Multiphase Flow
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Lecture No. # 28
Analysis of Specific Flow regimes- Slug Flow (condt.).

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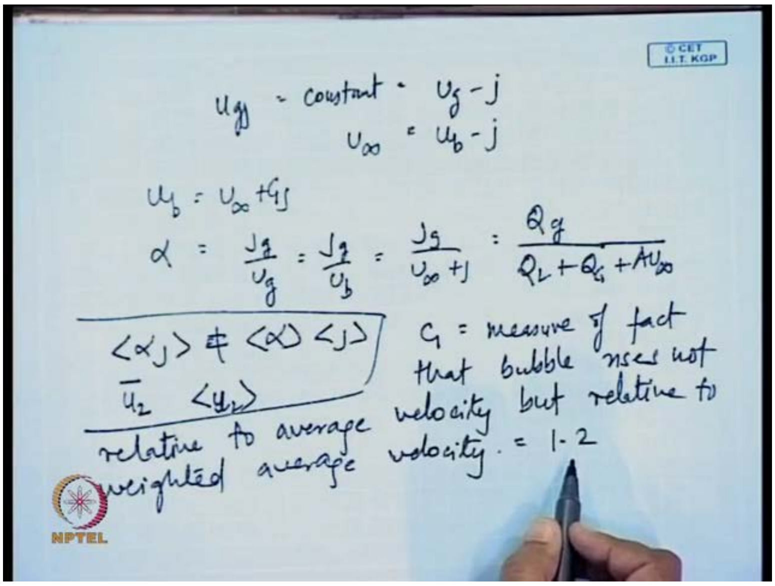

Well to continue with the distortion on the analysis of the slug flow pattern. So till yesterday what we did we found out that for the slug flow pattern the relative velocity or the drift velocity U G j that was a constant and this was nothing, but equal to U G minus j where the since all the gases confined as your slug flow if I show you the idealized version which I had mentioned in the in the last class yeah. So so if if you see transparency of the where I shown you the units cell for your slug flow analysis in a vertical tube we find that the this slug flow pattern all the gas we assumed was confined as as Taylor bubbles. So therefore U b and the velocity of the bubble can be taken as equal to the velocity of the gas in the slug flow pattern. So therefore this is equal to U v minus j and we found out that U G j was constant and therefore this will be equal to the velocity of the single bubble when it is raising in a stationary liquid or this is equal to U G at j equal to zero which can be defined as U infinity isn't? Just again I will like to remind you that this infinity have cause a different nomenclature as compared to the U infinity which we have referred in the bubble flow pattern as well as in the drift flux model ok. So from there we found out that U v this is equal to U infinity plus j isn't or in other words the thing was that in terms of suppose we would like to write it down. So therefore the this U d was U infinity plus j and accordingly alpha we would find it out as this was nothing, but j G by U G which is again j G by U b which is nothing, but j G by U infinity plus j in terms of the volumetric flow rates of the individual phases this gives us Q G by Q L plus Q G plus a U infinity. So, till this much I believe we had done in the last class. Now that time itself I had told you that the situation is not as simplified as I have mentioned it there has to be several corrections under different flow circumstances. For example, first thing which comes to he mind is we had assume at the bubble raise velocity is equal to U infinity plus G ok.

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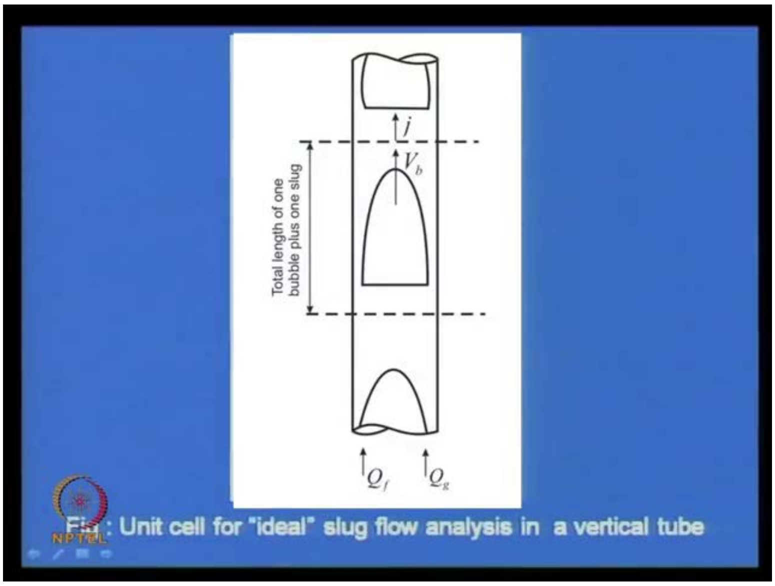

What do we assume that the entire in the front of the bubble this particular liquid the entire liquid flows at the same average velocity j, but usually it does not happen for (()) liquid slug can have a flat velocity profile, but otherwise we will be having some sort of a velocity profile and due to this existences of this velocity profile since the bubble tip is directed at the center of the tube. So, there will be a tendency of this bubble tip to raise with respect to the center line velocity of the liquid slug rather then the average volumetric flux of the liquid at the cross section ahead of the bubble correct. So therefore depending upon the velocity profile what we would like to do we would like to introduce particular correction since C 1 j when C 1 accounts for the effect of the or rather rather is it is a measure of he fact that the bubble does not simply move relative to the average liquid velocity, but a weighted average velocity. You remember when ever doing the slug model how had introduced the term c 0 whether whether if you remember I do not know whether you remember it I had shown you alpha j is not equal to the average value of alpha and the average value of j. So therefore the ratio was taken as a constant why because the average value is not always important the weighted average and from there we are introduced the weighted average velocity which is different from the sorry which is different from the cross sectional average velocity. So from those particular things if you remember we find that correction C 1 has been introduced here where C 1 it is nothing but it is the measure of the fact it is measure of fact that bubble raises not relative to average velocity, but relative to weighted average velocity. So therefore we have to introduce correction factor now remember that drift flux model also we are introduced a correction factor c 0 now both this these factors this c 0 and c 1. They more or less they have the same sort of a meaning and they also usually take up the same sort of a value usually we find c 1 it is equal to1.2, but remember that all though the physical reason reasoning behind the derivations at same for c 1 and c 0 but basically we had different parameters it is just like your volumetric flux and your velocity normally both superficial velocity and volumetric flux they have the same mathematical expression. But there meanings are completely different and if we really refer to a three dimensional vectorial form then they would be having different sort of inter peter physical interrpitation its just like that both of them are volumetric flow rate divided by the total cross sectional area, but the definitions are completely different one means the volume flow rate per unit cross sectional area and the other is the velocity if its that phase would have if it would have flowed alone in the pipe accidently for one dimensional flows both of them take the same mathematically expression. Its sort of this the other thing which should come to your mind is see suppose we are having Taylor bubbles more or less close to one another suppose these bubbles they are more or less close to one another in that case what happens there is a weak effect behind this particular bubble why this weak comes this down wards. So in liquid fill it has to mix with the liquid flux which is traveling upward as a result as I result week region is created and therefore, lot of small bubbles are shared away from the tail and this becomes an ideated mixture ok. Now when these bubbles are not sufficiently placed away from one another Then definitely the the visiting bubble that is going to influence the subsiding bubble and succeeding bubble what it tries to do it tries to travel faster, so that it can reach the proceeding bubble smaller bubble and then become a bigger bubble. So, therefore, there is another effect also which we should consider that effect is the wake effect of the preceding bubble I remember one thing in this (Refer Slide Time: 00:31) if there is a weak effect then in that case we see U G j or in other words that cannot be made equal to U infinity why because U infinity was the velocity of a single bubble in a stationary liquid column. Now the velocity which a single bubble would travel might not be equal to the velocity which it would have. Even number of bubbles are placed one above the other and there are travelling. Because each bubble would try to influence the succeeding bubble due to the weak effect they would try to travel faster. So therefore U G j is not strictly constant as we had assumed it to be why it is not strictly constant two things one is that it this j is or rather the bubble does not raise with respect to j but respect to some particular weighted average. Secondly that this is this particular U infinity may not be applicable for the present situation for a number of bubbles are flowing one above the other and there is a weak effect on the bubble. So therefore this also be corrected for a correct and accurate expression of U d for therefore, this is also corrected by a particular correction factor is C 2 ok where C 2 if I write it down the C 2 is the measure more or less you will get these in books, but probably not in the form that I am teaching you in the in the books its slightly more happens that what I have felt. So, if you fallow the notes and then refer to he book probably it will be simpler for you to fallow it ok. So therefore we find that C 2 it is a measure of change in relative velocity measure of change in relative velocity due to approaching velocity profile

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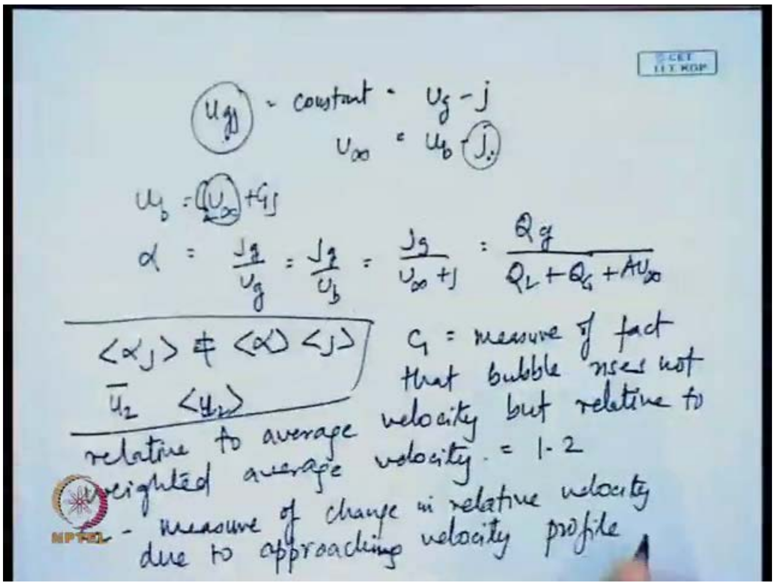

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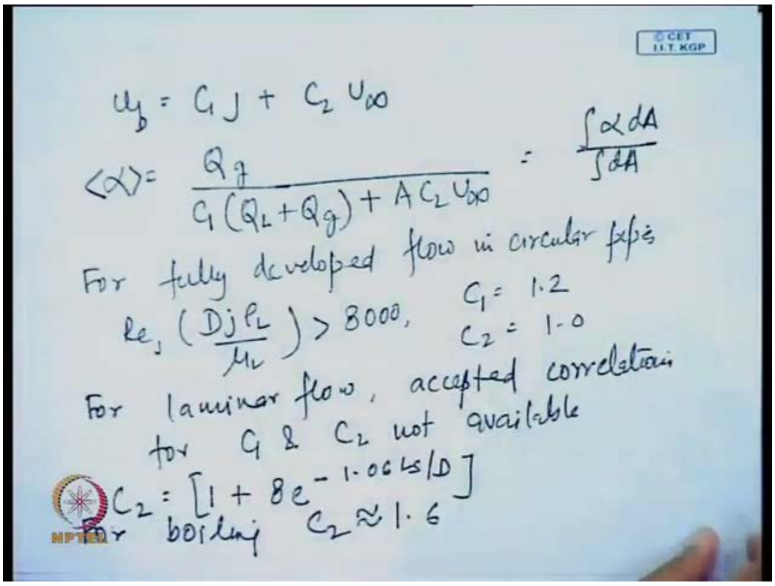

Ok. So, therefore, we find that usually what e find is. So, therefore, finally, this U b this becomes U b this becomes C 1 j one plus C 2 U infinity. Now (Refer Slide Time: 09:22) if we substitute this expression of U b into the expression of alpha then in that case what do we get we get alpha this is equal to Q G by C 1 Q L plus Q G plus A C 2 U infinity this is the corrected expression isn't, when we account for the weight effect of the or rather the measure of the change in relative velocity due to the approaching velocity profile as well as the constraint the fact that bubble raises not with respect to the average velocity. But with the weighted average velocity. So this is the final expression that we get. Now usual it has been observed that for fully developed flow in circular pipes when R e j R e j means remember this is the liquid flow at velocity j or on other words this is d e j rho L by U L and this is weighter than 8000 normally we get C 1 equals to 1.2 and equals to 1.0 that means, under that conditions we assume that more or less the bubbles are sufficiently placed away from each other or in other words the liquid slugs between the intermediate Taylor bubble the large enough such that the each bubble does not influence the motion of the succeeding one. So under such circumstances what fully turbulent flow fully developed in circular pipes usually we have C 1 which is given as one point two the same value as C 0 and C 2 equals to 1.0. And for laminar flow laminar flow we find that the U d is still not very well developed and accept the co relations for C 1 C 2 not available the U d still not well developed an accepted co relations for C 1 and C 2 there are not yet available. One particular thing is usually what people say is that C 2 it is depends upon the spacing of the bubbles. So therefore it should be a function of liquid slug length isn't. So if it is a function of liquid slug length people have proposed expressions this thought one plus eight e to the power minus 1.06 L s by d where your L s is liquid slug length or in other words it is the bubble separation length ok. People have also observed that what the case of boiling C 2 equals to one point six ok. Laminar flow when it is turbulent flow fully developed circular pipes then C 2 equals to 1.0 laminar flow accepted cold relations are available. But more or less we can take C 2 as this and for that particular case usually the velocities parameters parabolic. So therefore we can assume C 2 C 1 to be equal to 2.0 as well and sent flow boiling takes place for boiling yeah for boiling we can take people have assumed C 2 to be equal to1.6 ok. So these are the different values which people have assumed and accordingly people have tried to modify the expression of alpha in order to find out rather in order to substitute this particular alpha. In fact, I should have written it in this particular form in this it is actually the cross sectional average alpha where this is nothing if you remember this is nothing, but integral alpha d A by integral d A ok. So therefore this is the cross sectional average value of alpha the corrected value can be obtained from this particular form ok. Now this alpha if we substitute it in the pressure drop expression then we should get an expression of the pressure drop to a rather to expression to predict pressure drop for the flux flow pattern ok. Now under normal circumstances if you remember how did we start the derivation we started the derivation by considering negligible wall shear stress ok from that only we had found out that j two one is independent of Reigh is the function of alpha only what the did flux model as a result U G j is a function of alpha only. Entire thing was started if you remember in the d flux model from what gravity dominated flow situations where wall shear stress and neglegable for vertical yet in the vertical slug flow for slug flow in vertical is quite a reasonable assumption isn't.

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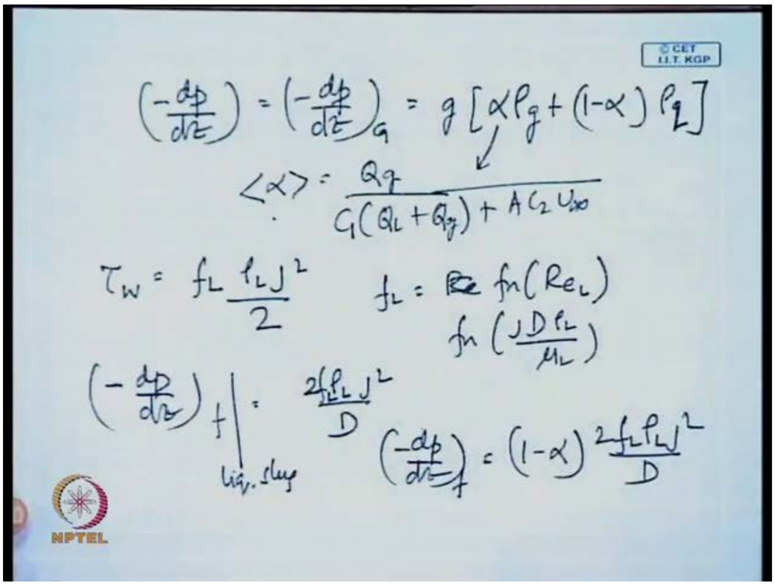

So therefore we find that the pressure gradient in this particular case this should be minus d p d z this should pre dominantly consider or consists of the gravitation pressure gradient which is nothing, but g into alpha rho g or alpha rho two whatever you may write do one minus sorry alpha rho L where this alpha can be given as I have already told you this is nothing, but the area average this is nothing, but C 1 Q L plus Q G plus A C 2 U infinity. So therefore in order to find alpha which can be substituted in the pressure gradient expression we need to know U infinity and how to know U infinity that we have already discussed all the other things if you find they are all input parameter ok. So for most of the cases we find that the pressure gradient it comprises of the the gravitational pressure gradient under fit suitation we find that it can be found out from this particular situation. Now suppose wall shear stress is important under that condition what will we do if wall shear stress is become important. Now just try to remember(Refer Slide Time: 03:04) just try to observe the unit cell which I have shown in this particular slide now if wall shear stress are important then we find that there are wall shear stresses all through the pile [FL] in the Taylor bubble we did the liquid flow downwards as a liquid film agree. So therefore in this case wall shear stress will be acting in the upward direction again if you notice the liquid slug the wall shear stress the liquid slug flows up wall shear stress will be more dominating. So therefore it is not very easy to find out the wall shear stress in this particular condition. So what do what will be refer to do in this particular case since it is not very easy certain things we can assume, first thing which you can assume is lets see in the Taylor bubble region more or less what do we find we find that liquid film is very thin and this nose region is also quite smaller compared to the cylindrical portion in the nose region what happens the liquid it accelerates from zero to some particular value which is a function of the distance from the tip of the nose or in other words at any particular distance say h from the tip of the nose the liquid film velocity will be root over of two G h loss of motion isn't. And but movement it reaches it tail region this particular nose we dint its quite small after that it flows at a constant velocity attains a terminal film thickness and it flows down at a constant velocity again it is flowing down at a constant velocity that means, the wall shear it completely balances the weight of the liquid yes or no? It must completely balance the weight of the liquid due to fit a liquid becomes a free flowing film under this condition if it is if the wall shear stress is contributing to rather it is travelling to few balance the weight of the liquid that means, that it does not contribute to the pressure gradient or in other words we can assume that the contribution of the wall shear stress in the Taylor bubble region, where he liquid flows as the liquid is negligible and can be neglected and we can assume that the wall shear primarily arises in the liquid slug region where the liquid slug it flow as pure liquid upward.

Do you agree with me or anyone who wants me to repeat this particular part what happens you just try to see slug flow is occurring liquid Taylor bubble liquid slug Taylor bubble liquid slug which is occurring in a vertical pipe when it is occurring in a vertical pipe then, what are the components of pressure gradient normally we have gravitational pressure, gradient frictional pressure, gradient acceleration pressure gradient acceleration pressure gradient it arises only can there is a area change or there is a phase change initially under these two conditions srise or there is very rapid pressure change due to phase the compressible flow compressible fuel under goes volume change with distance under these conditions it arises the under normal circumstances. We can help you that acceleration pressure gradient can be neglected if the pipe is not very large if there is no phase change and the two fluids flow through a area of constant cross section agree. So therefore p dominant what are the p dominant gravitational nad friction when the pipe is vertical quite naturally the gravity pressure gradient is much more important. In fact, most of the time neglect friction pressure gradient vertical pipe only a single fluid is flowing through it frictional pressure gradient usually comes when the pipe is horizontal that we have already seen for single phase two phase flow sometimes its quite important. Because in this particular case your frictional pressure gradient arises not only due to the frictional between the fluid and the wall, but also due to the interfacial shear has been. Now vertical pipes under normal circumstances flux fluid is occurring we we can assume quite safely that the frictional pressure gradient is negligable at some times to the gravitational pressure gradient ok, under that circumstances what do we get we find then the total pressure gradient can be obtained from the expression that I have written down here if wall shear becomes important under that circumstance what do we do if wall shear becomes important then definitely we have to consider the frictional pressure gradient across the wall. Remember (Refer Slide Time: 03:04) one thing frictional pressure gradient changes the cross to wall since the frictional pressure gradient or rather the wall shear is different in the tailor bubble region and in the fluid fluc region. Its different not only magnitude, but also in direction very important. In fact, which usually does not occur elsewhere in one particular pipe which wall shear is changing this is very not a very common situation, what do we find we find that the wall shear in the Taylor bubble region it occurs in the upward direction that means, when direction in which the Taylor bubble will raising why because fluid film is flowing down wards here and in the liquid slug region it is in the downward direction because the liquid alug is going in an upward direction right now we find that the wall shear then finding out the wall shera or finding out the frictional pressure gradient is not at all good and very straight forward we have to find it out for this particular region for this particular region its become quite complicated, but fortunately we observe certain other things as well what do we observe. We find that for most of the cases the Taylor bubble can be approximated as a cylinder constant cross section which has a constant curvature in the flow direction isn't. Therefore and the gas density and viscosity negligible as compared to liquid density and viscosity agree therefore the bubble surface can be assumed to be a surface of constant pressure therefore the interfacial shear here can be neglected. So interfacial shear part gone in the Taylor bubble region what there crust is remaining wall shear now in this particular case thing one thing what is happening the liquid is flowing as a film. The fluid may be at the tip of the nose it has a zero velocity and then it accelerates downwards along the nose the film thickness increases as well as the velocity increases till at the interception of the nose and the tail the film thickness it becomes constant and the liquid film attains a terminal velocity. And then after that it falls at that particular constant velocity it behaves like a free falling film, more or less the Taylor bubble is reasonably long enough why if it is reasonably long enough. The cylindrical portion will be larger as compared to the your hemispherical portion if there is very small Taylor bubble then it comprises mainly of hemispherical portion just like cap bubbles under this condition this particular assumption does not hold it halts only the more or less long scalar bubble where we can assume that the Taylor bubbles are cylindrical just because the nose region is much smaller as compared to the tail region.

Agree it with me now under such circumstances what happens in the tail region the liquid becomes freely falling film when it becomes a freely falling film that means, it is not cated by any net unbalanced force. So under this condition what happens we know what are forcing acting on the liquid film it is the weight of the liquid film pulling it down wards and the wall shear it is pulling it upwards when it becomes a freely falling film. That means, when it falls at a constant velocity from mechanics we very well now that the that means, it is not acted by nay net unbalanced force or in other words the wall shear completely balances or the wall shear force completely balances the agreed with me, that is why if the wall shear is continuity totally two compensating the weight of the liquid film how can iot contribute to the pressure or the pressure gradient can it contribute. So therefore we can very straightly neglect the pressure gradient which arises due to wall shear in the Taylor bubble region you can very straightly neglect the particular portion and we have to consider only the wall shear or rather the frictional pressure gradient which arises in the fluid slug region. Liquid slug is just pure liquid. So therefore, they have the fractional pressure we will which arises can be found out from the single phase free dynamic itself yes or no. So therefore, for this particular condition what can we write we can write that tou w this is nothing, but f L U L j square by two in the liquid slug region where L is a function of sorry it is a function of r e L at j velocity or it is a function of j G rho L by U L agree isn't. So therefore, under this particular circumstance what is minus d t d z frictionally equal to just as I have done it in the previous class this is nothing, but two rho L j square by D. Yes or no sorry two x L rho L j square by d this is the frictional pressure gradient at the liquid slug only agree and we can assume that since the entire gas closure scalar bubble liquid slug occupy a portion one minus alpha of the pipe value yes or no. Therefore the minus d p d z frictional for the entire pipe this should be equal to one minus alpha into two f L rho L j square by D yes or no.

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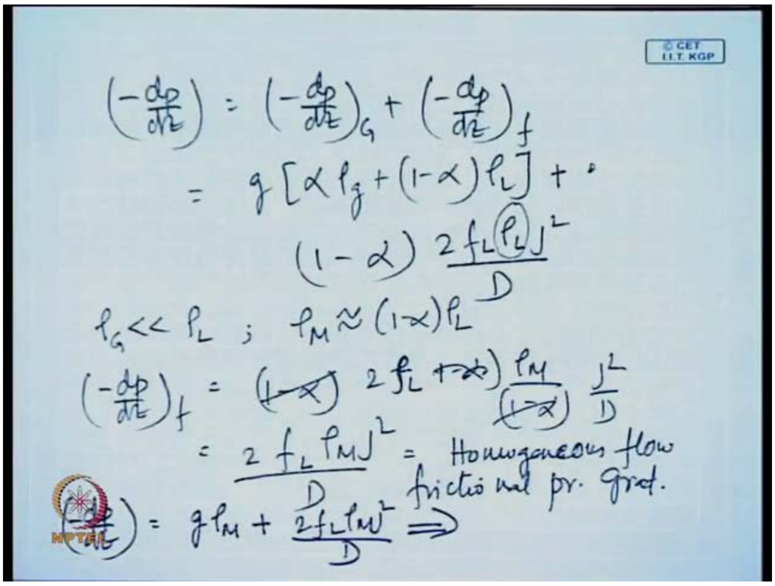

Agreed, if this is true therefore what does what happens the expression of the total pressure gradient the total pressure gradient therefore, it becomes minus d p d z which is equals to minus d p d z gravitational plus minus d p d z frictional it is nothing but equal to G into alpha rho G plus one minus alpha rho L plus your I will write it down plus one minus alpha two f L rho L j square by d can I write it in this particular form or not yes or no agreed. Now let us see if we can do something else we what do we know we know rho G is much less than rho L do we know this do we know this if we know this then can we write down rho m it is almost equal to one minus alpha into rho L we can do it. So therefore in place of this particular rho L we can substitute with rho L by one plus alpha why I want to do it is will tell you. So therefore your frictional pressure gradient this comes one minus alpha two f L one minus sorry sorry sorry sorry U f L rho m by one minus alpha j square by d can I write it in this particular form or in other words what does it become. It becomes two f L rho m j square by d does it become this and have you ever come across this particular expression are you aware of this particular expression what is this what is this particular expression it is the frictional pressure gradient that we had obtained from the homogenous flow theory correct. So therefore we find that this is the homogeneous flow friction homogenous flow frictional pressure gradient frictional pressure gradient where you have to remember one thing in this frictional pressure gradient how it is different from the homogenous flow value that we get the alpha which we used to calculate rho m that alpha has to be calculated from this particular equation. This will give us value which is different from the homogenous flow values which we obtained for under this flow condition is it clear to you. So therefore what do we find the final expression minus d p d z is G into rho m plus two f L rho m j square by d the same expression that we had got for the homogenous flow model thus we have to remember that how it is different from the homogenous flow model it is different just in one way that the alpha which is used to calculate rho m where this is the expression of rho m the alpha which is used that has to be obtained (Refer Slide Time: 15:18) from this particular expression which has be obtained from the slug flow pattern using the concepts of drip flux model correct. Now how you know that whether you have to consider the frictional pressure gradient or not what you suppose to do you know the flow condition inlet flow pipe diameter everything just to assume that what will be the pressure gradient if the flow was homogenous definitely for slug flow the pressure gradient will be less or closer to it. So what you do you first find out the frictional pressure gradient for the homogenous flow model then you compared it with the total pressure gradient or the gravitational pressure gradient if it is much less you forget about the frictional pressure gradient you simply assume that the to a to pressure gradient comprises of total pressure drop comprises of the gravitational pressure drop proceed the long method. If you find nine it has got the substantial amount friction contributes substantial to the pressure drop then in that case what you have to do you have to use the alpha which you have found out for the slug flow model then that alpha has to be used and accordingly the frictional pressure drop has to be found out and it has to be included here agreed and now what about the acceleration pressure drop remember one thing more or less when you do not know anything then what you have to do we have to these have to things that we know; that means, we have to assume suppose it was homogenous flow model then what could have been the acceleration pressure gradient usually as I told you for round pipes constant cross section no phase not a very large pipe it will not be very much, but remember one thing you have to be very very cautious when you use the expression of the acceleration pressure gradient as obtained from the homogenous flow model to find out the chocking conditions where slug flow model definitely you will not get the correct chocking condition for the slug flow situation this particular analysis quite tough since they do not have much time I will not be going into finding out the acceleration pressure drop for slug flow situation if any of you interested can discuss it after the class ok.

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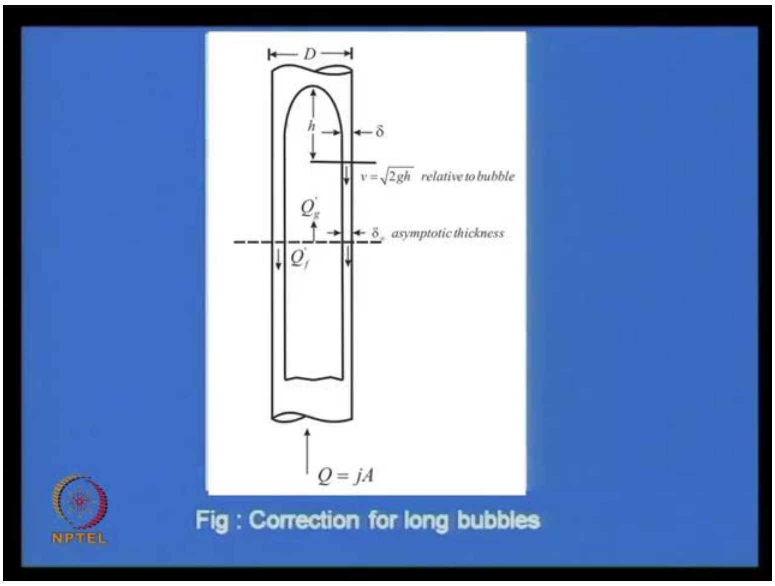

Fig : Correction for long bubbles

So therefore this is the way by which we found out the void fraction and the pressure gradient for the slug flow pattern now one particular correction I forgot to mention what is that correction suppose the bubbles are very large do you anticipate any other error might come in which has to be corrected for and then the corrected alpha some how it will effect alpha and it will effect pressure gradient if the bubbles are very long do you anticipate anything of that sort for long Taylor bubble say I got a sorry what do you thing will happen when we have long Taylor bubble just try to imagine when you have long Taylor what happens a good amount liquid will be flowing as films on the as angular films between the bubble and the wall often a good amount of the liquid will be flowing what do we always assume?

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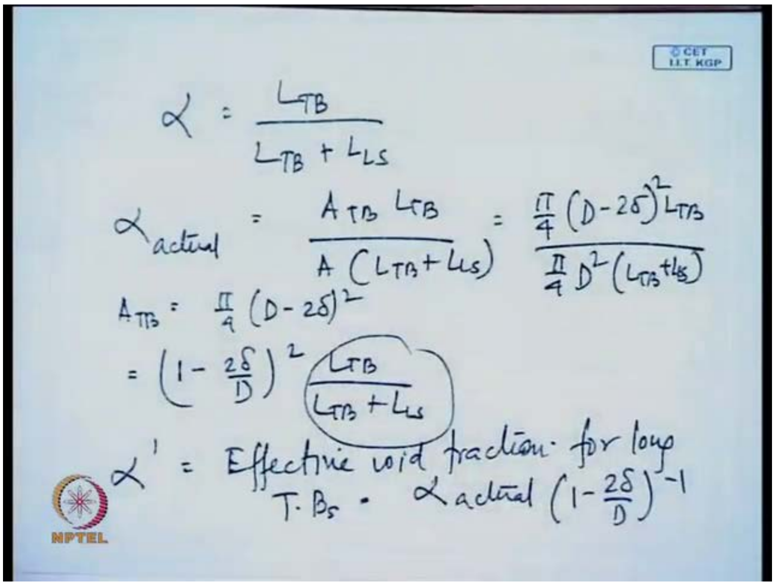

We assume that absence of Taylor bubble has occupied the entire cross section we assume that liquid slug occupy one minus alpha portion of the pipe can you assume that at that time there are two things that you have to think one is liquid a good amount of liquid flows as a film along the wall this is number one. So, this influences the calculation of alpha right and what is the other thing if a good significant amount of liquid flows as film then that significant amount of liquid will tha liquid that much amount less will flow through the liquid slug isn't. So, therefore, normally what do we do normally we how do we calculate alpha we assume that more or less the Taylor bubble it occupies the entire cross section alpha it is calculated by the relative lengths which have been occupied by the Taylor bubble and the liquid slug region. Now, can we do this now can we do this can we write that alpha is equal to this we cannot do this now why because in this particular case the actual alpha this is what A T B L T B by A into L T B plus L L s try to understand this very well you you can find it in books, but concept please try to understand do you agree with me what is A T B suppose the liquid film it is offer constant thickness delta one thing I assume that for long Taylor bubble it is much more easier to assume that the Taylor the nose region is negligible as compared to the tail region isn't. So one thing we can very safely assume that the liquid film attains A or rather it flows as a constant thickness film as a free falling film which attains a terminal velocity at a particular fixed thickness. Let us take the thickness or the film thickness to the delta then in that case what is A T B equal to A T B will be equal to pi by four into d minus D delta whole square yes or no or in other words this is going to be pi by four D minus two delta whole square L t D by pi by four d square L T B plus L L s yes or no do you agree with me [FL] if this is true then this can be written down as your one minus two delta by D whole square L T B by L T B plus L Ls can I write down the actual alpha see when the bubbles are not at all actual alpha is given by this, but now since the bubble is long and the liquid occupies significant portion as liquid films therefore, the alpha actual is this into this you agree with me now try to understand one particular fact what is it now we know that if you observe this again we find in this particular film region (Refer Slide Time: 33:39) again by this same logic the wall shear or there is no particular pressure drop here isn't. This particular this entire portion can be assumed to be a region of constant pressure as I have already discussed this does not contribute to the pressure gradient you agree with me if this is not contribute the pressure gradient then we can very well assume or we can imagine that the liquid film here it almost behaves like a gas and and this entire surface is maintained as a constant pressure and in this particular case isn't. So therefore actually the alpha should be calculated from this particular expression, but we have to remember one thing what has happen this L L s effected just because some amount of liquid from here has gone to the film and it flowing as film.

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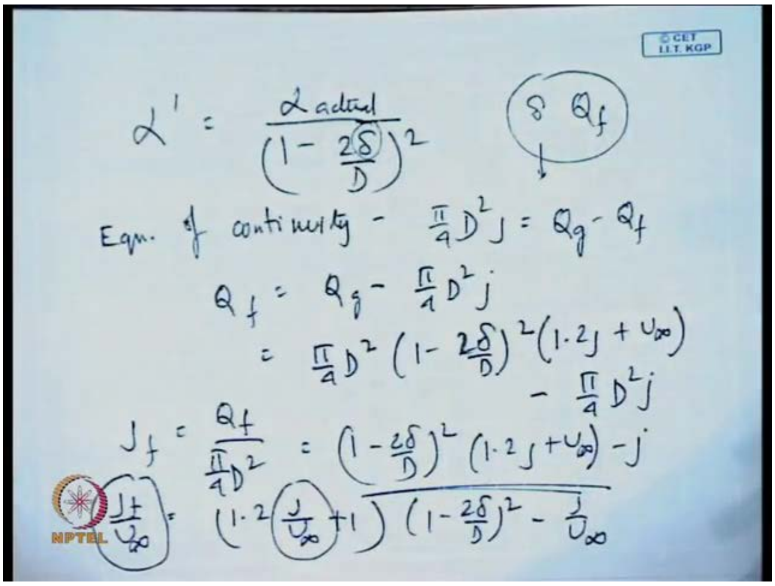

If this particular portion is significant then L L s will be reduced by a significant portion what amount will it be reduce it will be reduce by a amount which is propotional to this two delta by d isn't. So, therefore, if the effective alpha prime what is this effective void fraction which should be considered for long Taylor bubbles effective void fraction for long Taylor bubble what is this this is this particular portion this particular expression. So, therefore, this is equal to the actual expression into one minus two delta by D whole to the power of minus one or in other words I will write it down here your alpha prime this is equal to alpha actual by one minus two delta by D whole square yes or no. You tell me whether you have understood this particular portion or not have you understood this particular portion yes or no it is clear. So, therefore, how to find alpha under this particular condition in order to find out the effective void fraction not the actual one and the effective void fraction you have to remember two things that since the good amount of liquid is now flowing as film it is going to effect the liquid slug length, but no matter how much liquid flows as well that will be a region of constant pressure and therefore, the entire team a can be assume to be as for gas mass or constant pressure flowing there right. So, therefore, under this particular condition alpha prime can be obtained as I have written down here. But but to find out alpha prime what do you need to know the liquid (fill in thickness) in order to find out liquid fill in thickness you also need to know the flow rate of the liquid in the fill isn't. So for finding two particular unknown so you need two equations usually one equation you can get from the free in folic fill-in theory from there you can get particular equation from where you will get the other equation from equation of continuity at this particular portion yesterday somebody was asking that actually we should consider Q f flow in down somebody was asking me. So therefore your asking. (Refer Slide Time: 33:39) So, therefore, when the fill-in thickness is while significant we have to do it there we did not do because you was very thin. So therefore in this particular case while one equation you can obtain from fill-in theory from other equation from where you can obtain from equation of continuity (Refer Slide Time 33:39) equation of continuity from therefore, what do we know that the total volumetric flux across any pipe cross section should be constant or in other word the total volumetric flux here should be equal to the total volumetric flux here what is the total volumetric flux here it is nothing, but pie by four b square into j do you agree with me this then should be equal to Q G minus Q f in the Taylor bubble region agree. So from here what do we get we get Q f is nothing, but Q G minus pie by four D square by j agree or In other words this is for Q G we can write pie by four d square one minus two delta by D whole square and this in to one point j two plus U infinity this entire thing is to Q G yes or no this minus pie by four D square by j right from equation of continuity for the total volumetric flow across any pie cross section we can get this particular part. So from therefore what do we get we get j f what is that equal to Q f by pie by four D square correct. So this will be equal to one minus two delta by D whole square 1.2 j plus U infinity minus j just note this particular expression or in other words if you write it in terms of j f by U infinity just to make it non tangential because here to make it fond of making something non dimensional. So therefore in that particular case it becomes (vocalized noise) something on this sort ok. So we find that we can correlate j by U infinity with j by U infinity j by U infinity you already know. So, therefore, we can find out j f once we know U infinity. So, therefore, once we can find out j f we can find out Q f and then there is another equation from fall-in folic theory. So, from these two equations we can find out j f and delta once we can find out delta you can find out effective lambda prime sorry alpha prime which can be used in case the bubbles are long agree. So, these were all corrections which we could not in corporate for vertical slug flow vertical slug flow completed what about horizontal slug flow in a horizontal pipe (Refer Slide Time: 33:39) if we take up slug flow pattern do you anticipate that the same equations that we derive. So , long for the last two class that the same thing is going to happen in this particular case also why were is the same pipe just we have created in many horizontal (vocalized noise).

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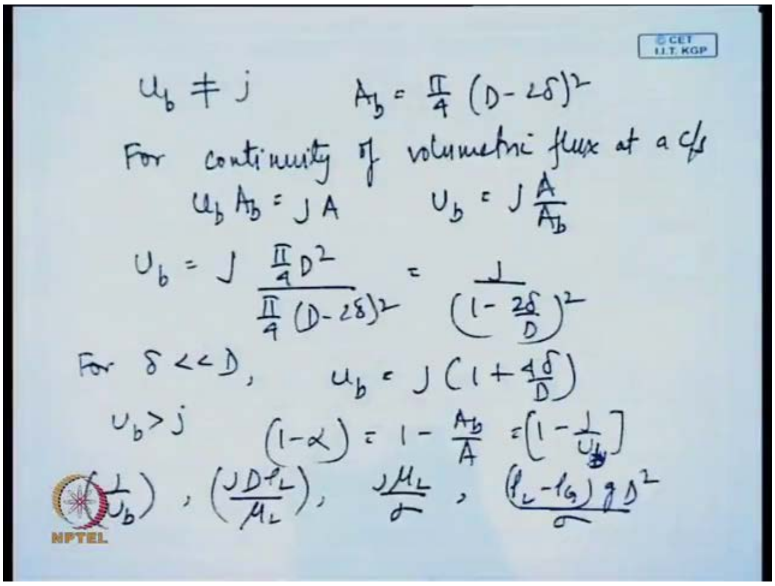

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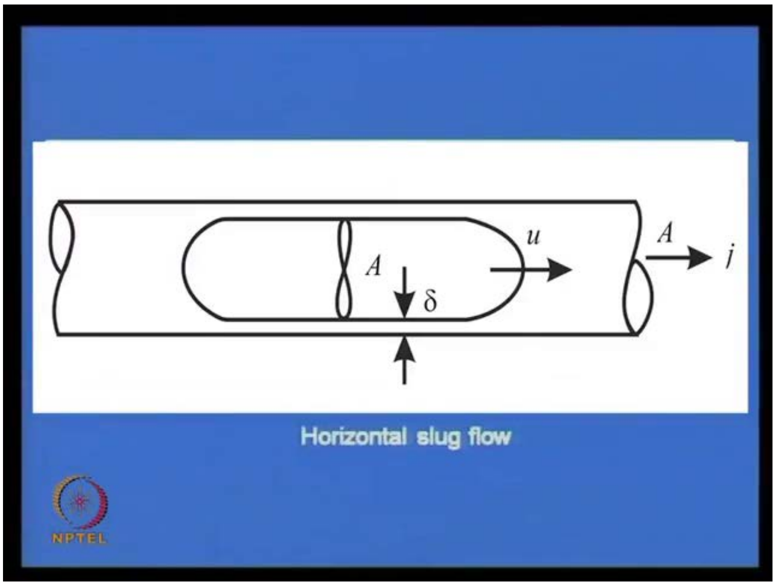

Vocalized noise the first thing you have to think that this particular vertical pipe the bubble was raising due to bouncy here the bouncing effect is not there at all. So therefore, the total dynamics becomes different in this particular case, but since there is no bouncy can you say were then delta leys were relative velocity between bubble and the liquid that you cannot say this particular case also the bubble velocity U b that will not be equal to j. So therefore the aliysis of horizontal slug flow which I have shown in this particular transparency this is going to be completely different as compare to the your vertical slug flow here we find that there is no drift slugs due to bouncy effect therefore, see the entire analysis is there based on U infinity do you understand what we did we found out U G j equals to constant that u G j was equal to U infinity. How did that U infinity come because the bubble used to raise in an vertical due due to bouncy. So here basic unit infinity concept is not there at all, but U b is not equal to j. So therefore how to find out U b in this particular case let us see that in this particular case what can we do to find out U b since there is no particular relative velocity his there is no particular this bouncy effect. So therefore we can assume that the liquid fill-in at the wall here that is more or less stationery because no particular pressure difference the bubbles are in constant pressure. So therefore since there is no particular difference gradient we can assume that the liquid fill-in between the bubble and the wall that is more or less constant that we can assume. So for this particular case what (Refer Slide Time: 44:42) we know we know that A b (vocalized noise) that is equal to pie by four into d minus two delta whole square that have doing for a long time, again from continuity of volumetric flux just I have done for the case of long bubble. So for continuity of volumetric flux (vocalized noise) at the cross section what do we know we know that for continuity U b A b must be equal to j in to A yes or no we have made one particular assumption we have assume the one bubble to be access metric in electrical order horizontal flow usually that does not happen particularly for low flow rates, but since the liquid fill in thickness is not very different we have made this particular assumptions here. So (vocalized noise)for from this what do we get we get U b A b this is equal to j in to A or in other words we need to find out U b stinted. So this U b can be written down as j in to A by A b were ever you do not understand you tell to do repeated or in other words this can be written down as j in to pie by four D square by pie bye four D minus two delta whole square yes or know or in other words this can be written down as j by one minus two delta by D whole square we can write it down usually what do we do we know delta is much much less than d we know it for that particular situation you have been doing for a long time this can be written down in this particular form yes or no (vocalized noise) fine. So for delta much much less than or the fill-in thickness is legible as compared to the diameter of the two the bubble raise velocity can be expressed in this particular form from here what do we get from here we get that U b is greater than j from this expression we get this and this automatically proves that the bubble has a velocity creative to the average volumetric flux or related to the liquid. slug do you agree with me anybody who wants me to repeat any part of this or in other words what is one minus alpha equal to this is nothing, but one minus a b by a what is a b by A A b by A equals to j by U b. So this is equal to one minus j by sorry one minus j by U b isinted. So therefore what do we find that in order to find out alpha (vocalized noise) again we can what what is important U infinity less than U infinity U b is important again U b it can b defined in terms of certain dimensionless parameter considering the forces which are acting on the bubble flux (vocalized noise) again we can say that gas viscosity they are negligible So, what are the important forces and by balance of those forces. What are the important groups that we get same thing will be their in initial is going to be importance substantial is going to be important your viscosity going to be important or only very important. So, we find the dimensional analysis (vocalized noise) in the absence of effects due to gas viscosity and gas initial if we neglect these two things. So their fore bubbles which move independently that is they are not falling into each other's way or in other words the liquid flux are in large for such circumstance we find what are the important dimensionless parameter which cover this one dimensionless parameter definitely j by U b fine because we have already got it that decides this is nothing it is just the liquid velocity is slug divided by the bubble velocity definitely one liquid slug lots number is going to come here Ok that has to be important which is j d row L by mu L this is going to be something very important here then viscous force substantial forces to they have to be important. So j mu L by sigma this another important thing viscous force by substantial forces and when will this slug flow will not happen when bouncy becomes important and the entire gas moves up. So therefore in order to decide the limit of slug flow pattern your bouncy by surface tension this particular term also has to be important mostly will find them these three terms governing at the limit of the slug flow will be governed by this particular term, do you agree with me if you notice these terms you find them more less these three terms particularly they more or less correspond to the dimensionless groups which we had obtain for vertical slug flow.

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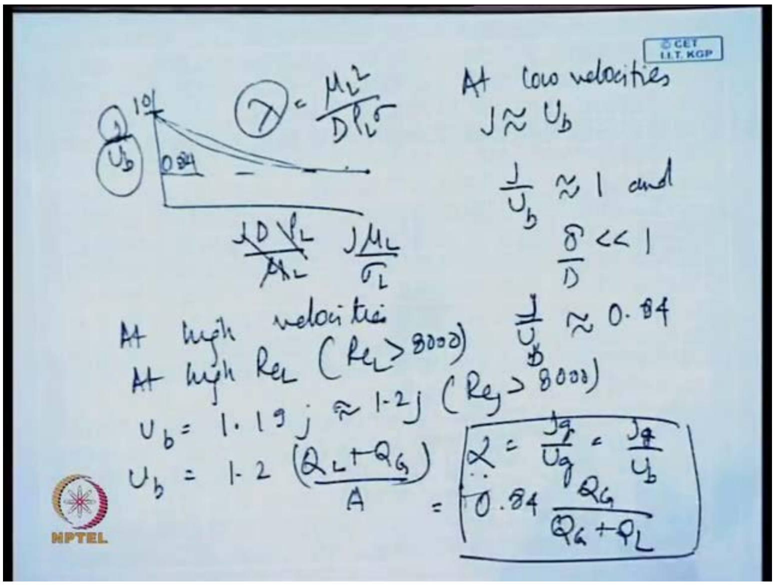

There are also it was bouncy by (()) bouncy by viscosity bouncy by initial more or less these terms are also respond to an identical balance of forces which we have been obtain for the vertical slug flow case as well. So therefore we find that again what we would like to do we would like to remove j from one particular thing. So we can combine these two to form a property liquid property group and usually we find that more or less this j by U b that is brought as the function of j d row L by U L then it is plotted as a function of this sorry very sorry very sorry it is plotted as j mu L by sigma L were one particular parameter lambda a lambda is equal to mu L square by D row L sigma this particular lambda is obtained (Refer Slide Time: 44:32)by eliminating j from these two terms now this particular graph is usually obtain from this particular graph from the type of graph we get we try to find out we know j. So we try to find out U b from here once we can find out U b from here (vocalized noise)then we can (Refer Slide Time: 44:32) we can find out one minus alpha and then we can go to predict the pressure gradient right now certain things have to remember in this particular case(vocalized noise) what are those certain things usually we find that a graph which has been obtain for very low velocity (vocalized noise)when the velocity is low then j is almost equal to U b isinted when the velocity is very low then j is almost equal to U b in other words j by U b that is almost equal to one and if j by U b is equal to one then in that case delta has to be much much (Refer Slide Time: 44:32) less than one do you agree with me. So at no velocity is what happens j is equal to U b almost this becomes this implies delta much less than d ok So, we find that at low velocities j is almost equal to U b and delta by D is much much less than one and at high velocities what happens when the velocity is high then definitely j by U b cannot be equal to one we now observe that high velocity or in other words at high resoles number r e L greater than eight thousands for this particular situations we have found usually j by U b that reduces to a value of 0.84 clear this people have obtain by experiment usually to plot this graph we find that this graph start from 1.0 and end and it reaches the accentuating at about 0.84. So, if j by U b is equal to 0.84 then in that case what is U b U b it is almost equal to 1.19 j correct which is close to about 1.2 j the value which is obtain for horizontal sorry vertical slug flow this is obtain for r e j greater than eight thousand correct. So, for those particular cases U b this is equal to 1.2 Q sorry Q L plus Q G by A isinted what is alpha it is j G by U G or this is equal to j G by U b correct. So therefore what is this equal to this will naturally be equal to zero point eight four Q G by Q G plus Q L do you agree with me yes or no this horizontal slug flow part is it clear to you what did we do I this particular case we found U b is not equal to j and and how we need to find out U b because decides alpha finds one minus alpha this alpha and one minus alpha has to be substituted in the pressure gradient calculations. (Refer Slide Time: 44:32) So how to do this we have to find out j by U b again we resolve to dimensionless analysis and we find that usually we work with these three particular dimensionless group were one of the contents is U b end one of them is just a property group from their people are found out when velocity is low then both the particular velocity is or rather both the liquid bubble flow at the same velocity or more or less j can be approximated to equal to U b which automatically implies this is possible only when the liquid fill-in thickness is very small this accurse at low velocity when velocity is high; that means, the liquid number is high under that particular case. We people have observed that the j by U b reduces to ratio of point eight four which gives you U b equals to almost about one point two G if this substituted in the execution of alpha we find that alpha can be obtain by this particular expression now this alpha is substituted in to the pressure gradient expression then we can get accurate pressure gradient expression, but remember in this particular pressure gradient is not very simple why because it primarily comprises of the frictional pressure gradient then it does not comprises of anything else if you have to calculate the frictional gradient you need to know the relativity lance of the liquid bubble sorry Taylor bubbles as well as the liquid slugs because with the same particular velocity we can have different combinations of lance of the liquid slug and the Taylor bubble so unless you know that difficult to find out the frictional gradient. So these are the few things which I wanted to say I also wanted to cover the annular flow pattern probably I will not get the time for it from the next class you will be studding boiling and condensation and after that I will be taking up few more class on the measurement system in two face flow if time permits at the end. Then I will come back to the angular flow pattern and discuss about few portion of it or else you can refer to web course to find out the details of horizontal slug flow the pressure drop part as well as the annular flow pattern thank you very much.