this will talk latter on. Let us say that the concentration between bulk and on the membrane surface is related by a partition factor. And this patrician factor let say  $m_i$  for the  $i^{\text{th}}$  species.

So therefore,  $\frac{C_{im}^F}{C_i^F}$  is equal to  $m_i$  is equal to  $\frac{C_{im}^D}{C_i^D}$  where  $C_i^F$   $C_i^D$  are bulk concentrations in feed and dialysate. That means, we are simply assuming a write now that the bulk concentration in the feed side and the

dialysate side. And the surface concentration of solute of the feed side dialysate side is related by the simple partition coefficient or partition factor. So, therefore, using this definition you can relate what is the rate of transport for the solute from the feed side to the bulk from the feed side to the dialysate side. In terms of bulk concentrations; that means, you are going to replace  $C_{i,m}$  in terms of  $C_{i,F}$  and  $C_{i,m}$  in terms of  $C_{i,D}$ .

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$$N_i = \frac{D_{im}}{L} m_i (C_{i,F} - C_{i,D})$$

$$m_i = 0.5 \text{ for urea soln. \& regenerated cellulose}$$

$$D_{im} \approx 10^{-11} \text{ m}^2/\text{s}$$

For Phenol,  $m_i = 0.01$  (PE membrane)  
 $= 17.5$  (Ethyl cellulose is memb. matl.)

$\frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$

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So, therefore,  $N_i$  is nothing but  $D_{im}$  by  $L$   $m_i$   $C_{i,F}$  minus  $C_{i,D}$  why you are doing this because this concentration are measurable. You can measure the concentration because you can take a sample and do a measurement, do an analysis either spectrophotometrical or reflective index or whatever so, these are measurable quantities. So, if you know the value of  $D_{im}$  if you know the value of  $m_i$ , if you know the membrane thickness is you measure the bulk concentration, dialyzer concentration. You can estimate what will be the molar flux how much, what is then unit of this will be Kg per meter square per second.

If concentration is the expressed in Kg per meter cube this will be molar per meter square per second if concentration expressed in molar per meter cube moles that will simply indicate. Why this very important? This is important simply because, if you know the membrane area if you can calculate this thing in  $N_i$  if you know the membrane area because that is basically macroscopic power right. If you know the membrane area you

can calculate. How much solute will be transferred from the feed side to the dialysate side per unit time.

So, if you have a target that after remove in 1 over let say 10 milligram of urea in what will be the membrane area and going to select, that will give you directly design aspect. The value of  $m_i$  is 0.5 for urea solution and regenerated cellulose that mean if you membrane material is regenerated cell cellulose. And if you have a talking about urea acquire solution the value of  $m_i$  is 0.5 and  $D_{im}$  will be in the order of  $10^{-11}$  meter square per second. How to calculate that  $D_{im}$  the? It is very easy to calculate diffusivity in the bulk, but you have to find out diffusivity in the membrane phase that will be quit complicated.

How to do? That will see later on another estimate expression for phenol  $m_i$  is equal to 0.0 1 if poly ethylene is membrane material. So, it becomes it is a function of the membrane material and this equal to 17.5 if ethyl cellulose is the membrane material. So, there is certain literature value the available for a particular solute and membrane material combination.

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Determination of  $D_{im}$

$$D_{im} = \frac{D_{ib} \epsilon}{\tau} F_{drag} \quad \checkmark$$

$D_{ib}$ : Diffusivity of 'i' th species in the bulk. ✓  
 $\epsilon$  = membrane Porosity ✓  
 $\tau$  = tortuosity Factor ✓

$F_{drag}$  = Drag coefficient  
 Solute rad.  $r_i$  and pore rad.  $r_p$  ✓

$$= 1 - 2.1 \left( \frac{r_i}{r_p} \right) + 2.09 \left( \frac{r_i}{r_p} \right)^3 - 0.95 \left( \frac{r_i}{r_p} \right)^5$$

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Now, let us look into, how this  $D_{im}$  that be calculated determination of  $D_{im}$  because that is very important. So, if you like to calculate the solute flux you can you can measure the concentration in the bulk as the fix side. In the all in the dialyzer side both of the experimentally measurable. If you know the literature of the value you can know the

value of what the what will be the value of  $m_i$ , but you should know the value of  $D_{iB}$  as well as that the diffusivity of the solute in the membrane phase. That is there is the relationship given  $D_{im}$  is equal to  $D_{iB}$  epsilon divided by tau and multiplied by a drag factor drag factor coefficient  $F_{drag}$ .

$D_{iB}$  is nothing but diffusivity of  $i$ th species in the bulk epsilon is membrane porosity, tau is the tortuosity factor because all the force and state is cylindrical force. There will be tortuosity force and will be interconnected and the drag coefficient  $F_{drag}$  can be return as let say particle radius, solute radius is  $r_i$ . And pore radius is  $r_s$  may be given by the expression  $1 - 2.1 r_i / r_s$  rest plus  $2.09 r_i / r_s$  whole cube minus  $0.95 r_i / r_s$  rest to the 0.5. The some kinds of relationship are provided if you know the radius of the solute and radius of the pore.

Now, using this definition of  $D_{im}$  it is very difficult because all the parameter is the mentioned that is in those and involved in the definition of this quantity extremely difficult to measure. For example, the tortuosity factor in cannot directly measure the tortuosity factor they are some thumb rules the value will be in between 2 to 4 similarly, the membrane porosity it is extremely difficult to find out the membrane porosity. So, and diffusivity of  $i$ th species in the bulk may be measurable or may be available literature. There is quit simpler on the other hand the solute radius can be measurable.

But the pore radius is very difficult to because the pore size is will not to be will be on the single pore size. So, there will be distribution of the pore size so, you must be take a talking the about in the average pore size. So, again that is difficult so, anyway if some out you can measure this quantities 1 can use this relationship in ordered to determine the value of  $D_{im}$  typically, the value of  $D_{im}$  will be at least 2 to 3 orders of magnitudes less. If you have a bulk diffusivity  $10^{-11}$  meter square per second the  $D_{im}$  will be in the order of  $10^{-12}$  for  $10^{-13}$  meter square per second.

So, that idea should be should not be  $10^{-26}$  that should be the idea that 2 to 3 order of magnitude, 1 to 3 order of magnitude will be less in the membrane phase compare to the bulk phase. On the other hand the parameters involved in calculation of the diffusivity through the membrane phase is complicated that their difficult. So, their other so, order to avoid that what one can do one can conduct a

separate set of experiment like the How it determine the permeability? How it determine the real attention of the solute? Exactly the same way we can conduct the batch dialysis experiment.

And by looking into the concentration profile one can estimate the value of  $D_{im}$  that will see later on. So, there so therefore, you see that when in the case of that dialysis like the membrane filtration like we have discussed earlier reverse osmosis, ultra filtration etcetera. The batch separation although it is the watch performance it is very important. Why this important? To extract some of the interesting and important kinetic parameter. So, will stop here and will continue the next class.