Novel Separation Processes
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Module No # 01
Lecture No # 12
Membrane Separation Processes (Contd.)

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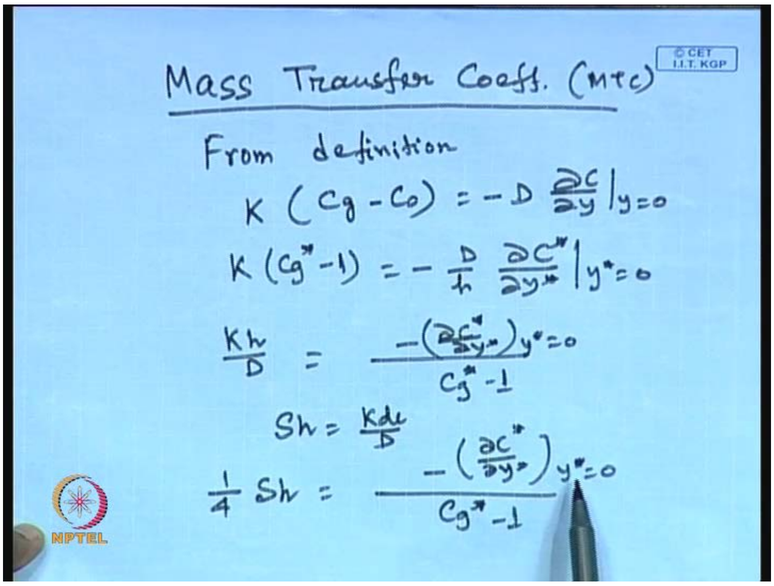

Mass Transfer Coeff. (MTC)

From definition

$$K\,(C_g - C_0) = -D \left.\frac{\partial C}{\partial y}\right|_{y=0}$$

$$K\,(C_g^* - 1) = -\frac{D}{h} \left.\frac{\partial C^*}{\partial y^*}\right|_{y^*=0}$$

$$\frac{Kh}{D} = \frac{-\left(\frac{\partial C^*}{\partial y^*}\right)_{y^*=0}}{C_g^* - 1}$$

$$Sh = \frac{K d_e}{D}$$

$$\frac{1}{4} Sh = \frac{-\left(\frac{\partial C^*}{\partial y^*}\right)_{y^*=0}}{C_g^* - 1}$$

Well, next we will consider the mass expression of mass transfer coefficient, how it takes care in case of how it looks like in-case of gel layer control filtration; that is very important parameter, design parameter and we come to it once again. So, mass transfer coefficient is very important design parameter as per as the membrane operation design of the membrane system is concerned. So, let us first write the expression or definition from definition, we start the expression of mass transfer coefficient; K times C g minus C naught. If if you remember it was C m minus C naught in the earlier case now in this case C g minus C naught minus D del C del y at y is equal to 0.

So, you can make it non-dimensional. So, you divide both-side by C naught. So, this becomes, K times C g star minus 1 is equal to minus D by h del C star del y star and this is becomes, y star is equal to 0 right. So, you you will be having K h by D is nothing but minus del C star del y star at y star is equal to 0 divided by C g star minus 1. And you can you can replace h by D by 4 and define the Sherwood number. What is the Sherwood number? Nothing but K times D equivalent by diffusivity. So, you will be getting 1 by 4 Sherwood is equal to minus del C star del y star at y star equal to 0 divided by C g star minus 1. So now, what you can you can do? You can, we have we have already they expression of del C star del y star in terms of y star delta excreta. And, evaluate that profile at y star is equal to 0 and substitute, that here.

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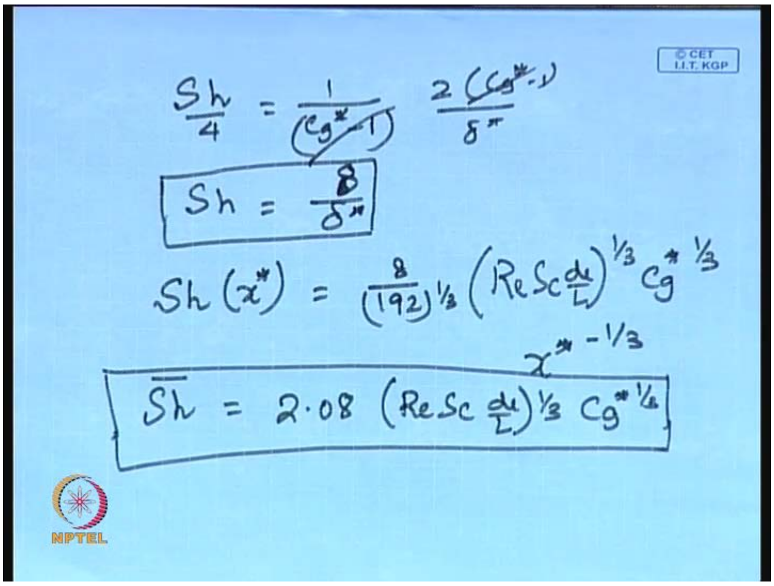

$$\frac{Sh}{4} = \frac{1}{\cancel{(C_g^* - 1)}} \frac{2(C_g^* - 1)}{\delta^*}$$

$$\boxed{Sh = \frac{8}{\delta^*}}$$

$$Sh(x^*) = \frac{8}{(192)^{1/3}} \left(Re\,Sc\,\frac{d_e}{L}\right)^{1/3} C_g^{*\,1/3} x^{*\,-1/3}$$

$$\boxed{\overline{Sh} = 2.08 \left(Re\,Sc\,\frac{d_e}{L}\right)^{1/3} C_g^{*\,1/4}}$$

So, if you do that what you will be getting is that Sherwood divided by 4 is equal to 1 over C g star minus 1 two times C g star minus 1 divided by delta star. We will be cancelled out. And, you can we know the expression of what is Sherwood number? Sherwood number just, becomes 8 by delta star. And we know, we have already got an expression of delta star order the thickness of boundary layer, as a function of x star and we are going to substitute is there. So, if you do that you will be getting a profile of Sherwood number as a function of x star. And, this becomes, 8 by 192 rest to the power 1 upon 3 Reynolds smite d e by L rest to the power 1 upon 3 C g star rest to the power 1 upon 3 x star to the power minus 1 upon 3.

Now, we can do a, you know length averaging you can integrate this equation out over the whole length from 0 to 1. And, it is basically integration of this quantity from 0 to 1 and, these returns value of 1.5. So, you will be getting 2.08 times Reynolds smite d e by L rest to the power 1 upon 3 times C g star to the power 1 upon 3. So, that is the expression of Sherwood number or mass transfer coefficient in-case of gel layer control filtration where the under the two-dimensional analysis when the mass transfer boundary layer is still developing. So, that gives the more or less and overview of how to obtain the permeate flux? Or Sherwood number mass transfer coefficient in case of steady set system. But the most realistic case in case of gel layer control filtration will be that gel layer will be depositing over the membrane surface. At constant concentration C g star or C g. C g star is nothing but C g by C naught at C g.

Then, it is thickness is growing with time. And, the growth because more particles will be will be arriving on the membrane surface it will be decreasing it is thickness. So, that will be offering more resistance against the solve in flux and it will be decreasing. And, because of the presence of the turbulence outside the channel, the thickness the growth of these thicknesses of boundary layer of the of the gel layer will be arrested and will be getting a steady state. So, more and and in-case of osmotic pressure control filtration, will could have done this analysis of the full transient. We could we did not do that why we did not do that? Because it has been compute that the transient in on osmotic pressure analysis, osmotic pressure filtration system exists only for a few seconds. Let, say 10 seconds maximum 1 minute. But in an in an actual operation will be of a duration of 4 hours, 6 hours, 8 hours. So, whatever is happening in within first few minutes or seconds, it is in material. Right you are looking for the final you know filtration characteristics.

So, therefore, we did not do the transient analysis in case of osmotic pressure control filtration. On the other hand transient analysis we did for the batch cell only. For the stirred cell or the cross flow cell we did not do that. On the other hand in-case of gel layer control filtration in a rectangular cell or a stirred cell the scenario is just different. Here, the gel layer controlling filtration will be having much more prominent transients. It is transient may exist for 10 minutes it can exist for several hours. Depending on the characteristics of the gel forming particle, porosity of the gel layer or cake layer are how they are behave in?

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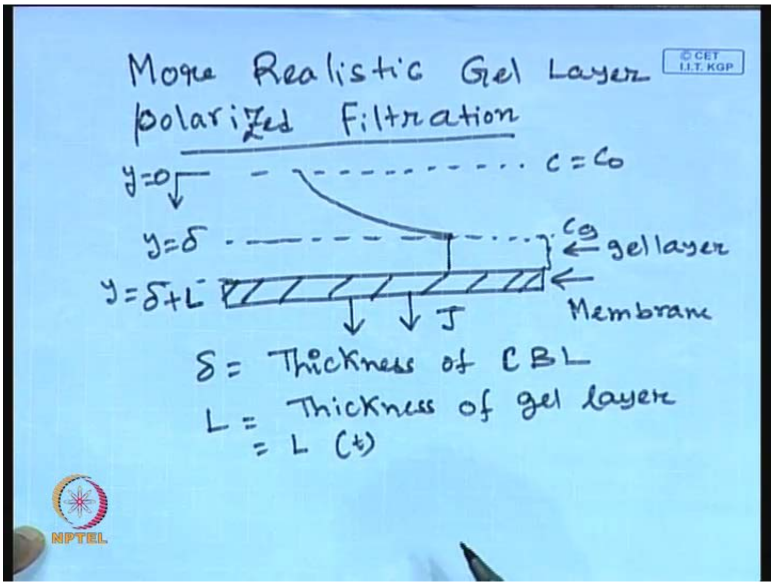
More Realistic Gel Layer polarized Filtration

δ = Thickness of CBL

L = Thickness of gel layer
$= L(t)$

So, therefore, more realistic case of gel layer control filtration will be the transient analysis that we are going to do now. In this case, will be considering gel layer as a as nothing but a cake layer. And, the scenario will be something like this. This is the membrane. And, this is the membrane. And, this is nothing but the gel layer. That is growing over the membrane surface. And, beyond the gel layer there is the concentration boundary layer. And, how is concentration boundary layer is form? Because C is equal to C naught here. C is equal to C gel here. So, therefore, there is an these C gel is basically at least 5 to 6 times sometime in some cases 50 times of the bulk concentration.

Because of this stiff concentration gradient there will be existence of a concentration boundary layer. And, over this the gel layer concentration is same. The concentration in this layer is always same at C g. So, therefore, we can start our coordinated system from the middle. Let, say at when it is bulk it is y equal to 0. And, it is y is equal to delta here. Delta is the thickness of concentration boundary layer. And, here it is at the membrane surface it is y is equal to delta plus L; L is the thickness of the gel layer. So, delta is thickness of concentration boundary layer or mass transfer boundary layer. And, L is thickness of gel layer produced by the gel forming particles. So, therefore, now now you see, having a particular turbulence in your system; that means, if you if the Reynolds number is fixed. At the stirred speed is fixed, your delta will be a sole function of Reynolds number.

So, this thickness will be fixed. On the other hand the gel layer thickness will be growing in time in most of the cases. So, it will be delta will be. So, L will be a function of time. So, in this case we are talking about a moving boundary condition. Right it is a moving boundary problem. This boundary is always moving always increasing with respect to time of operation. So, this is the system and will be having a permeate flux out from the membrane. So, therefore, what we can do? We can do mass value we can write down the solute balance equation in various domains of this of this you know system.

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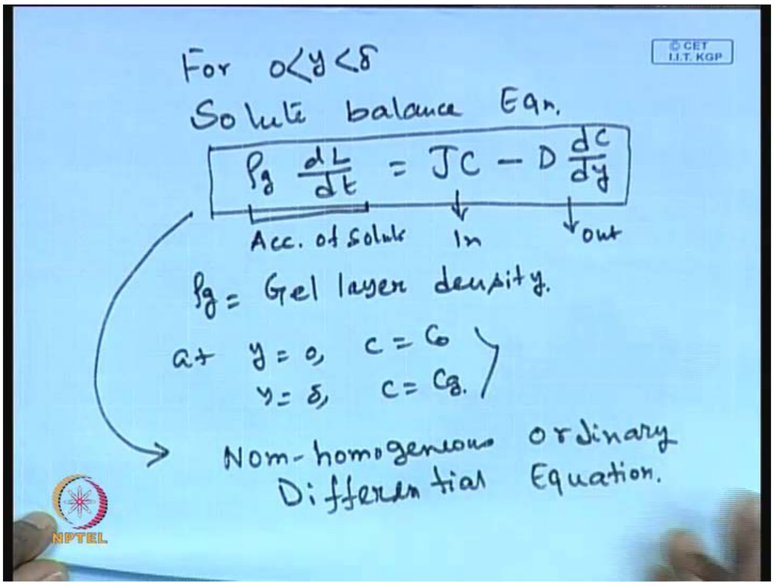

For $0<y<\delta$

Solute balance Eqn.

$$\boxed{\underbrace{\rho_g \frac{dL}{dt}}_{\text{Acc. of Solute}} = \underbrace{JC}_{\text{In}} - \underbrace{D\frac{dc}{dy}}_{\text{out}}}$$

ρ_g = Gel layer density.

at $y = 0$, $c = C_0$

$y = \delta$, $c = C_g$.

Non-homogeneous ordinary Differential Equation.

So, you write down the for y line between 0 to delta; that means, within the concentration boundary layer. The solute balance equation equation will be rho g d L dt is nothing but JC minus D dc dy. what is this? This is nothing but the accumulation term. Accumulation is equal to in minus out. Total accumulation of solute; this is in and this is out, because Cg is more than C naught. So, there will be backward backward seek end diffusion. And what is roe g? Rho g is nothing but the gel layer density or density of the particles forming the gel. And, boundary condition at y is equal to 0 here. C is equal to C naught and y is equal to delta, C is equal to C g.

Now so, within this boundary we have to integrate this equation out to remove the y. Now, what is this equation? This equation is nothing but non-homogeneous ordinary differential equation. How do solve ordinary differential equation? There are several techniques. You can concern you can break this equation in to two parts.

The homogeneous solution and the particular integral and, then you can get the solution and at the two up. So, you can you know you must have studied all the trick mathematical techniques in first year. Use, that and carry out these and carry the integration over y, over these two boundaries from 0 to delta.

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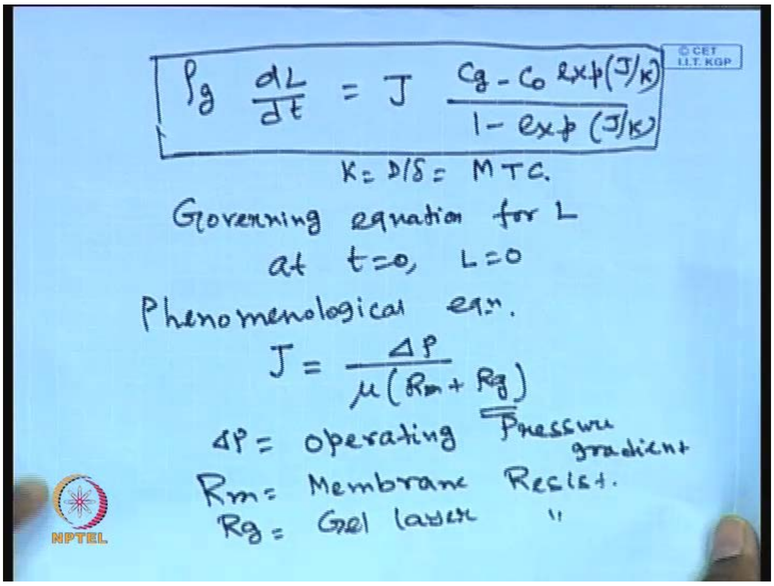

$$\boxed{\rho_g \frac{dL}{dt} = J \frac{C_g - C_0 \exp(J/K)}{1 - \exp(J/K)}}$$

$K = D/\delta =$ MTC.

Governing equation for L

at $t = 0$, $L = 0$

Phenomenological eqn.

$$J = \frac{\Delta P}{\mu (R_m + R_g)}$$

$\Delta P =$ operating Pressure gradient

$R_m =$ Membrane Resist.

$R_g =$ Gel layer "

And, whatever you will be getting is that you will be getting rho g times d L dt is nothing but J times C g minus C naught exponential J by K divided by 1 minus exponential J by K. And K is nothing but D by delta is the mass transfer coefficient. So, these gives the so, these gives the governing equation equation for L the thickness. How thickness grows with time? With the initial condition that is at t is equal to 0, L was equal to 0; obviously, when at the start of the operation the gel layer thickness was equal to 0. Now, the story is not finished here, because J is also a function of time. You have to establish how J is varying with time? So, for that what what will assume that if you remember the schematic have done in the beginning of the class. That there is a concentration boundary layer, there is a gel layer and there is a membranes membrane. So, gel layer resistance and membrane resistance they are put in series.

So, we can write the phenomenological equation to connect that. Phenomenological equation and what is the phenomenological equation? That is flux is proportional to the driving force or flux is equal to driving force divided by resistance. And there are two resistance here; mu R m plus R g, where delta P is the operating pressure gradient.

R m is membrane hydraulic resistance. Call it membrane resistance. R g is gel layer resistance. Now, one can ask why we did not consider the membrane resistance in the steady state analysis. And, we are we are answered already discuss that issue. That when it becomes initially, because gel layer does not form form the very beginning.

So, initially you have only membrane resistance becomes dominant. After, some time gel layer becomes almost equal equivalent to its magnitude then it predominant. Now, when the steady state is reached gel is basically the most prevailing resistance and it will be dominating one. Now, these gel layer resistance will consider these gel layer is nothing but a cake sort of layer. It is a cake layer.

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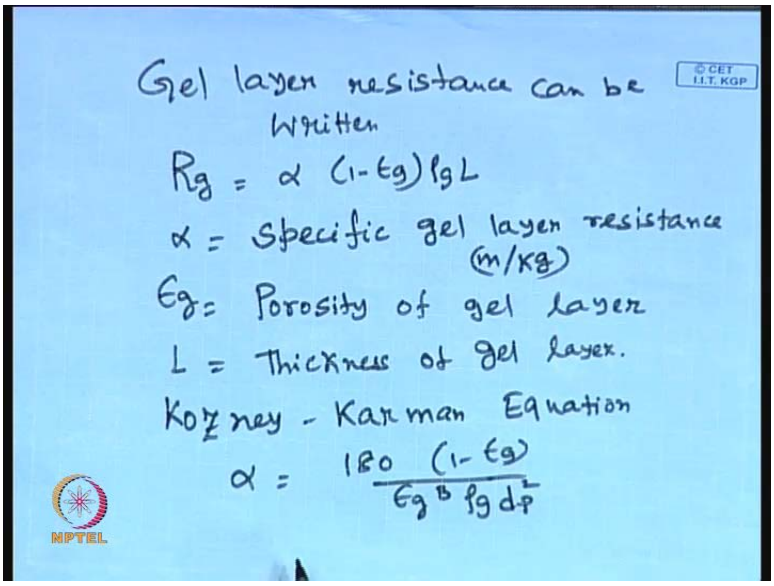

Gel layer resistance can be written

$R_g = \alpha (1-\epsilon_g) \rho_g L$

α = Specific gel layer resistance (m/Kg)

ϵ_g = Porosity of gel layer

L = Thickness of gel layer.

Kozney - Karman Equation

$$\alpha = \frac{180 (1-\epsilon_g)}{\epsilon_g^3 \rho_g d_p^2}$$

So, gel layer resistance can be written as, R g is equal to alpha 1 minus epsilon g Rho g times L. What is alpha? Alpha is known as the specific gel layer resistance. Now, where from you get this equation? This equation obtain by a solute balance and if you if you look into the filtration theory. In fact, you should study that in earlier in some earlier course. Look into the filtration theory; there the definition of specific gel layer resistance, porosity gel density, excreta will be given. And, it has the unit of meter per K g. And epsilon g is nothing but porosity of gel layer. Rho g is the gel layer density that we have already mentioned, and L is the thickness of gel layer. Now, alpha thus, specific gel layer resistance is a function of gel porosity and particle of the gel forming material. And, it can be the simplest one can one can have an estimate of alpha in terms of particle diameter and porosity that is known as the Kozney- Karman equation. If you ever do the filtration theory you will come to know all these characteristics equations.

So, alpha can be written from Kozney- Karman equation. That alpha is nothing but 180 times 1 minus epsilon g divided by epsilon g cube rho g dP square. Again, it is it is a empirical relationship. Alpha can be; that means a specific gel layer resistance or cake layer resistance can be expressed in terms of gel porosity, gel diameter of the gel forming particle, density of the gel gel forming material by this expression. So, if you know epsilon g if you know particles diameter then you calculate the value of alpha. Now, typically this alpha is the function of delta P and depending on the characteristics one can have whether, the cake layer or gel layer is compressible or whether it is incompressible. Now, let us you what we get?

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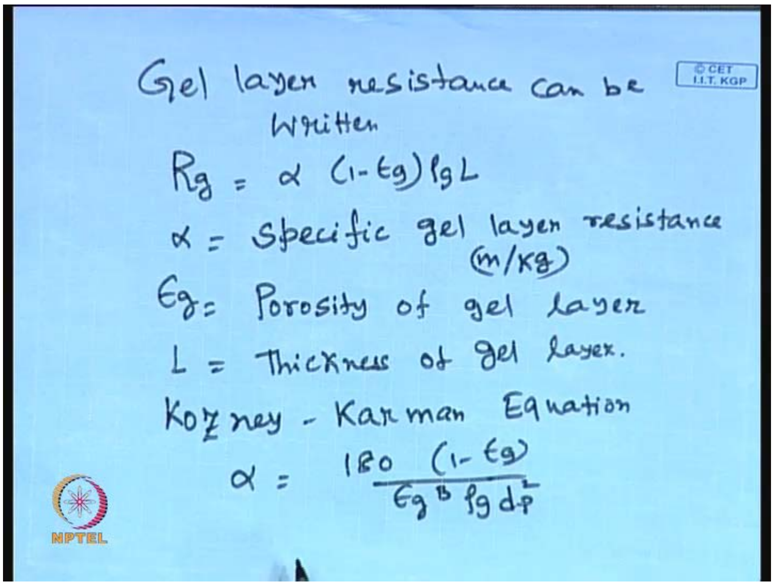

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α = Specific gel layer resistance (m/kg)

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L = Thickness of gel layer.

Kozney - Karman Equation

$$\alpha = \frac{180 (1-\epsilon_g)}{\epsilon_g^3 \rho_g d_p^2}$$

So, we have the expression of. So, our our aim is to find J as the function of time as will be delta p divided by mu R R m plus R g and what is R g? R g is nothing but alpha 1 minus epsilon g Rho g times L. This alpha can be calculated from the def from the fundamental equation of Kozney- Karman equation by epsilon g and D p; that means, from the fundamental property of gel porosity and particle diameter one can calculate the value of alpha. And then, one can go to the expression of gel forming gel layer thickness that is J by Rho g times C g minus C naught exponential J by K divided by 1 minus exponential J by K. With the initial condition at t equal to 0 and L is equal to 0.

Now, we have then so, we have expression of J there. So, you substitute in this expression J is nothing but a function of L. So, you can substitute there and you can substitute J is the function of L J is the function of L. So, the right hand side of this equation is nothing but d L dt is some function of L. So, this is an ordinary differential equation and with in with in initial value problem. So, it is an IVP problem. IVP ODE. So, it is it is an initial value problem. Ordinary differential equation and this can by solve by talent by by Newton's by R-K four method. May be, Round gutta four method can be used to numerically integrate this equation or you can use the Euler's equation or you can use other Adam equation you can use various equation you know algorithms to solve these equation ordinary differential equation. So, in the process what you get? In the process, will be getting L as a function of time. L as the function of time, once you get L as the function of time, you can get J as the function of time.

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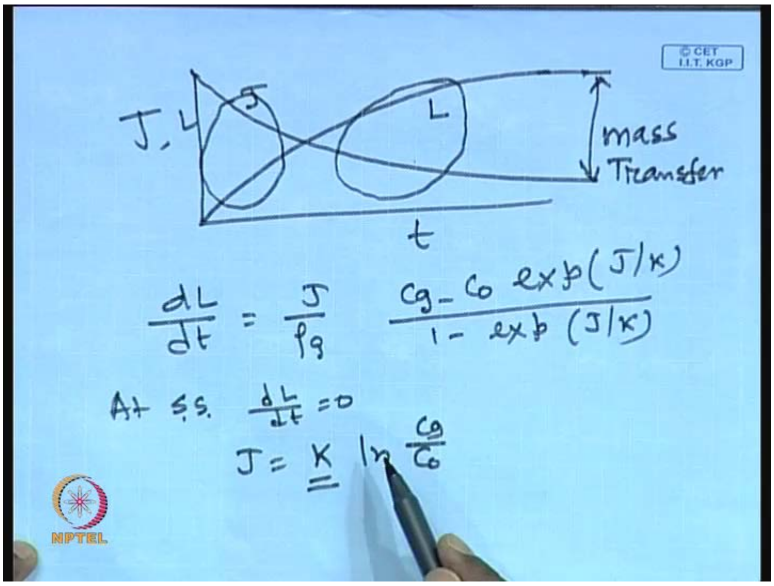

$$\frac{dL}{dt} = \frac{J}{\rho_g} \frac{c_g - c_0 \exp(J/k)}{1 - \exp(J/k)}$$

At s.s. $\frac{dL}{dt} = 0$

$$J = \underline{\underline{K}} \ln \frac{c_g}{c_0}$$

And, typically if you plot, the two plots are just opposite J as a function of time it will be something like this and L as the function of time it will be something like this. This is J and this is L. Gel gel layer thickness. And of course, you will be reaching a steady state. And, this steady state will be depending on mass transfer. So, this is the mass transfer dominated region. This is the in in between it is it will be gel dominated region, and here initially it will be membrane dominated region. So, there are three resistances one is the mass transfer another is the gel another is the membrane. So, initially you will be having the membrane dominated region. Then you will be having the gel dominated region and when the steady state reached will be getting the mass transfer dominated region.

And if you look into the expression of d L d t, I am just writing it once again d L dt will be J by rho g C g minus C naught exponential J by K divided by 1 minus exponential J by k. So, if you talk about the steady state; this is the transient operation if you talk about the steady state operation. Now, at steady state your d L dt will be equal to 0 right. So, what you get back? You get back the film theory equation that is, J equal to K l n C g by C naught right. So, at the steady state you will be getting J the steady state equation that you are obtain from the earlier analysis we have just turn in the earlier class that is J is equal to K l n C g by C naught. Now, what is the expression of K will be you will be putting? It will be either you will be putting the Leveque's equation that is 1.85 Reynolds smite D e by L rest to the power 1 upon 3 or 2.08 Reynolds smite D e by L rest to the power 1 upon 3. In fact, the second formulation will be more correct.

So, the steady state in the at at the end of transient in the gel layer control filtration initially you will be having the transient, and this initial means it is not few seconds, it may be few hours. So, it is very important here at the end whenever you will be getting the steady state you have you will get the steady state flux from whatever you have done earlier analysis. Now, if you look into the various parameters. Now it is very difficult to get the exact parameters like; alpha that is the gel layer specific gel layer resistance, epsilon g that is the porosity of the gel layer and particles are particle diameter. Now, there is a point in this particle diameter dP simply because we are we are using the Kozney-Karman equation from the filtration theory quantify alpha of the specific gel layer resistance or cake layer resistance.

But this particle diameter dP in the Kozney- Karman equation is valid if you have herds fears right. If you have herds fears spherical particles then you will be having the hand of of particles particles particular cell namely may be 1 micron 0.1 micron whatever it is. When most gel most of the cases in membrane filtration when you are talking about the gel layer controlling filtration for example, poly vinyl PVP poly vinyl pyramiding**,** PVA poly vinyl alcohol, or pectin. They are not spherical particles. What is what is Pectin? Pectin is nothing but a Polysaccharide it is a big chain. Polysaccharide chain, Proteins may be some of the moving (()) albumin it will be having ellipse side shape. You can considering it is a equivalent fear. But whatever, these Polymers like, poly vinyl alcohol there will not spherical shape like pectin. It it will be a polysaccharide. It is not spherical shape.

So, in this cases you will be having you will be considering that whenever we are quantifying alpha is equal to 180 1 minus epsilon g divided by epsilon g cube Rho g dP square; that means, we are assuming the gel layer resistance is equivalent to the resistance of equivalent spherical particles. So, dP is nothing but the equivalent spherical diameter of these particles. So, in most of that cases in membrane filtration whenever, talking about the gel the particles are not spherical in nature. So, therefore, we consider the equivalent spherical diameter of this particle. Now, this is basically a point of contention of the issue how to determine these parameters? It is very very important we determine these parameters for how to do that? So, for that there are you you would like like if you if you remember the example of real retention membrane permeability what you did? These parameters membrane performance parameter we did we conducted separate set of experiments to estimate them.

Like, L p the membrane permeability we conducted experiments we distilled water. And, determine the value of L p. Similarly, we conducted the different separate set of experiments under high turbulence to estimate the value of real retention. Similarly, we have to estimate a different set of experiments to estimate various parameters in the gel layer control filtration. Now, will look into those experiments or theory involved in in those experiments where we can identify? How to estimate these different parameters? Those will be appearing in the equation of the gel layer control filtration.

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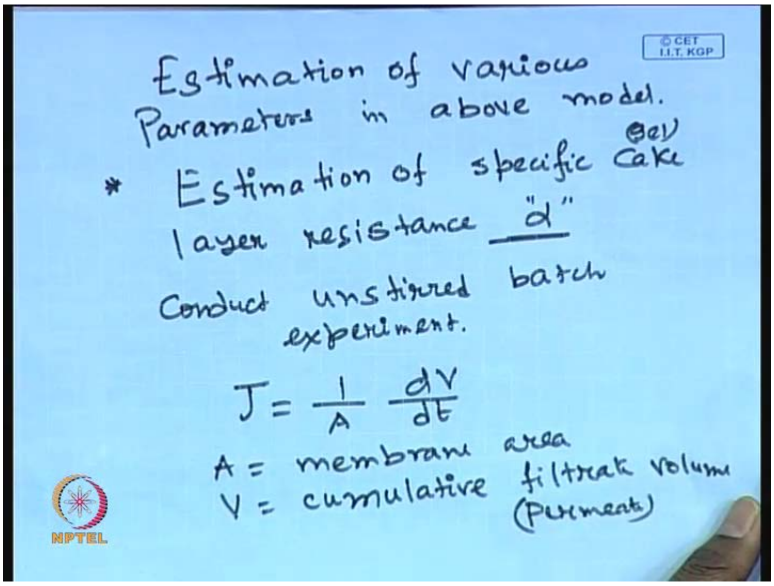

Estimation of various Parameters in above model.

* Estimation of specific cake (gel) layer resistance "α"

Conduct unstirred batch experiment.

$$J = \frac{1}{A}\frac{dV}{dt}$$

A = membrane area

V = cumulative filtrate volume (Permeate)

The first parameter now next, will look the estimation of parameters. Above model means the model that we have just discussed. First parameter is estimation of specific cake layer resistance alpha. Cake or gel. How to estimate the specific cake layer resistance alpha? If you remember that alpha is basically appearing when you when you are writing the expressing the R g of the gel layer resistance in terms of length. So, for that what you have to do? We have to conduct one unstirred batch experiment like earlier. There is there is very important the that is important of conduction of unstirred batch experiments. So, to estimate alpha will will conduct unstirred batch experiment with this gel forming solute. And, this is very simple it is a it is a cell it is a it is a cylindrical cell where you just feed the material in case the set the pressure by using nitrogen cylinder something.

And let, the system go as it is and measure the permeate flux we need not to control anything anywhere this is the very this is the simplest experiment with any any laboratory. So, conduct unstirred batch experiment to estimate the value of alpha how to do that? That you are just mentioning. The permeate flux J is written as 1 by a d V d t. What is A? A is nothing but the membrane area. And, what is V? V is the cumulative volume of filtrate; the water. That you are getting the permeate or this filter is nothing but the permeate. Cumulative volume that means, you are as time you just monitor the how much you are you are collecting with time? So, just monitor b by V versus t delta. Just measure it there is very simple either you can put the collect the permeate on a dicker or graduated cylinder and monitoring monitor the volume that is collected or if you do not want to do that just put it on a on a on a balance.

And see, the you know the weight that is getting and it is almost water. So, you know the density of the water you can get how much volume you are collecting cumulative volume you are collecting. Now, you now will go back to the phenomenological equation that is J is equal to delta p divided by mu R m plus R g and for a unstirred batch cell we just write J as 1 by A d V dt is equal to delta P divided by mu R m plus R g.

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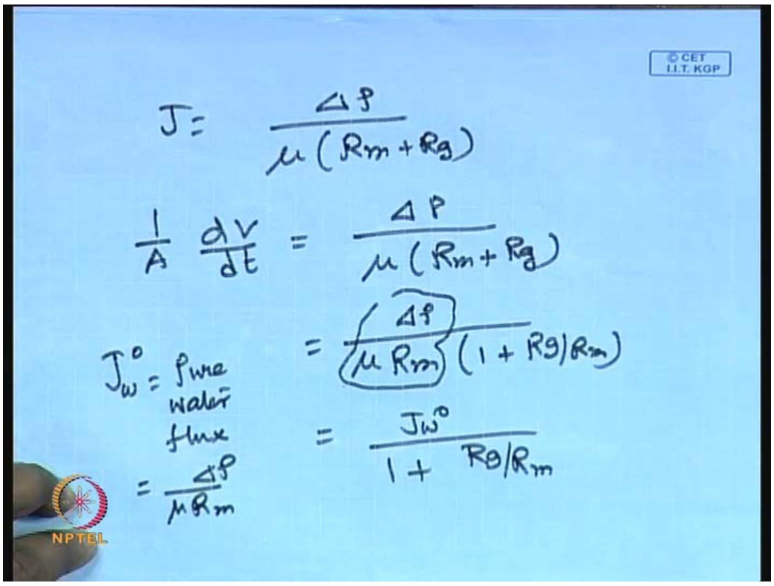

$$J = \frac{\Delta P}{\mu (R_m + R_g)}$$

$$\frac{1}{A}\frac{dV}{dt} = \frac{\Delta P}{\mu (R_m + R_g)}$$

$$= \frac{\boxed{\Delta P}}{\boxed{\mu R_m}\left(1 + R_g/R_m\right)}$$

$$= \frac{J_w^0}{1 + R_g/R_m}$$

J_w^0 = pure water flux $= \frac{\Delta P}{\mu R_m}$

So, you can rearrange it as delta P you can take R m out. So, this becomes 1 plus R g by R m. And, this is nothing but if you remember this is nothing but pure water flux. You have you have different earlier pure water flux. That is delta P by mu R m. So, you can write it as J w 0 divided by 1 plus R g by R m. Now, what you can do? Once, you write that then you can you can do a solute mass balance in the gel layer.

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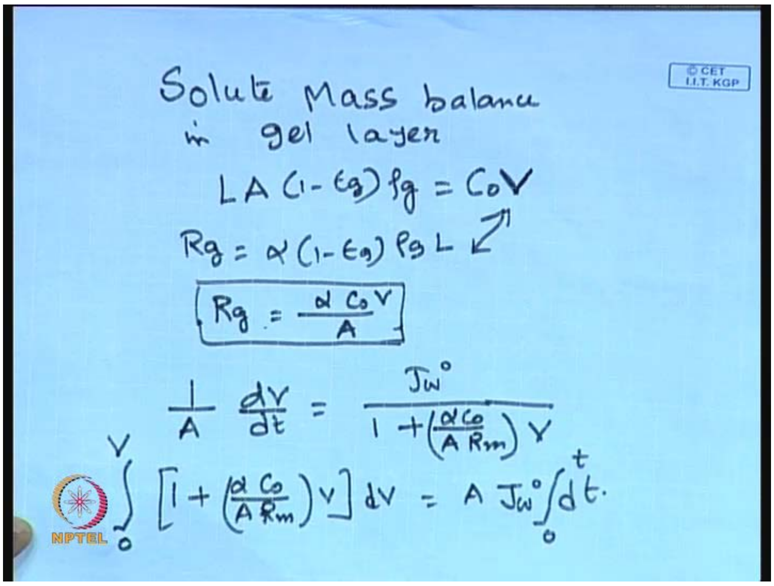

Solute Mass balance in gel layer

$$L A (1-\epsilon_g) \rho_g = C_0 V$$

$$R_g = \alpha (1-\epsilon_g) \rho_g L \swarrow\!\!\nearrow$$

$$\boxed{R_g = \frac{\alpha C_0 V}{A}}$$

$$\frac{1}{A}\frac{dV}{dt} = \frac{J_w^0}{1 + \left(\frac{\alpha C_0}{A R_m}\right) V}$$

$$\int_0^V \left[1 + \left(\frac{\alpha C_0}{A R_m}\right) V\right] dV = A J_w^0 \int_0^t dt.$$

You do a solute mass balance in gel layer what is that? That means, at any point of time what is the mss of the solute that is that is arrested that is present in the gel layer. What is that? L times A is the volume of the gel layer multiplied by 1 minus epsilon g because epsilon g is the porosity that much volume of volume fraction is occupied by water. So, rest of that; that means, 1 minus epsilon g is the volume fraction occupied by the particle. So, L times A times 1 minus epsilon g will be the volume of the gel forming particle? Right It should be multiplied by the density by of the gel layer of the gel forming particle roe g. That will be giving you the total mass of the solute that will be present in the gel layer. And, where from you got?

Because you are you are getting it from C naught times V. What is the V? V is the cumulative volume that you have collected at the particular volume. So, that much so, volume V was coming with the concentration C naught. So, C naught V is the total amount of solute that was that is arrested in the gel layer and from the definition of the geometric of the gel layer and their characteristics of the gel forming material we can get the mass of feed. So, they will be equal. So now, you can you can invoke the definition of gel layer resistance. R g was is equal to what is that? Alpha into1 minus epsilon g Rho g times L. So, I can write R g is nothing but alpha times C naught times V over A.

So, this L can be substituted as R g divided by alpha 1 minus epsilon g roe g. So, combining these two we can get R g or gel layer resistance as a function of gel layer specific gel layer resistance and cumulative volume. Now, I am going to put in the phenomenological equation that we have derived earlier. If you do that you will be getting 1 by A d V dt is equal to J w 0 divided by 1 plus R g by R m right. 1 plus R g by R m I am going to write R g as this. So, you will be getting alpha C 0 by A R m times V right. Now, everything all the dependent variable will be in terms of V. So, basically what I did? I just replaced L in terms of V. So, just take it down the other side and you will be getting 1 plus alpha C 0 by A R m times V D V A J w 0 times d t. Now, we can integrate it out; that means, time t from 0 to t time t is equal to 0 there is no filtrate volume you are at term t is equal to t you are getting volume V.

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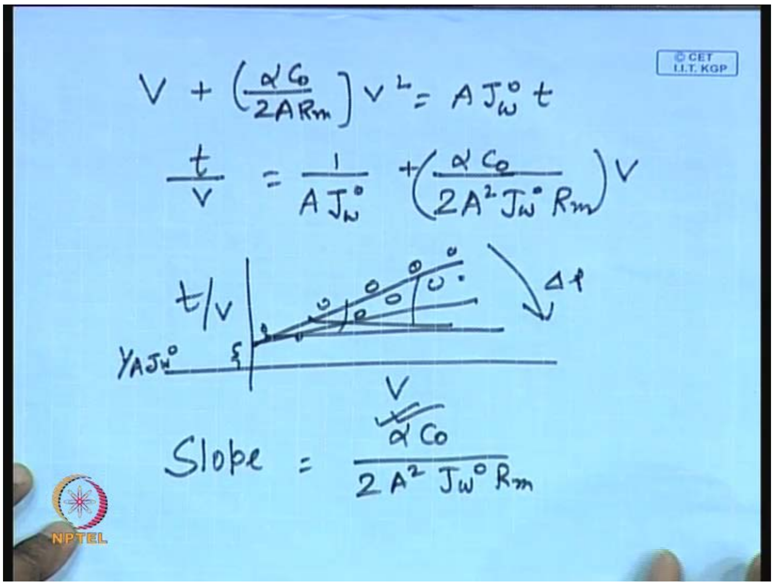

$$V + \left(\frac{\alpha C_0}{2AR_m}\right)V^2 = AJ_w^0 t$$

$$\frac{t}{V} = \frac{1}{AJ_w^0} + \left(\frac{\alpha C_0}{2A^2 J_w^0 R_m}\right)V$$

$$\text{Slope} = \frac{\alpha C_0}{2A^2 J_w^0 R_m}$$

So, from 0 to t and 0 to V you can integrate this equation out and see what you get? What you get is will be getting V plus alpha C naught 2 A R m V square is equal to A J w 0 times t. So, that is the expression you will be getting after doing the a after carrying out the integration. Now, you can rearrange this equation and these will be t by V is equal to 1 over A J w 0 plus alpha C 0 2 A square J w 0 times R m times V. Now, what is equation says? This equation says that if I plot t by V versus V it will be giving a straight line. The intercept will be given by this quantity the slope is given by this quantity. Right so, will be getting a straight line something like this.

So, the intercept is given by A 1 over A J w 0 and slope is given by this; that means, you do a you conduct a batch cell experiment just harmful just feel up the cell with the with the solute the solution. And then, pressurize it leave it leave the solution at at the particular pressure go on monitoring what is the cumulative volume? That collected at at as a function of time. Then, plot t by V versus V will be a getting a straight line from the slope you will be getting the value of alpha. Because C naught is known to you, you know the membrane area that is basically pi R square. You know the initial permeability the pure water flux that is delta p by mu R m and you know the R m. So only, unknown in the slope is basically alpha.

So, from the slope; this slope is nothing but alpha C naught divided by 2 A square J w 0 R m. And, from this equation, you can get the value of alpha. Now, we conduct the experiment for a different delta P you will be getting the value of alpha corresponding to the delta P. Now, you can set the operating pressure third one and carry out this experiment and plot these. So, you will be getting alpha corresponding to that particular delta P. In fact, if you plot you will be getting the variation of you will be get different slopes with the variation of delta P something like this.

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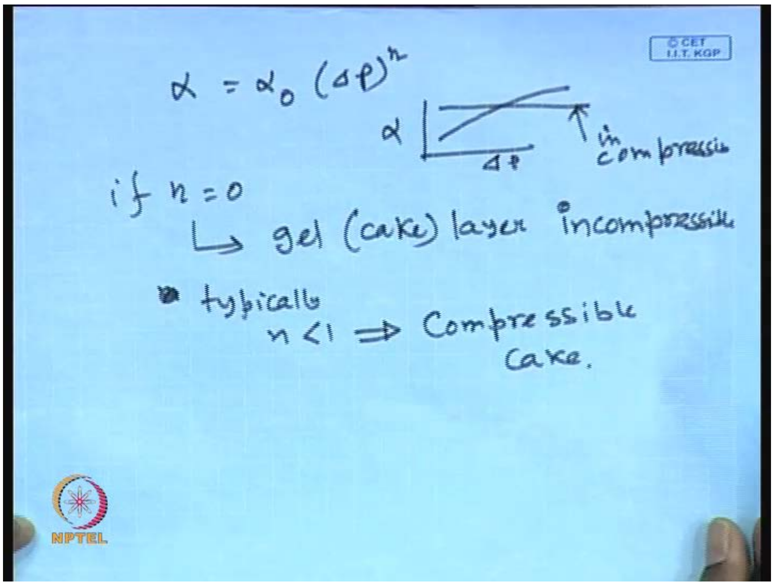

$$\alpha = \alpha_0 (\Delta P)^n$$

if $n = 0$

↳ gel (cake) layer incompressible

- typically $n < 1 \Rightarrow$ Compressible cake.

So, therefore, by conducting the batch cell experiment under various pressure one can get alpha as a function of pressure; that means, for for various pressure you can get the value of alpha then you plot the value of alpha versus delta P. It it may be a plot something like this so; that means, alpha can be express as a function of alpha naught times delta P rest to the power n. If this n is equal to 0 then alpha is invariant with delta P. In that case, the cake layer or gel layer is incompressible; that means, the pressure does not any effect on it it will not complex, does not any effect on it. So, in that case this is incompressible. If n equal to 0 the gel layer or cake layer is incompressible for n typically generally this n becomes slightly low less than 1 typically n is less than 1 and it; that means, it is a compressible cake.

So, those are different. So, one can… So, estimation of alpha is very clear to how to how to estimate the alpha. So, basically what you are going to do? It is it is very simple you are conduct then you are going to conduct the experiment in a batch cell and monitor how much volume cumulative volume you are going to get as a function of time? That means, you are just collect the permeate on a on a let say balance and just keep on monitoring the what is the time and this is the volume what is the time and this is the volume. So, V versus t delta and plot t by V versus V. You will be getting a straight line from the slope you will be you will be calculating the value of alpha or specific gel layer resistance. Next, we see how to estimate the value of epsilon g porosity of the gel layer and particle diameter.

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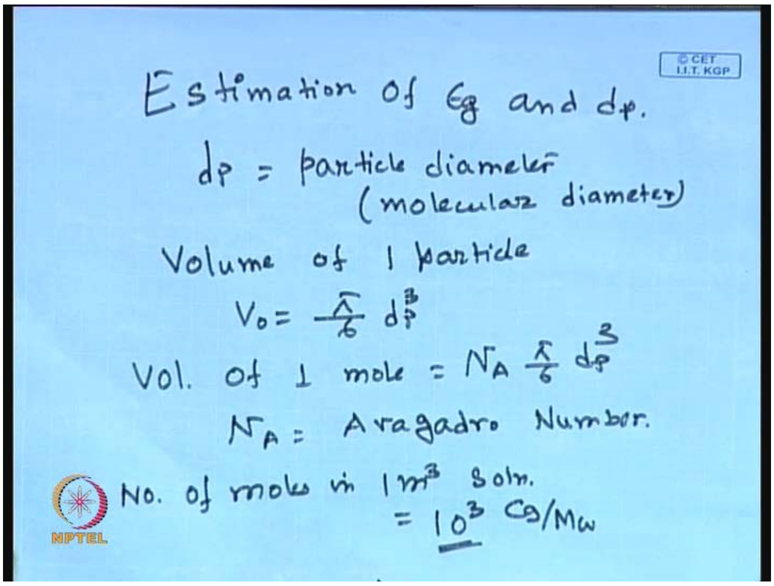

Estimation of ϵ_g and d_p.

d_p = particle diameter (molecular diameter)

Volume of 1 particle

$V_0 = \frac{\pi}{6} d_p^3$

Vol. of 1 mole $= N_A \frac{\pi}{6} d_p^3$

N_A = Avagadro Number.

No. of moles in 1 m^3 soln. $= 10^3 C_g/M_w$

Now, we we do a derivation here small derivation. Let us, consider dP is the particle diameter is that nothing but the molecular diameter. So, volume of 1 molecular 1 particle is what? Let us, consider is a spherical particle. There is a easiest way to start with volume of once particle will be nothing but V naught is nothing but pi by 6 dP cube 4 or

5 cube pi by 6 dP cube. Now, volume of one mole will be how much? You should be multiplied by the Avogadro number. N A pi by 6 dP cube, where N A is nothing but the Avogadro number; number of moles in one meter cube solutions let say, you need volume is one meter cube is nothing but 10 cube multiplied by the C g divided by the molecular weight. C g is the gel layer concentration in Kg per meter cube. So, if you divided by molecular weight, you will be getting number of moles and this factor 10 cube comes out because this one meter cube solution you have taken.

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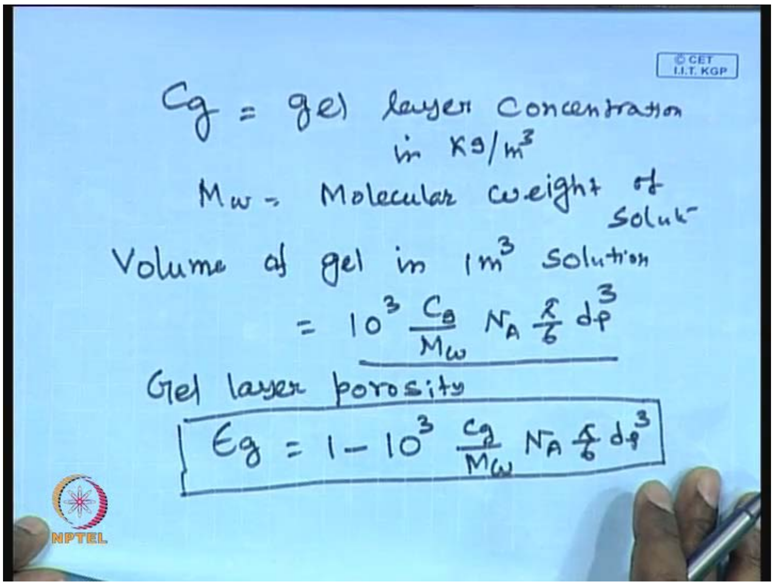

C_g = gel layer concentration in Kg/m^3

M_w = Molecular weight of solute

Volume of gel in 1 m^3 solution

$$= 10^3 \frac{C_g}{M_w} N_A \frac{\pi}{6} d_p^3$$

Gel layer porosity

$$\boxed{\epsilon_g = 1 - 10^3 \frac{C_g}{M_w} N_A \frac{\pi}{6} d_p^3}$$

So, we write the these quantities C g is nothing but the gel layer concentration in Kg per meter cube that is important; that is why this factor 10 cube is coming; M w is the molecular weight of solute. Then volume of gel in 1 1 meter cube solution is how much? Volume of gel in 1 meter cube solution is nothing but 10 cube C g by M w multiplied by N A pi by 6 dP cube; that is the volume of gel in 1 meter cube solution. Now, what is the porosity, definition of porosity? Porosity is the nothing but the fraction of volume that is occupied by water; that means, epsilon g is nothing but 1 minus this this is the solid volume occupied. So, the liquid liquid volume will be 1 minus that 10 cube C g M w N A pi by 6 dP cube.

So, this is the relationship of the porosity of the bed and the particle diameter, equivalent particle diameter if you are talking about a non sphere or not not hard sphere particles.

Now, we have the definition of alpha that is alpha is specific cake layer resistance that is related to the epsilon g and dP through Kozney- Karman equation right.

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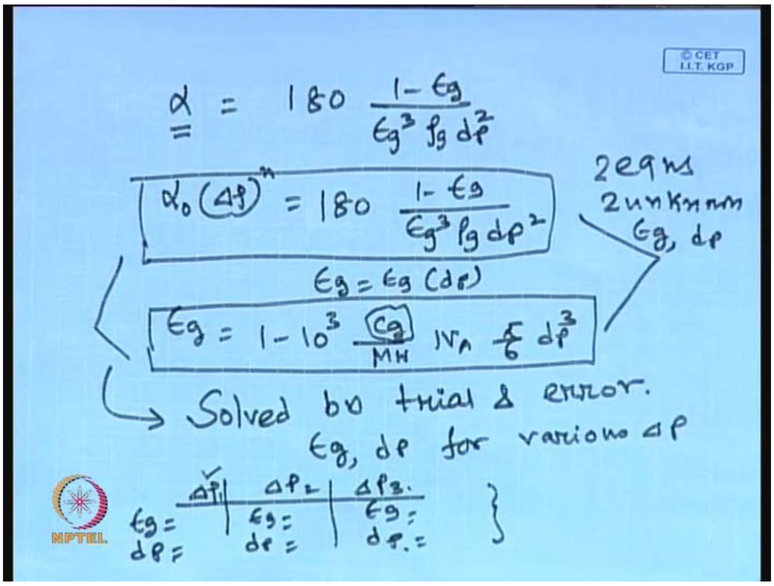

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$$\alpha = 180 \frac{1-\epsilon_g}{\epsilon_g^3 \rho_g d_p^2}$$

$$\boxed{\alpha_0 (\Delta P)^n = 180 \frac{1-\epsilon_g}{\epsilon_g^3 \rho_g d_p^2}}$$

2 eqns
2 unknown
ϵ_g, d_p

$$\epsilon_g = \epsilon_g (d_p)$$

$$\boxed{\epsilon_g = 1 - 10^3 \frac{C_g}{M_W} N_A \frac{\pi}{6} d_p^3}$$

Solved by trial & error.
ϵ_g, d_p for various ΔP

ΔP_1	ΔP_2	ΔP_3
$\epsilon_g =$ $d_p =$	$\epsilon_g =$ $d_p =$	$\epsilon_g =$ $d_p =$

If you write down that alpha is nothing but 1 over 80 1 minus epsilon g divided by epsilon g cube rho g times dP square; and we have already seen, this alpha can be a function of delta P if it is compressible. So, alpha can be written as alpha naught by delta P to the power n. We know how to determine the value of specific gel layer resistance by conducting batch cell experiments under different pressures. Once, you do that you can feed a something like this, alpha naught delta P to the power n is equal to 180 1 minus epsilon g epsilon g cube Rho g times dP square and write just now we have derived a relationship of a epsilon g as a function of d p. What is that? The epsilon g is equal to 1 minus 10 cube C g molecular weight NA pi by 6 dP cube. This is one equation, this is another equation.

So, for a particular delta P, the whole left hand side of this equation is known to you, and the right hand side you will be having combination of epsilon g and d p. Rho g rho g will be known. And this equation gives your relationship of epsilon g and dp if the value of gel concentration is known, and that will be known. So, you will be having two equations, two unknown. What is unknown? Epsilon g and dp. So, these two equations can be solved by trial and error. You can use neutral absent solved by trial and error and you can estimate epsilon g and dP for various delta P; that means, you can you can prepare a table of delta P for a particular delta P what is the epsilon g and d p? For the particular delta P what is the epsilon g and d p?

So, delta P 1, delta P 2, delta P 3 epsilon g dP so, various operating pressure these bed porosity and particle diameter will be change; obviously, these method these method if you if you remember these method is valid for the non herds fear solutes; that means, what I mean by that suppose, you are having a you are having a pectin gel and and they are not herds fear right they are polymeric. So, by by changing the pressure the gel will be compressed the characteristics will be changed. By suppose, you are you are the solute is fixed; that means, the size will be fixed. But if it is the herds fear then we are not going to going to adapt these method if, they are herds fear we are going to adapt these method because in that case the particle diameter is fixed. The porosity will be fixed.

So, everything will be fixed, but in most of the membrane filtration process we are not talking about the herds fear, we are talking about the polymers, we are talking about the polysaccharides, we are talking about the gel not forming of solutes, but by the by the high viscous liquids. In that case, the equivalent particle diameter and equivalent porosity at the particle operating pressure can be can be can be estimated by this method. So, there is a there is a certain difference between the herds fear and this is non herds fear case. So, what you are meaning by that? That means, delta P 1 you are dP will be something in delta P 3 under the delta P 3 operating condition dP will be something else; that means, these weight will be having an equivalent resistance of solutes having uniform particle diameter dP 1 under the pressure delta P 1. Same bed under the different operating pressure delta P 3 will be having the equivalent diameter dP 3 may be. Because the particle diameter particles are not herds fear. That not have actual dimension all the time; they will be compressible. So, that is the thing.

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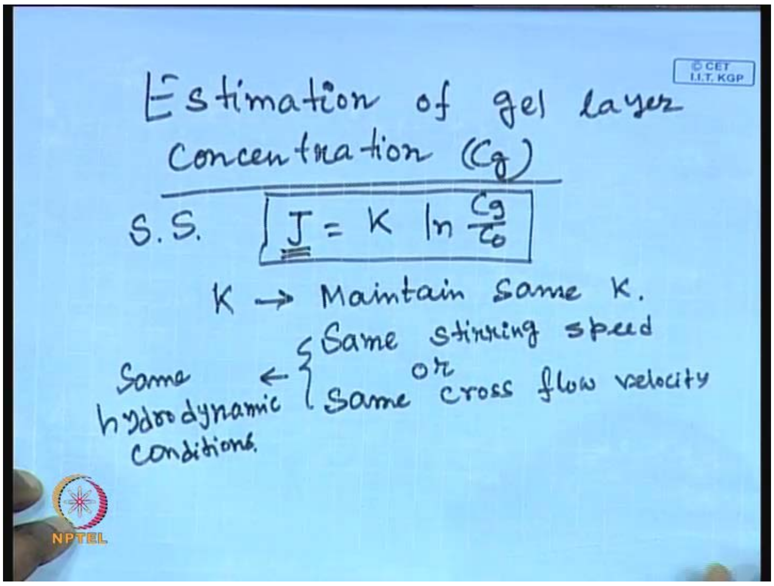

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Estimation of gel layer concentration (C_g)

S.S. $\boxed{\underline{J} = K \ln \frac{C_g}{C_0}}$

K → Maintain same K.

Same hydrodynamic conditions ← $\begin{cases} \text{Same stirring speed} \\ \text{or} \\ \text{Same cross flow velocity} \end{cases}$

Now, you have estimated all the parameters except one parameter that is the how to estimate the gel layer concentration? That is very vital. For estimated estimation of gel layer concentration. If you remember the the permeate flux under the steady state J is equal to K times l n C g by C naught. This is the expression film theory expression of permeate flux under the steady state operation. Now, what we can do? We do not know what is the value of c gel? What will do? We take different concentration various concentration of the solute. And carry out the filtration experiment and wait for the steady state being reached. Now we we keep the same mass transfer coefficient. We maintain the same mass transfer coefficient how to maintain that? If we maintain same hydro dynamic in the channel the mass transfer co-efficient will be same; that means, we are going to conduct number of experiments under the same stirring speed or same cross flow velocity.

That means same hydro dynamics. So, we conduct let say, 10 experiments 10 10 complete experiments by taking 10 concentrations; may be 1 1 k g in one case C naught may be let may be 10 K g per meter cube in other case C naught may be 20 K g per meter cube another case may be 25 K g per meter cube and may be 40 K g per meter cube like that. We consider we we conduct experiment separate independent experiments by considering various concentrations of the feed solute and under the same hydrodynamic conditions. And, measure and and in a every experiment every experiment wait for the steady state to be reached. May be, after half an hour may after one hour. Once steady state is reached then, we measure the steady state permeate flux.

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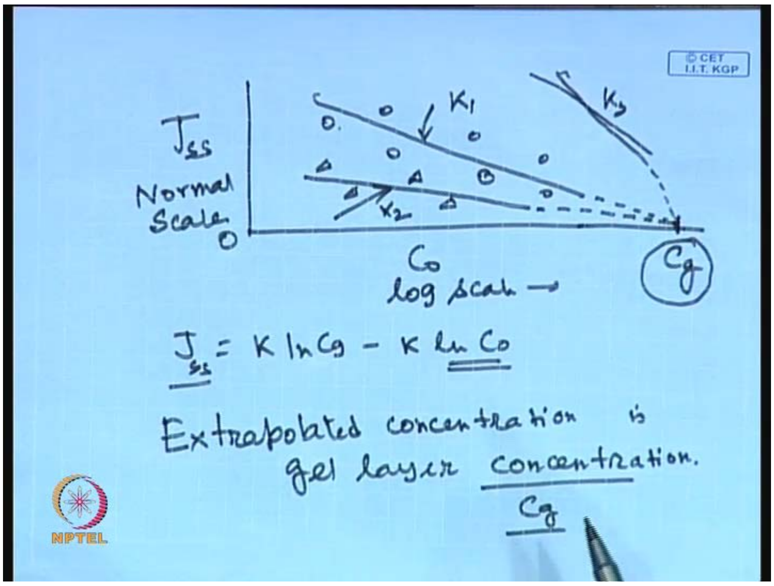

$J_{ss} = K \ln C_g - K \ln C_0$

Extrapolated concentration is gel layer concentration.

C_g

Now, we plot the steady state permeate flux versus feed concentration in a same (()) plot. J steady state versus concentration this will be normal, this will be logarithmic scale. And, if you look into the expression J is equal to K l n C g minus K l n C naught. So, the plot of J is versus l n C naught is suppose to be a straight line with a decreasing slope because there is a minus sign present. So, you are going to get a curve something like this. This may be the experimental points of the steady state flux and 10 different concentrations. You will be getting A getting A almost a straight line of decreasing slope. Now, you are going to extra polite these thing you are going to extra polite these thing and the point the point where J becomes 0 that particular concentration becomes gel layer concentration C gel. The all these 10 experiments have been conducted with the same mass transfer coefficient.

You can conduct these 10 experiments with the different mass transfer coefficient or difference stirring speed or different cross flow velocity if you do that. Then, will be K 1 then you are going to get another straight line may be del triangle triangle triangle and all these experiments are conducted with the mass transfer coefficient K 2. Then, if you can extra polite they will converged almost at the same gel point. Similarly, you can conduct experiment for a different mass transfer coefficient let say K 3 and they will be converging on the same gel point. So, that is how the gel layer concentration will be evaluated. So, if you if you plot this in a semi logarithmic plot the concentration that will be extra polite will be the gel layer concentration, concentration as C g.

Now, so, basically you have to conduct number of experiments independently and evaluate and and analyze those data and independently estimate the gel layer concentration. So, in these gel layers model the several parameters those will be appearing. So, will be having tools or method to estimate them separately, independently and then, you can use those data and can find the we can can use the gel layer controlling filtration model under the steady state condition. So, so I will stop here. So, we will continue these thing in the all these discussions in the next class thank you.