Multiphase Flow
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Lecture No. # 07
Simple Analytical Methods

Well, very good morning to all of you. So, what we were doing till the last class was I was trying to define a set of nomenclatures to you or I was trying to unified all the nomenclatures that will be coming. So, first I had defined a large number of nomenclatures which we have also encountered in our study of single phase flows. Just the total numbers of parameters are more here and as usual I have defined the face properties by denoting them with subscripts of one, two for the total property at the mixture property it is t p and subscript t p. In several books you can find m as the subscript, but usually I prefer it as t p. And, when interfacial I is the subscript in that way we had defined.

After that we had also defined a few properties which do not know whether we have it here or not.

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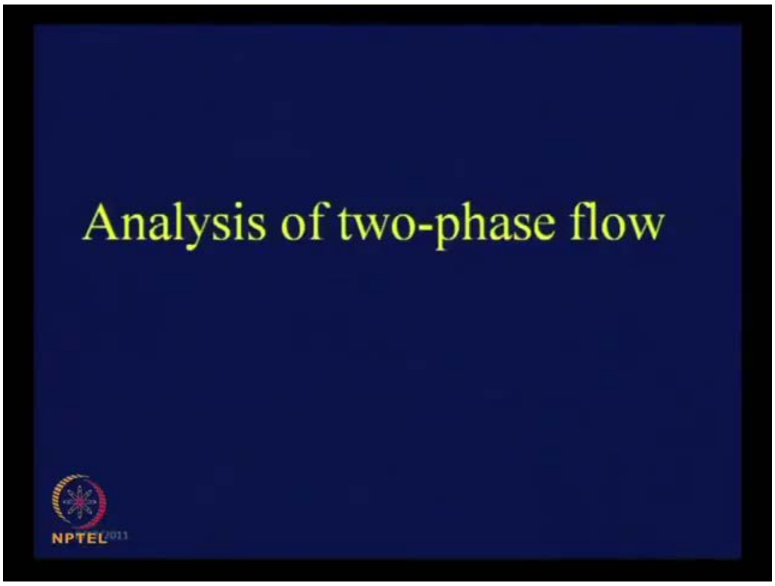

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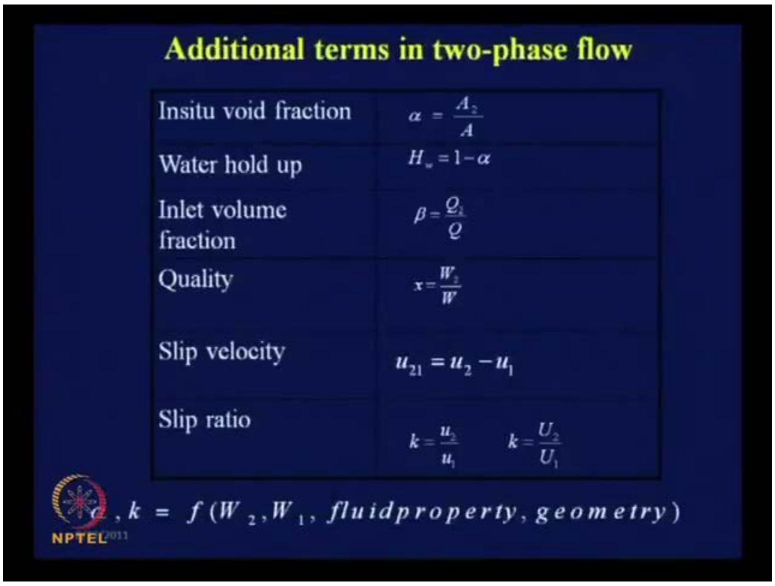

A few properties which were unique to two phase flow situations. I have almost completed this particular portion; the only thing which is left till now is three properties I would like to define volume flux of course, I have already defined. So, it is the volume flux, drift velocity and drift flux. Normally we will not be using these things, but for special models probably, we we will be using these particular situations.

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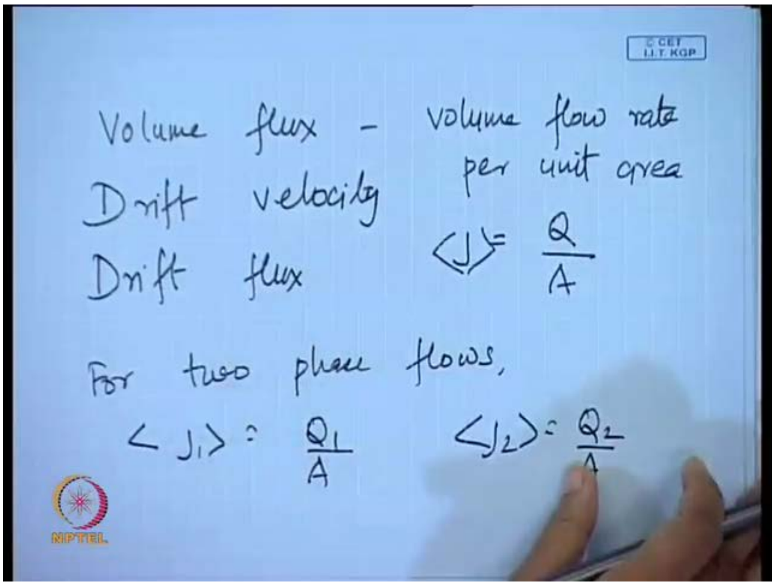

So, there are three properties which volume flux ofcourse, I had already defined then it is the drift velocity and it is the drift flux.

These three properties were the properties which we will be defining and then we are going to finish this nomenclature; we will be going for the analysis part. Now, as far as the volume flux is concerned; as we have already defined, it is simply the volume flow rate per unit area. This I have already told you and this is J equals to Q by A; usually it is the average J that we find. And for two face flow situations, naturally for everything we have to give certain subscripts. So, it is J 1 which is equal to Q 1 by A, J 2 equals to Q 2 by A. Now all these things they denote the average volumetric flux.

When there is no variation across the cross section, then at least the local flux and the average flux they are the same. So, therefore, under normal circumstances we are not going to put this brackets every time, it is implied that when we are speaking we are speaking about the average fluxes. And under normal circumstances, since we will be dealing mostly with one dimensional flow situations, our local fluxes and the average fluxes are going to be the same thing. So, they denote the basically whenever we put those brackets just like single phase flow situations they usually denote the cross sectional average values and usually we omit them. And there are certain other things

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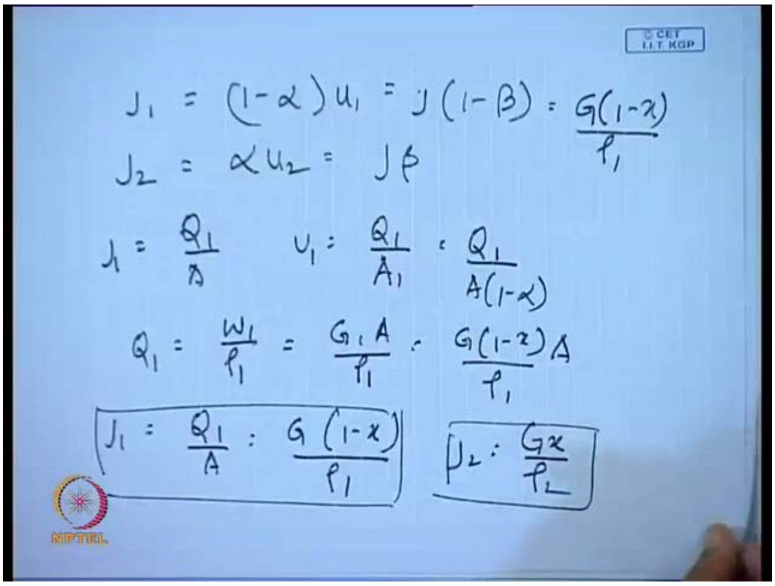

for example, how are these component fluxes related to your local component concentration as well as the local velocity.

That means local component concentration means alpha and local velocity U. How are J 1, J 2 related to the alpha and U 1, U 2? So, therefore, just from the definition you can know that since your J 1 is nothing but Q 1 by A and your U 1 is nothing but Q 1 by A 1 which is in other word Q 1 by A into one minus alpha. So, therefore, from here we know very well J 1 it is nothing but one minus alpha into U 1; J 2 is alpha into U 2. So, this is the relationship between the volumetric flux and the local component concentration, and the local component velocities. And, how are they related to your instead of alpha? How is it related to beta?

Any idea, how is it related to beta? J into one minus beta; please come prepared with the nomenclatures otherwise, it is going to be difficult and this is equal to J beta. It it is just the inlet flux or in other words suppose, I would like to relate the mass flux and the volume flux. So, how are these two related the mass flux and the volume flux we know this is nothing but equal to G into one minus x by row one.

Because we know Q 1 is nothing but W 1 by row one it is nothing but G 1 A by row one or in other words this is G into one minus x by row one and therefore, J 1 is nothing but Q 1 by A. So, this is going to be G into one minus x by row one; similarly, J 2 equals to G x by row two. So, therefore, this relates the overall mass flux and the or the component volumetric fluxes. So, by this relation we have related the volumetric flux where the void fraction volumetric flux where the inlet void fraction or the inlet volumetric composition and this is volumetric flux with the total mass flux.

Or in other words with the component fluxes if you see, how the volumetric fluxes and the mass fluxes of individual components are related?

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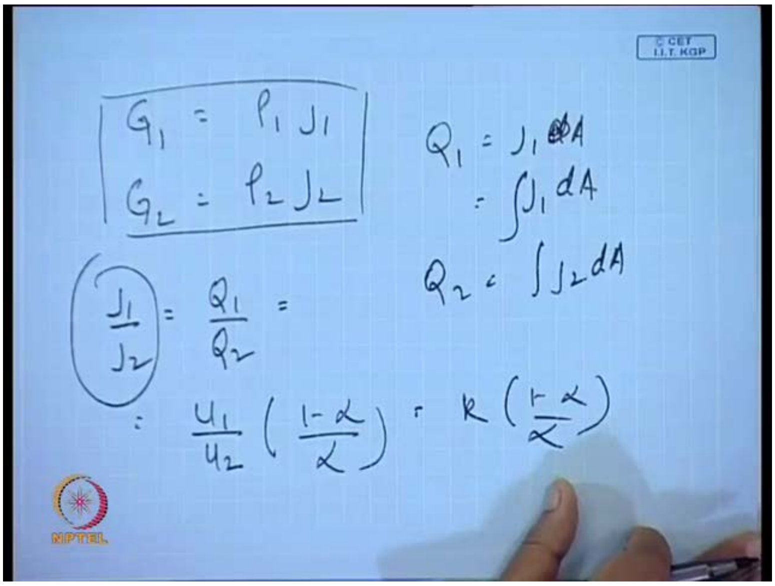

that is quiet natural it is nothing but G 1 equals to rho one. They are simply by substituting whatever we have learnt, but remember one thing usually we can write it down as Q 1 equals to J 1 into A, but the accurate expression is going to be something of this sort when there is a cross sectional variation of the void fraction; Q 2 naturally in the same way we can write it in this particular manner; when J 1 J 2 vary across the cross section we can write it in this particular way. So, these are the relationships which relate your component mass fluxes and the component volumetric fluxes.

Whenever you are given any particular derivations, please start from the basic definitions and then start deriving it is going to be easier for you. And as far as I am concerned, if you just start midway and then do something some marks will be deducted that you keep in mind well. So, therefore, in the similar way suppose we would like to relate say J 1 by J 2 with the slip ratio or the slip velocity; whatever it is. So, therefore, this is nothing but equal to Q 1 by Q 2 agreed and this is again nothing Q 1 as I have already written down here.

Here I have written down, Q 1 it can be written down as a into one minus alpha into U 1. So, if we write it down in this particular manner and we substitute it then we know that this is equal to U 1 by U 2 into 1 minus alpha by alpha. So, therefore, in this particular way or in other word this is simply K into one minus alpha by alpha where K is the slip ratio. Why we are trying to trying to connect all these things? Because this is a measureable parameter. So, therefore, if we can connect then if data are available on alpha we can find out K or the vice versa. Now this was about the volume flux.

Volume flux also probably, you have heard two other terms they come up just because of the different velocities the two phases have; now these two are the drift velocity and the drift flux

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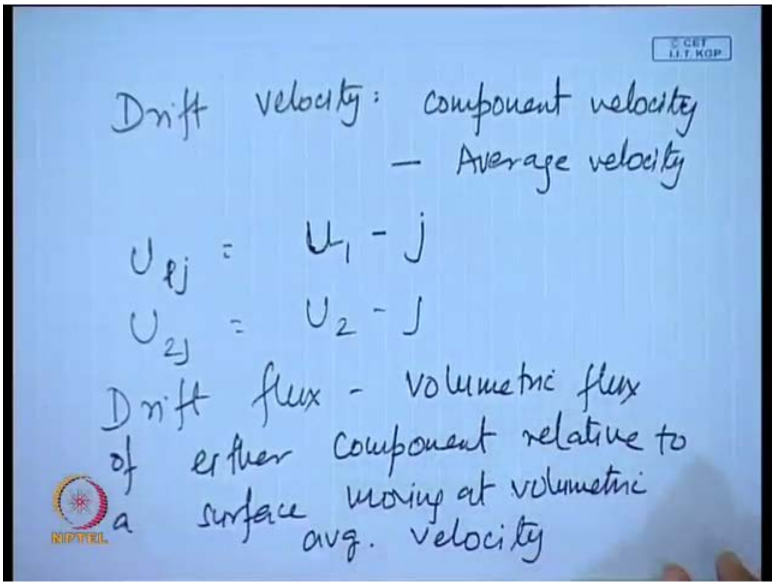

Now, the drift velocity if you take it up; drift velocity it is nothing but equal equal to [FL] this is nothing but equal to the component velocity minus the average velocity. So, what is the component velocity? How do we denote the component velocity? What is the nomenclature that I have used for the component velocity?

U one and the average is nothing but equal to j. So, the drift velocity it can be defined as U 1 J this is equal to U 1 minus J U 2 J this is equal to U 2 minus J. Now, next comes the drift flux; now the definition of drift flux this is the volumetric flux of either component these definitions you will get in wallace or cochlear or any particular book you are going to get these definitions. Volumetric flux of either component relative to a surface moving at volumetric average velocity; this is the actual definition. So, mathematically how we can define this?

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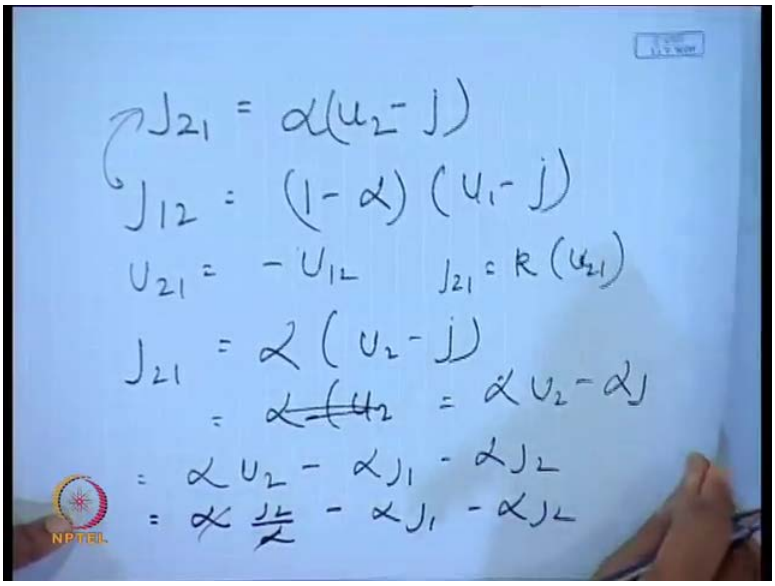

This can be defined is usually taken as J 2 1 and this is as I have told you volumetric flux of either of the component. What is the volumetric flux of either of the component? Say it is alpha into U 2 minus alpha into J. So, therefore, it is alpha into U 2 minus J; this is J 2 1 and J 1 2 equals to 1 minus alpha into U 1 minus J. So, these three are the definitions which I wanted to say volume flux, I have already told you and after that it is a drift velocity a drift velocity which is nothing but the component velocity minus the average velocity and then is the drift flux which we have defined it as the volumetric flux of either component related to a surface moving at the average velocity.

Now, remember one thing this drift flux is very important. In fact, we will be developing a drift flux model using this particular concept. So, therefore, it is quiet important. Now this drift flux it arises just because the two fluids are flowing at different velocities. All these relative things the slip the slip ratio this the relative velocity, the drift flux, the drift velocity everything arises just because the two fluids or the two phases are are at different velocities. So, therefore, we would like to derive or we would like to obtain expressions relating the drift flux and the relative velocity. And also we would like to see what is the relationship between J 2 1 and J 1 2? This is also another thing which we would like to see.

Why because just we know that for relative velocity what do we know U 2 1 equals to minus U 1 2. Similarly, what is the relationship between these two? And definitely, how is because J 2 1 arises as a result of say relative velocity. So, how is J 2 1 related to K 2 1? This is the thing which we would like to find out. Now, again let us start from the basic definition and let us see how we can do it.

J 2 1 as I have already defined, it is alpha into U 2 minus J. We can just write it down as alpha U 2 minus alpha J; it is this J is nothing but J 1 plus J 2. So, therefore, this can be written down as alpha J 1 minus alpha J 2; or in other words we can write it down as y plus this is minus alpha J. So, J is J 1 plus J 2 accordingly, I have written it down in this particular fashion. So, therefore, this can be written down as in that words with it is alpha U 2; this can be written down as alpha into J 2 by alpha minus alpha J 1 minus alpha J 2. So, therefore, we can write it down as,

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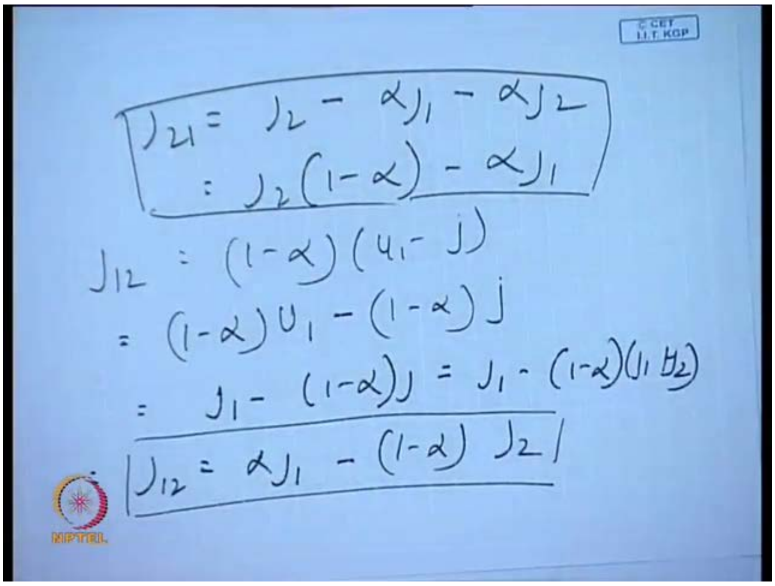

I will just repeat the expression once more here, J 2 1 this was equal to J 2 minus alpha J 1 minus alpha J 2 or in other words this is J 2 into 1 minus alpha minus alpha J 1. This is the relationship between the drift flux and the component volumetric fluxes. Now what about J 1 2 1 minus alpha into U 1 minus J? Again let us proceed in the similar fashion; let us break it down then we get it in this particular form and from here what we can do is we can just write it down as J 1 minus one minus alpha into J which is nothing but equal to J 1 minus one minus alpha into J 1 plus J 2.

Or in other words J 1 2 can be written down as alpha J 1 minus 1 minus alpha J 2. Just compare this particular expression with this particular expression and tell me what is the relationship between J 1 2 and J 2 1? J 1 2 equals to minus J 2 1.

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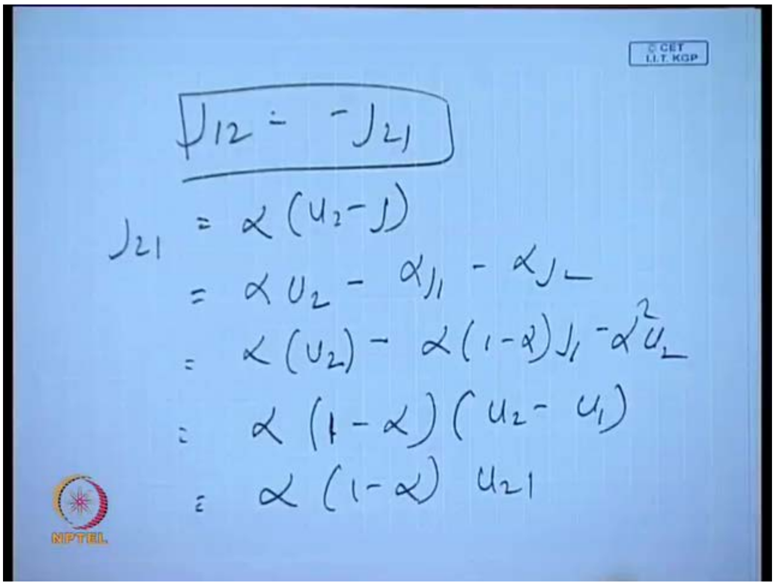

So, therefore, what do we what do we arrive from here; we find out J 1 2 equals to minus J 2 1. This symmetry is very very important; this should be kept in mind and this remember just like relative velocity there is such a type of symmetrical relationship. So, when J 2 1 increases, J 1 2 decreases and so on and so forth they are equally magnitude, but opposite in sign.

So, therefore, if J 1 2 acts in one particular direction; J 2 1 is going to act in the other direction; just in a opposite direction they are equally magnitude, but opposite in sign. Now, the relationship between the drift flux and the relative velocity. Again again let us start from the basics; the basic equation J 2 1 this is equal to again let us start from the basic relationship. In the previous case what we had done? We had substituted J as J 1 plus J 2; then we had substituted J 1 J 2 etcetera and we had tried to do this. In this particular case, since we want to find the relationship between J 2 1 and U 2 1. Naturally, this j has to be substituted in terms of J 1 J 2; J 1 J 2 have then to be expressed in terms of U 1 U 2; only after that we can get a relationship otherwise, it is not going to be possible.

So, for this particular case what we do? We can write it down just as alpha U 2 minus alpha J 1 same thing that we had done in the previously or in other word this can be written as alpha U 2 expressing J 1 in terms of alpha it is alpha into 1 minus alpha J 1 minus alpha square U 2; or in other words we can write it down as alpha into 1 minus alpha U 2 minus U 1; or in other words we can write it down as alpha into 1 minus alpha U 2 1. Where U 2 1 is nothing but the relative velocity of phase two with respect to phase one.

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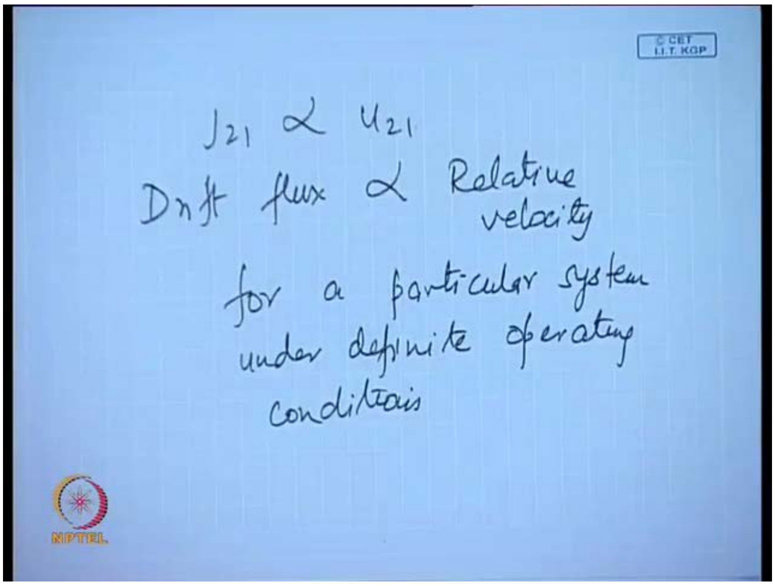

So, from this particular expression what do we get? We get J 2 1 is proportional to U 2 1 or in other words in words if you write it down, it is drift flux is proportional to relative velocity for a particular system under a particular condition; that means, we assume alpha to be constant. So, therefore, this is proportional to a relative velocity for a particular system under definite operating conditions. So, therefore, this thing has to be kept in mind; drift flux, drift velocity will be dealing in greater details; when we will be doing the drift flux model, but this was all and this completes the chapter of nomenclature. So, we had defined the set of nomenclatures that I will request you to revise the entire thing before we proceed further.

Now, initially what we did I tried to explain to you what is two phase flow? Why at all you have opted for this subject? And why at all you should study the subject? Next we had found out that the basic difference between single phase and two phase flow arises;

just because the two phases they have a wide variety of distributions. So, unless we understand the distributions, we cannot do much about it. So, we next took up a quiet detailed description of the different flow patterns which are encountered during vapour-liquid, gas-liquid, liquid-liquid, gas-solid. As well as three phase flows and also the changes which we encounter; when the two phase flow situation encounters any sort of a pipe fitting including bents, t-junctions, contraction, expansion orifice and so on and so forth.

Next, we started the analysis of two phase flows. Now, in order to analyze two phase flows what I felt was initially we would have to cover mostly the entire definitely I could not cover the entire range of nomenclatures; some things might come up in the process of discussing the analytical models which we will be discussing, but more or less the major set of nomenclatures which we are going to use. Now, after this comes the analysis of two phase flows.

How to analyze? It is nothing, it is just a fluid flow phenomena. So, therefore, it is going to obey the basic equations of two phase flow. So, we are going to do the same thing just like single phase flow; we will be writing the equation of continuity, equation of momentum, equation of energy; energy equation I had forgotten to derive the first day today after the just before the class ends I am going to derive it for you. So, we have to write those three sets of equations and then we have to apply suitable constitutive relationships the boundary conditions, the initial conditions and we have to solve them.

Whatever you have done so far, the same thing we are going to do. Just the number of equations are more; there we had one momentum equations, here if we have to consider the two phases separately there has to be two momentum equations. If we consider a mixture of two phases then definitely we will be having one momentum equation, but in that case that momentum equation will comprise of several mixture properties or two phase properties which are not easily measurable parameters.

So, therefore, we need additional constitutive relationships to determine this mixture properties. And in this way, the analysis of two phase flow will go on. Now, just like single phase flow we would first like to discuss what are the different methods of analyzing two phase flow? And then depending upon the time available we will be taking one model after the another.

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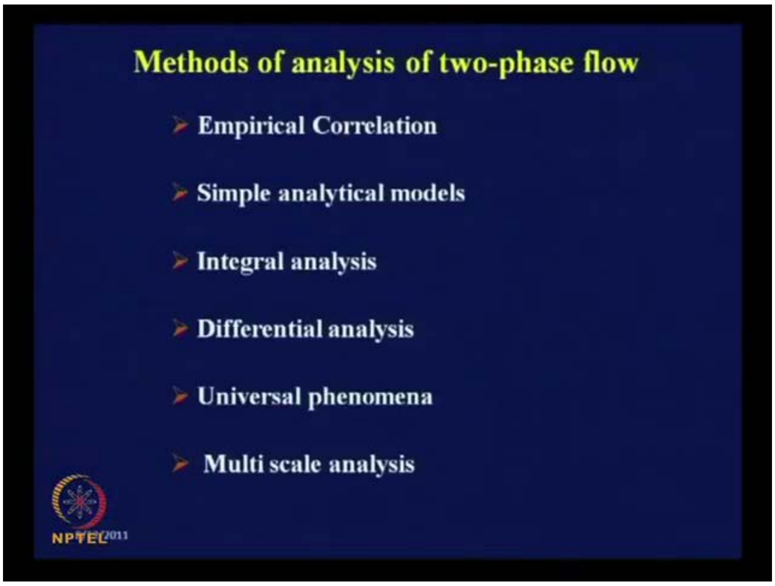

So, let us see the methods of analysis of two phase flow. So, just like single phase flow when we do not understand anything of physics; we do not know what to do; what we do? We go for empirical correlations. And you know that even in single phase flow, there are several occasions where we use this empirical correlations. In fact, for turbulence the number of empirical correlations are most blasius equation and then the other equations to find out f as a function of r e, many of them are empirical equation.

The very well known dittus boelter equation is an empirical equation. So, when we do not understand the physics when we do not know what to do about it, but we know that well this parameter is influenced by this this set of parameters then what we do we try to derive a relationship between the output and the input parameters either by some dimensional analysis or by grouping those parameters on the basis of some particular logic, and then the exact functional form between the input and the output is obtained from a large amount of experimental data; this is a very common approach.

So, naturally in two phase flow also the simplest thing which we can do is empirical correlations. And some empirical correlations are very widely used; we will just be touching on those empirical correlations because initially for everything there was of empiricism definitely I will not go for this.

Just like in heat transfer you have to study dittus boelter equation. So, in the same way, in this particular case also maybe the lockhart and martinelli correlation which is a very well known correlation maybe; like that one or two correlations we will be touching upon.

So, therefore, the basic thing which we usually do is that we from large from the experiments which are used for investigating the phenomena. A large amount of experimental data are collected and then based on these experimental data the empirical correlations they are derived either by dimensional analysis or by grouping of several variables on a logical basis. Now, what are the main advantages of empirical correlation? Firstly, the main advantage is it is very simple to use and the other thing is without understanding the physics more or less, we can use it within a particular range of application.

Whenever you use an empirical correlation, you will see there is a range of application which is given; quite natural beyond the range the physics changes. So, this same particular correlation cannot be used. So, therefore, the main advantages of using this are firstly, it is easy to use and secondly, this can be quiet accurate within the limits under which this has been developed. And what are the major disadvantages here? Since it is developed with very less inside into the physics of the flow, we do not have much idea how to improve the correlation.

So, therefore, this is one of the major problems in using this particular correlation and the second thing is, if we use it indiscriminately without using any logic; wherever we have a correlation; whenever you have to find out heat transfer coefficient be it forced convection, be it free convection use dittus boelter; will you get accurate result? You are not going to get it. So, therefore, if you use indiscriminately then it can lead to several erroneous results. These things you already know this is nothing special for two phase flow; these are the problems inherent in empirical correlations itself, but nevertheless when we cannot do anything we resort to this at least something is better than nothing.

Next what we can do next is the simple analytical models. This simple analytical models if you see these models they do not take into account the exact flow of distribution or they do not take into account the exact topology of the flow, but they can be very useful for predicting design parameters and with a minimum computational effort. Now what are the simple analytical models? We assume some sort of a distribution say for example, we assume that the two phases are intimately mixed with one another; if that is the case then the two phases do not manifest their presence separately while they are flowing.

The the entire thing manifests itself as a mixture with suitable average properties. So, we can treat the two phase mixture as a single pseudo fluid with suitable average properties and accordingly we can use simply single phase flow equations in order to predict the hydrodynamics of two phase flow under homogeneous flow conditions. So, under the simple analytical models the thing which we have is firstly, there are different distributions. We can assume that the two phases are intimately mixed; we can assume that they are totally separated and they interact just at the interface. These are the two extremes that we can assume.

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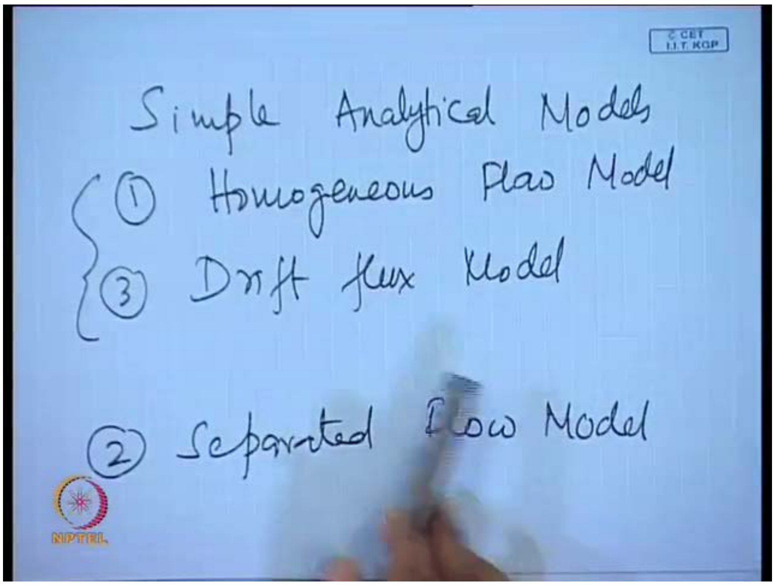

Accordingly, the simple analytical models they include one is the homogeneous flow model and the other extreme we have is the separated flow model. Now in homogenous flow model, what we have done? We assume that the components they are intimately mixed. So, that none none of them can manifest their properties separately and therefore, the entire hydrodynamics can be predicted by suitable average properties. For under certain circumstances, this can be very accurate.

We will be dealing with the simple analytical models in much greater details and you you can very well understand that when it is a completely dispersed flow. Under that circumstances, homogeneous flow model can give accurate results. But remember one thing unless we have very high phase velocities or at least one particular phase has a very high velocity, there will always be a slip which will be comparable to the mixture velocity.

So, therefore, assuming that the two phases see the first assumption when you make when you are telling that the two phases are flowing as a pseudo fluid is that the two phases are flowing at the same velocity; moment the velocity becomes different naturally they start manifesting their individual characteristics. So, therefore, the first thing is both of them are moving at the same velocity, but under normal circumstances under practical applications this happens under very few circumstances. For most of the cases there has to be a relative motion between the two phases. So, therefore, to improve the predictions of the homogeneous flow model what we have to do? We have to incorporate the effect of this particular relative velocity and modified expressions of the homogeneous flow model.

Now, for that what we can do? We incorporate the effect of relative motion by the concept of drift flux. And therefore, the improvement of the homogeneous flow model is the drift flux model which is another this drift flux model this is an improvement of the homogeneous flow model which incorporates the relative motion between the two phases and this is usually achieved by introducing the concept of drift flux which we have done in such greater details just shortly; and this will be discussed separately.

So, therefore, generally these two they are used for mixed flow and dispersed flow, if you remember the the classification of flow pattern which I had done during my flow pattern discussion. At one extreme there was dispersed flow the other extreme there was separated flow, and in the middle there was a range of mixed flow. So, for dispersed flow for very high phase velocities homogeneous flow model, but as the phase velocity start getting little less then we find the slip comes in and gradually even in dispersed flow the relative motion has to be included; then when we entered the mixed flow regimes at times it happens that if you incorporate the drift flux model we get a much better prediction as compare to the homogeneous flow model even if we have not considered the exact distribution; just by considering the drift flux model the predictions are improved drastically.

So, therefore, drift flux model is for greater applicability and for a large number of occasions this is a very very useful concept. And the other extreme what we have we have stratified flow; we have annular flow where the two phases they do not mix at all definitely, if you use say homogeneous flow model for it you are not going to get very good results that is quiet an expected situation. So, for this particular case we have the separated flow model. Here what we do? We write separate continuity, separate momentum, separate energy equations for the two phases and we take into account that the two phases they interact at the interface.

If we do not consider the interaction, then we are just considering simply single phase flow of either of the phases. Definitely, they will just be single phase flow equations; they will not be accurate. We do it under certain circumstances, but definitely the results are not very accurate because unless we consider the interactions it cannot be a two phase flow situation. So, therefore, for very accurate results or for reasonably accurate results we are suppose to write down the equations of continuity momentum energy for the two phases and we have to incorporate the inter phasic interactions between them by using suitable constitutive relationships.

So, these are the three simple analytical models we will be dealing with all three of them in great details and one by one we will be dealing with them. And then then you will be understanding more about how we have incorporated the two face flow characteristic into single phase flow equations and develop more or less accurate prediction results for predicting the hydrodynamics. Basically we will be doing the hydrodynamics and then if time permits we will go for the heat transfer characteristics as well.

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Now, after simple analytical models, the next thing which we have are integral analysis simply just like your single phase flow analysis.

In single phase flow, where do we use this integral analysis any idea? What do we do in this integral analysis? We assume some form of the of a distribution of concentration and some form of the distribution of your velocity profile etcetera, and then after that what we do?

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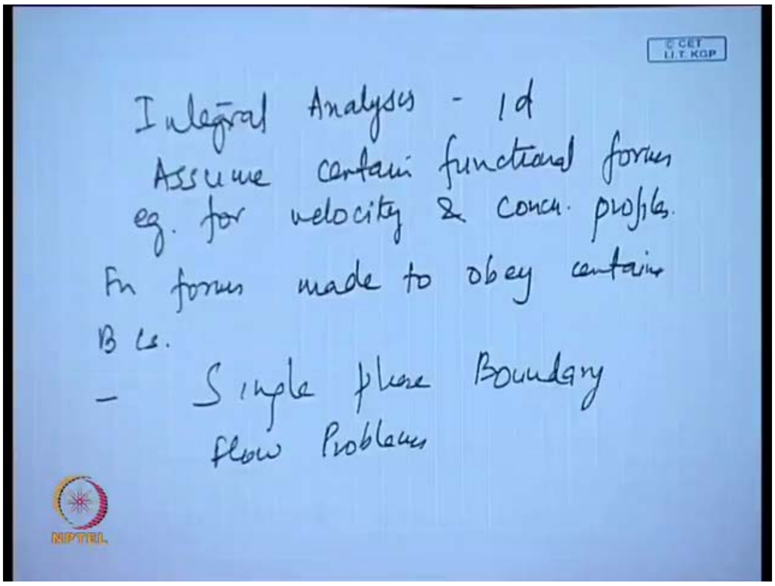

So, integral analysis usually it is a one dimensional flow analysis and what we do? We first assume certain functional forms certain functional forms for example, say velocity and concentration profiles. And then these particular functional forms, they are made to obey certain boundary conditions and accordingly they are integrated and we we get the total thing.

This particular thing where we we use it quiet frequently in single phase flow analysis, we use them for the single phase boundary flow problems. There we assume some sort of a parabolic profile or certain things for velocity and accordingly after that we try to assuming that r equals to 0; u equals to 0; r equals to capital r u equals to u max or something we integrate it and then we finally, get the velocity profile. Same thing it happens here also. For more accurate analysis what we do?

After integral analysis, we go for the differential analysis. Here, we have

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noted it down in the form of the increasing amount of complexities.

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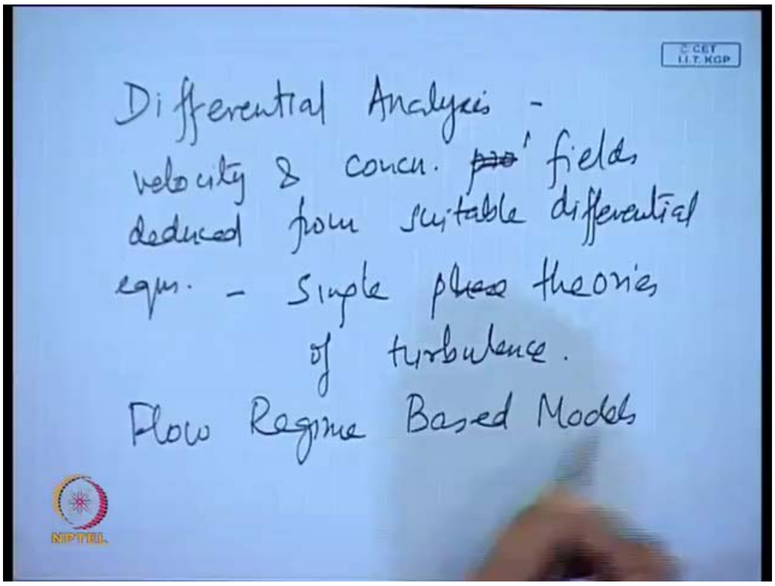

So, in differential analysis again the same thing we do; velocity and concentration fields velocity and concentration fields they are deduced from suitable differential equations. Then, these suitable differential equations they they we assume that they follow one dimensional flow idolization; they are written for time average quantities usually sometimes more sophisticated theories may even consider temporal variations.

This way encounter in single phase theories of turbulence is differential analysis . So, therefore, we find that this differential analysis, it is usually the one dimensional both integral and differential analysis we usually use it for one dimensional analysis. So, integral analysis it is performed for two phase flow situations just like it is performed for single phase flow situations by assuming certain functional forms which describe maybe the velocity or the concentration profiles these functions they are made to satisfy certain boundary conditions and then they are integrated, and then we get the entire profile.

See, just as it has been done for single phase boundary flow problems. Differential analysis slightly more more complex and more accurate; we assume the velocity and concentration fields which are deduced from suitable differential equations and then some usually way; we assume time average quantities, but temporal variations can also be considered for more sophisticated theories and just if it if you draw analogy we use this for single phase theories of turbulence. The next thing which we usually do which I forgot to mention here that is the flow regime based models.

See, So far whatever we were discussing, we were not considering the exact topology of distribution. Empirical correlation you were not considering anything. Homogeneous flow of theory we assumed that they were intimately mixed. Next here, integral analysis, differential analysis we were not exactly considering how the two phases are distributed.

So, therefore, the next approach which I forgot to mention in my slide it is the flow regime based models. Here what we do? We consider the exact flow distribution; how the two phases they are distributed? Accordingly seeing the flow distribution, we write the equations of the conservation equations equations of continuity momentum energy etcetera. Then we solve them depending upon the topology, we decide the boundary conditions we apply them and then we try to solve them.

So, this is a much more logical approach. Now, remember one thing, usually what is our approach? Just like single phase flow theory, what we would like to do? We would like to take up say more or less simplified thing maybe simple analytical models. There using more complex theories, we would like to introduce suitable correction factors or modifications to that particular theory. So, that the predictions are improved. Now, these correction factors; these particular modifications they can be obtained empirically as well if we really do not know the physics how to incorporate them. For example, in the churn flow situation, we hardly know anything about the physics, it is so random so chaotic.

There probably, if you have to maybe we have derived a model and and we find that the it is not very well predicting the experimental results, but if we increase the values by say a factor of k or something it is better. We do not have any clue of how to predict k. What we do? We take a large number of experimental data, we run them and now-a-days we have got many other good things; we have a n n genetic algorithm and those things. So, we can we can run them and we can find that well. With this particular range of data over this particular range, this value of K as a correction factor if it is included then the predictions are much improved.

So, those things either we can take them from from empiricism if nothing is available or we can use more sophisticated theories to to incorporate them. So, then the chances of improve prediction increases. So, usually it is sort of a pyramidal sort of a thing where the simplest thing is at the base and gradually we go to more and more sophisticated things, and usually the the more sophisticated models are usually they are not used as such; they are used just as a tool to improve the prediction of the simpler models. So, that is a much more logical approach, because if we use differential analysis or maybe more and more complex things maybe the computation etcetera they become so difficult, that it becomes unmanageable for us.

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So, usually we do not go for that the thing which I had written here is the multi scale analysis. This multi scale analysis is again you consider all the types of variations whatever variations are going in different different scales and to try to do it. So, usually our approach will be that we will try to keep the model as simple as possible and then we would try to introduce correction factors or modifications as the case maybe maybe for those particular things we will consider the physics and we will use the more complex models to improve the predictions of our simple models. And then I have mentioned one more thing which is the universal phenomena.

In the slide I have mentioned, it is a class of very powerful techniques which is based on universal phenomena that are independent of flow regime analytical model or the particular system also. Typical examples, maybe they various theories of say wave motion or the extimum techniques for obtaining the locus of limiting behavior of a system and so on and so forth. This we shall not be dealing with this much in our present course. So, well with this particular thing I am going to end the basic introduction to two phase flow. Whatever, I had as in introduction two phase flow that I am going to end and we are going to start simple analytical models we will be starting with the homogeneous flow theory.

And then gradually we will be going to drift flux model to the before that one thing I would like to do. I had told you that I had forgotten to mention or rather to derive the energy equation. I had derived the single phase equation of motion momentum equation. Energy equation is very simple. You tell me when a fluid is flowing through a pipe say water is flowing through the pipe, what are the different energy terms that we should be considering there?

Before I end the class let us start the homogeneous flow theory, I would just like one or two lines I will just derive it. It has to have otherwise it cannot move. Potential energy it must must have a position of its own; then since it is flowing pressure energy has to be there very correct; then internal energy has to be there and then if there is some amount of heat input usually work done is zero.

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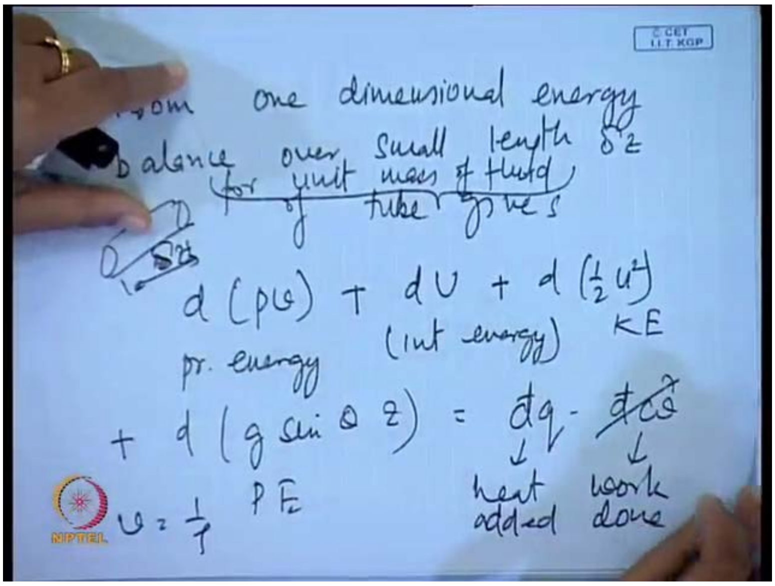

So, therefore, we can write down the energy equation in this particular way.

Say from one dimensional energy balance over a small length delta z; just like we had done for the same figure if you remember; the the same particular figure we are taken on delta z; the same same thing we are going to do; delta z of tube. So, what are the different types of energy we have? p v, this is the pressure energy plus internal energy plus this is for unit mass of the fluid that I am doing; for one dimensional energy balance was small length delta z for unit mass of fluid let me mention it of tube this should be here.

For unit mass or fluid this gives this thing internal energy then this is nothing but kinetic energy and we have the potential energy this is equal to dq minus; this is remember capital q is volume flow rate; this is small q. So, this is the heat added; this is the work done we never use it just denote it I have given a small w because capital w is mass flow rate, but this for most of the cases this is equal to 0.

Just note one thing, all these dI have written just d and these d are marked with dash. None of you have done my thermodynamics class then you would understood I am very very fussy about differentiating between inexact and exact differentials. So, therefore, just note this thing. So, where this v as you know this is nothing but equal to one by rho. So, therefore, this is the total energy balance equation which you can write down for a single phase flow situation

Now here we know what is this d u equal to the internal energy; we can do two things.

One is we can combine this two and we can write it down as d u plus d p v d h; this is one thing.

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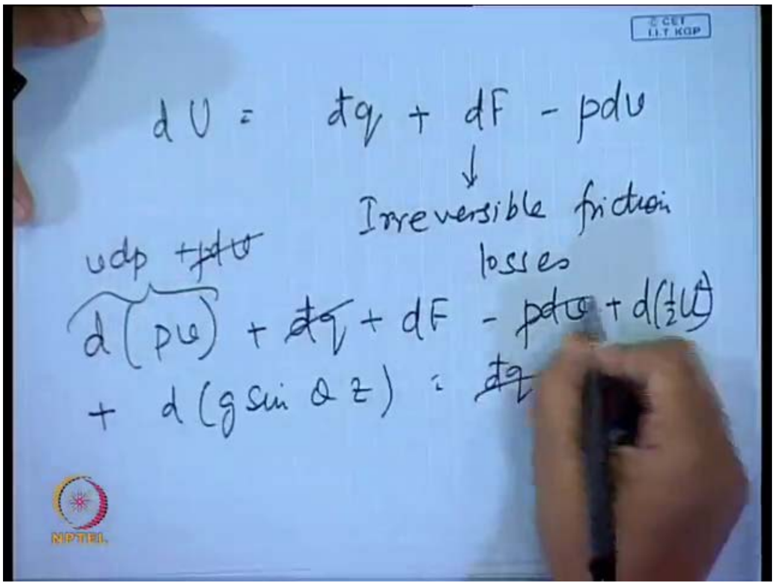

The other thing is we can also express dU as dq plus dF minus p dv, where this d f it arises from irreversible friction losses. So, therefore, these three are the components of du. Now, if we substitute this particular equation in the basic energy balance equation; then what do we get if in this particular equation instead of this du we write it down as say dq let me do it is it getting very messed up; leave it I will rewrite it once more .

So, what I had written it down, dp v plus instead of du I will be writing dq plus dF minus p dv plus d of half u square plus d g sin theta z equals to dq; this cancel out; this can be written down as v d p plus p dv. So, these also cancel out.

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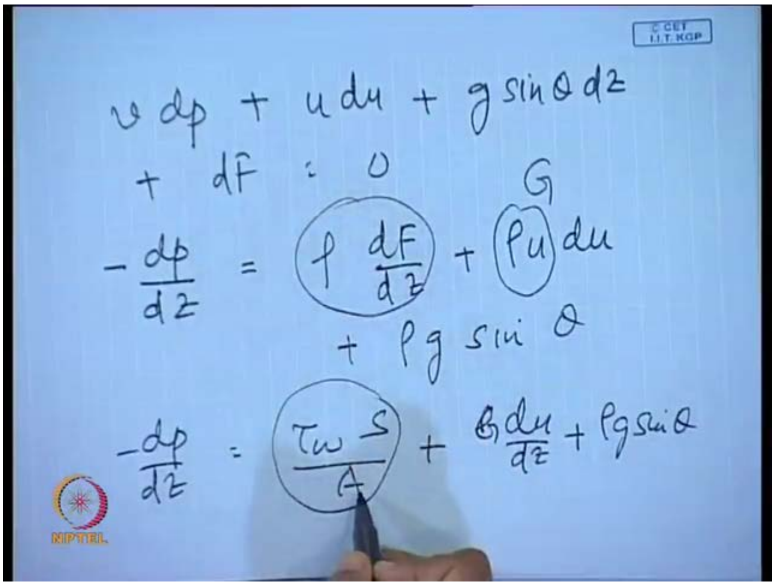

So, finally, what we get we get an expression as v dp plus u du plus g sin theta d z plus dF equal to zero. You also must have obtained the same particular derivation. Or in other words if we divide throughout by dz, we get something like see if you are getting this or not minus dp dz we take dp dz to this particular side then we get minus dp dz we can divide throughout by v or in other words v is nothing but one by rho. So, therefore, this becomes rho dF dz plus rho u d u plus rho g sin theta yes or no fine.

Now, if we compare this particular expression; what is this rho u equals to? It is nothing but G. Now, if you remember from momentum equation the final expression which we had obtained what was it it was minus dp dz equals to tau w S by A plus g du dz plus rho g sin theta. Remember, from the basic momentum equation we had derived this. So, therefore, if both of them express pressure gradient and pressure is the property of the system then therefore, each term must be corresponding to one another. So, therefore, we find that all the terms are corresponding except this term and this term. This gives you the wall friction and this as I have told you f is nothing but the irreversible frictional losses.

From where does the irreversible frictional losses come when only a single phase fluid is flowing through water flowing through a pipe naturally from the friction nothing else. So, therefore, it is quiet obvious that these irreversible pressure losses they arise to friction and therefore, rho dF dz is equal to tau w S by A. For single phase flow it is not a problem, but remember one thing when we go for two phase flow situation even under homogeneous flow condition irreversible frictional losses will arise for the interaction between the two phases.

So, therefore, this should comprise of not only the friction between the wall and either of the fluid, but also between the fluid and the fluid. So, therefore, these two need not necessary be equal for two phase flow situations, but they are definitely equal to single phase flow situations. So, therefore, this completes everything that I had to tell you about the introductory portion of multiphase flow. So, from the next class we are going to start the homogeneous flow model, it is the simplest flow model and you can very well expect what we are going to do there? Simply we are going to write the single phase flow equations maybe with average properties, but under that circumstance also the situation will not be as simple as the single phase flow situation even under such a simplified situation as well.

So, we will be seeing what are the additional things that we have to consider even for the simplest case of two phase flows and we will be proceeding accordingly.

So, thank you very much.