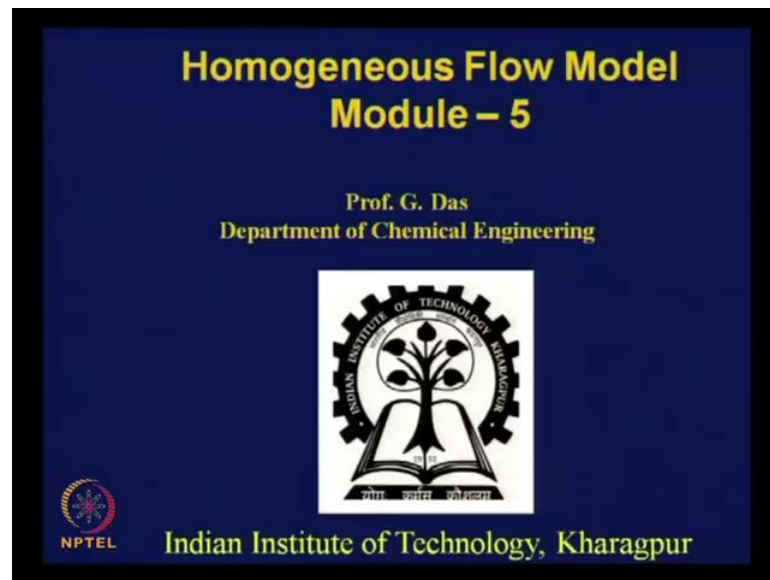


Multiphase Flow
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Module No # 01
Lecture No # 08
The Homogeneous Flow Theory

Very good morning to all of you. So, today we are going to start the homogeneous flow theory the simplest possible analytical flow model which We are going to discuss we are going to start it today.


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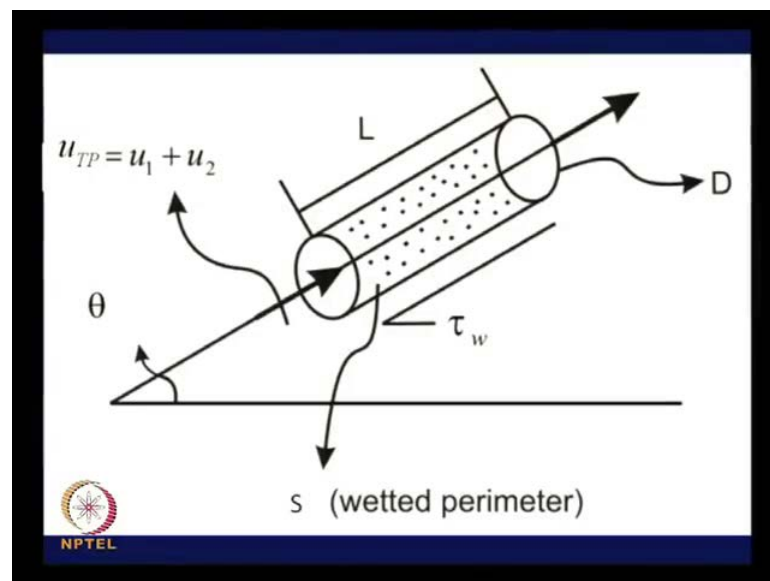
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Homogenous flow model

- Assumptions-
 - Two fluids are uniformly mixed and moving as a pseudofluid at the mixture velocity
 - The slip velocity between the two phases negligible which implies that both the fluid are moving at an average velocity
 - Attainment of thermodynamic equilibrium between the phases



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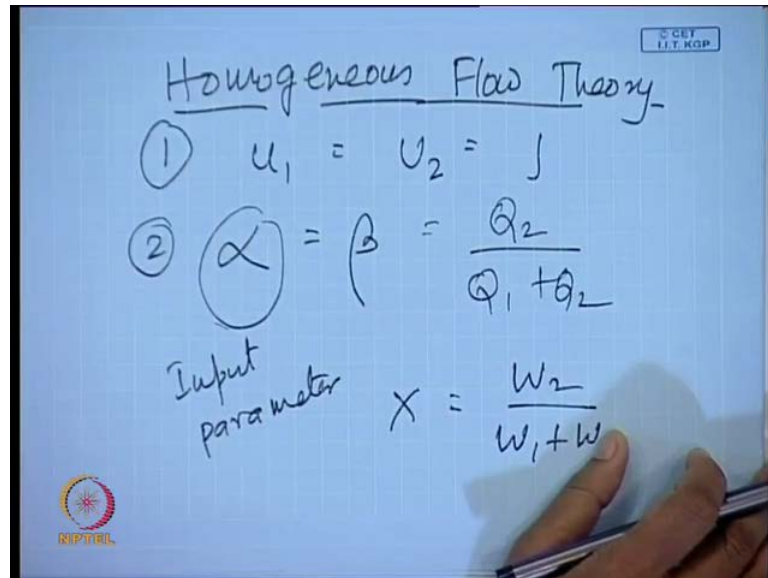
So, here what do we assume we assume that the 2 phases they are thoroughly mixed up with one another they are completely mixed up with one another. So, that this can they can flow as a as a pseudo fluid.

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And what are the basic assumptions of this particular model naturally 2 fluids are uniformly mixed and they are moving as a pseudo fluid at the mixture velocity. So therefore, this shows that both the fluids this automatically implies that the slip velocity

between the 2 phases it is negligible and on other words the 2 fluids are moving at the same average velocity.

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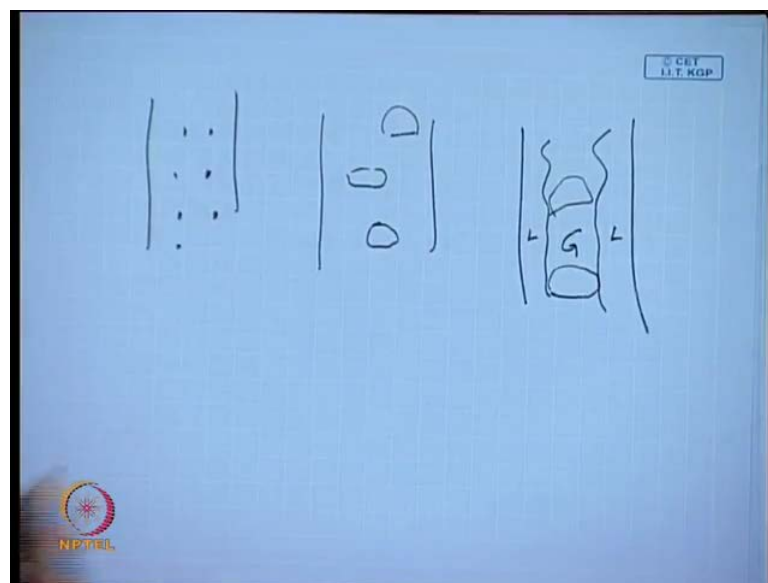
What does it imply? It implies the first thing which it implies is number 1 that the 2 fluids are moving at the same velocity. Which is the mixture velocity or in other words u_1 equals to u_2 equals to j agreed this is the first thing that it implies. The automatically the next thing which it implies it, that since there is no slip between the phases due to the slip α was not equal to β you remember. So, the next thing which automatically implies is α equals to β or in other words α is nothing but equal to Q_2 by Q_1 plus Q_2 . So, what does this imply? This implies that under these circumstances α becomes a measurable input parameter.

If we know the flow rates of the 2 phases which is very easy the inlet flow rates you just have 2 rotameters or any particular flow meter. You can find out the inlet flow rates moment, you know the inlet flow rates we can find α . So, now under such circumstances we find I will just put the heading here although you know what I am teaching. So, you see that we have come for the very drastic simplification under this particular case. And what has it has this given us? This has given us that the inlet void fraction and the insitu void fraction has become equal or in other words the void fraction or the insitu composition is nothing but a measurable parameter.

So therefore, in this particular case just remember just for recapitulation your suppose there is evaporation condensation sorry condensation or boiling them. In that case we can obtain the quality from the mass fraction the inlet mass fraction and the void fraction from the inlet volumetric flow rate. And finally, the third assumption this is simply attainment of thermodynamic equilibrium between the phases this has to happen. Unless there is any thermodynamic equilibrium between the phases there will be heat transfer, momentum transfer, there will be energy transfer, there will be mass transfer moment. Those things happen then there obviously under such circumstances we cannot have the homogeneous flow assumption because the entire thing has to be uniformed throughout.

So, therefore, the three assumptions remember mathematically if we express them it is just that u_1 equals to u_2 which is equal to either u or j . j is a better thing we can relate it as with volumetric flux so, we call it j other thing is α equals to β . And then attainment of thermodynamic equilibrium, these are the basic and very drastic assumptions which govern the homogeneous flow theory.

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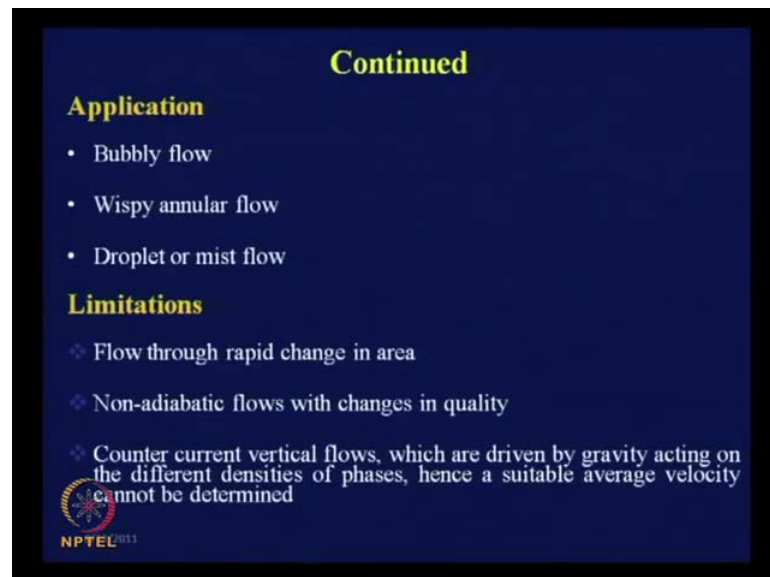


So, under what circumstances are they applicable, naturally they are applicable for bubbly flow situations it is not exactly bubbly I should say dispersed bubbly where the bubbles they are dispersed uniformly. If you remember when I had drawn the pictures this should be corrected as dispersed bubbly that means, the entire thing in the I mean it sort of this particular appearance. This particular appearance this is the correct thing

moment the bubbles become larger and they get deformed the non uniformity goes away under this particular condition also the homogeneous flow model does not apply.

But we apply it see how much deviation is there and then we suggest some suitable correction factors. So therefore, the ways of application are firstly, the bubbly flow, droplet flow, mist flow it is a same thing almost wispy annular when the film is very thin, but wispy annular is not a very correct thing. Actually in wispy annular what you should be doing is it is a very interesting situation actually we have a liquid core, we have gas and then here we have big chunks of liquid.

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Continued

Application

- Bubbly flow
- Wispy annular flow
- Droplet or mist flow

Limitations

- ❖ Flow through rapid change in area
- ❖ Non-adiabatic flows with changes in quality
- ❖ Counter current vertical flows, which are driven by gravity acting on the different densities of phases, hence a suitable average velocity cannot be determined

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As a matter of fact I will tell you that bubbly and wispy annular they are not very different. Other than the fact that in bubbly the entire thing looks like, this in wispy annular a very thin liquid film can be observed at the walls it is very difficult to differentiate between the 2 and their ordinary laboratory conditions. I will be requesting amit to show them bubbly and wispy annular and just tell them how difficult it is to differentiate between the 2. So, under such circumstances more or less we can use the homogeneous flow assumption.

Now, let us see under which circumstances we cannot use it at all, even if the two phases they are completely mixed with one another I agree, but it is flowing may be it is flowing through a orifice or a very short nozzle under that circumstances. What do you expect is going to happen? Completely mixed flow, but it is flowing may be through a very short

nozzle or may be through an orifice, orifice means that abrupt contraction. Then what is going to happen? Rapid acceleration will take place.

Under this rapid acceleration then we cannot have the 2 phase flow situation this I have already written down here. So, this is 1 thing where we cannot have. And for the rapid acceleration what is the problem let the 2 phases accelerate, why cannot we use it? Because under such circumstances there is a drastic reduction or drastic change in pressure moment there is a drastic change in pressure flashing occurs, moment flashing occurs thermodynamic equilibrium is no longer valid clear to all of you.

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So, therefore, whenever we encounter any particular situation where there is a flow through a short nozzle or may be through an orifice where there is rapid acceleration. Flow through rapid change in area under such circumstances since the equilibrium condition is not valid we do not have your homogeneous flow theory. So therefore, certain specific cases I would like to read out flashing of steam water mixture through short nozzles and orifices, we cannot use the homogeneous flow theory. Next say bubbling in super heated liquid why because in this case we have to consider bubble nucleation, bubble growth and things like this. Then super cooled vapor that also condensing in high velocity stream.

So, these are the things if you can note down you can note down the situations where this is inapplicable due to rapid acceleration and pressure changes I have just mentioned this, but I have not given this specific examples. Flashing of steam water mixture through short nozzles and orifices bubbling, in super heated liquid or super cooled vapor condensing in high velocity steam. And particles of solid propellant burnt in nozzle of rocket engines, under this particular situation there is rapid acceleration rapid pressure changes and we cannot use it.

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Then suppose we have non adiabatic flow with changes in quality, all these things moment all these rapid changes and quality etcetera rapid change in pressure etcetera take place thermodynamic equilibrium. The last assumption, which I had shown in the previous slide if you remember this assumption is no longer valid. There is another very

interesting situation where we cannot use homogeneous flow theory. That is suppose you have counter current flows even if it is a bubbly flow situation you have counter current flows. Then in that case the flow is driven by gravity or vertical flows and in this case the gravity it acts on the different densities of the 2 phases, moment gravity acts on the different density of the 2 phases.

In this particular case can you define a suitable average velocity, do you understand? Because in this particular case say the gas is rising up at say u_2 the liquid is coming down at say u_1 . What will be your average velocity? u_2 minus u_1 . So, here a suitable average or in other words can you tell that u_1 equals to u_2 equal to j , definitely you cannot say. So, even if it is may be a mixed sort of an appearance also or counter current flows we cannot use the homogeneous flow assumption remember these things. So, these two circumstances just remember whenever there are rapid changes in changes in say area pressure etcetera. Nonadiabatic flow situation and for counter current flows these are some of the limitations.

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Continued

Continuity $W_{TP} = \rho_{TP} u_{TP} A$

Momentum $W_{TP} \frac{du_{TP}}{dz} = -A \frac{dp}{dz} - S\tau_w - A\rho_{TP} g \sin \theta$


$$\rho_{TP} = \alpha\rho_2 + (1 - \alpha)\rho_1$$

$$-\left(\frac{dp}{dz}\right)_f = 2f_{TP} \rho_{TP} \frac{u_{TP}^2}{D}$$

$$-\left(\frac{dp}{dz}\right)_f = \frac{2f_{TP} G_{TP}^2}{D\rho_{TP}}$$

$$-\left(\frac{dp}{dz}\right)_f = \frac{2f_{TP} G_{TP}^2}{D} (v_1 + xv_{12})$$

$$-\left(\frac{dp}{dz}\right)_a = G_{TP} \frac{du_{TP}}{dz} - \left(\frac{dp}{dz}\right)_a = G_{TP} \frac{d}{dz} \left(\frac{W_{TP}}{A\rho_{TP}} \right)$$



So, now let us go into the basic derivations do not look into the slide we are going to derive it, but I have put down everything in the slide just for your convenience so, that we can you can refer to it. But for the time being we will not be looking through the slide we will be deriving them ourselves and we will see how the derivations are different from what we have derived for the single phase flow equations.

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Continuity $W = \rho u A$ (single phase)

$$W_{TP} = \rho_{TP} U_{TP} A$$
$$\rho_{TP} = \frac{Q_2}{Q_1 + Q_2} \quad U_{TP} = \frac{Q}{A}$$

So, let us start with the continuity equation. For single phase flow what is the continuity equation? It is nothing but the conservation of mass. So therefore, total mass is w that is equal to $\rho U A$ this is for single phase. Here what should I write? Here same thing the total mass flow rate has to be constant. Is not it? So therefore, we can write down the individual mass phases might vary, but the total one has to be constant. So, therefore, this has to be equal to $\rho_{TP} U_{TP}$ or you can write it down as j into A , we can write it in this particular way where we know that ρ_{TP} it is nothing but equal to as I have already mentioned. And this U_{TP} is nothing but equal to Q by A . So therefore, we find all of these are measurable parameters fine.

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Momentum Equ.

$$\left(-\frac{dp}{dz}\right) = \frac{S}{A} \tau_w + \rho g \sin \theta + G \frac{du}{dz}$$

[Single phase flow]

Two phase homogeneous Flow

$$\left(-\frac{dp}{dz}\right) = \frac{S}{A} \tau_{wTP} + \rho_{TP} g \sin \theta + G_{TP} \frac{du_{TP}}{dz}$$

For only liquid flow - $\frac{DG}{\mu L} = \frac{DU_L \rho_L}{\mu L}$

Next if we go to the momentum equation, momentum equation if you remember the last class also just we had derived it. What was the expression for single phase flows? It was minus $d p d z$ that was equal to S by A τ_w wall shear stress this was the derivation for single phase flows.

For 2 phase, (no audio from: 12:54 to 13:02) 2 phase homogeneous flow. What should I write it down? Pressure gradient this will be equal to wetted perimeter by cross sectional area that is the pipe so, no problem. This has to be 2 phase agree, now we cannot $2 \tau_w$ is equal to τ_w 2 phase. Why we cannot say? Because τ_w 2 phase that is $(())$ a rather any wall shear stress it is a function of friction factor. If it is a function of friction factor, it is a function of reynolds number, if it is a function of reynolds number then try to understand or try to remember for only liquid flow. What is reynolds number? It is $D G$ by μL or in other words $D U_L \rho_L$ by μL .

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$$\left(-\frac{dp}{dz}\right)_f = f_n \quad (f)_{\text{single phase}} = f_n(Re)_{\text{single phase}}$$

$$\left(-\frac{dp}{dz}\right)_{f_{TP}} = f_n (f)_{\text{two phase}} = f_n \frac{(DG)_{TP}}{\mu_1}$$

$$\mu_{TP} = f_n(\text{Compos--})$$

For 2 phase flow then naturally what happens the frictional pressure gradient $d p d z$ frictional for single phase flow, this is nothing but a function of your friction factor single phase, which is a function of Reynolds number single phase. For 2 phase flow situation just remember 2 phase this will be a function of friction factor 2 phase, which is a function of Reynolds number 2phase. Definitely this Reynolds number 2 phase and Reynolds number single phase they both of them firstly, I do not I really do not know how to define 2 phase flow Reynolds number.

But even if we define it also then it should be the this one is equal to $D G$ by μL , in this particular case D is fine diameter of the pipe $G T P$ is also fine the total flow rate, but this definitely has to be $\mu T P$. Is not it? Where $\mu T P$ it will be a function of composition and things like that. So, definitely $\mu T P$ can never be equal to $\mu 1$ or μL whatever it is. So, therefore, we (()) we find out that even if you assume homogeneous flow theory, but then also your frictional pressure gradient for the 2 cases will not be the same agree.

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So therefore, to continue with this here we have instead of S by $A \tau W$ we have τW 2 phase then here it has to be $\rho T P g \sin \theta$. Is not it? Plus $G T P d U T P d z$. Where this is the frictional pressure gradient, this is the gravitational pressure gradient, this is the acceleration pressure gradient.

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Energy balance eqn.
 $dq = dh + d\left(\frac{1}{2}u^2\right) + d(gz \sin \theta)$
↳ unit mass of single phase flow

For two phase flow
 $dq = W_{TP} d\left(dh_{TP} + \frac{u_{TP}^2}{2} + g \sin \theta z\right)$

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Similarly, if we take up we will just do the 3 equations and then we will expand the momentum equation to find out how to find out the pressure drop or the pressure gradient from known input parameters. So, energy balance equation if we take up same thing that we had done in that particular case it was dq was equal to dh plus d half u square just in the previous class we had defined it. This particular case also it is going to be dq suppose this was for unite mass of single phase flow.

For 2 phase flow what do we get? 2 phase flow this is going to be dq equals to if it is not for unit mass then we will put a $W_{TP} d\left(dh_{TP} + \frac{u_{TP}^2}{2} + g \sin \theta z\right)$. This is the energy balance equation that we can write if we are having some sort of a heat transfer may be some sort of condensation evaporation, then probably we will have to use the energy balance equation.

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Now, if we consider these equation what do we find the momentum equation and the continuity equation the momentum equation I have already shown, and in the continuity equation.

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So, we find what are the things that we are suppose to determine rho T P which we can find it out U T P that also it is an input parameter. If we take up the momentum equation well let me write it down once more some sort of over writing has taken place there.

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The image shows a handwritten derivation on a blue background. At the top, the equation is:
$$-\frac{dp}{dz} = \frac{S}{A} T_{w, TP} + \rho_{TP} g \sin \theta + G_{TP} \frac{dU_{TP}}{dz}$$
 Below this, the friction factor term is expanded:
$$\left(-\frac{dp}{dz}\right) f_{TP} = \frac{2}{D} f_{TP} \rho_{TP} U_{TP}^2$$
 This is further simplified to:
$$= \frac{2}{D} f_{TP} \frac{G_{TP}^2}{\rho_{TP}} \left[\chi U_L + (1-\chi) U_G \right]^2$$
 A definition for the parameter χ is given as:
$$\chi = \frac{w_L}{w_{TP}}$$
 The NPTEL logo is visible in the bottom left corner.

So, therefore, this is going to be this is equal to say minus d p d z frictional or I will put it as in this particular way itself. (No voice: 18:40 to 18:47) this is the expression which we had obtained. Now, in this particular scene already I have told you how to define rho T P, this I have told you how to define so, S A etcetera you already know. So therefore, as I have told you that let us take by take up term by term and see how it can be expressed in terms of measurable parameters. So, initially let us take up the frictional pressure gradient.

So, this minus d p d z f 2 phase naturally, it is for the 2 phase flow situation how can we suppose we take the analogy from single phase flow equations. Then we know that this frictional pressure gradient can be expressed in terms of friction factor and other things. What was the basic definition I do not know whether you remember it or not this is frictional factor rho T P U T P square this equation definitely you should be remembering. Is not it? So, or in other words this can be written down as 2 D f T P G T P square by rho. Can we write it down in this particular term? The we are replacing this with G T P square by rho T P.

Now, as I had already mentioned previously this 1 by ρ_{TP} this can also be written down as 2 by $D f_{TP} G_{TP}^2$. And this particular term this can be defined in terms of quality. So, this can be expressed in this particular form and again x as I have already mentioned this is nothing but we $(())$ this is also a input parameter.

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$$\begin{aligned} \left(-\frac{dp}{dz}\right)_{f_{TP}} &= \frac{2}{D} f_{TP} G_{TP}^2 [U_1 + x U_2] \\ &= \frac{2}{D} f_{TP} \rho_{TP} U_{TP}^2 \\ &= \frac{2}{D} f_{TP} G_{TP}^2 \\ \left(-\frac{dp}{dz}\right)_g &= \rho_{TP} g \sin \theta z = [\beta \rho_L + (1-\beta) \rho_V] g \sin \theta z \end{aligned}$$

So therefore, if you substitute this then from there. What do we get? We get the frictional pressure gradient for the 2 phased flow situation, this is obtained as 2 by $D f_{TP} G_{TP}^2$ and this particular term.

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This particular term this can be written down as v_1 plus $x v_2$. Why I have written it in this particular form? Because if we have to find out the properties, using your mollier chart steam tables etcetera. Then from there it is very easy to get v_1 v_2 and so, on and so, forth. So therefore, this can be written down in this particular form, then if we take up the we find that here everything we know except this f_{TP} term this is the only unknown other than that all the terms we already know. Is not it? There is 1 other form by which also we can derive it that also I can do it. This is before I do this I had started it with sorry $\rho_{TP} U_{TP}^2$ the basic if you see from where I had started it. So, this then I had substituted it in terms of G_{TP}^2 by ρ_{TP} .

Instead of that we can also write this term as $\rho_{TP} \int U_{TP} dz = G_{TP}$ or G . So, we can write it down as $\frac{2}{D} \frac{f_{TP} G_{TP}^2}{\rho_{TP}^2} \int U_{TP} dz = J$. I have already said whatever way you wish you can write it down it is just depending on the availability of the data in which form you would like to express. So, we can either take up this particular form or we can take up this particular form whatever form you take you find all the terms you know except the equivalent friction factor fine. So, this was all I had to say about the frictional pressure gradient.

Let us take up the gravitational pressure gradient. How did we express the gravitational pressure gradient? It is simply the mixture density. So, this you can write it down as ρ_2 by ρ_1 or whatever form if you want you can write it down and you can express it. Even you can write it down as $\beta \rho_2 + (1-\beta) \rho_1$ whatever you wish this is something not very important. This can be done in a straightforward manner.

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Now, let us take up the acceleration pressure gradient. acceleration please remember these terms the final expressions which you have obtained for the frictional pressure gradient, the gravitational pressure gradient.

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$$\begin{aligned}
 \left(-\frac{dp}{dz}\right)_{acc} &= G_{TP} \frac{dU_{TP}}{dz} \\
 &= G_{TP} \frac{d}{dz} \frac{W_{TP}}{\rho_{TP} A} \\
 &= G_{TP} W_{TP} \frac{d}{dz} \left(\frac{1}{A \rho_{TP}}\right) \\
 &= G_{TP} \frac{W_{TP}}{\rho_{TP}} \left[\frac{d}{dz} \left(\frac{1}{A}\right)\right] + \frac{d}{dz} \frac{1}{\rho_{TP}} \\
 &\quad - \frac{1}{A^2} \frac{dA}{dz} \quad \quad \quad \frac{G_{TP} W_{TP}}{A} \frac{d}{dz} \left(\frac{1}{\rho_{TP}}\right)
 \end{aligned}$$

$x u_z + (1-x) u_1$

Suppose we take up the acceleration pressure gradient.(no audio from: 24:46 to 24:53) How did we define it? $G_{TP} \frac{dU_{TP}}{dz}$ now why does acceleration take place in this

particular case can you tell me? If there is no heat transfer nothing under adiabatic flow condition, can you tell me why this acceleration pressure gradient happens? This happens see a flow is occurring. Why is flow occurring? There must be a pressure gradient if the pressure gradient is under normal circumstances under normal circumstances we find that the pressure drop or the pressure gradient is not so, very drastic that it will affect the specific volumes of the 2 phases.

But if the pressure drop is very drastic then naturally what is going to happen your the density of the fluids are going to change particularly the compressible flow fluid. Is not it? May be it is a air water flow the air sorry water density will not change, but the air density is going to change. Is not it? So, remember one thing particularly if 1 of the phases is a gas phase, then we can have acceleration pressure gradient how important or how high it is going to be that will depend on the circumstance. Under normal circumstances may be the pressure drop is not so, very high that it is going to affect the density of the gas phase.

But there will be an acceleration pressure gradient, whenever there is a 2 phase flow particularly if 1 of the phases is a gaseous phase this particular thing we do not say for single phase liquid flow. So therefore, this particular the this acceleration pressure gradient this arises just because 1 of the phases is compressible or in other words considering compressibility of the phases. And there is one more thing if there is an area change. So therefore, this $U T P$ in order to evaluate $d U T P / d z$ what we have to do we have to express it in terms of those particular quantities which will be affected due to with axial distance or which will contribute to your acceleration pressure drop.

So, how can we express it? We can write it down as d/dz of again basic definition. Can we do this? This particular term this is constant the things which will be affected with distance just because pressure changes with distance at this term and this term. Is not it? So therefore, we can write this down as (no audio from:27:38 to 27:45) Can we do this? Now, you perform this particular integrations sorry this particular differentiations and what do you get you get $G T P W T P$ and if you perform this particular differentiations then in that case what you get is by $\rho T P d/dz$ of $1/A$ plus d/dz of $1/A$ sorry very sorry plus very sorry $G T P W T P$ by $A d/dz$ of $1/\rho T P$ we can write it in this particular form.

Now, if we do this from here what do we get $\frac{d}{dz}$ of $\frac{1}{A}$ is nothing, but $-\frac{1}{A^2} \frac{dA}{dz}$. So, this we can substitute in place of this and if we have to do this how can we write down $\frac{1}{\rho T P}$ is this is nothing, but equal to $\frac{v}{T P}$. So, this can be written down as $x v^2$ plus $1 - x v$ yes or no.

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The whiteboard shows the following derivation:

$$\left(-\frac{dp}{dz}\right)_a = -\frac{G^2_{TP}}{\rho_{TP} A} \frac{dA}{dz}$$

$$+ G^2_{TP} (u_2 - u_1) \frac{dx}{dz} + x \frac{du_2}{dz}$$

$$+ (1-x) \frac{du_1}{dz} \rightarrow u_1 = f(p)$$

$$\left(\frac{du_1}{dp}\right) \frac{dp}{dz}$$

$$\left(\frac{du_1}{dp}\right) \left(\frac{dp}{dz}\right)$$

There are also some scribbles and a small logo in the bottom left corner of the whiteboard.

So, we can substitute them and then we can get a actual expression of acceleration pressure gradient do it. And then let us see what you are going to get? So, from this particular situation the thing which we get is, it is specifically equal to if you do it and you find it out this is specifically equal to $-\frac{G^2_{TP}}{\rho_{TP} A} \frac{dA}{dz}$. Is not it? 1 of the A has been taken up here 1 of the A has been taken up with $\frac{W}{T P}$ to give you $\frac{A}{A^2}$ and 1 of the A are remaining. And the other term if you do it then the thing which you are going to get is $G^2_{TP} v^2$ minus $v^1 \frac{dx}{dz}$ plus $x \frac{dv^2}{dz}$ plus $1 - x \frac{dv^1}{dz}$.

Just see whether you are getting all these 4 terms or not 4 rather 5 terms because the thing is here if you write it in this particular form both x might vary as well as v^2 might vary x varies. Only when there is a heat transfer and v^2 varies when there is an abrupt or rather a very rapid increasing pressure. So, to keep matters normal we will be considering all possible things that can happen. So, therefore, from here if you differentiate we get $\frac{dx}{dz} v^2$ or $v^2 \frac{dx}{dz}$ plus v^1 sorry minus $v^1 \frac{dx}{dz}$. And then

we get $v_2 \frac{dx}{dz}$ sorry $x \frac{dv_2}{dz}$ and $2 \frac{v_1}{D} \frac{dz}{dz}$. Is not it? So, I have combined all of them and I get this is the total expression.

Now, in this expression you see. Why is this particular v_1 varying with z ? It is varying just because it will vary only if v_1 is a function of pressure. So therefore, if that is so, then this can be written as $\frac{dv_1}{dp} \frac{dp}{dz}$ yes or no, usually this term is negligible because v_1 is taken as the liquid phase. And similarly, this term it can be written down as we can write it down in this particular form. So therefore, you substitute in this particular total expression of $\frac{dp}{dz}$ which we had obtained in this particular total expression, you substitute the frictional pressure gradient from that expression which we had derived.

Frictional pressure gradient can be substituted from this particular expression then the acceleration pressure gradient it can be substituted from this particular expression. And the sorry the gravitational pressure gradient can be substituted and the acceleration pressure gradient that can be substituted from this particular expression. We find that we get the acceleration pressure gradient only when there is a rapid change in pressure due to with the specific volumes change or if we have a change in area.

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The image shows a handwritten derivation on a blue background. At the top right, there is a small box containing the text "© IIT KGP". The derivation starts with the equation:

$$-\frac{dp}{dz} = \frac{2}{D} f_{TP} (v_1 + x v_{12}) + f_{TP} g \sin \theta$$

Below this, there are two lines of terms that are subtracted from the right-hand side:

$$- \frac{G_{TP}^2}{f_{TP} A} \frac{dA}{dz} + G_{TP}^2 \left[v_{12} \frac{dx}{dz} + x \frac{dv_{12}}{dp} \frac{dp}{dz} + (1-x) \frac{dv_1}{dp} \frac{dp}{dz} \right]$$

The final equation is obtained by combining the terms:

$$-\frac{dp}{dz} = \frac{2}{D} f_{TP} (v_1 + x v_{12}) + f_{TP} g \sin \theta - \frac{G_{TP}^2}{f_{TP} A} \frac{dA}{dz} + G_{TP}^2 v_{12} \frac{dx}{dz} \frac{1}{1 + G_{TP}^2 \left[x \frac{dv_{12}}{dp} + (1-x) \frac{dv_1}{dp} \right]}$$

In the bottom left corner, there is a logo for "NIPIT IIT KGP".

So, if you substitute all of these in the total expression of minus $\frac{dp}{dz}$, just substitute them and then tell me what is the final expression that you are getting? Substitute them let us see what is the final expression that you will be getting final expression? That we will be getting is minus $\frac{dp}{dz}$ this will be equal to $\frac{2}{D} f_{TP} v_1$ plus $x v_{12}$ plus let

me keep it as rho T P. Because it is a input parameter [FL] sorry the z will not be there very sorry this I had written it earlier also because, this is the pressure gradient where none of you pointed out this mistake to me where did I make that mistake.

Initially also probably I have I cannot find it now, but anyhow z will not be there I did not mention it fine. So, this plus this is the other term due to area change then we have those particular terms due to quality change and due to (no audio from 34:29 to 34:40) So, this is the total expression for the pressure gradient.

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$$-\left(\frac{dp}{dz}\right)_g = g \cos \theta \frac{1}{(v_1 + xv_{12})}$$

$$\frac{dp}{dz} = \frac{\frac{2f_{TP} G^2 (v_1 + xv_{12}) + G^2 v_{12} \frac{dx}{dz} - G^2 (v_1 + xv_{12}) \frac{1}{A} \frac{dA}{dz} + \frac{g \cos \theta}{(v_1 + xv_{12})}}{1 + G^2_{TP} \left[x \frac{dv_2}{dp} + (1-x) \frac{dv_1}{dp} \right]}$$

For $x=x(h,p)$

$$\frac{dp}{dz} = \frac{\frac{2f_{TP} G^2 (v_1 + xv_{12}) + G^2 \frac{v_{12}}{h_2} \frac{dh}{dz} - G^2 (v_1 + xv_{12}) \frac{1}{A} \frac{dA}{dz} + \frac{g \cos \theta}{(v_1 + xv_{12})}}{1 + G^2_{TP} \left[x \frac{dv_2}{dp} + (1-x) \frac{dv_1}{dp} + v_{12} \left(\frac{\partial x}{\partial p} \right)_h \right]}$$

Now, let us take all the pressure gradient expressions to the left hand side. Under that circumstance what do we get? We get an expression as (no audio from: 35:05 to 35:12) this is the expression and this divided by because everything has been taken to the left hand side. So, this divided by G square two phase this is the final expression that we get for calculating the pressure gradient when two phase flow is occurring under homogeneous flow condition. The entire thing has been written down in this particular slide and this is the final expression which we have derived. Now, if you observe this particular expression. What do you get? You find out that more or less all the terms are input parameters except f T P.

So therefore, the next thing which we would like to do is to discuss methods by which we can express the 2 phase frictional pressure gradient in terms of certain input parameters or in terms of whatever we know in terms of or in for the case of single phase

flow. So, we cannot idealize 1 particular definite unique method of defining $f T P$ we will be discussing the different methods. And other than $f T P$ if you observe both the numerator and the denominator you find that other terms which we have the pipe diameter the mass flux even the quality change with length this can be obtained from enthalpy balance.

If at all there is a heat balance otherwise this term cancels off and the other thing the phase physical properties the taper if there is a taper then we have this term otherwise this term also cancels out. So therefore, more or less everything else is a you can obtain everything else from known input parameters. This we have done for the situation we have considered there is an area change. Any doubts, you want to ask? $\cos \theta$ is written there that probably I have taken it with respect to the vertical, but it should have been $\sin \theta$ because every time I consider from the horizontal very true that is very true. This is not very correct in my slide I have written it down as $\sin \theta$ there.

Now, remember one thing under what circumstance have I derived this I have derived that well there is a there is a taper due to which there is a d/dz . And I have assumed that there is a frictional pressure gradient, there is an gravitational pressure gradient and there is a quality change due to heat input or output. So therefore, please remember in this particular equation that I have shown your d/dz this x I have assumed this varies only with z and this variation occurs due to heat transfer.

Tell me one thing if I have significant flashing there is a heat input also and there is significant flashing with it. So therefore, then what happens? If there is significant flashing as well then in that case along with pressure your x should also change. Is not it? So therefore, under that circumstance for this particular expression, this particular expression what have I assumed

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$x = x(h)$
 For significant flashing
 $x = x(h, p)$
 $\frac{dx}{dz} = \left(\frac{\partial x}{\partial h}\right)_p \frac{dh}{dz} + \left(\frac{\partial x}{\partial p}\right)_h \frac{dp}{dz}$
 $\frac{\partial h}{\partial z} = \frac{h_{12}}{h_{12}}$
 $\frac{dx}{dz} = \frac{1}{h_{12}} \frac{dh}{dz} + \left(\frac{\partial x}{\partial p}\right)_h \frac{dp}{dz}$

I have assumed x is the function of enthalpy only. So, if I know the enthalpy balance I can find it down. For significant flashing if there is significant flashing, under that condition will x be a function of enthalpy only under that condition it will be a function of both enthalpy as well as pressure. So therefore, under that circumstance then in that case dx/dz that cannot be obtained from enthalpy balance alone. You agree with me? So, under that circumstance dx/dz can be expressed as dx it has to be expressed in terms of h as well as in terms of p .

So, it will be dx/dh at constant p dh/dz plus dx/dp at constant h dp/dz . You agree with me? So therefore, if that is there then instead of the dx/dz that I have mentioned here this has to be replaced by this whole term. Do you agree with me? And then what is this term can you tell me, what is this particular term, any idea? At constant pressure how enthalpy changes with quality latent heat of vaporization? So therefore, this is nothing, but h_{12} or in other words this is nothing, but equal to $1/h_{12}$. So, therefore, for that circumstance we can write down dx/dz it is nothing, but $1/h_{12} dh/dz$ plus dx/dp at constant h dp/dz can we do this yes or no.

So therefore, this is the acceleration pressure gradient in presence of flashing. So, for that what happens if there is a taper there is an area change, if there is rapid pressure changes the specific volume of either both or either of the fluids might change. And then if there

is significant flashing then enthalpy will also change with sorry not enthalpy your quality will also change with pressure.

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So, instead of the $d x d z$ which I had shown you instead of this particular $d x d z$ term we need to substitute it with this particular expression. So, if we do it then what do we get if we do it then you can do it for yourself you can please derive all the things that I am doing in your when you go home because it is very important that you know these derivations otherwise it is going to be very difficult. Because gradually you find that expressions are getting larger and larger. So, unless you derive them regularly it is going to be very difficult for you.

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So, we find that if we do it then finally, the expression of pressure gradient it has the frictional pressure gradient if you compare the 2 it has the acceleration pressure gradient due to area change. It has the gravitational component and along with this it also has the 1 particular term which shows how enthalpy changes with $d z$. And more importantly if you observe the denominator you will find instead of considering the specific volume changes with pressure. Apart from this we are also considering how quality changes with pressure? So therefore, we find that the expression in this particular case it becomes much more involved.

So, one thing which has to be discussed here the 2 aspects which I would like to discuss in the derivation which I have made firstly, in the derivation we find that other than $f T P$ we know everything. So, next we will be discussing the methods of finding out $f T P$ and the other thing is the significance of the denominated term which I have obtained. If you observe the denominator you find that I presume I had derived compressible flow situations for you. Is not it? So, after this class we will be going a little for compressible flows because I do not I am not very sure how much you know about it. So, for some of you it may be a repetition.

Small amount of compressible flows I will be deriving and once or rather I will be discussing once we discuss that then the denominator which I had obtained while deriving the pressure gradient for compressible flows. That will be clear to you and from

there by drawing analogy we can know how this particular denominated term. How it is important or what does it signify? So, that the next class I will be discussing some amount of compressible flows then we will be arrive at the significance of the denominated terms. Which we had derived under that circumstances and then automatically from analogy we can understand.

How this particular denominated term is important? But before that today I would like to start some discussion on the estimation of f_{TP} because that is the only bottleneck in this particular expression.

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Evaluation of f_{TP}

For low quality vapor-liquid mixture $f_{TP} = f_{l0}$

$$f_{l0} = fn(Re_{l0})$$

$$Re_{l0} = \frac{DG_1}{\mu_1}$$

$$-\left(\frac{dp}{dz}\right)_f = \frac{2}{D} f_{TP} G_{TP} J_{TP} = \frac{2}{D} f_{TP}^2 v_{TP} = \frac{2}{D} f_{TP} G_{TP}^2 [xv_2 + (1-x)v_{12}] = \frac{2}{D} f_{l0} G_{TP}^2 [xv_2 + (1-x)v_{12}]$$

$$\phi_{l0}^2 = \frac{-\left(\frac{dp}{dz}\right)_{TP}}{-\left(\frac{dp}{dz}\right)_{f0}} = 1 + x \frac{v_{12}}{v_1} = -\left(\frac{dp}{dz}\right)_{f0} \left[1 + x \frac{v_{12}}{v_1}\right]$$

For high quality vapor-liquid mixture $f_{TP} = f_{g0}$

Now, what a can be the different techniques for deriving f_{TP} now first thing is suppose it is low quality vapor liquid mixture that means, vapor is in a very small amount under that circumstances

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① $f_{TP} = f_{LO} = f_n(Re_{LO})$
 $Re_{LO} = \frac{DG_1}{\mu_1}$ For low quality v-l flow
 $= \frac{DG(1-x)}{\mu_1}$

② $f_{TP} = f_{G0}$ For high quality v-l flow


③ $f_{TP} = f_n(Re_{TP})$ $Re_{TP} = \frac{DG_1}{\mu_{TP}}$

I can write it down here f_{TP} it is nothing but equal to f_{LO} we write. What is this f_{LO} ? This means that there is a friction factor when liquid only flows through the pipe. Since the vapor quality is very less than in that case we assume that your frictional pressure gradient or the frictional pressure it arises primarily due to the flow of the liquid phase means of the 2 phase mixture. So, under that circumstance we can assume that f_{TP} is equal to f_{LO} . f_{LO} means when liquid of the mixture is flowing alone in the pipe. So, this f_{LO} . What it is? It is a function of Re_{LO} . What is this Re_{LO} , can you tell me?

This is nothing, but D the mass flux $U\rho$ is a mass flux, mass flux of the liquid phase only that means, G_1 agreed by μ_1 or in other words this is DG into $1 - x$ by μ_1 . So, if we take that then from this particular slide you see that the frictional pressure gradient instead of this particular f_{TP} we can express it in terms of f_{LO} . And therefore, under such circumstances your frictional pressure gradient can be predicted from this particular form itself. So, we can derive it from this particular form.

Now, 1 thing I would like this is 1 particular thing that we can do, if we are having a very high quality vapor liquid mixture for that situation we can write down f_{TP} equals to f_{G0} . So, for high quality and low quality this can be 2 circumstances for most of the cases it is neither a very high quality mixture nor a very low quality mixture both of them can be in comparable amounts.

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$$f_{TP} = f(\text{Re}_{TP}, \epsilon / D)$$
$$\text{Re}_{TP} = \frac{DG_{TP}}{\mu_{TP}}$$
$$\text{Re}_{TP} = \frac{DG_{TP}}{\mu_{TP}}$$

To find μ_{TP} for a suspension of fluid sphere at low concentration

$$\mu_{TP} = \mu_1 \left[1 + 2.5\alpha \frac{\mu_2 + \mu_1}{\mu_2 + 2.5\mu_1} \right]$$

If that circumstance happens then in that particular situation, usually what we do? We can write it as a function of 2 phase reynolds number. Where this 2 phase reynolds number can be expressed in terms of $D G_{TP}$ by μ_{TP} . This is a much more realistic expression as compare to these this particular expression and this particular expression. But the problems which arises with the expression or the problem which arises by expressing f_{TP} as a function of 2 phase reynolds number is moment we have a 2 phase reynolds number then a 2 phase viscosity has to be defined.

So, this 2 phase viscosity in it actually it has to be a function of the composition of the 2 phase mixture. Is not it? So, what sort of function will it be it will not be a very straight function because remember. How this viscosity term has emerged? It has emerged due to the shear. So, it cannot be a very straightforward function just like density.

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So therefore, the problem with the third definition there are 3 definitions, 1 is this the this is for low quality vapor liquid flows, then we can also assume this is equal to this for high quality vapor liquid flows. And the third thing is f_{TP} is a function of reynolds number, where reynolds number is $D G$ by μ_{TP} . these are the 3 ways by which we can define and we find that there are problems with this, there are problems with there for very idealize situation and here the problem is to define the μ_{TP} .

So, in the next class we will be discussing different ways of deriving μ_{TP} number 1, number 2 we will be finding out that for all the cases. That we have done we can express the 2 phase frictional pressure gradient under homogeneous flow condition in terms of the single phase pressure gradient. When either it flows as liquid only or it flows as gas only. This is a very conventional way of doing in 2 phase flows and this particular expressing 2 phase pressure gradient in terms of single phase pressure gradient. They are usually done in terms of 2 phase multipliers they are basically the correction terms which have to be incorporated 2 single phase pressure drop expressions in order to obtain 2 phase pressure gradient.

So, we will be discussing the 2 phase multipliers and we will be discussing different ways of defining μ_{TP} and so, on and so, forth. So, in the next class we will be continuing this particular discussion thank you very much.