Novel Separation Processes
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Lecture No # 10
Membrane Separation Processes (Contd.)

Good morning everyone. So, we will be looking into the modeling of unstart batch experiments. Now, unstart batch membrane by separation processes; and in the last class we have elaborated what is the importance of this unstart batch experiments, and we will why we will looking at two model them, and how we can generate a large number of data in a short period of time. So, what we did we again we wrote down the concentration species balance equation within the mass transfer boundary layer, and in this case the concentration is a function of time as well as space in the y direction. So, we go reduce the equations into and analyze the equations and found out that a similarity solution can we useful in this case because the governing equation is a parabolic partial differential equation, and one of the boundary condition is residing at infinity.

So, in this case we can have a similarity solution like the cross flow cell cross flow experiment, and we can identify made the equation non dimensional and we formulated the boundary conditions as well and made them non dimensional. Then what we did we did an order of magnitude analysis at the edge of the concentration boundary layer and found a similarity parameter in this particular case. And in this class we will be getting the so, what is the next procedure, next procedure is replacing the partial derivative in terms of similarity derivative of similarity parameter. And we expect the partial differential equation will now reduce to the governing ordinary differential equation and the we look into the solution of the ordinary resultant ordinary differential equation.

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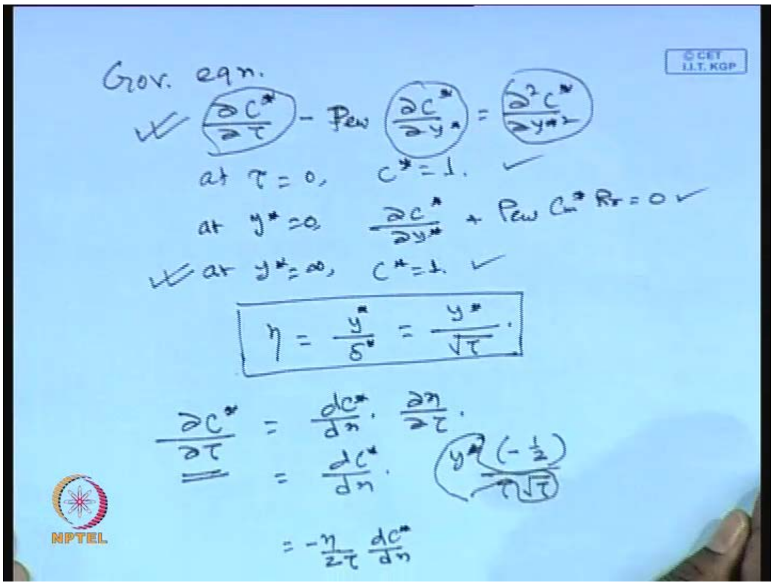

Gov. eqn.

$$\frac{\partial C^*}{\partial \tau} - Pe_w \frac{\partial C^*}{\partial y^*} = \frac{\partial^2 C^*}{\partial y^{*2}}$$

at $\tau = 0$, $C^* = 1$.

at $y^* = 0$, $\frac{\partial C^*}{\partial y^*} + Pe_w C_m^* R_r = 0$

at $y^* = \infty$, $C^* = 1$.

$$\boxed{\eta = \frac{y^*}{\delta^*} = \frac{y^*}{\sqrt{\tau}}}$$

$$\frac{\partial C^*}{\partial \tau} = \frac{dC^*}{d\eta} \cdot \frac{\partial \eta}{\partial \tau} = \frac{dC^*}{d\eta} \cdot \frac{y^*\left(-\frac{1}{2}\right)}{\tau\sqrt{\tau}} = -\frac{\eta}{2\tau}\frac{dC^*}{d\eta}$$

So, you should remember the governing equation that we have written in the last class for the transient or the time varying infiltration system is del C star del tau is equal to minusPe w, del C star del y star is equal to del square C star del y star square; and we wrote down the initial and the boundary condition at time tau is equal to 0 that is my initial condition; C star is equal to 1 that is C is equal to C naught; at y star is equal to 0, we have the mixed boundary condition that is the convective flux towards the membrane and the diffusive flux away from the membrane will be equal del C star del y star plusPe w C m star times real retention is equal to 0; and at y star is equal to infinity, we had C star is equal to 1.

Now, these boundary condition gives the initial condition; this is the boundary condition lowest to equal to zero; and these are another boundary condition at y star is equal to infinity, constituting the mass transfer boundary layer is really very small order of magnitude are less compare to the actually dimension of the system. So, it is in fact, on the order of 10 to the minus 6 micron meter. Now, this governing equation is a parabolic partial differential equation, and this boundary condition is infinity. So, we can have a similarity solution, and indeed we can have a similarity solution, we have analyze a situation and found out the similarity parameter in this case as y by delta, which is nothing but, y star make it non dimensional; so, it becomes y star by root over tau. So, this is the similarity parameter we have derived in the last class. So, what is left now, the left is basically to get the derivative of this partial derivative of the original, in terms of the original coordinate system to be expressed, in terms of this similarity parameter or combine variable parameter, eta. So, instead of two independent variables y star and tau, you will be having a single variable eta and our resultant equation will be boiled down into an ordinary differential equation.

So, let us look into the these derivatives. So, del C star del tau will be nothing but, del d C star d eta times d eta del tau. So, this becomes d C star d eta and del eta del tau will be y star will be remaining as it is tau to the power minus half right. So, it will be minus half to tau to the power minus three by two. So, it will be nothing but, tau root over tau. Now this becomes y star by root over tau; this becomes nothing but eta; so, this becomes eta minus eta by two d C star d eta, and you will be having another tau here; so, it will be you will be having eta by 2 tau d C star d eta. So, this partial derivative with respect to tau can be expressed in terms of the total derivative with respect to eta. Now, we will look into the other derivatives del C star del y star and del square C star del y star square.

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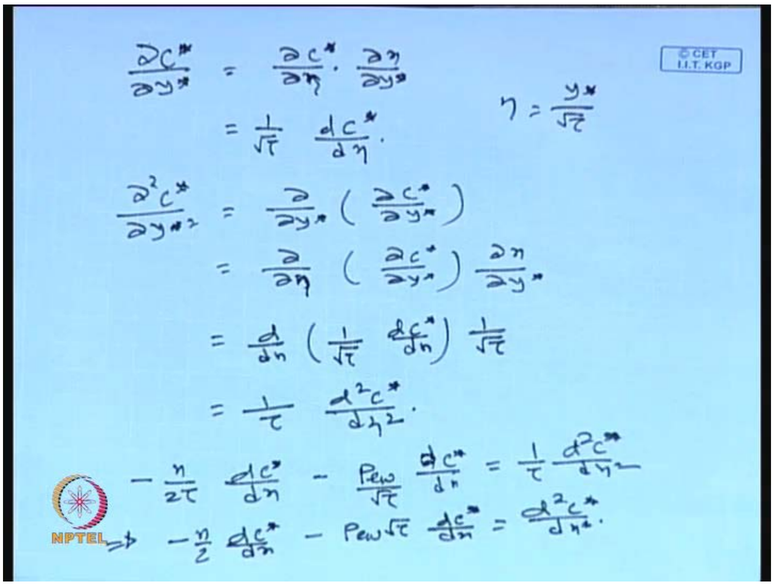

$$\frac{\partial C^*}{\partial y^*} = \frac{\partial C^*}{\partial \eta} \cdot \frac{\partial \eta}{\partial y^*}$$

$$= \frac{1}{\sqrt{\tau}} \frac{dC^*}{d\eta}. \qquad \eta = \frac{y^*}{\sqrt{\tau}}$$

$$\frac{\partial^2 C^*}{\partial y^{*2}} = \frac{\partial}{\partial y^*}\left(\frac{\partial C^*}{\partial y^*}\right)$$

$$= \frac{\partial}{\partial \eta}\left(\frac{\partial C^*}{\partial y^*}\right)\frac{\partial \eta}{\partial y^*}$$

$$= \frac{d}{d\eta}\left(\frac{1}{\sqrt{\tau}}\frac{dC^*}{d\eta}\right)\frac{1}{\sqrt{\tau}}$$

$$= \frac{1}{\tau}\frac{d^2C^*}{d\eta^2}.$$

$$-\frac{\eta}{2\tau}\frac{dC^*}{d\eta} - \frac{Pe_w}{\sqrt{\tau}}\frac{dC^*}{d\eta} = \frac{1}{\tau}\frac{d^2C^*}{d\eta^2}$$

$$\Rightarrow -\frac{\eta}{2}\frac{dC^*}{d\eta} - Pe_w\sqrt{\tau}\frac{dC^*}{d\eta} = \frac{d^2C^*}{d\eta^2}.$$

So, del C star del y star will be del C star del eta del eta del y star and if you remember eta is nothing but, y star by root over tau. So therefore, del eta del y star will be nothing but, one by root over tau and C star is now suppose to be a sole function of eta; so, this partial derivative will be replaced by a total derivative tau d C star d eta; and what is del square C star del y star square, del square C star del y star will be nothing but, del del y star times del C star del y star; now, I replaced del del y star by del eta del eta del y star del C star del y star remains same this becomes del eta del y star using the general of the differentiation, and what is this del C star del y star it is already expressed in terms of d C d eta.

So, this becomes d d eta this becomes total derivative, because concentration is now a sole function of eta. So, root over this will be root over tau d C star d eta and del eta del y star will be again one over y root over tau. So, this becomes one over tau d square C star d eta square. So, that way we can express all the partial derivatives in terms of the single variable eta; now, you can substitute in the governing equation this becomes what.

So, you just write it del C star del eta becomes minus eta by 2 tau d C star d eta minusPe w del C star del y star becomes one over root tau d C star d eta one over root over tau d C star d eta; and this becomes d square del square C star del y star square will becomes one over tau d square C star d eta square; so, multiply both side by tau and let us see what you get, this becomes minus eta by 2 d C star d eta minus P e w root over tau d C star d eta is equal to d square C star d eta square. Now, we can combine this two terms and this ordinary differential is second order ordinary differential equation can be written in a neat form.

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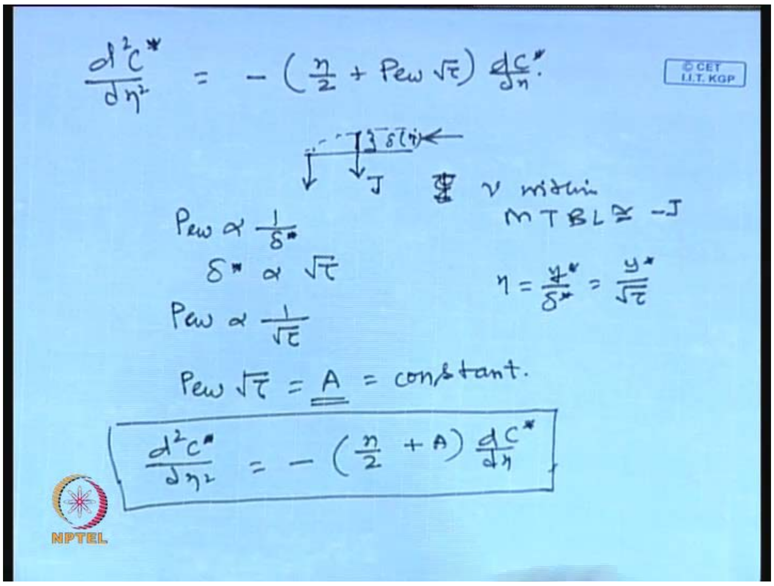

$$\frac{d^2C^*}{d\eta^2} = -\left(\frac{\eta}{2} + Pe_w\sqrt{\tau}\right)\frac{dC^*}{d\eta}$$

v within MTBL ≅ −J

$Pe_w \propto \frac{1}{\delta^*}$

$\delta^* \propto \sqrt{\tau}$

$Pe_w \propto \frac{1}{\sqrt{\tau}}$

$\eta = \frac{y^*}{\delta^*} = \frac{y^*}{\sqrt{\tau}}$

$Pe_w\sqrt{\tau} = \underline{\underline{A}}$ = constant.

$$\boxed{\frac{d^2C^*}{d\eta^2} = -\left(\frac{\eta}{2} + A\right)\frac{dC^*}{d\eta}}$$

And that becomes this d square C star d eta square is equal to minus eta by 2 plus P e w root over tau d C star d eta. So, the governing equation becomes a second ordinary differential equation now this if you remember the permeate flux that is the permeate flux that you are getting out of the membrane will be pretty small and these will be and the thickness of the concentration boundary will be also it is very thin compare to the any physical dimension of the system. So therefore, we assume that the y component velocity in the concentration boundary layer will be same as that is valid at the wall; although, it is an approximation by it will be a valid approximation there; that means, v within the mass transfer boundary layer, within mass transfer boundary layer will be nothing but, approximated as minus J.

So therefore, and we have seen that this value of the permit flux or the solvent flux will be inversely proportional to the thickness of this delta, less be the thickness of the concentration boundary layer more less resistance it will be offering to against the solvent flux; so, you can expect a higher permeate flux at the beginning of the you know at lower period of term now in this case delta will be function of tau time; so, when the thickness becomes more the permeate flux will be small; so, which is inversely proportional to that; so therefore, non dimensional permeate flux will be inversely proportional to delta star, and delta star is directly proportional to root over tau, that you have already seen because, the definition if you look into a definition of eta; eta is nothing but, y star by del star and the and del star is proportional to t to tau to the power half. So, it will be one by root over tau. So, delta star is directly proportional to root over tau and therefore, P e w will be nothing, but it is inversely proportional to root over tau; that means P e w times root over tau is A constant, and we call this constant as A.

So, in terms of this new constant now we rewrite our governing equation as d square C star d eta square is equal to minus eta by 2 plus A d C star d eta; now, we have to solve these equation at the same time, we have to write down the boundary conditions in terms of eta as well; now, since this is an these are second order ordinary differential equation with respect to eta; we must be having two boundary conditions on eta to solve this governing equation into ordinary differential equation.

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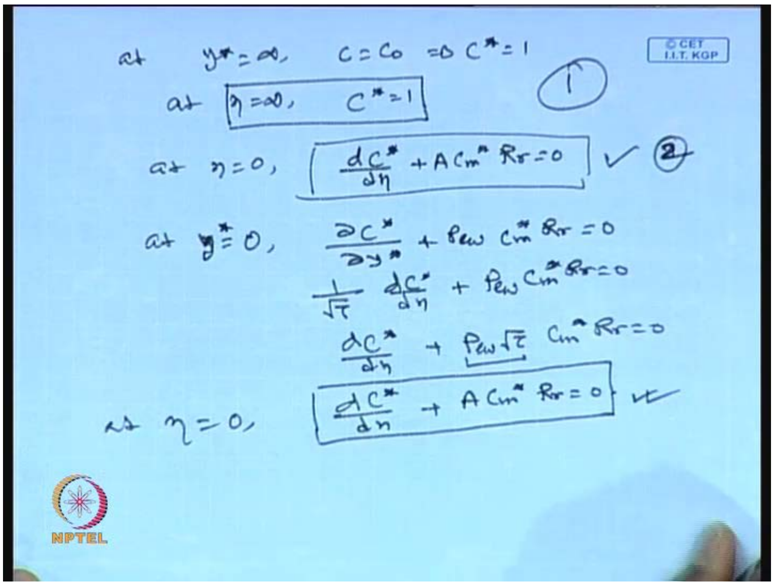

at $y^* = \infty$, $C = C_0 \Rightarrow C^* = 1$ ①

at $\boxed{\eta = \infty, \quad C^* = 1}$

at $\eta = 0$, $\boxed{\frac{dC^*}{d\eta} + A C_m^{*} R_r = 0}$ ✓ ②

at $y^* = 0$, $\frac{\partial C^*}{\partial y^*} + Pe_w C_m^* R_r = 0$

$\frac{1}{\sqrt{\tau}} \frac{dC^*}{d\eta} + Pe_w C_m^* R_r = 0$

$\frac{dC^*}{d\eta} + \underbrace{Pe_w \sqrt{\tau}} C_m^* R_r = 0$

at $\eta = 0$, $\boxed{\frac{dC^*}{d\eta} + A C_m^* R_r = 0}$ ✓✓

So, at eta at y star is equal to infinity C was is equal to C naught; so, C star is equal to 1; so therefore, and at eta is equal to infinity, we have C star is equal to one that is one boundary condition expressed in terms of eta, and the other boundary condition was at eta is equal to 0; at eta is equal to 0, you had d C star d eta plus A C m star Rr is equal to 0; how did you get this equation, this equation can be obtained if you look into the governing equation, the original governing equation at eta equal to 0; that means, let us clarify everything, and let us do a couple of steps derivation; so, that will be then things will be clearer at y star is equal to 0, we had the original boundary condition was as del C star del y star plus P e w C m star Rr equal to 0, that was the original boundary condition; now, if we express del C star del y star in terms of eta, we have already derive that one over root over tau d C star d eta, del C star del y star can be written as one over root over tau d C star d eta, we have already looked into a derivation P e w C m star Rr is equal to 0; so, d C star d eta plus P e w root over tau C m star Rr is equal to 0 and this is the constant A that we have already defined earlier.

So, get this expression, which is the boundary condition at eta is equal to 0; y star equal to 0 means eta equal to 0; so, that is how you obtain this boundary condition in term of eta, and these two boundary conditions this is number 1, this is number 2 that defines the you know solution of the governing equation that whatever we have expressed earlier in terms of eta; so, this gives the derivation of the how you obtain these boundary condition at eta is equal to 0; now, we will be in a position to get the solution of concentration profile.

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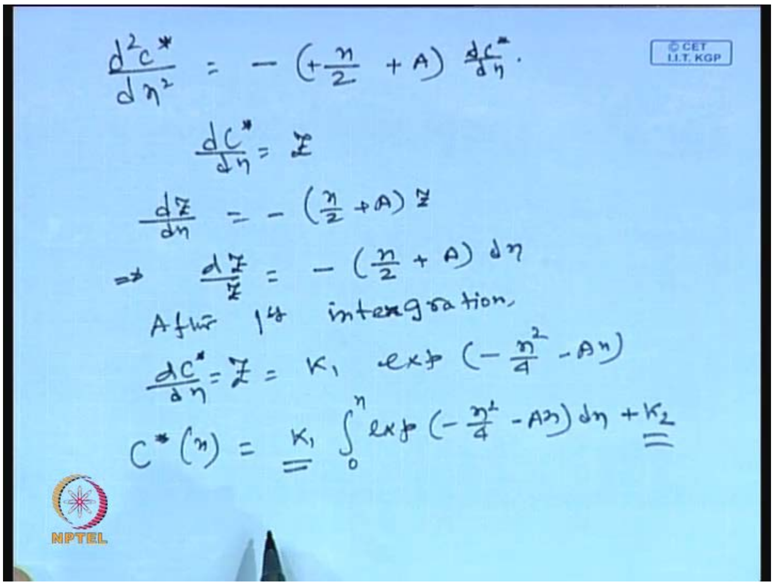

$$\frac{d^2C^*}{d\eta^2} = -\left(+\frac{\eta}{2} + A\right)\frac{dC^*}{d\eta}.$$

$$\frac{dC^*}{d\eta} = Z$$

$$\frac{dZ}{d\eta} = -\left(\frac{\eta}{2} + A\right)Z$$

$$\Rightarrow \frac{dZ}{Z} = -\left(\frac{\eta}{2} + A\right)d\eta$$

After 1st integration,

$$\frac{dC^*}{d\eta} = Z = K_1 \exp\left(-\frac{\eta^2}{4} - A\eta\right)$$

$$C^*(\eta) = \underline{\underline{K_1}} \int_0^{\eta} \exp\left(-\frac{\eta^2}{4} - A\eta\right)d\eta + \underline{\underline{K_2}}$$

So, let us write down the governing equation one more time discuss is C star d eta square minus eta by 2 plus A d C star d eta; now, you consider d C star d eta is another variable Z; so, this becomes d Z d eta minus eta by 2 plus A and Z; so, this becomes d Z by Z is equal to minus eta by 2 plus A d eta; so, the one first integration will give you l and Z; so, in terms of the after first integration, what you get is, you get Z is equal to k 1 exponential minus eta square by 4 minus A eta. So, the k 1 is the first constant of integration.

So, what is Z, Z is nothing, but d C star d eta; so, one more integration will result you the expression of C star as a function of eta, what is the expression of C star, C star will be k 1 exponential minus eta square by 4 minus a eta d eta plus k 2, and what is this integration, integration is basically over the variable eta; so, the minimum value of eta is eta equal to 0 and so, it this integration basically runs from 0 to eta implicitly; now you have the two boundary conditions that we have just discussed in the last slide and we using these two boundary conditions, one can evaluate these two constants of integration.

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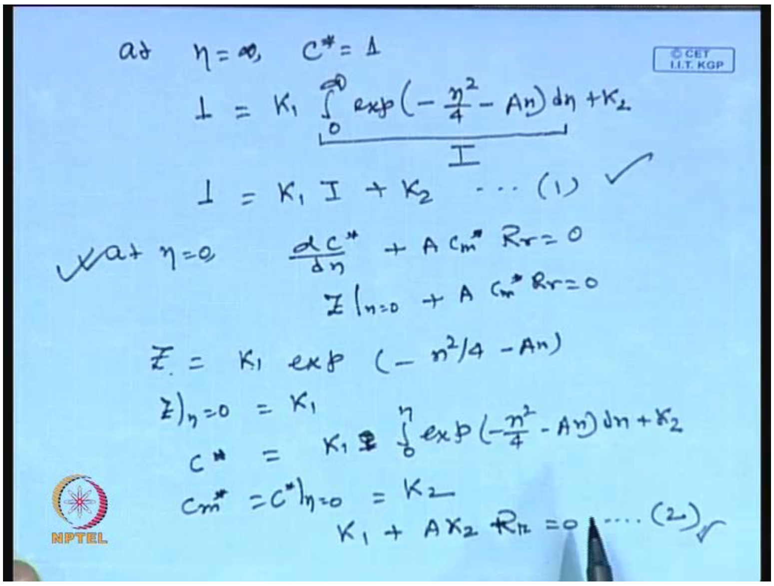

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at $\eta = \infty, \quad C^* = 1$

$$1 = K_1 \underbrace{\int_0^\infty \exp\left(-\frac{\eta^2}{4} - A\eta\right) d\eta}_{I} + K_2$$

$$1 = K_1 I + K_2 \quad \cdots (1) \checkmark$$

$\checkmark$ at $\eta = 0$ $\quad \frac{dC^*}{d\eta} + A C_m^* R_r = 0$

$$Z|_{\eta=0} + A C_m^* R_r = 0$$

$$Z = K_1 \exp\left(-\eta^2/4 - A\eta\right)$$

$$Z)_{\eta=0} = K_1$$

$$C^* = K_1 \int_0^\eta \exp\left(-\frac{\eta^2}{4} - A\eta\right) d\eta + K_2$$

$$C_m^* = C^*|_{\eta=0} = K_2$$

$$K_1 + A K_2 R_r = 0 \quad \cdots (2) \checkmark$$

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Now, let us look into the boundary conditions, and evaluate the constants of integration k 1 and k 2; the first constant of first boundary condition was at eta is equal to infinity; C star is equal to 1; that means one you just put this condition in your solution, one becomes k 1 0 to infinity exponential minus eta square by 4 minus A eta d eta plus k 2; now, again if you know the value of A then these integral is known to you, because it is a definite integral, and once you know the value of A, because that is a constant it must be assuming some value; and how to evaluate the value that we will see later on; once you know the value of A then you can this definite integral can be evaluated by using any you know Trapezoidal rule or Simpson's rule putting the upper limit as you know some higher numbers may be 10 15 something; and how to decide 10 15, I think we have discussed earlier just put any number and you were getting a result that just change the number, just increase the number arbitrarily let us say from 10 to 12 and change, and look into the result if the result does not change in the decimal third order to fourth decimal place; that means 10 is good enough as infinity for this particular problem.

So, let us indicate this integral as I; so, this becomes k 1 I plus k 2, this is equation number 1 then we put the other boundary condition, the other condition was at eta is equal to 0 you had d C star d eta plus A C m star Rr is equal to 0, and what is d C star d eta, d C star d eta is nothing but, Z the variable Z you have defined Z evaluated at eta is equal to 0 plus A C m star Rr equal to 0; now, let us look into what is Z at eta is equal to 0, if you look into the expression of Z, the expression of Z was k 1 exponential minus eta square by 4 minus A eta; so therefore, Z evaluated at eta equal to is nothing but k 1, this exponential term becomes one; so, Z at eta is equal to 0 is nothing but k 1, and what is C m star, if you look into the expression of C star, that was k 1 I k 1 0 to eta exponential minus eta square by 4 minus A eta d eta plus k 2; and what is C m star, C m star is nothing but, C star evaluated at y star equal to 0 and y star equal to 0 is nothing, but eta equal to 0; so, C m star is nothing but C star evaluated at eta equal to 0, and once it is eta equal to 0; that means, this upper limit becomes 0; so, this integral will be then from 0 to 0; so, whole integral will vanish; so, this is nothing but the other constant k 2.

So, once you get that then you can write the boundary condition of at eta equal to 0 at d C star d eta at eta equal to 0 is nothing but k 1; so, this will be k 1 plus A C m star will be nothing but k 2 Rr equal to 0; so, you will be getting the second equation from the second boundary conditions; now, there are two equations and two unknowns k 1 and k 2 to be evaluated; once these two constants evaluated, then you will be getting the full concentration profile within the mass transfer boundary layer; now, in the next slide we will be doing the derivation of k 1 and k 2 from this two equations.

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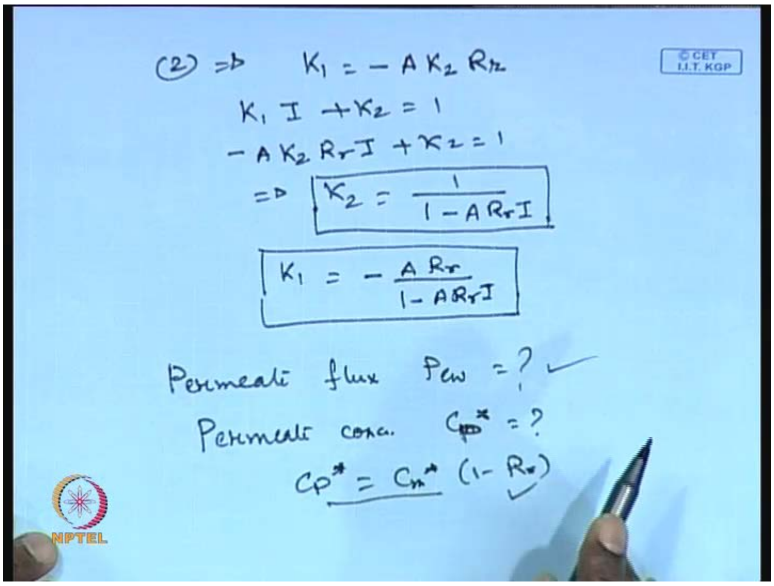

(2) $\Rightarrow$ $K_1 = -A K_2 R_r$

$K_1 I + K_2 = 1$

$-A K_2 R_r I + K_2 = 1$

$\Rightarrow \boxed{K_2 = \frac{1}{1 - A R_r I}}$

$\boxed{K_1 = -\frac{A R_r}{1 - A R_r I}}$

Permeate flux $P_{ew} = ?$ ✓

Permeate conc. $C_p^* = ?$

$\underline{C_p^* = C_m^*} (1 - \underset{✓}{R_r})$

So, from equation two we will be getting k 1 is nothing but minus A k 2 Rr; so, you just put into the first equation that will be k 1 I plus k 2 is equal to 1; so, k 1 is minus A k 2 Rr I plus k 2 is equal to 1; so therefore, k 2 will be nothing but, one by one minus A Rr I; so, this is the expression of k 2; and what is k 1, k 1 is nothing but minus of A Rr divided by one minus A Rr I; so, this is the expression of k 1; so, using these two constants k 1 and k 2 and knowing the value of a one can evaluate the concentration profile within the mass transfer boundary layer; now, but our aim is to basically evaluate what is the permeate flux, what is non dimensional permeate flux, and what is the permeate concentration C P star; and C P star can be obtain if you can evaluate the C m star the membrane surface concentration through the relationship is C P star in nothing but, C m star times one minus real retention; if you know the value of real retention that can be you know there are several ways to evaluate the value of real retention of a particular membrane solute system from various independent set of experiments, that we have discussed already earlier.

So, once you know the C m star and P e w from this expression you can calculate the permit concentration as well; so, you can predict the system performance at every location of time point; so, we have to evaluate the concentration profile at eta is equal to 0 that is the C m star, what is the expression of C m star now, if you look into the derivation earlier C m star was nothing but the constant k 2.

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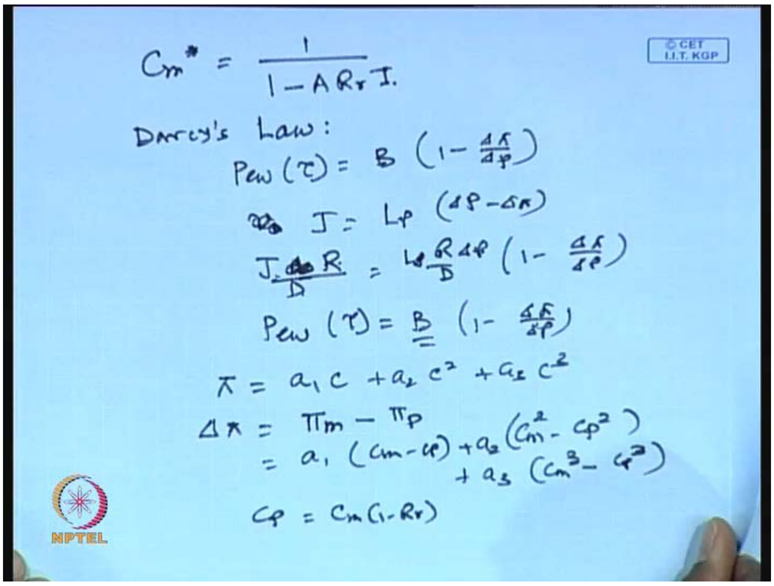

$$C_m^* = \frac{1}{1 - A R_r I}.$$

Darcy's Law:

$$Pe_w(\tau) = B\left(1 - \frac{\Delta\pi}{\Delta P}\right)$$

$$J = L_p(\Delta P - \Delta\pi)$$

$$\frac{J R}{D} = L_p \frac{R \Delta P}{D}\left(1 - \frac{\Delta\pi}{\Delta P}\right)$$

$$Pe_w(\tau) = \underline{\underline{B}}\left(1 - \frac{\Delta\pi}{\Delta P}\right)$$

$$\pi = a_1 c + a_2 c^2 + a_3 c^2$$

$$\Delta\pi = \pi_m - \pi_p$$
$$= a_1(C_m - C_p) + a_2(C_m^2 - C_p^2) + a_3(C_m^3 - C_p^2)$$

$$C_p = C_m(1 - R_r)$$

So, write down the C m star is nothing but constant k 2 is equal to 1 over 1 minus A Rr I; and we have the Darcy's law will be valid as the solvent flux equation through the porous medium for every point of time, and if you remember the non dimensional form of Darcy's law will be P w tau B into 1 minus delta pi divided by delta P; how you obtained this equation, this equation obtained by the non dimensional of the Darcy's law; if you forget, I can do a couple of steps and you can rebrush your memory that Darcy's law was nothing but J is equal to del P minus del pi, and you just make it non dimensional J d e by D, in this case d e equivalent will be nothing but the radius of the cell because it is the unstart back cell; so, it is a cylindrical cell with their inner radius is R lets say; so, L p R by D make it non dimensional; so therefore, take delta Pout.

So, take delta Pout these becomes delta pi by delta P; so, what is J R by d, this is nothing but P e w non dimensional P e w this is valid at every tau, and L p R del P by D is a constant B and 1 minus del pi by del P; these are non dimensional parameter that takes care of operating pressure diffusivity solid diffuse solid property operating conditions membrane property and of course, the geometry property R.

So, and what is delta pi, if you remember pi is nothing but, we had an expression pi is equal to nothing but a 1 C plus a 2 C square plus a 3 C cube. So, we have the expression of pi as a polynomial of concentration; so, what is delta pi, delta pi is nothing but the osmotic pressure difference between the membrane surface and in the upstream and the permeate stream. So, delta pi will be nothing but a 1 C m minus C p plus a 2 C m square minus C p square plus a 3 C m cube minus C p cube; so, know you can omit, you can replace Cp in favor of C m by doing this exercise, Cp is nothing is but C m into 1 minus real retention; if you substitute this the delta pi will become a sole function of membrane surface concentration.

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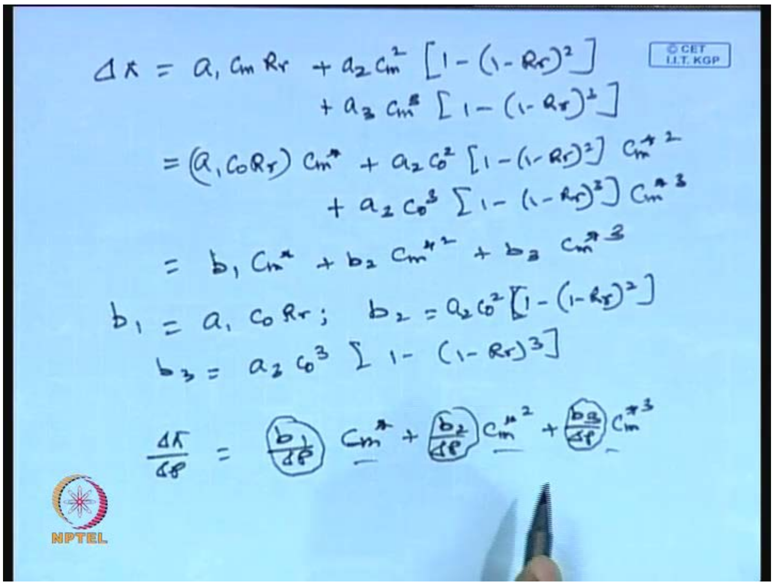

$$\Delta\pi = a_1 C_m R_r + a_2 C_m^2 \left[1-(1-R_r)^2\right] + a_3 C_m^3 \left[1-(1-R_r)^3\right]$$

$$= (a_1 C_0 R_r) C_m^* + a_2 C_0^2 \left[1-(1-R_r)^2\right] C_m^{*2} + a_3 C_0^3 \left[1-(1-R_r)^3\right] C_m^{*3}$$

$$= b_1 C_m^* + b_2 C_m^{*2} + b_3 C_m^{*3}$$

$$b_1 = a_1 C_0 R_r ; \quad b_2 = a_2 C_0^2 \left[1-(1-R_r)^2\right]$$

$$b_3 = a_3 C_0^3 \left[1-(1-R_r)^3\right]$$

$$\frac{\Delta\pi}{\Delta P} = \boxed{\frac{b_1}{\Delta P}} \underline{C_m^*} + \boxed{\frac{b_2}{\Delta P}} \underline{C_m^{*2}} + \boxed{\frac{b_3}{\Delta P}} \underline{C_m^{*3}}$$

So, let us do that and see what we get, what we will be getting is that delta pi will be nothing but, a 1 C m Rr plus a 2 C m square, this becomes 1 minus 1 minus Rr square, this becomes a 3 C m cube 1 minus 1 minus Rr whole cube; and now, make C m non dimensional in favor of the bulk concentration; so, this becomes a 1 C 0 Rr times C m star, this becomes a 2 C 0 square 1 minus 1 minus Rr square times C m star square plus a 3 C 0 cube 1 minus 1 minus Rr whole cube C m star cube, and this becomes b 1 times C m star plus b 2 times C m star square plus b three times C m star cube, and your b 1 will be nothing but a 1 C 0 Rr, your b 2 will be nothing but a 2 C naught square 1 minus 1 minus Rr whole square, and b 3 will be nothing but a 3 C 0 cube 1 minus 1 minus Rr whole cube.

So, these are (( )) of to express delta pi in terms of C m star only, and 1 by delta pi by delta P will be nothing but, b 1 by delta P plus delta P C m star plus b 2 by delta P C m star square plus b 3 by delta P times C m star cube, and now this becomes a non dimensional number, this becomes a non dimensional number, this becomes a non dimensional number, and all the variables are non dimensional. So, whole things becomes non dimensional, and in a sense your osmotic pressure difference can be expressed in terms of C m star only and that is the way how the derivation goes. So, what you have now, let us write down the governing equations whatever you are the relevant equations in one place that will be better otherwise the whole thing becomes clattered.

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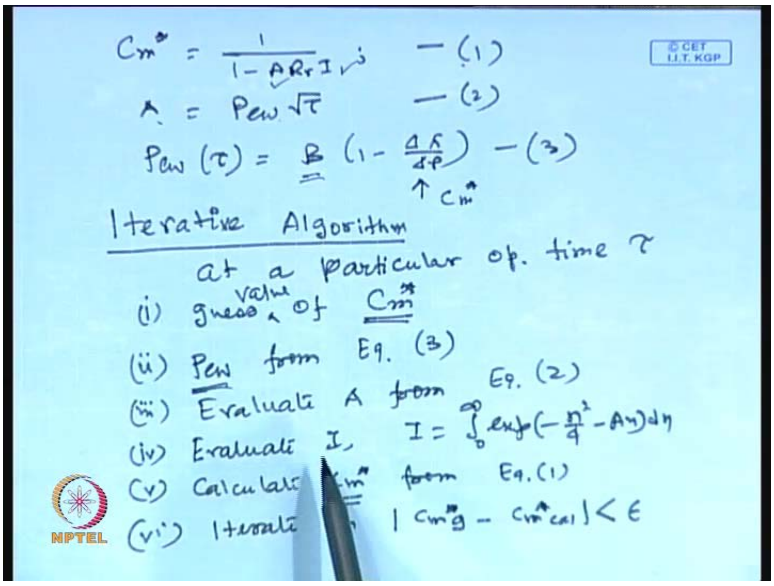

$C_m^* = \frac{1}{1 - A R_r I_v}$ — (1)

$A = P_{ew}\sqrt{\tau}$ — (2)

$P_{ew}(\tau) = \underline{\underline{B}}\left(1 - \frac{\Delta\pi}{\Delta P}\right)$ — (3)

$\uparrow C_m^*$

Iterative Algorithm

at a particular op. time τ

(i) guess value of C_m^*

(ii) P_{ew} from Eq. (3)

(iii) Evaluate A from Eq. (2)

(iv) Evaluate I, $I = \int_0^\infty \exp\left(-\frac{\eta^2}{4} - A\eta\right)d\eta$

(v) Calculate C_m^* from Eq. (1)

(vi) Iterate $|C_{m,g}^* - C_{m,cal}^*| < \epsilon$

So, the relevant equations are first equation is C m star as one minus A Rr I and what is A becomes A is nothing but, if you remember A is nothing but Pw root over tau; and what is the Darcy’s law, Darcy’s law is nothing but P e w, at any point of time becomes B times 1 minus delta pi by delta P and this is a function, sole function of C m star.

So, that’s it. Now, using an appropriate algorithm one can do an iterative calculation, and can get the permeate flux and permeate concentration at every time point, and you’ll be getting a profile as a function of type how. So, we use the following iterative algorithm, what is the iterative algorithm, the at a particular operating time, operating time let us say 5 second 10 second or whatever particular time tau you do the following, step number 1: guess the value of C m star, once you guess the value of C m star we can calculate P e w from equation number 3, what is equation 3 from here, this is let us say 3 2 1.

So, you know the value of C m star, because you have already guessed it; and this is a sole function of C m star; delta P s the operating pressure operating condition that is known to because you are setting it; B is a parameter that is all the values are known to you membrane radius, cell radius and so for. So, you can get the value of P e w, once you know the value of P e w from step number 3 is from equation 2 you know the time point, you know the value tau, you know the P e w; so, you can get the value of A. So, evaluate A from equation 2; once you do that then you can you are in a position once you know the value of a you are in a position to evaluate the definite integral I; evaluate I from its definition, I is equal to 0 infinity exponential minus eta square by four minus A eta d eta. So, you know the value of A substitute there; so, everything is known eta is a dummy variable; so, just evaluate the definite integral I; so, once you know the value of I then you can go back to equation number 1, because I is now known to you, A is now known to you, Rr I already determined. So, you can calculate C m star from equation 1, and check whether this calculated value is close to that guess value or not.

So, next one is just iterate, on the absolute value of C m star guess minus C m star calculated is less than epsilon or not; if not then you guess a new value of C m star likewise this loop has to be iterated for a particular time point and what you do next next you have an increment of time tau that is a tau plus delta and re iterate this equation this this steps, but in the second step in the second step you should have a judicial guess of C m star the judicial guess is basically the converged value of C m star at the earlier time point then it will be the the the iteration of the convergence will be faster for the next time point likewise what in the likewise it will be continued it will be continued and what is the final outcome of this exercise final outcome is that you will be getting various a profile of C m star as a function of tau you will will be getting a profile ofPe w as a function of tau.

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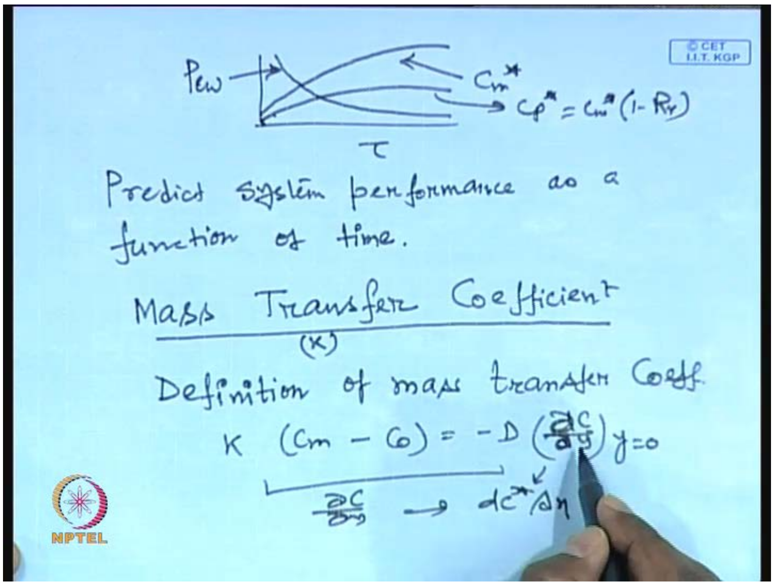

Predict system performance as a function of time.

Mass Transfer Coefficient

(K)

Definition of mass transfer Coeff.

$$K\,(C_m - C_0) = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

$$\frac{\partial C}{\partial y} \rightarrow dc^*/d\eta$$

So, out of it you’ll be gettingPe w as a function of tau and this is this isPe w and this is C m star as a function of tau once you know the C m star is a function of tau you can get a value of CPstar as a function of tau through the relationship CPstar is equal to C m star into one minus real retention. So, the this way one can get the one can predict the performance system performance as a function of time in an unstart batch cell experiment and why you are conducting the unstart batch cell experiment thatI think you have a discussed that issue earlier and that that is the way.

Now, one maybe being a chemical engineering maybe interested what is expression of a mass transfer coefficient in this case now now this mass transfer coefficient k is a vital parameter as far as the design of any a chemical engineering system is concerned where the mass transfer is involved. So, we start with the definition of mass transfer coefficient k times C m; that means, C at y equal to 0 minus C naught is equal to minus d d C d y del C del y at y is equal to 0 that is the definition of the mass transfer coefficient.

Now, what do we do I think we have done this exercise earlier in the case of rectangular geometry we have to replace del C del y in terms of d C star d eta in terms of non dimensional similarity parameter eta and before before that you have to make these equation non dimensional. So, we we first we make then make the equation non dimensional then we put the value of the governing equation the the the derivative in terms of eta .

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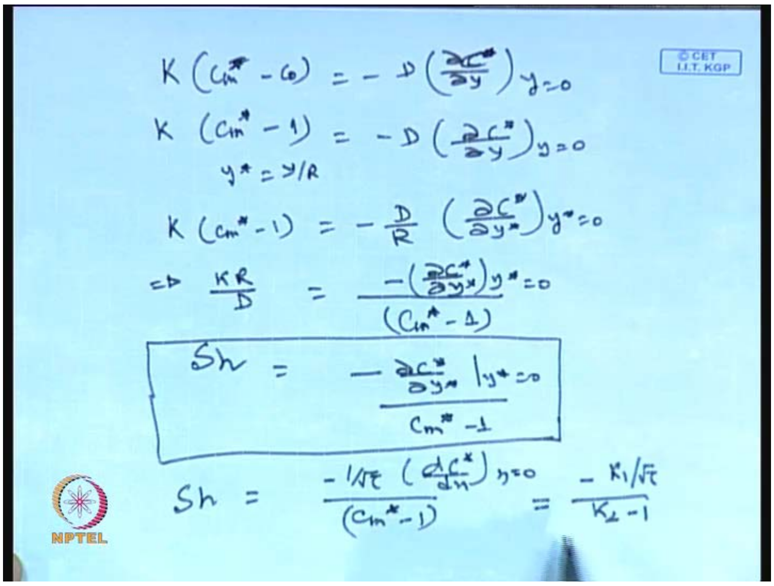

$$K\left(C_m^* - C_0\right) = -D\left(\frac{\partial C^*}{\partial y}\right)_{y=0}$$

$$K\left(C_m^* - 1\right) = -D\left(\frac{\partial C^*}{\partial y}\right)_{y=0}$$

$$y^* = y/R$$

$$K\left(C_m^* - 1\right) = -\frac{D}{R}\left(\frac{\partial C^*}{\partial y^*}\right)_{y^*=0}$$

$$\Rightarrow \frac{KR}{D} = \frac{-\left(\frac{\partial C^*}{\partial y^*}\right)_{y^*=0}}{\left(C_m^* - 1\right)}$$

$$\boxed{Sh = \frac{-\frac{\partial C^*}{\partial y^*}\Big|_{y^*=0}}{C_m^* - 1}}$$

$$Sh = \frac{-1/\sqrt{\tau}\left(\frac{dC^*}{d\eta}\right)_{\eta=0}}{\left(C_m^* - 1\right)} = \frac{-K_1/\sqrt{\tau}}{K_2 - 1}$$

So, k C m star minus C 0 is equal to minus d del C star del y del C del y this there is no star here at y is equal to 0 now you divide both both side by C 0. So, this become C m star C m star minus 1 is equal to minus d del C by C 0. So, it become del C star del y at y is equal to 1.

Make y star as y by R. So, y is nothing, but r times y star y star. So, k C m star minus one d minus d by R del C star del y star now this becomes y star is equal to 0 and what is this kR by D is nothing, but minus del C star del y star at y star is equal to 0 divided by C m star minus one and this quantity kR by D is nothing, but the sherwood number now there is a neat expression of sherwood number in terms of non dimensional parameter and if you now put the expression of del C star del y star in terms of eta and this becomes sherwood is equal to minus 1 over root over tau d C star d eta at eta is equal to zero divided by C m star minus 1.

Now, we have done this exercise earlier just few minutes back that d C star d eta d eta equal to 0 and C m star in terms of the integration constants and this becomes minus k 1 by root over tau divided by k 2 minus 1 just just put the put the value of d C star d eta eta equal 0 we have done that probably few minutes back and C m star is nothing, but k 2. So, it becomes k 2 minus 1 now you put the expression of k 1 and k 2 because you have evaluated the k one and k two in terms of ARr and I and substitute there and see what you get what you get what you will be getting is like that sherwood is nothing, but one over I times root over tau.

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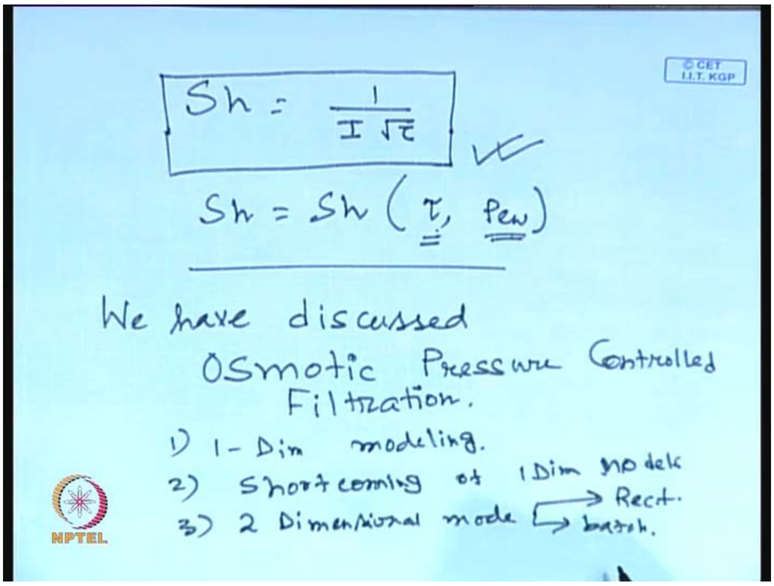

$$\boxed{Sh = \frac{1}{I\sqrt{\tau}}}$$

$$Sh = Sh(\underline{\underline{\tau}}, \underline{\underline{Pe_w}})$$

We have discussed

Osmotic Pressure Controlled Filtration.

1) 1-Dim modeling.

2) Short comming of 1 Dim models

3) 2 Dimensional mode → Rect. → batch.

So, this is the expression you will getting at the end of it now in this expression. So, so in a sense sherwood number will be a function of two things that is observed here one is time another is if if you remember in the integration of I there is a value there is a constant a and a is nothing, butPe w times root over tau. So, this will be function ofPe w as well. So, the porosity the the permeate flux this permeate flux is will occur will occur because of the porous membrane present at the wall. So, porosity over of wall will be taking by this parameterPe w and the time variation is taken care care by the non dimensional quantity tau.

So, therefore, sherwood will be and if you if you look into sherwood sherwood will be a function of both permission permission as well as the time of operation and as it is evident from this expression the sherwood is inversely proportional to time; that means, as time progresses the mass transfer coefficient the sherwood number will go on decreasing; that means, mass transfer coefficient will be go on will will will go on decreasing because mass transfer coefficient is directly proportional to sherwood number there is a non dimensional version of mass transfer coefficient.

So, therefore, as time progresses the concentration boundary layer develops on the membrane surface it offers physically it means it offers more resistance against the solvent flux and and and the permeate flux and the mass transfer amount of mass transfer will be reduced since the amount of mass transfer will be reduced by by the mass the transfer coefficient and the permeate flux will be connected by the boundary condition at y equal to 0 if you remember. So, therefore, the permeate flux will also decrease as time progresses.

So, these maybe of interest to the chemical engineering engineer student people a chemical engineers to design efficiently a system. So, that goes the modeling part on calculation park/part os part of the osmotic pressure controlled unstart batch cell.

So, let us try to summarize whatever we have done till now we have discussed osmotic pressure controlled filtration. So, we have discussed one dimensional one dimensional modeling we have discussed the short coming of one dimensional models then we discussed the the the two dimensional model in both geometries a a rectangular cross flow cell there is a steady state process and in a batch cell there is an unsteady state process. So, both steady state and transient operations we have seen the modeling how to model the system.

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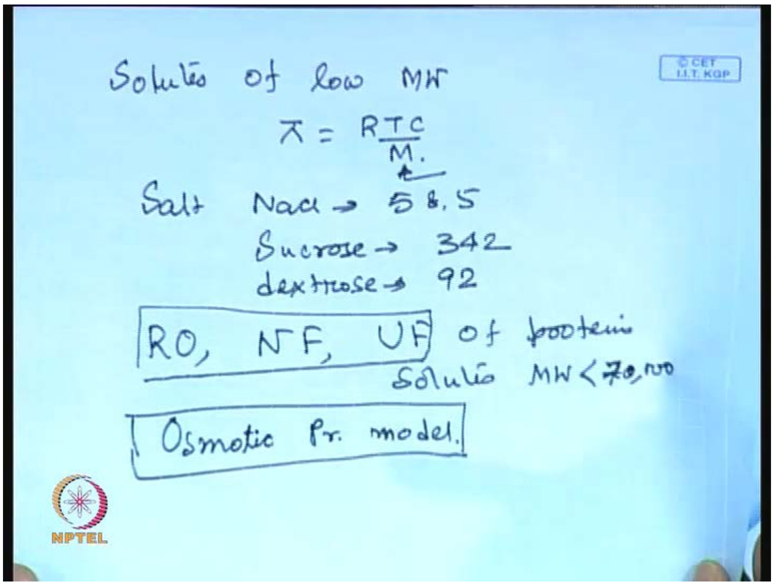

Solutes of low MW

$$\pi = \frac{RTC}{M.}$$

Salt NaCl → 58.5

Sucrose → 342

dextrose → 92

RO, NF, UF of protein

Solutes MW < 70,000

Osmotic Pr. model.

Now, let us discuss when this osmotic pressure controlled filtration becomes very important the osmotic pressure controlled filtration becomes very important when you are talking about filtration of solutes which will be having very high osmotic pressure solutes of low molecular weight solutes will not be having the osmotic pressure the solvent the solution will be having the osmotic pressure, but the solvent must be containing the solutes of low molecular because if you look into the definition of osmotic pressure the first coefficient is RTC by M. So, if you go for lower molecular solutes lower molecular typically you can you can correlate with the size that you you can talk about the smaller sized particles if you have solutes of smaller molecular less the osmotic pressure becomes dominant.

For example salt NaCl its is fifty eight point five sucrose three hundred forty two dextrose 92 I guess 92 or 42 or something like that.

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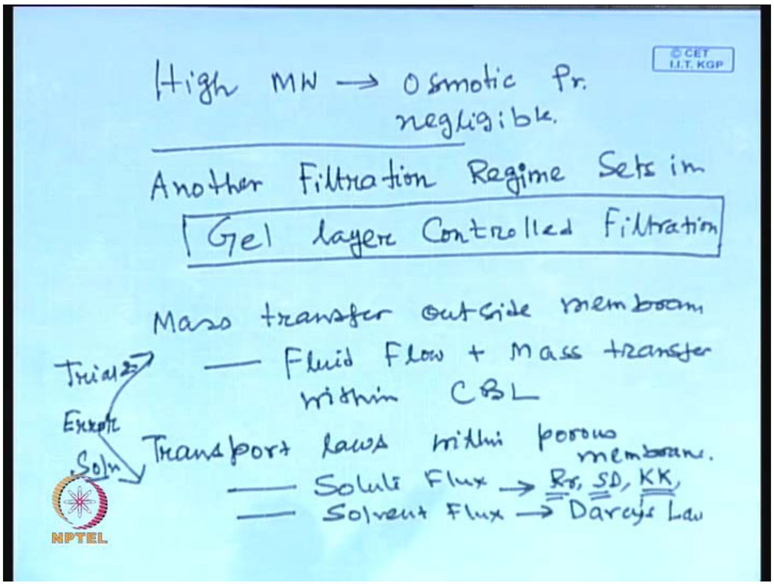

High MW → Osmotic Pr. negligible.

Another Filtration Regime Sets in

Gel layer Controlled Filtration

Mass transfer outside membrane

— Fluid Flow + Mass transfer within CBL

Transport laws within porous membrane.

— Solute Flux → RS, SD, KK,

— Solvent Flux → Darcy's Law

So, you will be having if you if you have the osmotic pressure in the range of around the molecular in the range of 1000 the osmotic pressure can be very important for the charged neutral solutes for the for the charged proteins which you will be having a high osmotic molecular for example, 70000,80000 but because of the charged charged effect and other interactions the osmotic pressure can be very high.

So, if you are talking of untrust solutes for example, polysaccharides dextran or things like that are very high molecular rate the osmotic pressure the denominator becomes very large in that case osmotic pressure will be negligible for most of the cases of reverse osmosis nano filtration and ultra filtration of proteins and solutes having molecular rate less than 20000 the osmotic pressure module becomes very important it can be even even for the putting which will be having a molecular at seventy thousand the osmotic pressure is very very very important because of the chartered effect and the other other effects.

So, you will we talk about the osmotic pressure model in those cases. So, if you remember starting from micro filtration to a nano filtration to reverse osmosis most of the operating conditions or the membrane separations they remain under the osmotic pressure controlled regime now for the solutes which have a very high molecular weight the osmotic pressure becomes negligible another filtration regime will set in that is known as the gel filtration or gel layer controlled filtration now jis/this gel filtration is what is gel filtration gel filtration is gel filtration can occur now before going into the gel layer filtration we can we can discuss the various you know aspects of osmotic pressure model that we have already discussed that we have seen the various osmotic pressure models for an and we have identified that the mass transfer outside the mass outside the membrane; that means, within the mass transfer boundary layer that has to be coupled or hooked up by the mass transfer or the the solute transport or solvent transport within the porous membrane and mass transfer outside boundary layer sorry outside membrane is solved by fluid flow plus mass transfer within concentration boundary layer that has to be coupled by the transport laws within porous membrane and these transport laws are typically the solute flux equation and solvent flux equation.

Now, this solvent flux is typically and generally and for all the cases is calculated by or or quantified by darcy’s law on the other hand the solute flux is given by either a definition of real retention or by the the solution diffusion model or kevin kachinski model modified models either that definition of real retention or a solution diffusion model that we have already discussed and modified version of the solution diffusion model that is the quedan kochalski model these are the various models that you are talked about the solute flux. So, by by by getting some definetion the appropriate definition of the solute flux and solvent flux these two has to have up to be coupled and you can get the system performance prediction and the the typical feature is you’ll be getting a trial and error solution the solution is always trial and error.

Now, if we let us go to the gel layer control filtration because it is another regime in the whole filtration system it is more rampant in case of micro filtration and in in at the upper upper range of alter filtration cases where the forces are much bigger.

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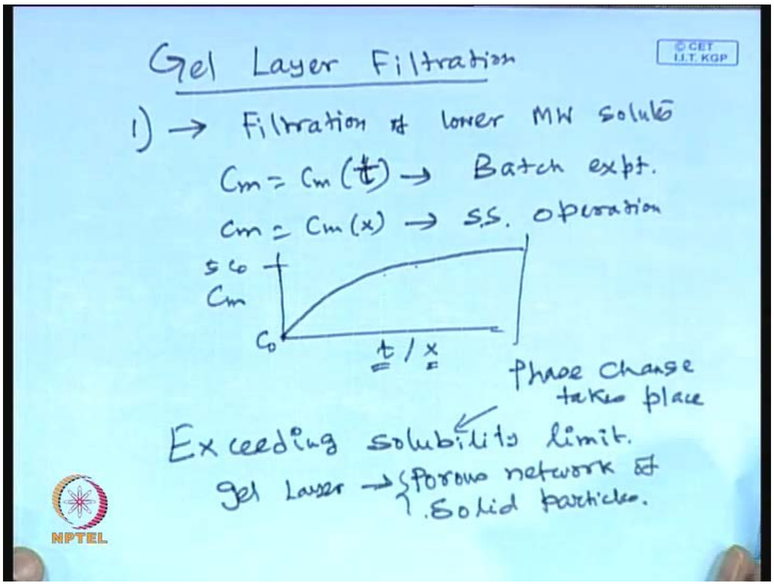

Gel Layer Filtration

1) → Filtration of lower MW solute

$C_m = C_m(t) \rightarrow$ Batch expt.

$C_m = C_m(x) \rightarrow$ S.S. operation

Phase change takes place

Exceeding solubility limit.

Gel Layer → { Porous network of
.Solid particles.

now there are two ways when gel layer filtration can occur in the case in case number one if you talk about any osmotic pressure controlled model basically you you are talking about filtration of lower molecular weight solutes. So, initially; that means, if you're talking about a batch cell operation or transient operation initially the filtration is osmotic pressure controlled if you talk about a steady state case for a for for initial length of the channel the ultra filtration is osmotic pressure controlled now as time progresses or the length of you know channel it goes on if you go down as you go downstream of the channel the membrane surface concentration increases; that means, C m is a function of tau or t or time of operation in case of batch experiment and C m is a function of X in case of steady state operation whereas, X is the length of the module now and in both the cases you have seen that the minimum value of membrane surface concentration is C 0 and it will be always increasing beyond C 0 and it keeps on increasing.

Now, if you plot C m versus t or x the curve will be typically something like this starting from C 0 to let us say five times C 0 or whatever now what happens if you talking about such a system which is basically an osmotic pressure controlled initially the membrane surface concentration increases with time for a batch scale batch batch system or as a function increases with respect to X in a steady state cross flow system and it may happen at the particular temperature it exceeds the solubility limit if it exits the solvability limit at that point onwards the whole system the the solute particles will be deposited as a solute particles over the membrane surface because the phase transformation will occur. So, there may be a phase change takes place; that means, you are exceeding the solubility limit in that case the formation of solute solute particle will will will be will lead to formation of a porous network of solute particles over the membrane surface that is known as the gel layer.

So, that leads to the formation of the gel layer, but since it is porous you can still expect some amount of flux solvent flux out of it some amount of g out of it, but that will be typically lower and in this case we would like to avoid that gel layer formation the second way of.

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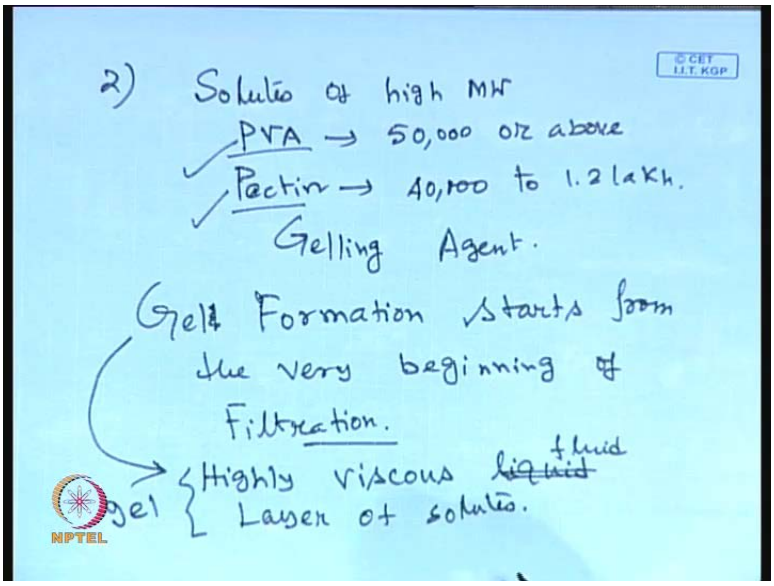

2) Solutes of high MW

✓ PVA → 50,000 or above

✓ Pectin → 40,000 to 1.2 lakh.

Gelling Agent.

Gels Formation starts from the very beginning of Filtration.

gel { Highly viscous ~~liquid~~ fluid
Layer of solutes.

So, that is that is that is one way of formation of gel layer and most common one is that first some solutes it is not osmotic pressure from the very beginning in that case we are talking about solutes of very high molecular rate if you're talking about filtration of solutes of very high molecular rate for example, polyvinyl alcohol which will be having a molecular rate around fifty thousand or above pectin pectin is a polysaccharides it is basically a you know it will be having it is a it is a polydispersed material; that means, it'll be having a molecular rate distribution let us say from forty thousand to 1 point 2 lakh it is a it is a polydisperse.

Now, things like the solutes like polyvinyl alcohol pectin they are well known gelling agent pectin is a polyvinyl alcohol is a polymer pectin is basically a polysaccharide that is available in any fruit juice and you would like to like to remove pectin from a citrus fruit juice because pectin forms a an or complex with protein and it destroys the fruit juice. So, one has to remove the pectin from the fruit juice for a clarified juice which will be having a high shelf life.

So, maybe alter filtration can be used for it. So, if you are talking about the solutes like polyvinyl alcohol pectin or things like that which are well known gelling agents the gel formation occurs from the very beginning of the filtration gel formation starts from the very beginning of filtration. So, there is no nothing like osmotic pressure controller and then the regime changes over to gel layer the gel formation starts from the very beginning of a filtration and you cannot avoid that what you can do you can you can. So, I said in the other earlier mechanism we said that we can you can avoid the gel layer formation.

So, you can set your operating convenience such that that formation can be avoided, but in this case gel layer formation cannot be avoided because, but but the operation has to be done because of the juice has to be clarified . So, therefore, one has to be very careful and set the tuning parameters of the operating parameters such that the gel layer formation becomes minimum and what is a definition of a gel layer it is not a solute you know layer it becomes it is a basically highly viscous liquid layer I should not say liquid it is a highly viscous fluid layer it flows stills it flows highly viscous fluid layer of solutes again it is porous it forms over the membrane surface and it is gel we we term it a gel in this particular case. So, in that we I stop here in the next class what we will do we will look into the modeling and the calculation of the gel layer control filtration thank you.