Multiphase Flow
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Lecture No. # 09
The Homogeneous Flow Theory (Contd.)

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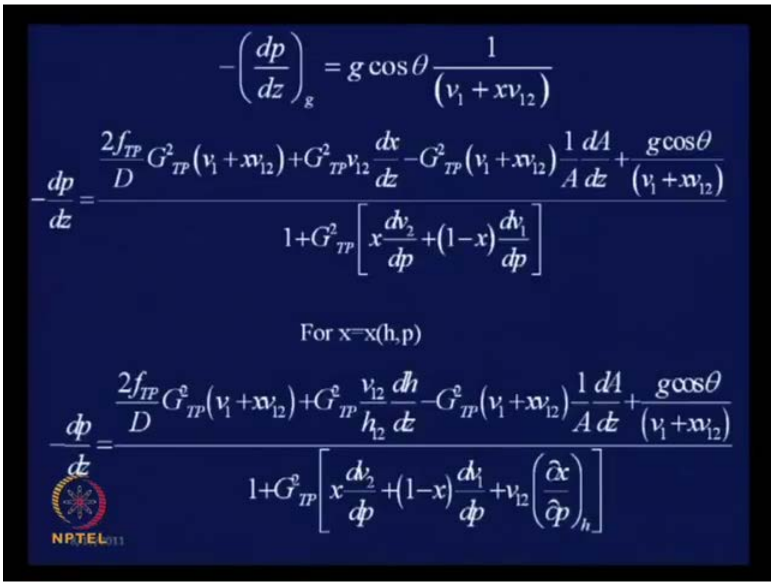

$$-\left(\frac{dp}{dz}\right)_g = g\cos\theta\frac{1}{(v_1 + xv_{12})}$$

$$-\frac{dp}{dz} = \frac{\frac{2f_{TP}}{D}G^2_{TP}(v_1+xv_{12}) + G^2_{TP}v_{12}\frac{dx}{dz} - G^2_{TP}(v_1+xv_{12})\frac{1}{A}\frac{dA}{dz} + \frac{g\cos\theta}{(v_1+xv_{12})}}{1+G^2_{TP}\left[x\frac{dv_2}{dp}+(1-x)\frac{dv_1}{dp}\right]}$$

For x=x(h,p)

$$-\frac{dp}{dz} = \frac{\frac{2f_{TP}}{D}G^2_{TP}(v_1+xv_{12}) + G^2_{TP}\frac{v_{12}}{h_{12}}\frac{dh}{dz} - G^2_{TP}(v_1+xv_{12})\frac{1}{A}\frac{dA}{dz} + \frac{g\cos\theta}{(v_1+xv_{12})}}{1+G^2_{TP}\left[x\frac{dv_2}{dp}+(1-x)\frac{dv_1}{dp}+v_{12}\left(\frac{\partial x}{\partial p}\right)_h\right]}$$

Well a very good morning to all of you. So, we will be continuing with our discussions of the homogeneous flow theory which was left. In the last class what we did was, we derived the pressure gradient expression which we can get from the homogeneous flow theory. And, we found out that the pressure gradient expression, it is more or less similar to the pressure gradient expression that, we obtain for the compressible fluid flows, we have on the denominator, there is one particular term, which refers to the frictional pressure gradient. Then, there is one term which refers to the gravitational pressure gradient. And two terms which refer to the acceleration pressure gradient, one term being due to the acceleration due to area change, the other being the acceleration due to the quality change.

And there is of course,, another denominator part which is; which resembles that single phase flows, single phase compressible flows and regarding the significance of the denominator, we will be discussing later. Probably, before that I would like to cover some basics of the compressible flow theory, because I find many of you, you are not very conversant with it, may be for some students it might be a repetition. But, then also I would prefer to cover may be I will take two lectures on it so that, some portion of the compressible flows can be covered. And after that I will be discussing the significance of the denominator term in this particular case.

And I had also mentioned that, for significant flashing, what we did was, the; we found out that when the significance flashing then, in that case, we cannot straightaway write that quality is a function of axial distance. In this particular case, we find that quality it is a function of enthalpy as well as pressure, because it is not a function of only enthalpy that, it can be obtained from heat balance equation then, there is flashing then, apart from enthalpy change or enthalpy balances, the pressure gradient also comes into being. And, under such circumstances, the momentum equation or the pressure gradient expression becomes slightly different.

Now, in this particular respect, I would also like to mention that, we have defined this on the basis that, quality is a function of enthalpy and pressure, it is not a function of anything else. But, remember one thing suppose, the kinetic energy effects they are also quite pronounced, under such circumstances we cannot write X is a function of h and p only. Under that circumstances, we have to perform a proper energy balance equation and from that energy balance equation probably, we would like to get it. Under that circumstances, we this; does not hold any longer X equals to X h p this does not hold.

And there, we have to perform an energy balance equation and from that energy balance equation, we have to define an effective thermodynamic path which can replace this particular expression X equals to X h p. So, this expression can be replaced by a actual thermodynamic path by which, the change has taken place and only after that probably, we can go for the actual expression of pressure gradient. Now, while defining a path, it might happen that, we can allow only a given fraction of the equilibrium quantity of vapor to be formed. Or maybe, we require a more thorough analysis in which the actual quantity can be related to the non equilibrium heat and mass transfer processes.

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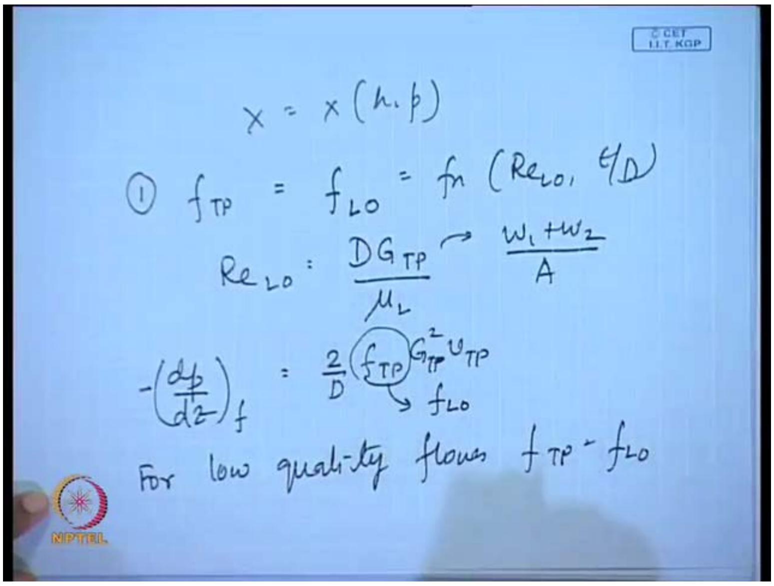

$$x = x(h, p)$$

$$① \quad f_{TP} = f_{LO} = fn\left(Re_{LO}, \epsilon/D\right)$$

$$Re_{LO} = \frac{D G_{TP}}{\mu_L} \quad G_{TP} \rightarrow \frac{w_1 + w_2}{A}$$

$$-\left(\frac{dp}{dz}\right)_f = \frac{2}{D}\,\underset{\searrow f_{LO}}{\boxed{f_{TP}}}\,G_{TP}^2\,v_{TP}$$

For low quality flows $f_{TP} \sim f_{LO}$

So, we will be doing a problem on it and there you will find that, if kinetic energy effects are quite pronounce then, in that case X as a function of enthalpy and pressure this does not hold any longer. And for that particular case, we have to go for something more elaborate, we have to write the actual energy balance equation, where the kinetic energy, potential energy all the terms, from there, we have to find out the proper expression of enthalpy. And that has to be substituted in the pressure drop expression and it has to be done accordingly.

Well the other thing, which I had mentioned in the last class was, apart from discussing the significance of the denominator term, we find that in this particular expression of momentum which is already present, in that particular expression, we find that the only term which we do not know is the two phase friction factor f T P this is the term.

And then I think, we has started discussion on the different methods of finding out f T P. Now, there is not a very standard method just like, we have in single phase flows, because this situation is not very standardized number one. And number two is that, it is quite expected that it will vary depending upon, the composition of the two phase mixture that can also happen is it not. What will happen for very low quality flows, might not happen for very high quality flows. So, therefore, different approaches have been used to define f T P for two phase flow under homogeneous flow conditions.

So, the different approaches if we write it down. In fact, I have got a slide on it as well. So, the first thing which we can do if you refer to this particular slide we find that, if it is a very low quality vapor liquid flow. Under that, condition what we can assume? We can assume that, if it is very low quality then probably, the mixture it behaves more like a liquid than a vapor is it not. So, under such circumstances, we can write f T P this is equal to f, when the entire mixture flows as liquid or in other words, we define it as f L O or f liquid only when the entire mixture it flows as liquid.

And naturally then this becomes a function of Reynolds number and of course,, the roughness factor which we all know where this Reynolds number this is defined as in terms of mass flux we would like to do just, because this is much more convenient. So, therefore, otherwise we have to write it as v u rho and all those things again rho, rho T P etcetera has to be given. So, therefore, we can write it in this particular term where the G T P is the is a measureable input parameter as I have already mentioned this is nothing but w 1 plus w 2 by A, is it not.

So, therefore, if we substitute it then, we find that the expression of pressure gradient, the frictional pressure gradient which was there. What was the expression of frictional pressure gradient? It was 2 by d f T P G square v T P is it not, this was the expression; G square T P also, this was the expression of the frictional pressure gradient which is encountered for gas liquid two phase flows. Now, for low quality flows, what we have assumed? We have assumed that, f T P equals to f L O. So, instead of this particular f T P, we can substitute it with f L O.

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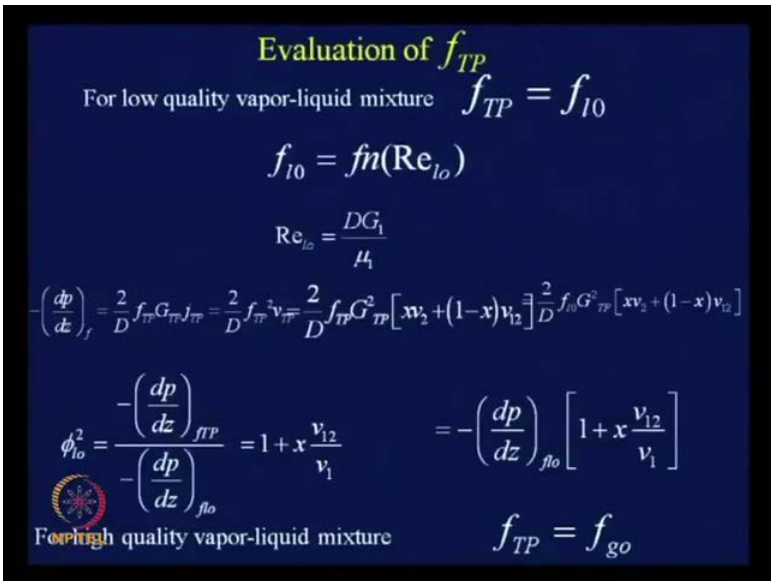

Evaluation of f_{TP}

For low quality vapor-liquid mixture $f_{TP} = f_{l0}$

$$f_{l0} = fn(\mathrm{Re}_{lo})$$

$$\mathrm{Re}_{lo} = \frac{DG_1}{\mu_1}$$

$$-\left(\frac{dp}{dz}\right)_f = \frac{2}{D} f_{TP} G_{TP} J_{TP} = \frac{2}{D} f_{TP}{}^2 v_{TP} = \frac{2}{D} f_{TP} G^2{}_{TP}\left[xv_2 + (1-x)v_{12}\right] \frac{2}{D} f_{l0} G^2{}_{TP}\left[xv_2 + (1-x)v_{12}\right]$$

$$\phi_{lo}^2 = \frac{-\left(\frac{dp}{dz}\right)_{fTP}}{-\left(\frac{dp}{dz}\right)_{flo}} = 1 + x\frac{v_{12}}{v_1} \qquad = -\left(\frac{dp}{dz}\right)_{flo}\left[1 + x\frac{v_{12}}{v_1}\right]$$

For high quality vapor-liquid mixture $f_{TP} = f_{go}$

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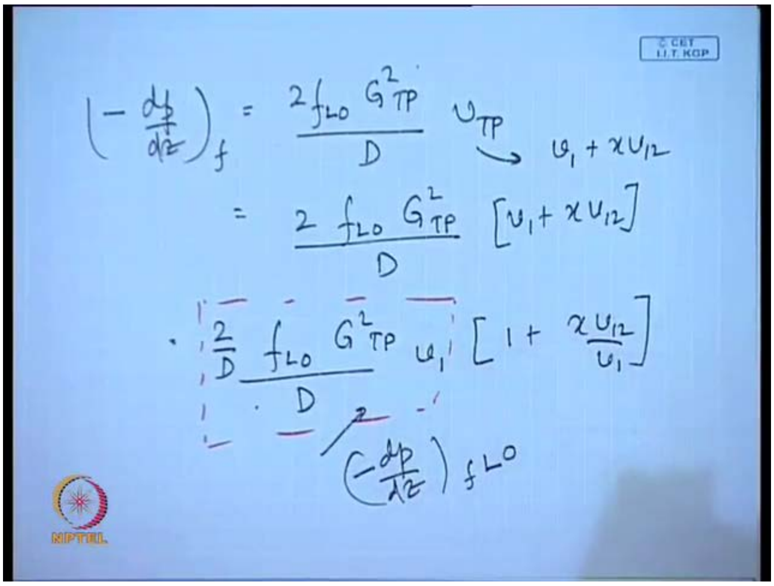

$$\left(-\frac{dp}{dz}\right)_f = \frac{2 f_{LO} G^2_{TP}}{D} \; v_{TP} \rightarrow v_1 + x v_{12}$$

$$= \frac{2 f_{LO} G^2_{TP}}{D}\left[v_1 + x v_{12}\right]$$

$$\frac{2}{D} \frac{f_{LO} G^2_{TP}}{D} v_1 \left[1 + \frac{x v_{12}}{v_1}\right]$$

$$\left(-\frac{dp}{dz}\right)_{fLO}$$

Under that circumstances what do we get? Under that circumstances, we get minus d p d z frictional, the frictional pressure gradient this can be expressed as 2 frictional for liquid only G T P by D v T P. Where, we know that this v T P is nothing but v 1 plus X v 1 2 is it not. Or therefore, this can be written down in this particular form so that all of them are measurable properties entirely in terms of measurable properties, we can write it in this particular form as well.

Slightly little more modification, we can express it in this form as well I have just taken out v 1 and then it becomes this form. Now, why have I written the third step from the second step, can you tell me? Moment I have expressed in it in this particular form what do we find? What is this particular expression, any idea? What does this signify? Any idea? What does this particular? It is a product of two terms basically now. So, you can tell me, what is the first term here, in this particular case? (No Audio From: 10:00 to 10:06) It is; it represents the pressure drop when the entire mixture flows as liquid through the pipe. Do you get my point.

Just note since, there is G T P here and v 1 that is the specific volume of the liquid phase. So, therefore, this term it signifies, the pressure drop when the entire mixture flows as liquid only through the pipe. Do you get the point. So, therefore, this can be substituted as, the frictional pressure gradient for liquid, when the X entire mixture flows as liquid only in the pipe. So, what does it; how did it; does it simplify the situation? What does it do? In that case, we find that, what we can do, we can just take liquid at the total mixture flow rate and we can allow it to flow through the same pipe. Under that, circumstance finding out the pressure drop or the pressure gradient becomes very easy, yes or no.

When only liquid is flowing through the pipe, you need not even measure the pressure drop you can just find it out from standard single phase flow equations. So, once you can find out the pressure drop for only liquid flowing or the entire mixture flowing as liquid through the pipe then, in that case, if we know that then along with that, a correction factor has to be incorporated. The second term, this particular term is the correction factor which is again expressed in terms of your known physical properties.

So, therefore, we find that if the mixture is low quality vapor liquid flows, where more or less the major quantity of the two phase mixture comprises of the liquid phase. Then, the simplest thing that, we can do is, we can simply estimate the pressure of only or the entire mixture flowing as liquid only through the pipe. And along with that, once we know it, we can just incorporate a correction factor, where the correction factor in this particular case is in terms of quality and physical properties.

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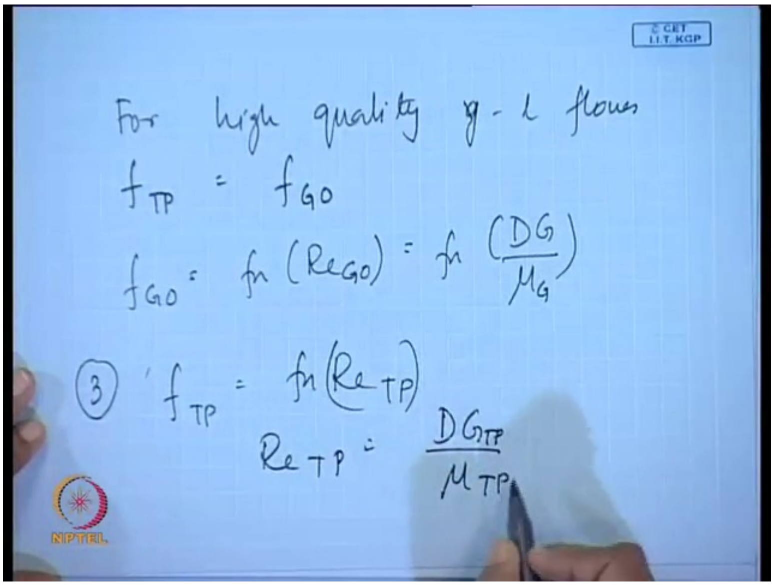

For high quality g - l flows

$$f_{TP} = f_{GO}$$

$$f_{GO} = fn\,(Re_{GO}) = fn\left(\frac{DG}{\mu_G}\right)$$

$$③\ f_{TP} = fn\,(Re_{TP})$$

$$Re_{TP} = \frac{D G_{TP}}{\mu_{TP}}$$

Specific volume is nothing but the physical properties. So, we can just incorporate a suitable correction factor to the expression of the single phase liquid flow, only pressure gradient. And, we can obtain the pressure gradient for the two phase flow situation, clear to all of you, when it was flowing as liquid or rather when we were dealing with low quality vapor liquid flows. Now, in a similar way, when we are dealing with high quality vapor liquid flows. For high quality vapor liquid flows, just in the same way that, we have done for low quality vapor liquid flows or in fact, gas liquid flows I should be mentioning. In that particular case also, we can very well write it down as f T P is f gas only means entire mixture is flowing as gas, any doubts you can clarify it from me.

Yeah, what? Where, we meet them? This D will be there. I am very sorry, I am very very sorry. Thank you very much, very sorry, yeah definitely, I have written it twice that I made a mistake that.

So, for high quality vapor liquid or gas liquid flows in a similar situation, we; what can we do? We can write down the rather, we can express the two phase friction factor, as the friction factors which would occur, if the entire mixture flows as gas only or the vapor only through the pipe. So, in this particular case again, we can write it down as f T P equals to f G O. Where, f G O it is a function of Reynolds number, again this is a function of D G by mu G, but mu G is more or less negligible in this particular case.

So, in this; in the same way we can write down agreed. But, remember one thing these two expressions, this particular expression and this particular expression, these two are applicable only for high quality, this is applicable just for high quality sorry low low quality vapor liquid flows. Now, just try to understand one particular thing, in this particular case see, it does not confirm to the limiting conditions. What are the limiting conditions that, we know? The limiting condition is that, the frictional pressure gradient must reduce to the gas so many pressure gradient, when X equal to 1, is it not.

This is a very standard thing that, in the two phase mixture when there is only gas. Suppose, you start from X equal to 0 to X equal to 1, X equal to 0 means, it is just liquid flowing, X equal to 1 means, it is just gas flowing. Now, if you put X equal to 1 in this particular expression, are you going to get the expression which we would obtain when only gas is flowing through the pipe. Just observe these expressions, you are not going to get the expressions when only gas is flowing in the pipe is it not. So, it does not confirm to one of the limiting condition and that is a very severe limitation, we can use it for only a very specific situation, we cannot extrapolate it by any particular means.

Same way if you take up, the other particular expressions which, we are derived for gas only; for high quality gas liquid flows, again we find that, if we extrapolate it then, we cannot obtain the situation for X equal to 0 or for only liquid flow. So, therefore, we find there is a gross limitation in using or rather in assuming, f T P as f G O or f T P as f L O, we can do it under very restricted conditions and therefore, we cannot rely on it, as a more or less even a generalize situation. So, for a generalize situation, what we will be doing, for a generalize situation in that particular case then, the best thing is you simply define a two phase Reynolds number which we had already discussed in the last class. And you just assume that, your two phase friction factor is a function of two phase Reynolds number, this can very well be done.

And how to define two phase Reynolds number, may be in the more or less in the same way as you define single phase Reynolds number. So, therefore, we can define Re T P as D G no problem in D and G or rather G T P by mu 2 phase. So, we find that, whenever we have to define an equivalent two phase friction factor, that has to be done in terms of a two phase Reynolds number and moment, we try to define a two phase Reynolds number. Again, we have come across, another average property which we did not define earlier that was the two phase viscosity. So, therefore, once more, we have to start our discussions on the different ways of predicting, the two phase viscosity of a of any particular two phase mixture or the effective viscosity of a two phase mixture or a multiphase system is it not.

Now, in this particular case also, let me tell you, we cannot arrive at a very straightforward situation. We can suggest several things which are applicable under different situations and as the case may be depending upon our circumstances, we will adopt the most suitable definition of viscosity which will suit our particular requirements, agreed all of you.

So, therefore, what are the different methods or rather what are the different expressions of two phase viscosity? Naturally, it will contain single phase viscosity and some sort of composition in it is expression. The best expression would have arrived, if we could have actually analyzed the flow field and then, from that analysis, we could have obtained, the expression of your two phase viscosity. But, unfortunately that, is not very simple that is quite complex one expression is available, but again that is subjected to several severe simplifying assumptions. For example, it is applicable for very low concentration of the discontinuous phase etcetera etcetera, when both of the concentrations are comparable definitely, we do not have any unified analytical expression for that fact.

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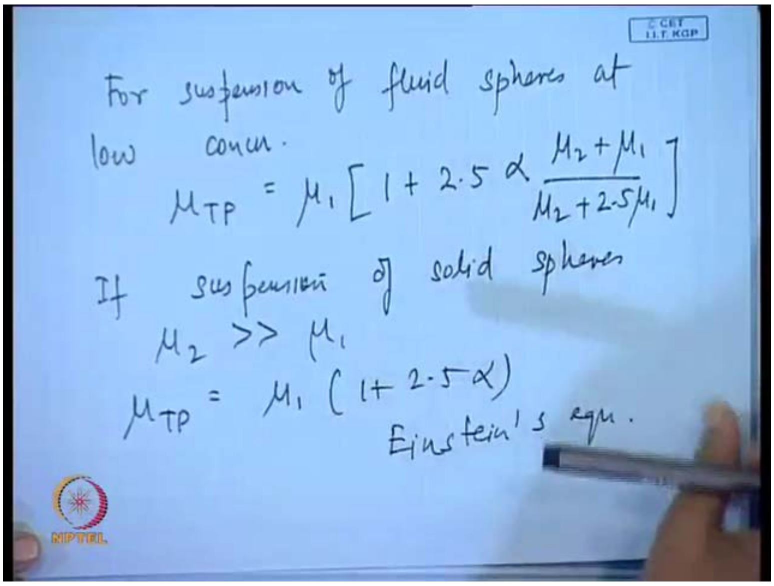

For suspension of fluid spheres at low concn.

$$\mu_{TP} = \mu_1 \left[1 + 2.5 \alpha \frac{\mu_2 + \mu_1}{\mu_2 + 2.5\mu_1} \right]$$

If suspension of solid spheres

$$\mu_2 >> \mu_1$$

$$\mu_{TP} = \mu_1 (1 + 2.5\alpha)$$

Einstein's eqn.

So, now, let us go to discuss what are the different expressions of your two phase viscosity. How to express mu T P in different forms? Now, as I was telling that, for suspension of fluid spheres at low concentration, we have generally mu T P, one particular expression it was obtained to alpha mu 2 plus mu 1 by mu 2 plus 2.5 mu 1, this is; this has been derived, when as the equivalent expression of the two phase viscosity for a suspension of fluid spheres at low concentration. For such a situation this particular expression which we find that, if suspension of solid spheres then, in that case what happens, if the suspension comprises of solid spheres then, under that case mu 2 is much much greater than mu 1 yes or no. So, therefore, under such circumstances mu T P becomes mu 1 into 1 plus 2.5 alpha, which is termed as the Einstein's equation.

In a similar way if the suspension consists of say bubbles in that particular case, your mu 1 will be much much greater than, mu 2, yes or no. So, just see for that case, what is the expression of your effective viscosity.

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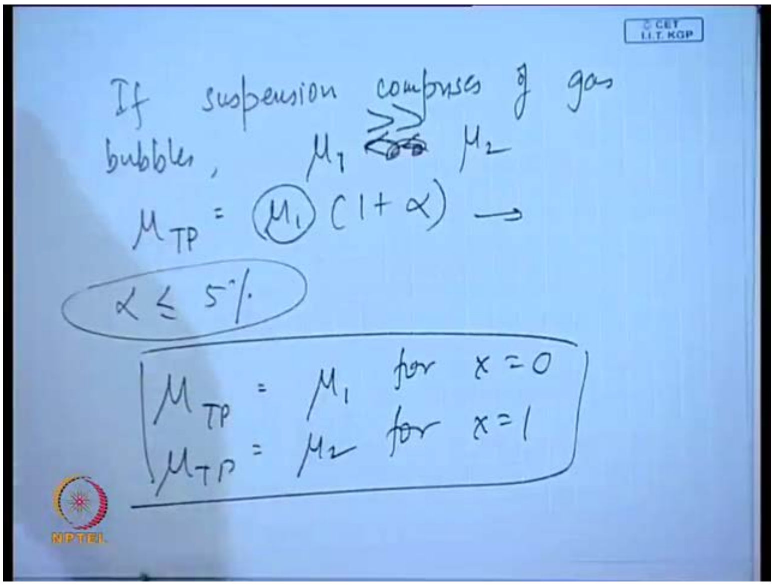

If suspension comprises of gas bubbles, $\mu_1 \gg \mu_2$

$\mu_{TP} = \boxed{\mu_1}(1+\alpha) \rightarrow$

$\boxed{\alpha \le 5\%}$

$\mu_{TP} = \mu_1$ for $x = 0$

$\mu_{TP} = \mu_2$ for $x = 1$

If suspension comprises of gas bubbles, such a circumstance where mu 1 is much much greater than mu 2. So, under this circumstance again, if you derive it you get mu T P will be equal to mu 1 into 1 plus alpha. Now, we know that both this particular equation as well as the Einstein's equation they both of them, they are valid for concentrations below 5 percent or in other words when alpha is less than equal to 5 percent, for such a situation only these two equations are valid.

But, for most of the circumstances, we will be dealing with a larger concentration of the dispersed phase. So, for such a situation what should we be doing? For such situations, numerous rheological models are available. Which one? alpha of; when alpha is; very sorry, mu 1 is much much greater than mu 2 very sorry. So, we find that, more or less both these expressions Einstein's equation and this particular expression they are applicable. When your solid or rather when the dispersed phase concentration or the second phase concentration is less than 5 percent. Such that, the viscosity change due to the presence of the second phase is very small.

If you observe both of them, the expressing terms of mu 1. So, therefore, naturally the viscosity change due to the presence of the second phase is negligible, under such circumstances only they are applicable. Now, the normal circumstances then, what to do? Analytical expressions are not available, several numerous rheological models are applicable to account for larger values of alpha and particles of different sizes shapes etcetera. It can happen the particles are of different sizes and shape, the alpha is a; of much larger concentration, for those particular situations several rheological models are applicable. Remember one thing, under such circumstances, the fluid might start behaving as a non newtonian fluid.

So, therefore, under those particular circumstances, we have to consider the certain facts. Number one is the fluid may start behaving as a non newtonian fluid. Secondly, the flow patterns might influence the expression of effective viscosity. So, under these circumstances, it is very difficult for us to provide or to propose a unified sort of an expression. So, several numerical rather sorry several rheological models they have been proposed, for such particular circumstances, keeping into mind that, they confirm to the limiting conditions under certain circumstances.

What are the limiting conditions? Limiting conditions are mu T P equals to mu 1, for X equal to 0, mu T P equals to mu 2, for X equals to 1. See, just because these two conditions who are not applicable, we had to shift from f L O f G O to f T P, is it not. So, therefore, we have to keep into mind that, whenever we are using any rheological model that, must confirm to the limiting conditions. Where mu T P or rather the two phase viscosity reduces to the liquid viscosity for no gas or vapor flow and the two phase viscosity reduces to the gas viscosity for only gas or vapor flow is it not. So, based on these particular criteria several empirical equations have been proposed by numerous authors and those particular equations are used as the case may be.

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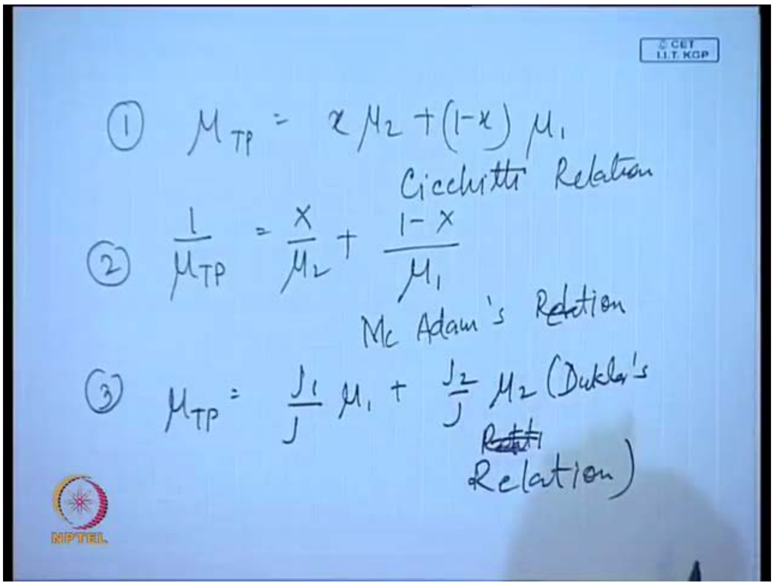

① $\mu_{TP} = x\mu_2 + (1-x)\mu_1$

Cicchitti Relation

② $\frac{1}{\mu_{TP}} = \frac{x}{\mu_2} + \frac{1-x}{\mu_1}$

Mc Adam's Relation

③ $\mu_{TP} = \frac{J_1}{J}\mu_1 + \frac{J_2}{J}\mu_2$ (Dukla's ~~Relati~~ Relation)

What are the equations? Let us see, the equations they express mu T P in terms of quality and mostly they express it in terms of quality. So, these expressions are one is, mu T P equals to a very straightforward expression, but it is not very much used definitely, because it does not give a very accurate result, this was proposed by the Cicchitti, it is known as Cicchitti Relation. The other equation which is much more used has been proposed by Mc Adam’s, which is expresses viscosity in terms of quality and single phase viscosity in this particular form, this is nothing, but the Mc Adam’s Relation sorry it is the Mc Adam’s Relation.

And the third one it expresses mu T P in terms of your volumetric fluxes, this is something of this sort it is known as the Dukla’s Relation. So, these are the relationships which are usually used out of this the Mc Adam’s equation, if nothing is mentioned in your test or when you have to analyze some two phase flow situation, flowing under homogeneous flow it is safest to use the Mc Adam’s relationship. So, therefore, the Mc Adam’s, if we use the Mc Adam’s relationship then, in that case you get something of this sort and there, we find that in this particular case.

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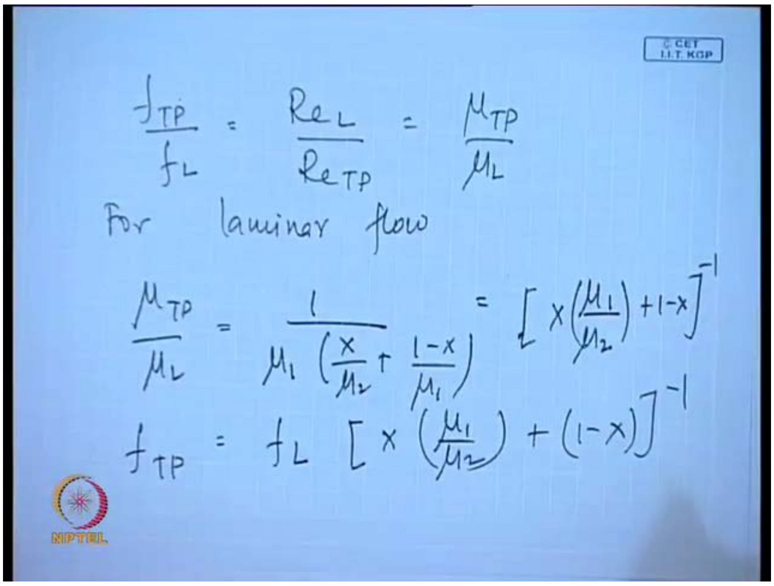

$$\frac{f_{TP}}{f_L} = \frac{Re_L}{Re_{TP}} = \frac{\mu_{TP}}{\mu_L}$$

For laminar flow

$$\frac{\mu_{TP}}{\mu_L} = \frac{1}{\mu_1\left(\frac{x}{\mu_2}+\frac{1-x}{\mu_1}\right)} = \left[x\left(\frac{\mu_1}{\mu_2}\right)+1-x\right]^{-1}$$

$$f_{TP} = f_L\left[x\left(\frac{\mu_1}{\mu_2}\right)+(1-x)\right]^{-1}$$

Suppose, we have the laminar flow pattern. So, for such circumstances, what we can do? We can assume that, your f T P by f L this is nothing, but equal to R e L by R e T P which is nothing, but equal to the mu T P by mu L. Now, this can very well be so, for laminar flow, we can use this particular expression. Now, suppose, we assume, rather we take the mc adam’s equation. So, if we take up the mc adam’s equation, then in that case mu T P by mu L this can very well be written down in this particular form X by mu 2 plus 1 minus X by mu 1 or in other words in a better still form, it can be written down as X mu 1 by mu 2 plus 1 minus X whole to the power minus 1 is it not, it can be written down fine. They are just the way in which we substitute that is the thing I would like to show you. Under such circumstances, how to express f T P in terms of known variables.

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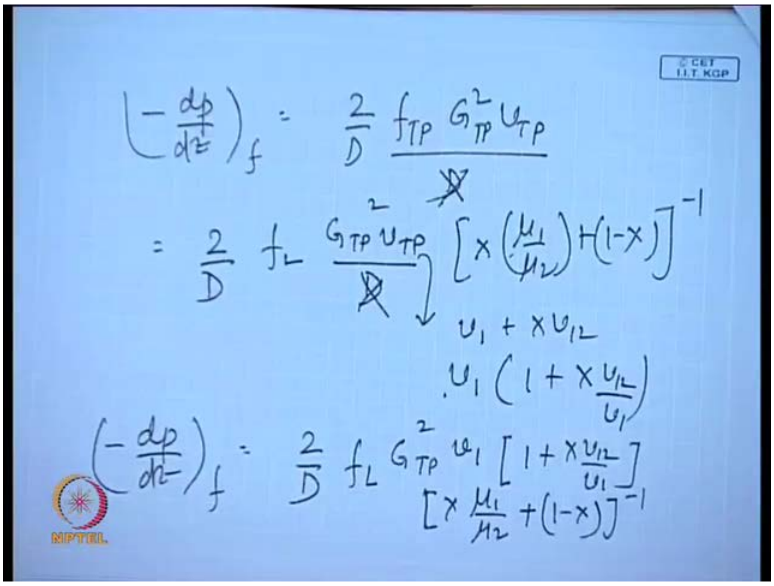

$$\left(-\frac{dp}{dz}\right)_f = \frac{2}{D}\frac{f_{TP}\,G_{TP}^2\,v_{TP}}{\cancel{D}}$$

$$= \frac{2}{D} f_L \frac{G_{TP}^2\,v_{TP}}{\cancel{D}}\left[x\left(\frac{\mu_1}{\mu_2}\right)+(1-x)\right]^{-1}$$

$$v_1 + x v_{12}$$

$$v_1\left(1 + x\frac{v_{12}}{v_1}\right)$$

$$\left(-\frac{dp}{dz}\right)_f = \frac{2}{D} f_L G_{TP}^2 v_1\left[1 + x\frac{v_{12}}{v_1}\right]\left[x\frac{\mu_1}{\mu_2}+(1-x)\right]^{-1}$$

So, therefore, once we can express mu T P by mu l this is the relationship between f T P and f L. So, therefore, we can write down as f T P it is nothing, but f L into X mu 1 by mu 2 plus 1 minus X whole to the power minus 1, yes or no, we can just make this particular substitution. Accordingly, what we can do? We can write down the frictional pressure gradient in this particular form, again the same thing 2 by D f T P G square v T P by D. Instead of f T P, we can substitute this particular equation.

So, therefore, if you write it down in this particular form what do we get sorry again I have written 2 d's very sorry 2 D it can be f L G T P square this should have been G T P square v T P by D X mu 1 by mu 2 plus 1 minus X whole to the power minus 1, we can do it in this particular form, very sorry [FT] I have to change it is really getting very bad. Actually, the point is usually I have a habit of writing down here, but once I am forgetting that I am writing very sorry and this v T P it is again nothing but as I had expressed it, it is v 1 plus X v 1 2 is it not Or in other words it can be v 1 into 1 plus X v 1 2 by v 1.

So, therefore, if we substitute again this in terms of the two phase specific volume then, we get the frictional pressure gradient as 2 by D f L just note the type of substitutions that, I am doing v 1 into 1 plus X v 1 2 by v 1 into X mu 1 by mu 2 plus 1 minus X whole to the power minus 1, can I write it in this particular form, when I have expressed the two phase or rather when I have expressed the two phase friction frictional pressure gradient in terms of a two phase Reynolds number.

So, in this particular case also I find that, I could actually break down, the frictional pressure gradient for two phase flow under homogeneous flow conditions as a product of two or three terms. That, the first term again, if you observe it this particular first term it is nothing but it gives you, the frictional pressure gradient for only liquid flow is it not. This gives you the frictional pressure gradient for only or rather for only liquid flow agreed. And, the other portion this part and this part, they refer to the correction, where in the correction, we find that, it is in terms of quality and again phase physical properties.

So, therefore, if we know the phase physical properties, if we know the mass fraction of the two phases. Then in that case we would be able to express the two phase frictional pressure gradient in terms of a single phase pressure gradient and some correction factor based on composition, single phase properties and so on and so forth. So, these are the different ways in which, usually the frictional rather usually a frictional pressure gradient is expressed. Now, remember one thing this whatever I have done, this was based on the basic supposition of laminar flow, for laminar flow this was the correction factor, is it not.

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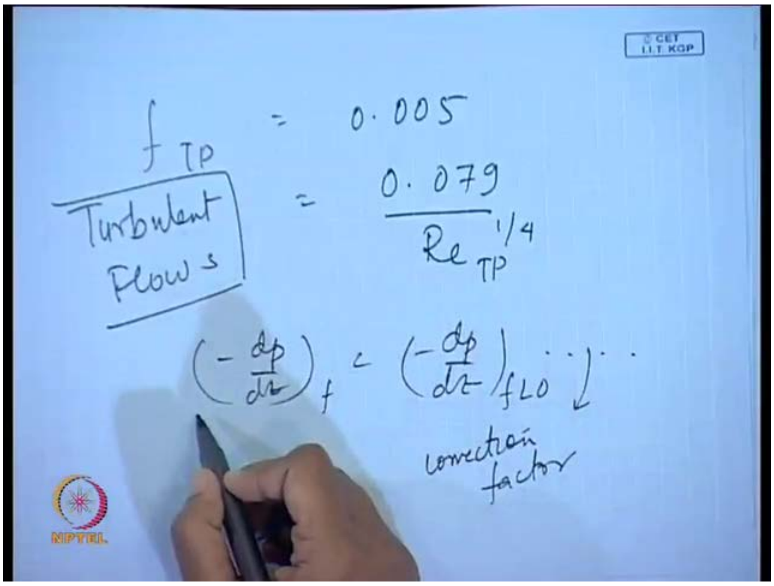

$$f_{TP} = 0.005$$

Turbulent Flows

$$= \frac{0.079}{Re_{TP}^{1/4}}$$

$$\left(-\frac{dp}{dz}\right)_f = \left(-\frac{dp}{dz}\right)_{fLO} \left[\ldots\right]\ldots$$

correction factor

Because, the basic thing I had started from this particular expression which is applicable for laminar flow depending on this, I had substituted and I had expressed f T P in terms of f L which relationship is valid only for laminar flow, agreed. Now, for turbulent flow what are different approaches that we can use, just the approaches which are available for single phase flows. What we can say? We can say, one is for turbulent flows, one is f T P equals to 0.005 you can take or maybe, we can take it is 0, it is a Blasius Equation.

Any of them, we can take and accordingly, we can substitute it and if we substitute it and if you proceed then, you will find that. In the similar way, you can you will be in a position to express their frictional pressure gradient as the frictional pressure gradient for liquid flow only. And some sort of a correction term or a modification term, which needs to be added to the single phase pressure gradient in order to obtain the two phase pressure gradient. So, therefore, what are the methods in short, if we discuss? What are the methods that, we have adopted? First thing is for low quality flows, we can assume f T P equals to friction factor as if the whole mixture was flowing as liquid number one.

Number two was for high quality flow the same thing. The other thing is when none of them can be assumed then, what we can do? We can simply assume that, well the two phase mixture all though it is flowing as a pseudo fluid. So, therefore, it has an equivalent frictional factor which is a function of the equivalent Reynolds number of the two phase mixture. Now, in this particular case, there was one advantage that the inlet and the insitive volume fraction of the compositions were the same, is it not.

So, therefore, what we can do? We can simply express, the frictional pressure gradient for two phases under homogeneous flow conditions as or in terms of a two phase friction factor which is a function of two phase Reynolds number. Remember normally for two phase flow Reynolds number is not a very good non dimensional or not a very good characterizing parameter. Because in Reynolds number, how to define the velocity and how to define the density and the viscosity, all these things have to be mixture properties and they have to be expressed in terms of suitable average of single component property, agreed.

So, therefore, the; this is the other thing that we can do, we can define a proper two phase Reynolds number and in terms of the two phase Reynolds number, we can define the two phase friction factors. Now, for that particular case, we find the only bottle neck is defining a proper or rather expressing, the two phase viscosity in terms of single phase components. Again the same that if it is high quality vapor liquid flows, we can do something, but in this case that, condition does not arise, because we have defined f T P and R e T P, for those particular cases where more or less both the parameters or rather both the phases are flowing under comparable compositions.

So, therefore, under this particular condition, what we can do? We can define R e T P and accordingly, we can define f T P for that, we need mu T P. Now, for mu T P what are the different expressions, we find that, mostly we have to rely on empirical correlation or rather empirical expressions, keeping in mind that, the empirical expressions confirm to the suitable; confirm to the practical limiting conditions. What are the practical limiting conditions? Naturally, when the mixture flows only as or rather when X equal to 0 the mixture is nothing but only liquid flow. So, under that condition mu T P has to be equal to mu liquid or mu 1, when X equals to 1; that means, it is just vapor or gas flow under that, conditions mu T P has to be equals to mu 2.

So, therefore, keeping these two limiting conditions in our mind, suitable or rather several researchers have proposed suitable expressions of two phase viscosity in terms of quality and single phase viscosities. Three expressions are used out of that, if nothing is mentioned the most commonly used expression is that due to Mc Adam’s, agree. So, therefore, under most circumstances we will use that. So, we have used it for the laminar flow condition and we have found out that, we can express the two phase frictional pressure drop, under homogeneous flow conditions. In terms of single phase liquid pressure frictional pressure drop, when the entire mixture flows as liquid only in the pipe and the particular correction factor.

Similar way we can use f T P or rather, we can express f T P just the way, we have expressed, the single phase friction factor for turbulent flow as well. What are the different approaches? You either assume a constant value 0.005 or whatever or we can use the blasius equation or some other more complex equation, we can do it in this particular case also. And you please substitute it and find out what the turbulent flow case for both the cases, for using a constant value of friction factor. As well as for using at the Blasius Equation, what is the equivalent correction factor to the single phase frictional pressure gradient? For liquid flow only which has to be incorporated in order to obtain the two phase frictional pressure gradient, is it clear to you what I mentioned.

What I meant was, I have done this for the laminar flow condition I would like you to do the same exercise for turbulent flow condition for both this as well as this. And then to derive the correction factor which will be applicable for the turbulent flow condition, this was the correction factor, which was applicable for the laminar flow condition. So, just find out, what is the appropriate correction factor for the turbulent flow condition under these situations, agreed.

Now, there is one more important thing that, I would like to say see very frequently, we would like to express two phase friction frictional pressure gradient in terms of a single phase pressure frictional pressure gradient and some suitable correction factor, why? Because, single phase pressure gradient is very well known, if the correction factor is not very large, it is just like maybe, we have introduced your momentum correction factor, kinetic energy correction factors in your expressions of kinetic energy and all those things in single phase flow.

You remember, alpha u square by 2 beta and alpha were the two correction factors sort of that. So, we would always like to stick to things, which we know very well and we; and to see whether by incorporating suitable correction factors, whether that particular value can be modified to give us the value which we require.

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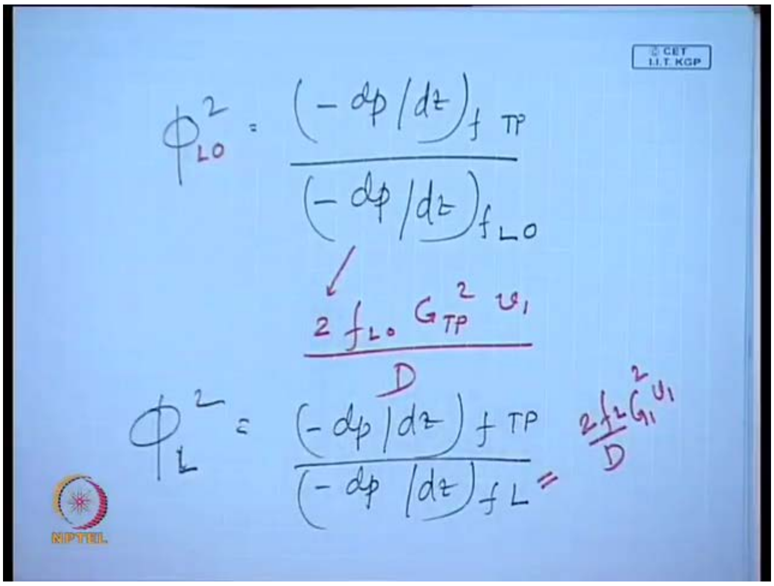

$$\phi_{LO}^2 = \frac{(-dp/dz)_{f\,TP}}{\underset{\swarrow}{(-dp/dz)_{f\,LO}}}$$

$$\frac{2 f_{LO} G_{TP}^2 v_l}{D}$$

$$\phi_L^2 = \frac{(-dp/dz)_{f\,TP}}{(-dp/dz)_{f\,L}} = \frac{2 f_L G_l^2 v_l}{D}$$

So, single phase frictional pressure gradient expressions are very well known, they can be obtained analytically without a need of much complexities. So, if we can define the two phase frictional pressure drop, in terms of single phase pressure drop and the correction factor then, it is much more convenient. So, there are two ways of defining the two phase frictional pressure gradient, one is by a suitable definition of the friction factor and the other is by a suitable definition of the correction factors.

Now, these correction factors they are defined in terms of phi square, these are known as two phase multipliers these; and they are simply they relate, the two phase friction factor d p d z frictional. Say for two phase, if you want, you can write it down, generally I have referred to two phase friction factor as just f in the previous slides, it is the ratio of this by the frictional factor when ether of the phases flow in the pipe. Depending upon, what flows in the pipe, we have different definitions of phi square. So, it is basically this particular correction terms, they are used to express the two phase frictional pressure gradient in terms of a related single phase pressure drop or the pressure drop when the mixture flows as liquid or vapor or whatever it is.

Now, remember in this particular case, we can define four different correction factors, how? One is, in terms of when the entire mixture flows as liquid, do you get the point which we have been doing so long. When the entire mixture flows as liquid. What is the expression of this particular frictional pressure gradient? This, frictional pressure gradient will be 2 f L O G square T P v 1 by D, which we have obtained so long.

This correction factor is known as phi square L O, do you get my point. There can be another correction factor which we can define as phi L square, which can be defined as the two phase frictional pressure gradient by the frictional pressure gradient. When the liquid which was there in the pipe now flows alone, do you get the point, whatever liquid was there try to understand the difference between the two.

For the first case what it was, the entire mixture flows as liquid, for the next case what is it? For the next case, this is 2 f L by D G 1 square v 1, just note the difference between this and this. And try to understand the correction factors that we have or the two phase multipliers as they are called; the these particular ratios they are expressed in terms of two phase multipliers. In other words, they mean that, the multiplying factor to the single phase frictional pressure drop which has to be multiplied such that, we get the two phase frictional pressure gradient.

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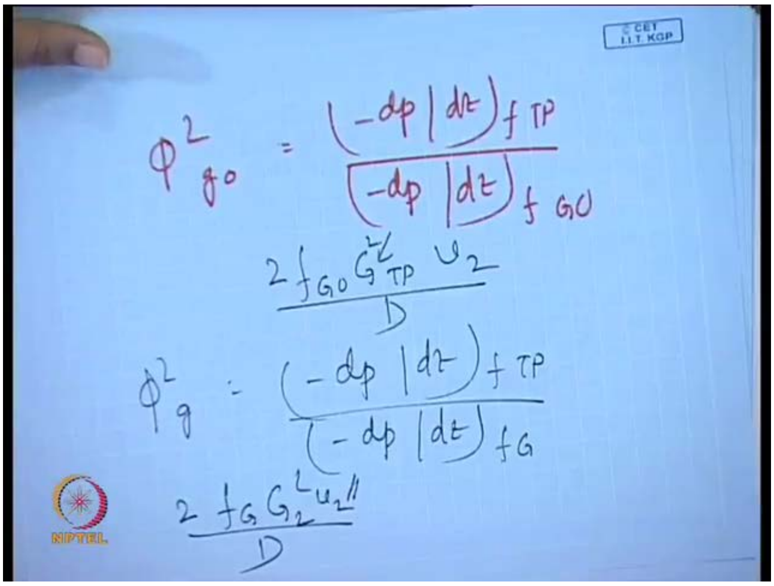

$$\phi^2_{go} = \frac{(-dp/dz)_{fTP}}{(-dp/dz)_{fGO}}$$

$$\frac{2 f_{GO} G^2_{TP} v_2}{D}$$

$$\phi^2_{g} = \frac{(-dp/dz)_{fTP}}{(-dp/dz)_{fG}}$$

$$\frac{2 f_G G_2^2 v_2''}{D}$$

Now, try to understand the situation, there can be number of situations. What are the situations? One is, when the entire mixture flows as liquid or when the entire mixture flows as gas. When the entire mixture flows as liquid then, naturally the mixture; the liquid is flowing, in terms or rather the whole particular mass flux has to be included. Similarly, for gas flow rate if you take phi square G O, this will be minus d p d z frictional two phase by minus d p d z frictional for gas only. Where this particular term it will be 2 f G O g square T P v 2 by D, is it correct or in other words, the entire mixture is flowing as either the vapor or the liquid. But, if you; other than that, there is another way of defining, where we do not assume that, the entire mixture is flowing as liquid. We assume, whatever liquid was there in the mixture that is not flowing alone in the pipe, is it clear to you.

Whatever, because see this is a much more realistic situation, because assuming the entire mixture two flow as liquid means, whatever gas was there that also you assume two flow as liquid. This L O phi square L O and phi square G O are useful for boiling condensation situations where, we start with a saturated or sub cool liquid and then, gradually your phase change occurs for those particular circumstances, we use phi square L O or phi square G O. But for other normal circumstances what we use? We use phi L square and phi G square, where phi G square again, if we define it this will be in terms of the gas phase in the two phase mixture that is flowing alone in the pipe, G 2 square v 2 by D.

If you understand the liquid phase things, the same way the gas phase things can be understood. So, be sure if whether you have understood it, otherwise I am going to define it once more. The difference between phi square L and phi square L O same is the difference between phi square G and phi square G O are the differences, clear you. Just note, these two expressions and then it is going to be clear. Once see, we would always like to express it in terms of a frictional pressure gradient which we can easily measure or rather which the analytical or expression which we already know.

Single phase pressure drops are very well known. So, we can define it, in terms of equivalent single phase flows. How to define the equivalent single phase flows? The equivalent single phase flows can be that the entire mixture flows as a single phase or whatever amount of liquid was there, that is now flowing alone in the pipe, do you get my point. So, therefore, in that case, this expression and this expression are not the same and therefore, phi square L O and phi square L needs distinguishing, clear to you this portion. Now, here, we find that apparently, if you see, we are going to find that, well the only difference is in the mass flux, here it is the two phase or the total mass flux, here it is just the liquid mass flux.

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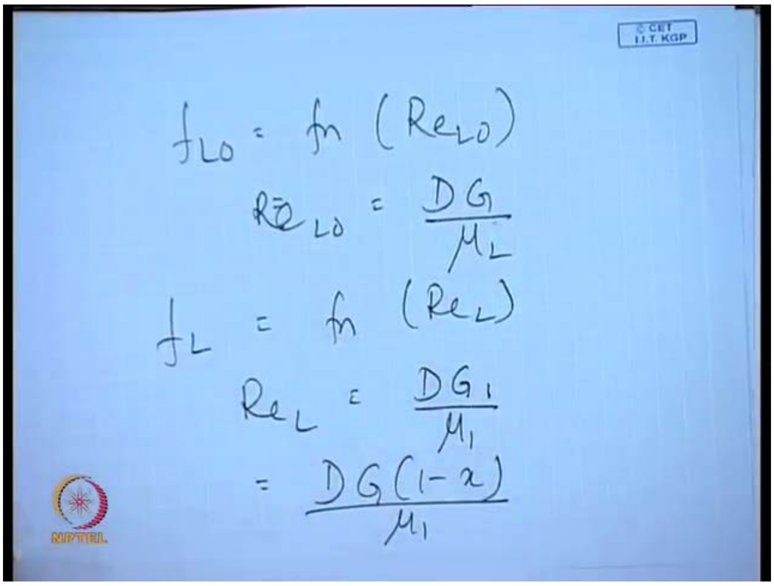

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$$f_{LO} = fn\,(Re_{LO})$$

$$Re_{LO} = \frac{DG}{\mu_L}$$

$$f_L = fn\,(Re_L)$$

$$Re_L = \frac{DG_1}{\mu_1}$$

$$= \frac{DG(1-x)}{\mu_1}$$

But if you note, closely you will find that, I have written one f L O and I have written one f L. That means, I have differentiated between the friction factor, when the entire mixture flows as liquid and only the liquid of the mixture flows alone in the pipe, can you tell me is it necessary to differentiate between f L O and f L? For both the cases just liquid is flowing through the pipe, is it not. And friction occurs only at the wall, for both the cases it is just the liquid which is in contact with the wall, was it necessary for me to differentiate between f L O and f L? Was it necessary? Why? Very true, because friction factor is a function of Reynolds number.

So, for the first case, we find f L O it is a function of R e L O agree, where R e L O is nothing but D G by mu L, f L function of R e L, where R e L is D G 1 by mu L or in other words equals in terms of input parameters it is this. So, therefore, keeping in mind, this probably while you are studying single phase flows it never occur to you that, even when, we are having only liquid flow depending upon the amount of liquid which is flowing also, the friction which occurs only at the wall changes, this is something which you are suppose to keep in mind. This did not probably affect you during your study of single phase flow, but this definitely is going to matter these small small things, which are nothing but single phase flow fundamentals. They will be affecting you more and more they will be influencing you more and more when you are going to study the two phase flow cases. So, these things please remember.

So, the two ways of defining the frictional pressure drop is, either to define a proper two phase friction factor or to define a proper two phase multiplier. Friction factor definition, usually for the general trend is to define it in terms of a two phase Reynolds number, for which we need a proper definition of two phase viscosity. And for the for your two phase multiplier, you can define it in any particular way, you can assume that the entire mixture is flowing as a liquid. Usually, we use phi L O square and phi L square, because usually we are used to dealing with this low quality or comparable quality flows.

And generally, the pressure gradient it is more influenced by the liquid component. So, therefore, you can assume, that the entire mixture flows as liquid this, we usually do you do for boiling experiments or entire mixture flowing as gas or vapor for condensation experiments. But when for usually for adiabatic two phase two component systems, usually we assume that, whatever liquid was there in the pipe that, now flows alone or whatever gas was there that now flows alone, because that is the much more realistic situation. And for that particular case accordingly, we can find out the single phase pressure gradient, when the liquid of the two phase mixture flows alone and then that, can be; once that is found out and the two phase multipliers certain correlations etcetera are available.

So, if you know those then, we can multiply the single phase frictional pressure drop with those particular multipliers and we can obtain, the frictional pressure gradient which we would encounter for two phase flow under homogeneous flow conditions, agreed. Now, more or less once, we know how to define f T P, all the expressions which we had everything which was there in this pressure drop expression, we know. And therefore, we are now in a position to find out the pressure gradient for two phase mixture under homogeneous flow conditions. The only thing which is remaining is the significance of this denominator term.

So, in the next class, I would like to cover some basics of compressible flow and after that once, we know the significance of the denominator which was obtained for single phase compressible flows, we would automatically be in a position to find out the significance of the denominator for two phase flow under homogeneous flow conditions well. So, well continue with this in the next class, where we will be doing some basics of compressible flow. Thank you very much.