Multiphase Flow
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Lecture No. # 06
Definitions and Common Terminologies (Contd.)

Parameters which are very particularly we would like to relate alpha the void fraction or one minus alpha the in-situ liquid holdup with certain measurable parameters. So, that using those relationships we can find out the non measurable quantities or the in-situ quantities from the measurable quantities. Primarily, we would be interested in trying to express either the void fraction or the in-situ composition, or the in-situ velocities, or may be the slip ratio, or the relative velocity in terms of phase physical properties, conduit characteristics; may be inlet volumetric fraction, inlet mass fraction if there is any heat transfer and so on and so forth.

So, in the last class I believe that we had almost completed our listing the basic definitions.

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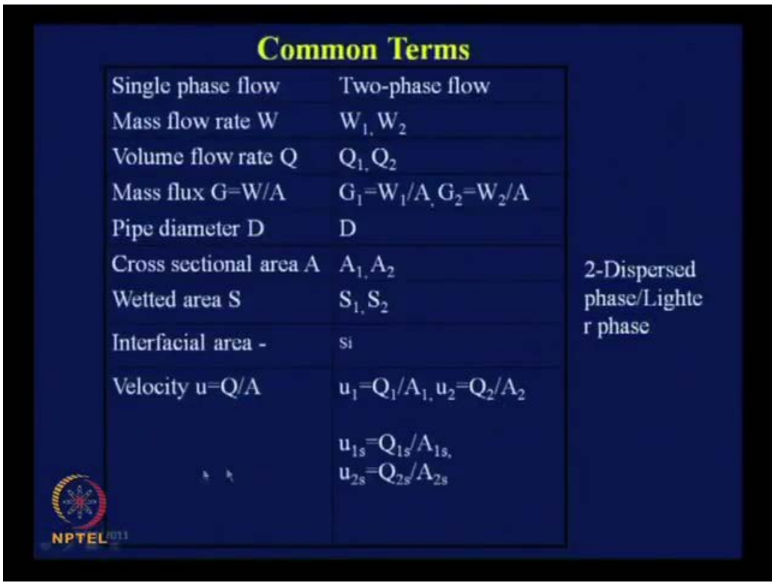

Common Terms

Single phase flow	Two-phase flow
Mass flow rate W	$W_{1,} W_2$
Volume flow rate Q	$Q_{1,} Q_2$
Mass flux G=W/A	$G_1=W_1/A_, G_2=W_2/A$
Pipe diameter D	D
Cross sectional area A	$A_{1,} A_2$
Wetted area S	$S_{1,} S_2$
Interfacial area -	si
Velocity u=Q/A	$u_1=Q_1/A_{1,} u_2=Q_2/A_2$ $u_{1s}=Q_{1s}/A_{1s,}$ $u_{2s}=Q_{2s}/A_{2s}$

2-Dispersed phase/Lighter phase

So, what we had found is that more or less, we had defined mass flow rate as w and I had already told you that instead of only w in single phase flow we come across W 1, W 2 and W t t p also. The total two phase mixture which has not been mentioned here, but I had told it to you in the last class; then volume flow rate Q 1, Q 2. Please remember both the mass and the volume flow rates they are extremely important whenever we try to characterize single phase flows. In this case also it is very important, but along with this the in-situ volume composition and the in-situ mass fraction they are also equally important.

So, therefore, first we are defining the mass flow rate; you are use to defining it as m dot, but in our case we are going to define it as w, volume flow rate capital Q, mass flux which you have not come across much till date, but we will be using it quite frequently particularly when the cross sectional area remains constant. Quite frequently, we will be using it, it is just the total mass flow rate of any particular phase divide by the total cross sectional area. So, please remember G is an input quantity or it is a measurable parameter, because both W 1 and W 2 are input and A is the cross sectional area.

Then of course, pipe diameter, it is fine cross sectional area here it is just a is sufficient for single phase flow, but in this particular case we come across A 1, A 2 then the wetted perimeter which I should say that is S 1, S 2 and s i the interfacial area which is an added parameter; which comes in this particular case. Interestingly, we found that the velocity was an input parameter there and we define velocity as the volume flow rate divided by the cross sectional area or the mass flow rate W by rho A you know all these things.

But in this particular case even for two phase flow situation; that means, when we are just introduced one additional phase in the flow passage; we are in a position to define four different velocities which I have already discussed in the last class. We found out that the common definition are the in-situ velocity which is Q 1 by A 1, Q 2 by A 2. Now, since A 1 and A 2 they are non measurable parameters; they depend upon the phase physical property, the interfacial distribution, the conduit characteristics and so on and so forth.

Therefore, these two parameters they cannot be controlled by the experimenter or the designer. Therefore, they can at most be related to the in-situ composition. So, therefore, just to define because we are very much use to mention or to characterize a fluid with its velocity. Although, we usually measure flow rates; we are very comfortable in mentioning the velocity of fluids one meter per second. Hardly, we say it is one liter per minute. So, therefore, just to propose two particular convenient definitions of velocity, the definitions which can be treated as input parameters; we have defined two other velocities which we say as the superficial velocity.

u 1 s and u 2 s that we have defined it is slightly wrong; this A 1 s and A 2 s are incorrectly written; they should be a only. These are not correctly written please note down the corrections. So, therefore, we have defined them as this entire thing it is not correct. Please correct it;

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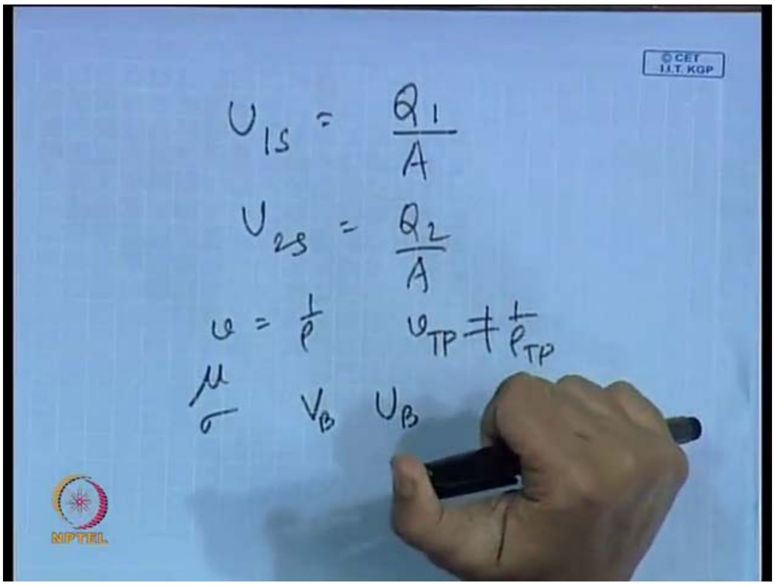

$$U_{1s} = \frac{Q_1}{A}$$

$$U_{2s} = \frac{Q_2}{A}$$

$$v = \frac{1}{\rho} \qquad v_{TP} \neq \frac{1}{\rho_{TP}}$$

$$\mu \qquad \sigma \qquad V_B \qquad U_B$$

this should be U 1 s equals to Q 1 by A, U 2 s equals to Q 2 by A; these are not correct. So, please keep them into mind.

So, these are the actual definitions. So, therefore, whenever we refer to a fluid in two phase flow; we will be finding in several journals in text books they mention the velocity of fluids; please remember the velocity they mention are the superficial velocities. We come across the superficial velocities, in-situ velocities; we find out only when there is a definite purpose for it, but for the purpose of characterizing of fluid we would like to do it or we do it conventionally by using the superficial velocity of the fluid. So, therefore, theses two extra definitions had come up. Apart from the definitions, which and other than this viscosity were is refer to as mu surface tension as sigma volume as v; when it is a specific volume it is small v which is nothing, but the one by rho.

So, in this particular case we have v T P which is definitely not equal to one by rho T P. So, in every class I mention just because it is very important. So, therefore, we have got a mu; we have got a sigma and usually when we refer to bubbles then probably the bubble volume it is refer to as V B, the bubble velocity is refer to as U B and things like that. So, apart from these particular definitions which you have already come across certain additional terms will be introduced just for defining the specific case when more than one fluid flows simultaneously through any particular conduit.

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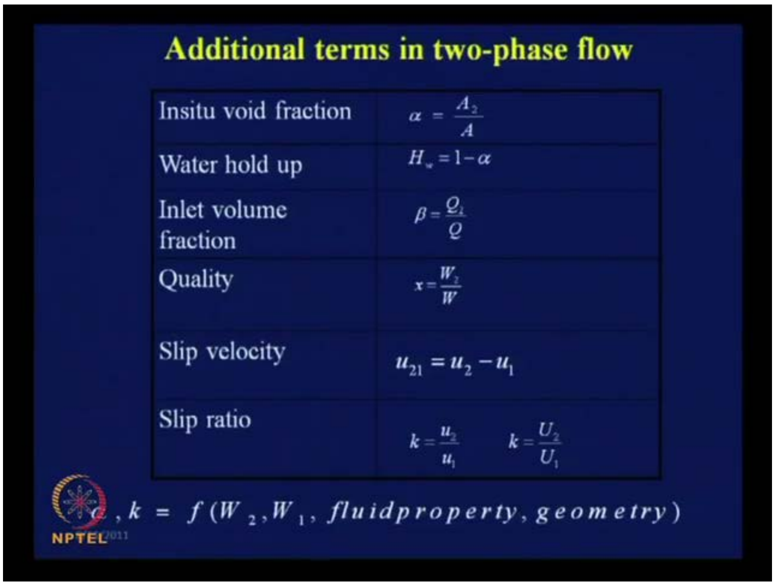

Additional terms in two-phase flow

Insitu void fraction	$\alpha = \frac{A_2}{A}$
Water hold up	$H_w = 1 - \alpha$
Inlet volume fraction	$\beta = \frac{Q_2}{Q}$
Quality	$x = \frac{W_2}{W}$
Slip velocity	$u_{21} = u_2 - u_1$
Slip ratio	$k = \frac{u_2}{u_1}$ $k = \frac{U_2}{U_1}$

$\alpha, k = f(W_2, W_1, fluidproperty, geometry)$

These additional definitions; firstly, it is the void fraction alpha, usually it is the volume fraction when the void fraction or the in-situ composition does not change with length under fully developed flow conditions; usually the area fraction and the volume fraction they are the same.

So, therefore, in any particular way we can define it; but the actual definition alpha is that volume occupied by the lighter phase or the volume fraction which is occupied by

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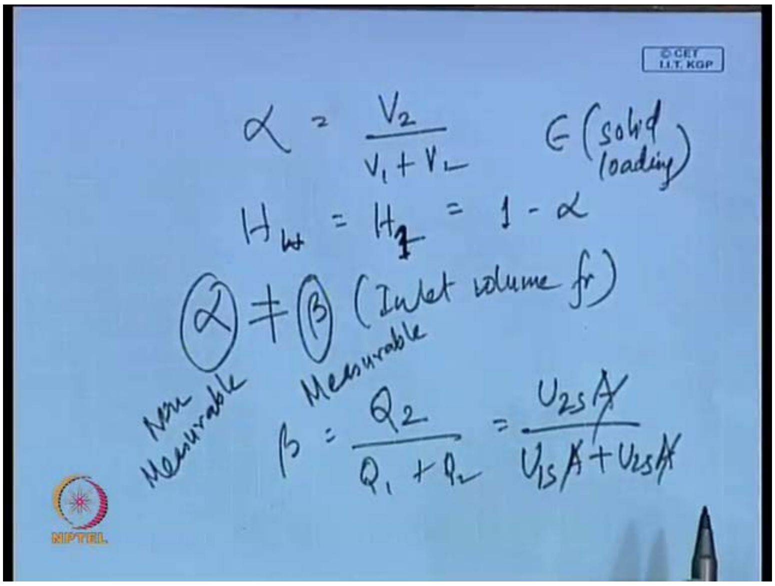

$\alpha = \frac{V_2}{V_1 + V_2}$ ϵ (solid loading)

$H_{ht} = H_1 = 1 - \alpha$

$\alpha \neq \beta$ (Inlet volume fr)

Non Measurable

Measurable

$\beta = \frac{Q_2}{Q_1 + Q_2} = \frac{U_{2s}\cancel{A}}{U_{1s}\cancel{A} + U_{2s}\cancel{A}}$

the lighter phase it can be given as V 2 by V 1 plus V 2. And, as I had mentioned in the last class it is H 1; it is the holdup of the heavier phase; it is usually equal to 1 minus alpha. And, I had already told you that alpha the in-situ fraction, it is not equal to the inlet volume fraction.

So, therefore, again this is a non measurable parameter not under the control of the experimenter or the designer, but this is very much of a measurable parameter and this beta it is equal to the inlet volume fraction which is naturally related to the volumetric flow rate; unlike alpha which is related to the in-situ volume occupied by the two phases. So, beta in terms of volume metric flow rate it will be Q 2 by Q 1 plus Q 2.

If you would like to refer beta in terms of velocity, then how should we define beta in terms of velocity? V 1 or U 1?

Alpha equals to V 2 by V 1 plus V 2, it is the void fraction or in the other words it is the in-situ gas fraction and therefore, usually it refers to the lighter phase. Of course, when it is solid liquid then under sub circumstances usually we refer to solid loading as epsilon unlike alpha. So, whenever we have solid loading we refer to as epsilon. But tell me if I would like to express beta in terms of velocity how should I do it? Is it U 2 or should it be U 2 s? The superficial velocity because Q 2 is nothing, but U 2 s A by U 1 s A plus U 2 s A and A is cancel out. So, therefore, beta can be related with the superficial velocities as well; definitely it cannot be related to the in-situ velocities.

So, I had defined beta and then I said particularly for condensation and boiling situations the quality of flow also changes; that means, when the flow is flowing up the total mass flow rate which

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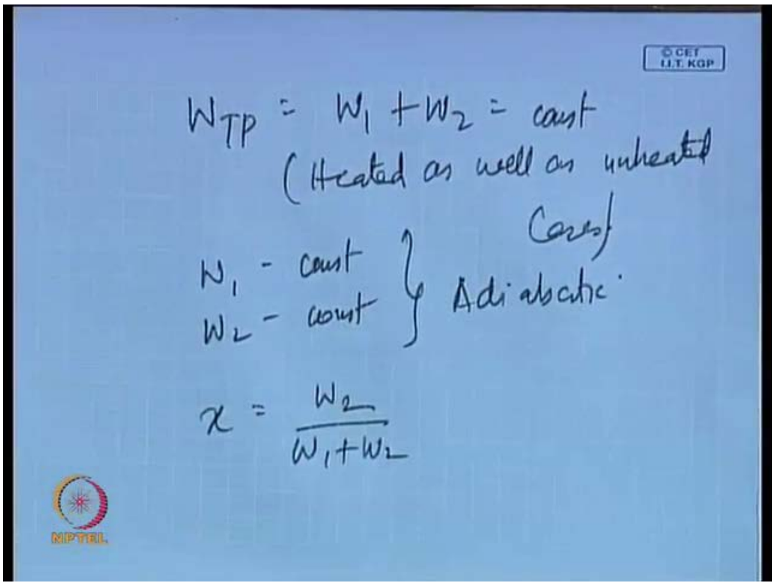

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$$W_{TP} = W_1 + W_2 = \text{const}$$

(Heated as well as unheated cases)

$$\left.\begin{array}{l} W_1 - \text{const} \\ W_2 - \text{const} \end{array}\right\} \text{Adiabatic}$$

$$x = \frac{W_2}{W_1 + W_2}$$

we can write it down as W T P equals to W 1 plus W 2; this is definitely constant; this is true both for heated as well as unheated cases.

But in unheated cases, what happens? In unheated cases, W 1 is also constant; W 2 is also constant under adiabatic conditions, but when we are considering heated condition then in that case W TP is constant, but W 1 and W 2 they change. And therefore, we find that throughout the channel, we find that the quality or the mass fraction of the two phases they keep on changing and from thermodynamics we would like to refer to this mass fraction as quality of flow.

So, therefore, we define x in the last class this was nothing, but equal to W 2 by W 1 plus W 2. But remember one thing alpha is not a measurable quantity, but x is definitely a measurable quantity; it can be measured from heat transfer or heat balance equations or enthalpy balance equations. In the last class we had already mentioned that if we know the heat flax if we know the amount of sub cooling of the inlet liquid, we can find out x; and in other words if both the liquid and the vapour are in equilibrium under that circumstances also we can find out x.

So, x is definitely not a non measurable parameter, it is a measurable quantity or it can be derived from measurable quantities unlike alpha.

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So, therefore, after the other things which we had mentioned, the two other things which arise due to the basic characteristics of two phase flow which distinguishes it from single phase flows.

What distinguishes it from single phase flow? As I have already told you, it is just not the presence of two phases which makes a flow situation two phase. It is the slip between the two phases or in other words it arises because the interface is influenced by the relative motion of the two phases. That is why, this distinguishes two phase flow as compare to single phase flow situations.

So, therefore, if there has to be a slip between the phases, then definitely we need something to quantify the slip. Usually, as I have mentioned yesterday we have quantified slip by two quantities; one is the slip velocity or the relative velocity and the slip ratio. Relative velocity,

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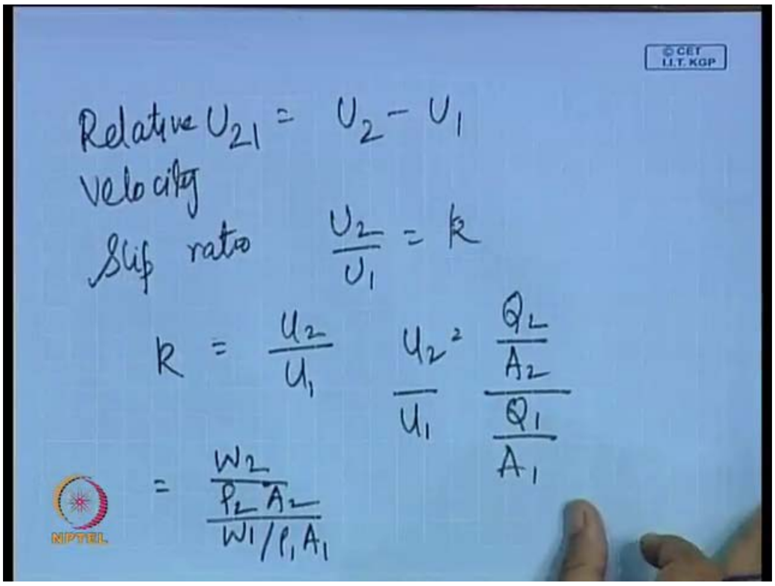

Relative velocity $U_{21} = U_2 - U_1$

Slip ratio $\frac{U_2}{U_1} = k$

$$k = \frac{U_2}{U_1} \qquad \frac{U_2}{U_1} = \frac{\frac{Q_2}{A_2}}{\frac{Q_1}{A_1}}$$

$$= \frac{\frac{W_2}{\rho_2 A_2}}{W_1/\rho_1 A_1}$$

we have defined it as U 2 one which is nothing, but equal to U 2 minus U 1 and this is the relative velocity, and the slip ratio which is nothing, but U 2 by U 1 which is known as k.

So, these two were the important parameters and these are interrelated U 2 U 1; this difference it arise due to the difference between alpha and beta. So, therefore, if we know k we can find out alpha; if we know alpha we can find out k. So, therefore, both of these parameters they are dependent upon the mass flow rates the properties of the two fluids the geometry and so on and so forth.

It is just to show that none of them are measureable; none of them can be controlled; we can just estimate them; we can just evaluate them, but we have to include them in any sort of analyses we would like to do for two phase flow situations. Now, in the last class I think I had discussed till this portion and then I had told you that suppose see if this is the situation, then you would definitely like to express either k or alpha; we would like to have a relationship between them. So, that if we know one, we can find out the other. Particularly, if there is condensation and evaporation then in that case we would like to have a relationship between alpha, x, k and so on and so forth.

So, whatever relationships we have derived from now, please make sure that you are going to go through all those derivations; you are deriving them in your hostiles today; you have to be very thorough with them before you proceed further. And remember one thing whenever you are given to derive any particular quantity, to derive any particular relationship please start from the basic definitions. Do not just assume anything, because the basic definitions are very important; very frequently I find that students they get tend to get confused whether G 1 was W 1 by A 1 or W 1 by A. So, these things are the basic.

So, remember the basic definitions whenever you are asked to derive any quantity start from the basic definition and then you proceed further for the derivation. For example, suppose I would like to derive say k in terms of quality and such other things. What is k equals to? It is nothing, but equal to u 2 by u 1. What is u 2? Can you tell me? From the basic definitions Q 2 by A 2; what is u 1? Q 1 by A 1. Now, A 1 A 2 they are definitely not measurable; if there is condensation boiling then in that case Q 2 Q 1 has does not have much meaning under that circumstances W 1 W 2 are better. Because at least if we know the quality, we can find out W 1 W 2. So, therefore, what we would like to do?

We would like to express Q 2 as W 2 by rho 2 A 2 and Q 1 as W 1 by rho 1 A1; or in

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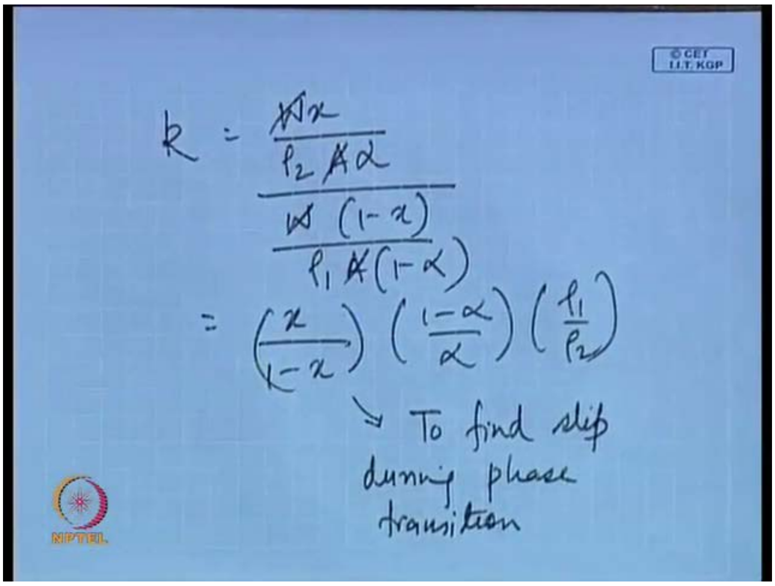

$$k = \frac{\dfrac{\cancel{W} x}{\rho_2 \cancel{A} \alpha}}{\dfrac{\cancel{W}(1-x)}{\rho_1 \cancel{A}(1-\alpha)}}$$

$$= \left(\frac{x}{1-x}\right)\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\rho_1}{\rho_2}\right)$$

$\searrow$ To find slip during phase transition

other words W 2 again we do not know, because W 2 will be changing along the length what we can find out from heat balance is the quality. So, therefore, what we can do? Instead of W 2, we can write it down as W x by rho 2; A 2 also we do not know; we can write it down in terms of A alpha.

Similarly, this can be written down in this particular way. And therefore, we find what is k equal to this these things cancel out. So, k we find it is nothing, but equal to x by one minus x one minus alpha by alpha rho 1 by rho 2; rho 1 rho 2 are known, x is known. So, therefore, if alpha can be found out, k can be found out or vice versa. This relationship is particularly very important to find slip during phase transition; this is a very important relationship.

So, in this particular way I will be doing some further problems; you will be getting some assignments in this particular way. We can derive certain non measurable quantities in terms of measurable quantities. And, but please remember whenever you are doing, you are suppose to start from

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the basic definition and then you are suppose to proceed in the way that I have done it; this is the way that you are suppose to do.

Now, before I go to derive or rather define certain other properties which are very important, I would like to spend some time on the estimation or the definitions of void fraction and liquid holdup. Why? Because it is the basic quantity of two phase flow just like mass flow rate, volume flow rate, volume fraction is also very important. It is not necessary that under all operating conditions you will be in a position to measure the volume fraction. See in the last class I had already explained you have to measure the total volume fraction.

What you have to do? Say two phase mixture is flowing through this particular pipe, you have to suddenly stop the flow here. Let the volumes separate out and then you have to measure. It is very evident that under practical situations in a huge plant you cannot afford doing this. So, very frequently it will not be possible for us to measure or else you can use some sort of a censor or something which will cover the entire cross section and it will measure the volume across this section, but again this becomes intrusive; this will disturb the flow. So, very frequently we find circumstances demand that we are not in a position to measure the volume fraction.

Maybe we are in a position to measure the area fraction; that means, the concentration across the area or the composition across any particular cross sectional area. It can happen that even measuring the area average void fraction is not possible. Under that condition, maybe we can measure it across the particular cord.

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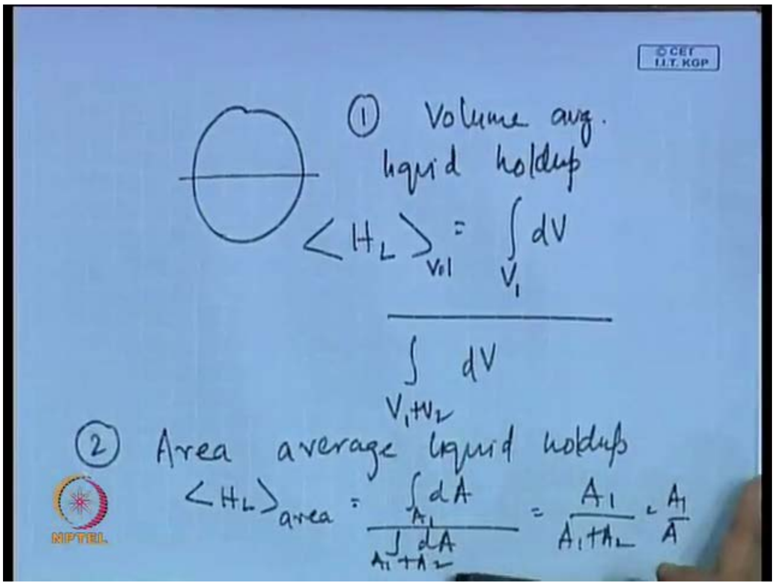

① Volume avg. liquid holdup

$$\langle H_L \rangle_{Vol} = \frac{\int_{V_1} dV}{\int_{V_1+V_2} dV}$$

② Area average liquid holdup

$$\langle H_L \rangle_{area} = \frac{\int_{A_1} dA}{\int_{A_1+A_2} dA} = \frac{A_1}{A_1+A_2} = \frac{A_1}{A}$$

Suppose, this is the pipe cross section either we can measure it across the entire cross section or maybe we can measure it across any particular cord. So, that we get the cordial average void fraction.

So, before I proceed, I would like to spend some time on the different definitions of holdup their mathematical expressions and just how to convert them into volume average void fraction, because that finally gives us the in-situ composition. So, as I have already told you the basic definition, it is definitely the volume average liquid holdup.

Now, how do we define this volume average liquid holdup? Usually the way we define it, this is given as mathematically we can express it in this particular term; it is integral V 1 d V by integral V 1 plus V 2 d V. This is the actual definition and this is the most useful definition of holdup in industrial designs; and this gives us the overall composing of the flowing mixture.

Now, when it is not possible to measure the volume average void fraction, the next which we try is the area average liquid holdup. When the flow does not change with length or in other words this is the volume average value for the infinitesimal length of the test section. When we consider infinitesimal length of the test section, then area average gives you the volume average. Or in another words, when the holdup does not vary with length of the conduit; that means, under fully developed flow conditions the area average gives you the volume average. So, we can measure either of them and we can use it for estimating or for expressing the in-situ volume fraction.

So, this can be written down as volume and this can be written down as area, and the mathematical expression it is equal to d A A 1 by integral d A A 1 plus A 2; or in another words this is nothing, but A 1 by A 1 plus A 2 where this is simply the cross sectional area.

So, therefore, we would first try to measure either the volume average or the area average. Now, for measuring area average the techniques are much more commonly available as compare to the volume average. What we can do? Across this particular cross section, we can measure the effective conductivity of the flowing mixture. Suppose, it is a air water mixture; we know the conductivity of pure water; we know the effective conductivity of air it is almost negligible.

Now, if you can measure the effective conductivity of the two phase mixture inside this, there we might get one estimate of the area average void fraction. We can also use optical techniques for this particular purpose. But let me tell you one thing, it is not as easily done as said; and at the end of this particular course, we will be spending some time on measurement techniques then we will be going into the details of the measurement techniques; how we can use the conductivity technique for measuring these? How we can use the optical techniques? What are the problems? And so on and so forth.

Because it is not as straight forward as I say that you just measure the effective conductivity; you know the conductivity of water; you know the conductivity of air. So, that you can find it out. It is not so, because the relationship is not linear, it depends upon the distribution. Definitely, the relationship between effective conductivity and void fraction that you would expect in bubbly flow cannot be same as the relationship between the two in annular flow. So, definitely the relationship between effective conductivity and void fraction will depend upon the flow distribution. Then finding out the effective conductivity is also not very straight forward.

Therefore, area average also it is not very easily done. And as a result, apart from volume average and area average,

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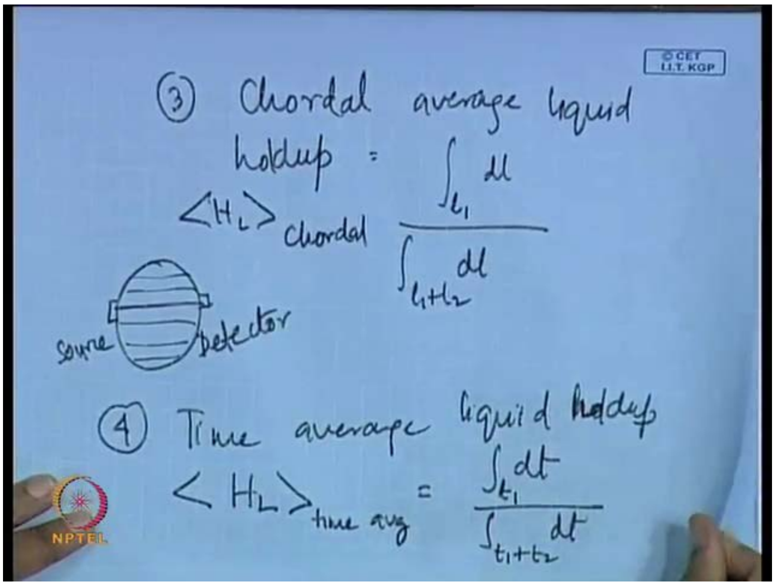

③ Chordal average liquid holdup =

$$\langle H_L \rangle_{chordal} = \frac{\int_{l_1} dl}{\int_{l_1+l_2} dl}$$

④ Time average liquid holdup

$$\langle H_L \rangle_{time\ avg} = \frac{\int_{t_1} dt}{\int_{t_1+t_2} dt}$$

the next definition which I would like to define is the chordal average void fraction or the chordal average liquid holdup. How do we define this? This is across a chord as I mentioned. So, therefore, this is integral l 1d l by integral l 1 plus l 2 d l. And this is defined as H L chordal. Now, how do we measure this? Suppose we have say any particular radiation maybe have gamma ray source or maybe any particular beta ray source, or maybe a optical source, or infrared source whatever it is.

So, therefore, if it sense are very narrow beam of light; say maybe we have the source here and we have the detector just on the diametrically opposite portion. And, if more or less along this particular chord we send the optical beam or whatever it is any particular very narrow beam across this. So, the amount which is atenuated or the amount which is absorbed; that we can find out; we know the initial strength; we know the amount which is transmitted both of them we can detect.

So, from these two we can find out what is the effective composition. We know what is the amount which is transmitted when the channel is full of water; we know the amount which is transmitted when the channel is empty or in other words it is full of air. So, therefore, and we know the amount which is transmitted under the present circumstances.

So, from there again with certain mathematical techniques etcetera we can find out the amount of the light ray which has been transmitted and it can be related to the void fraction. So, therefore, when the volume average and the area average are the not possible then we go for the chordal average void fraction. And it is usually converted to the area average values by two techniques. One is we can do extensive mathematical manipulations. We can convert this chordal average to the area average by extensive mathematics; this is one.

The other thing which we can do is we can keep on measuring the void fraction across different chords and then we can integrate the whole thing, and we can find out the chordal average void fraction. So, we can use either by the use of multiple beams or by mathematical manipulation we can do it. Here l means the length of the chord which is occupied by the gas phase and the length of the chord which is occupied by the liquid phase. Because along the entire chord, the entire thing cannot be filled of liquid or the entire length cannot be filled with the gas when the two phases are flowing.

Some portion will be filled with the gas phase; some portion will be filed with the liquid phase. Now, when it is completely filled with the liquid phase we know that the amount of light which has been transmitted; when it is completely filled by the gas phase we knew the amount that is transmitted. Now, when fifty percent or thirty percent of the length of the chord if it is occupied by the liquid phase.

We measure the amount which is transmitted and then from there we estimate what is the length of the chord which is occupied by the gas phase or the liquid phase. So, therefore, just like area average we find out the cross sectional area occupied by the gas phase or phase two, the cross sectional area occupied by phase one; in the same way in this particular case, we find out the length of the chord which we have selected the length of the chord which is occupied by the gas phase or phase two and the length of the chord which is occupied by the liquid phase or phase one.

So, what are the different things that we measured or rather we defined the volume average liquid holdup? Usually, we will be talking in terms of holdup and not void fraction because if we talk about holdup even for liquid liquid system it just refers to the volumetric composition of the heavier liquid.

Even when we are talking about liquid solid applications it refers to the volume fraction of the liquid phase. So, therefore, usually we will be referring to the liquid holdup instead of the void fraction for two phase flow situations. So, therefore, first we define the volume average holdup; next we define the area average holdup; next chordal average holdup and finally, suppose I tell you that I would like to measure the void fraction or the composition at a particular point.

Then under that case, what should I call it? Should it be a point average liquid holdup. What should it be? If I consider a point then can there be a faction of the point which is occupied by the gas phase, fraction of the point which is occupied by the liquid phase; that particular point will either have only gas or it will be occupied by only liquid. The thing which might happen is we can consider the time maybe we take a total time of measurement and among that time, how much of the time the point was occupied by the gas phase? How much of the time the point was occupied by the liquid phase?

So, if we measure it at a particular point; we can do it. There are several techniques maybe conductivity probes can be used, certain radiation scattering techniques can be used and so on. So, under those circumstances we get the time average void fraction. And measuring the time average void fraction, it is very important. Why? What we can do? We can measure the time average void fraction and if we do it at different points then in that case we can get actually the voidage profile. We can find out how the voids are distributed in the cross section? Do u get my point?

Till now, whatever measurements we were doing, we were just getting the average composition across a particular section of the test passage. We were not concentrating or we were not in a position to find out how this average value is distributed across that particular section. But remember one thing, it is very important to know the average composition, but at the same time it is also important to know how this particular phases are distributed. For example, suppose you want a mass transfer operation or a chemical reaction. So, therefore, definitely towards the center of the pipe maybe its bubbly flow we have more bubbles; towards the wall we have less bubbles.

So, therefore, where will the reaction be the fastest? Suppose, it is a gas phase control reaction, more amount of the reaction will occur in the centre; very less reaction or mass transfer is going to occur towards the walls. We would like to ensure a very uniform distribution or in other words we would like to ensure that the mass transfer occurs uniformly throughout; or we would like to find out where the conversion or the mass transfer is maximum and then we can incorporate that in our mathematical models.

So, therefore, it is not always sufficient to know the average composition; it is also very important to know the profile of distribution of the two phases. Because if we can play with the distribution maybe we can get enhanced recovery or our enhanced product which we want.

So, apart from these average values it is very important to know the time average void fraction which we can find out by noting the void fractions or rather by noting the in-situ compositions at different points across particular cross section. So, this gives us the next definition which is the time average liquid holdup. So, this is usually defined as integral d t t 1 by integral t 1 plus t 2 d t; or in other words it is better defined as

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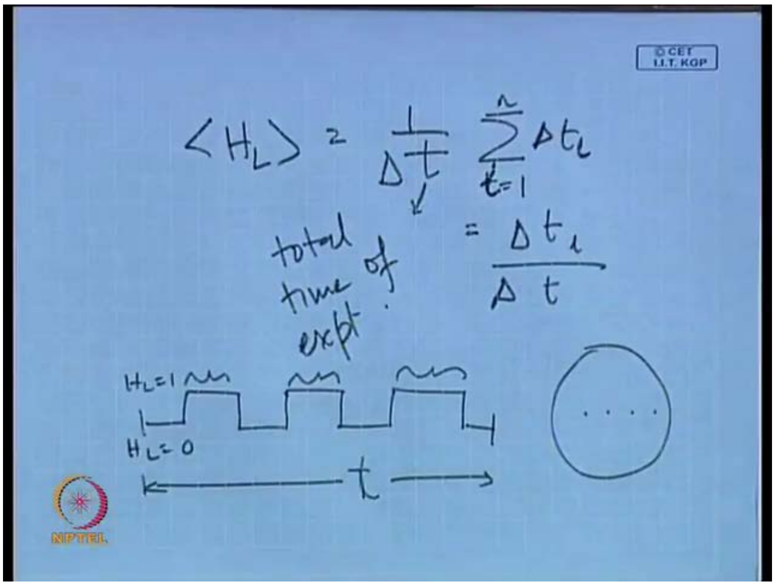

$$\langle H_L \rangle = \frac{1}{\Delta t} \sum_{t=1}^{n} \Delta t_l = \frac{\Delta t_l}{\Delta t}$$

because we cannot measure your infinitesimal quantities of time. So, therefore, this is better defined as 1 by delta t, where t is the total time of experiment.

So, therefore, 1 by t sigma i equals to 1 to n delta t l; or in another word this gives you delta t l by delta t the total time of experiments. So, therefore, what does it give you? Suppose you have taken a experiments or rather you have performed a experiment for say three minutes. Out of that three minutes, see when you are measuring at a point what you find? It is either going to give you a zero void fraction or unit void fraction.

Ideally, this is the ideal condition, but under experimental conditions you will find that the change is not so very instantaneous. But ideally what you should get? You should get some sort of a square waves where this gives you say H L equals to 0, this gives you say H L equal to 1. So, you take the measurements over this entire particular time, this gives you the total time of measurement out of that you find out for how long this H L was equal to 1.

So, this particular time divided by the total time gives you the time average void fraction; and if you know this particular say you know the time average void fraction at each and every point. So, from there you can find out the voidage profile or the distribution of the voids and accordingly you can play with it; or you can control it and you can manipulate it; or you can at least have an idea what it is. So, that you cannot do anything with it; you can at least incorporate it in your models. Definitely, if it is a parabolic sort of a profile and you assume a flat profile then definitely your results are not going to be accurate.

So, therefore, remember one thing after this whenever I refer to the liquid holdup or the void fraction or the solid loading whatever it is, I will be meaning the volume average value. But remember one thing, apart from the volume average value just because it is not always possible to measure the volume average value. So, therefore, we have other definitions like the area average, the chordal average and of course, the time average void fraction. But we always have to convert it to the volume average value or the area average value; and it is the area average or the volume average which gives us the actual volumetric actual in-situ composition of the two phase mixture. Usually area average and volume average we take as constant, because we assume that it does not vary with length. When it varies with length then definitely they cannot be considered as equal. For adiabatic fully developed flow conditions we assume both of them to be equal.

So, these are definitions regarding the different types of holdup definitions which we we can use. Now, we will be going for the other definitions which are left. Now, as whenever we do any particular analysis what we have to do? We have to first select coordinates; x y z are the common cartesian coordinates we use. Remember, in this particular case we have already used x for quality.

So, therefore, for our particular case what we shall be doing is the coordinate in the direction of flow, say suppose flow is occurring in this particular direction

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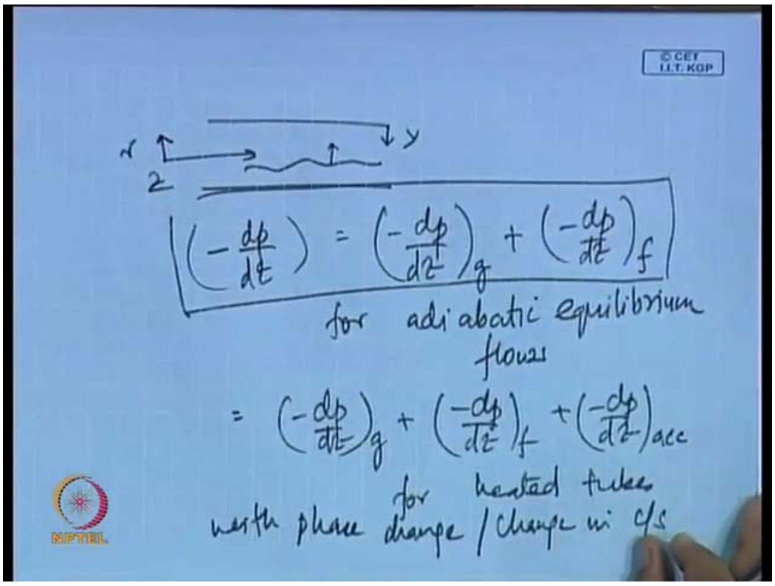

$$\boxed{\left(-\frac{dp}{dz}\right) = \left(-\frac{dp}{dz}\right)_g + \left(-\frac{dp}{dz}\right)_f}$$

for adiabatic equilibrium flows

$$= \left(-\frac{dp}{dz}\right)_g + \left(-\frac{dp}{dz}\right)_f + \left(-\frac{dp}{dz}\right)_{acc}$$

for heated tubes with phase change / change in c/s

the coordinate in the direction of flow will be taken as the z coordinate. Suppose it is circular pipe, from the axis we take the coordinate as r. So, along the axis of the conduit we take z is used as the coordinate and the radial distance from the axis of the pipe, it is usually r; and we use y when we have to measure the distance either from any particular boundary. Suppose from the surface if we have to measure the distance or maybe there is an interface from the interface we have to measure the distance then in that case we use y.

This is the usual coordinate system if you want you can refer to that coordinate in the directional flow as x as you are use to do it, but just to avoid unnecessary confusions I would like to specify this particular portion. In the direction of flow it is z and maybe perpendicular to the direction of flow from the tube axis in any particular tube we take it as the radial coordinate r; usually y is taken as the distance from any particular boundary it can be an interface; it can be the solid wall whatever it is distance from there is taken as y.

The other thing is rate of change of static pressure with distance; since this is the z direction. So, the rate of change of static pressure with this distance is naturally equals to d p d z. So, therefore, you must have noticed initially also when I did the momentum equation derivation for single phase and two phase I have referred to as d p d z. So, the reason is simply this and nothing else. So, we mention the rate of change of static pressure with distance as d p d z.

Now, naturally positive values they will denote a rise of pressure with axial distance. Whenever we have a positive value this shows pressure increase in a direction of flow. But always we know that the pressure falls in the direction of flow. So, therefore, we usually pressure gradient is denoted as minus d p d z. Now, we knew that minus d p d z this was equal to the for single phase flow; this is usually equal to the gravitational pressure gradient and the frictional pressure gradient; for adiabatic equilibrium flows.

This you have already derived you know it. And this is equal to the gravitational pressure gradient, the frictional pressure gradient as well as the acceleration pressure gradient comes into being when we have heated tubes and under that circumstances what happens? Under that circumstances either the liquid goes in the vapor phase or the vapor comes to the liquid phase; under that circumstance naturally one of the phases some portion of it is either accelerated or decelerated. So, therefore, for such situations we have the acceleration pressure gradient. But whenever, we are having heated tubes accompanied by phase changes it is no longer a single phase flow situation.

So, therefore, for heated tubes with phase change we have this acceleration pressure gradient, but under that circumstances it is no longer a single phase flow situation. Or we can also have this acceleration pressure gradient when there is any way to accelerate the flow; we can accelerate the flow by change of phase definitely; there is one other way to accelerate the flow change in cross sectional area. So, we can have this for heated tubes or for expansions, contractions etcetera; we can have this acceleration pressure gradient. But under normal circumstances for flow in pipes, for adiabatic equilibrium conditions, for single phase flow situations this is the expression of the pressure gradient.

And for two phase flow whether it is adiabatic equilibrium or whether it is accompanied by heat transfer

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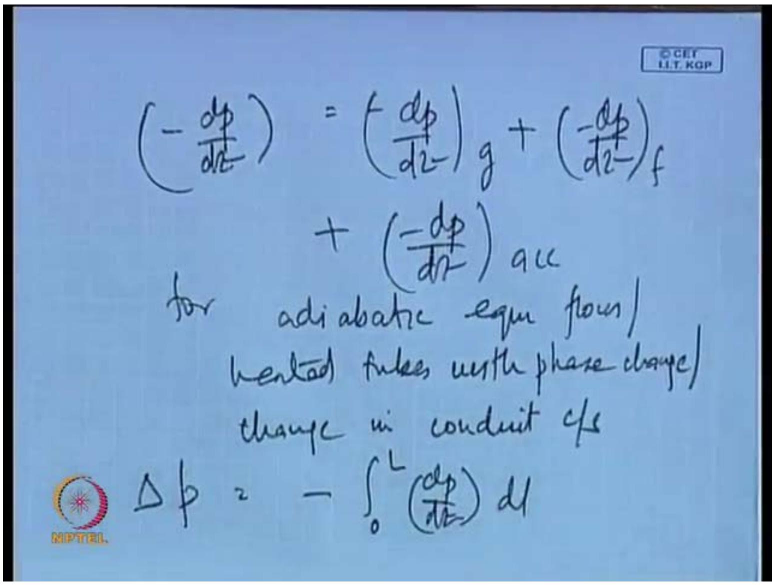

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$$\left(-\frac{dp}{dz}\right) = \left(-\frac{dp}{dz}\right)_g + \left(-\frac{dp}{dz}\right)_f + \left(-\frac{dp}{dz}\right)_{acc}$$

for adiabatic equ flows/
heated tubes with phase change/
change in conduit c/s

$$\Delta p = -\int_0^L \left(\frac{dp}{dz}\right) dl$$

the expression of pressure gradient, it always comprises of the three components for all circumstances; for adiabatic equilibrium flows heated tubes with phase change as well as change in conduit cross section; for all such cases, we do have an acceleration pressure gradient. But what will be the expression of this acceleration pressure gradient? How many terms it will comprise of? That will depend on what are the factors which contribute to the acceleration pressure gradient.

If there is an area change accompanied by a heat transfer definitely it will comprise of the requisite number of terms; under normal circumstances, it will just have one particular term. Why it will have that term? Because it is expected that when it is flowing at least if one of the phases is compressible like the gas phase then definitely with pressure the specific volume of the gas phase is going to change. So, as a result some sort of acceleration will be there.

But under normal circumstances if the pipe length is not very large, we can neglect the acceleration pressure gradient, but if in any problem during your exam you are neglecting the acceleration pressure gradient; at the end you have to calculate the pressure gradient and show that it is negligible compare to the total pressure gradient; it is less than say 10 percent or 5 percent that is why you have neglected it in calculating the total pressure gradient.

As engineers, we would not be very much concerned regarding the exactness or the high accuracy of the results; we would just like to get a more or less accurate results with not much of computational efforts. So, therefore, you are neglecting it, but you have to show under that circumstances that the acceleration pressure gradient was not more than ten percent or five percent whatever it is.

The other thing is apart from pressure gradient as engineers, we are also interested to find out the pressure drop; and the pressure drop it is quite evident this is nothing, but may be integral 0 to l d p d z d l. So, just remember that the pressure drop and the pressure gradient they have the opposite signs.

This you already know, but this was just to to remind you that this is the thing; that we usually prefer to.

Well, after this, there are certain other terms rather which I would like to discuss we will not be using them very frequently, but for certain simple analytical analysis we will be using them. In the last class, I had defined apart from mass flux, heat flux; I had defined something like the volumetric flux. Remember? How do we define the volume flux?

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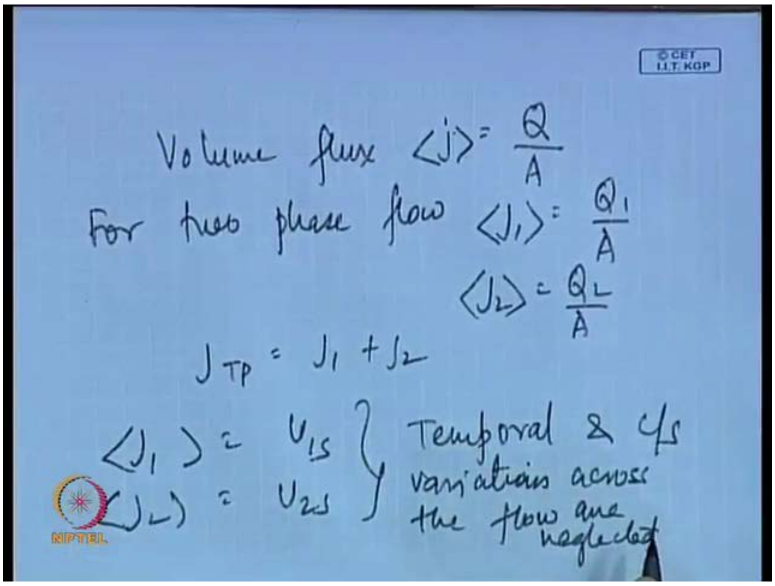

Volume flux $\langle j \rangle = \frac{Q}{A}$

For two phase flow $\langle J_1 \rangle = \frac{Q_1}{A}$

$\langle J_2 \rangle = \frac{Q_2}{A}$

$J_{TP} = J_1 + J_2$

$\left.\begin{array}{l} \langle J_1 \rangle = U_{1S} \\ \langle J_2 \rangle = U_{2S} \end{array}\right\}$ Temporal & c/s variation across the flow are neglected

It is small j; it is just the volumetric flow rate divided by the area; I had already defined it. Now, in this particular case, this was fine for single phase flows. Now, for two phase flow what do we get? We have J 1 just like G 1 this is equal to Q 1 by A; we have J 2 and we know the mixture volumetric flux it is nothing, but J 1 plus J 2.

Now, remember one thing, whenever I refer to this particular volume fluxes I assume that the total volumetric flow rate does not change across the cross sectional area. So, we assume that the volume flux is the area average volume. Under normal circumstances this should be this type. So, actually we are referring to the area average volume fluxes.

So, therefore, but under normal circumstances we assume that the volume flux they are more or less constant with area; and therefore, always we do not put this particular averaging signs. But when we are just writing J, J 1, J 2, we assume that they are just the area average values. So, please remember this one more thing I had expected this question from you; if you see the definitions you are going to find that they are similar to the definitions of superficial velocity.

So, therefore, you can always ask me what was the need of defining volumetric flux and superficial velocity? Now, here remember one thing; when we are considering one dimensional steady state flow situations both of them are the same, but in reality volumetric flux it is a vector quantity.

It is a vector quantity and for the present situation it will be used exclusively refer to the scalar component of flow. So, under the present circumstances volumetric flux and superficial velocity they have the same mathematical expression, but they do not have the same definition. Remember, superficial velocity was the velocity which the fluid would have had it flowed alone in the pipe. Volumetric flux is simply the volumetric flow rate divided by the total area.

So, by definition if you will find that by definition they are not the same and volumetric flux is a vector quantity, but for the present situation it will be used exclusively to represent the scalar component in the direction of motion along the pipe. And therefore, under such conditions it is numerically equal to the superficial velocity of the individual phase.

But the basic definitions are different they can be used interchangeably when temporal and cross sectional variations across the flow are ignored. Please remember that J 1 equals to U 1 s, J 2 equals to U 2 s; this can be used only when temporal and cross sectional variations across the flow are neglected; otherwise we cannot use it. For this one dimensional two phase flow under that circumstances, then how can we relate J 1 J 2 with the in-situ velocities?

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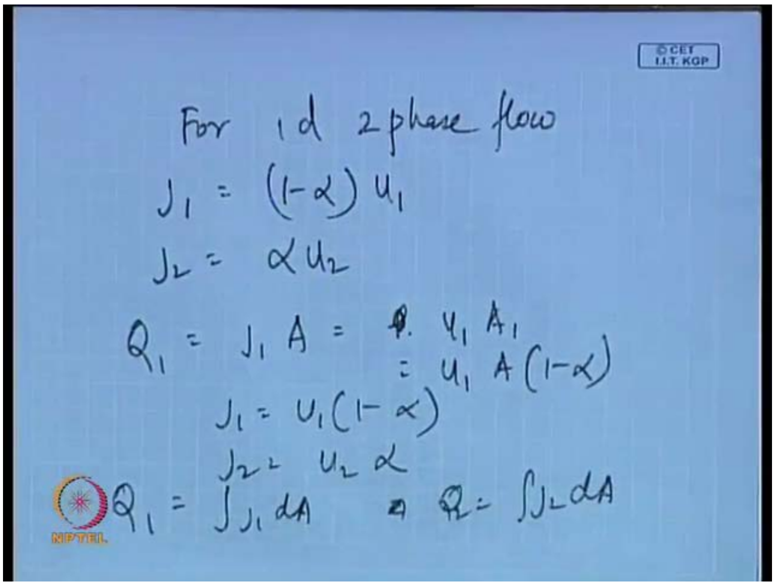

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For 1d 2phase flow

$J_1 = (1-\alpha) u_1$

$J_2 = \alpha u_2$

$Q_1 = J_1 A = u_1 A_1$

$= u_1 A (1-\alpha)$

$J_1 = u_1 (1-\alpha)$

$J_2 = u_2 \alpha$

$Q_1 = \int J_1 dA \quad Q = \int J_2 dA$

So, for one dimensional two phase flow we find the volumetric fluxes of the individual components can be related to the local component concentration and the local velocity in what way this is equal to 1 minus alpha into u 1, and this is equal to alpha into u 2.
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What is Q 1 equal to? From the basic definition which I have shown Q 1 equals to J 1 into A; this is equal to u 1 into A 1. What is A 1? u 1 A into 1 minus alpha. So, therefore, J 1 equals to u 1 into 1 minus alpha; same way J 2 equals to u 2 into alpha. So, therefore, your the volumetric fluxes they can be related to the local component concentration and the velocity by these particular relationships for one dimensional two phase flow; and the total local flux as I have already mentioned it can be expressed by this particular expression.

So, from here we can get two Q 1 as I have told you it is J 1 d A integral, Q 2 it is integral J 2 d A. So, therefore, when J 1 and J 2 they vary across the cross section when we are not considering one dimensional flows; then in that case this integral signs have to be given and the variation of J 1 and J 2 with area have to be incorporated in this mathematical expressions; and if J 1 and J 2 do not vary with cross section then it is J 1 A and J 2 A.

So, we will be continuing with a small amount of discussion which is left. After that tomorrow we are going to consider the simple analytical models and then after that we shall be taking up the simplest analytical model, the homogeneous flow model, and we will be analyzing two phase flows using that particular model.

Thank you very much.