Mutliphase Flow
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Lecture No. # 26
Analysis of Specific Flow Regimes

Good afternoon to all of you. So, till now whatever we have done, we have done the flow, we have done simple and first I have given you an introduction and then we have discuss the different flow patterns which are available, and after that I have discuss some way simple and analytical models. Now, among the analytical models for mixed flow patterns or for well depressed flow patterns, we have discussed the homogenous flow theory. And then I have said that when there is a relative motion between the phases which is a quite common occupancy, and under very rare circumstances only at very high phase velocities only, we find that the two phases are so intimately mixed that they behave as a single sudo fluid and the homogenous flow conditions applied.

Under most of the conditions, even when the two phases are very mixed together there is a relative motion between the two phases, why? Because the lighter phase we will naturally try to travel faster than the heavier phase. So, therefore, there is a split and under such conditions, we often have to correct or modify the homogenous fluid theory by incorporating the drift flux model. On the other hand the other extreme was the separated flow model therefore, what we found that it was mainly there for flow patterns where the two phases occupy separate cross sections of the tube for example, the animal flow pattern and the satisfied flow pattern are ideal examples of the separated flow pattern. So, under that condition we found that the two fluid model is very applicable, where the two phases are considered separately.

And separate mass momentums in energy equations are written for the, and then there are solved by considering suitable interaction between the interphase. For simplified matters we can ignore the interfacial interaction, and we can use several empirical correlations to find out the two phase pressure drop, where we assume that the two phase pressure drop primarily comprises of the frictional component. Or in other words, the two phase pressure drop it is primarily, it is means we are trying to find it out for horizontal pipes under conditions, where the acceleration and the gravitational pressure drops are negligible, so after completing the analysis of the homogenous flow model, the gift flux model, as well as the separated flow model. Now, we would like to take up specific flow patterns applied the simple analytical models, and see how we obtain the, or rather how we analyze the specific flow patterns.

Now, since they do not have much classes after this, maybe I will be able to give utmost three classes to this specific flow patterns, then you will be doing boiling condensation a good amount of it, and after that we will be doing the measurement techniques and instrumentation. So, since I do not have much classes, I will be trying to cover the analysis for bubble flow patterns and this slug flow patterns. Slug flow pattern I will do because it is a very unique flow pattern, it has got a periodic flow phenomena, so this analysis is slightly difficult. Well, the other thing the bubble flow pattern I will take up, because it is an ideal case of a mix flow pattern, I would also like to take up the annular flow pattern.

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But in that case I even if I do not have time I will leave it for your analysis part, or other for yourself study part. You please grow through it apply the two fluid model there, and accordingly you can analyze the annular flow pattern.

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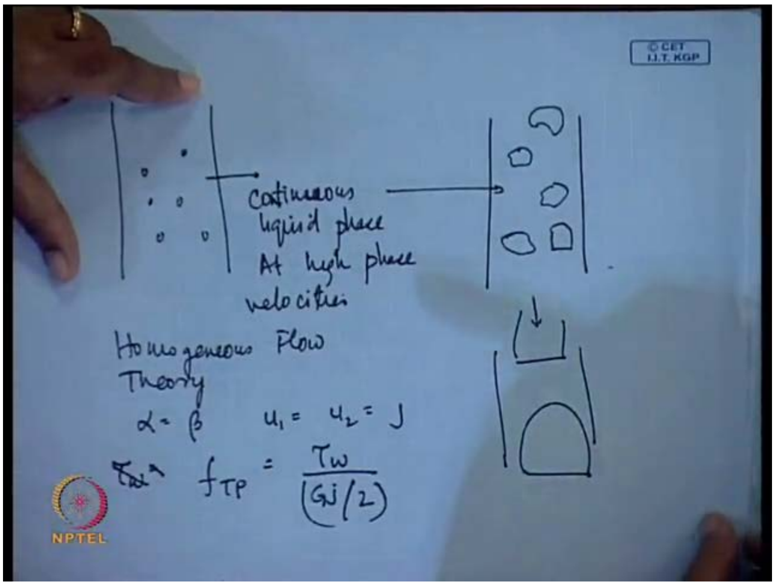

Now, lets us take up the bubble flow pattern. Now, for the bubble flow pattern what have we observe? It is basically, it has got a continuous liquid film, sorry a continuous liquid phase, I am sorry continuous liquid phase with bubbles dispersed in it. Now, this is the ideal most case where very small bubbles are dispersed, this occurs at high mixture velocity or the high phase velocities. Otherwise, we usually do not get such situation, we find that we have a liquid phase where bubbles of different sizes and shapes are depressed. Now, this sizes and shapes of the bubble they basically depend, maybe we have different sort of bubbles, and the sizes and shapes that primarily governed by the volume of the air, which we have injected by the relative proposition of the two phases.

Now, we find that in both the distributions it is characterized by a continuous liquid phase, where the dispersed phase is the gas phase, but the dispersed phase can take on a variety of shapes and sizes, and in the limit we find that gradually as we keep on increasing the gas velocity finally, we find that these bubbles (( )) and they form such type of tailor bubbles, and liquid slugs where we get the slug flow pattern. Now, it is quite evident that the analysis which will be applicable here may not be applicable here also. So, more or less we will be discussing the unified approach, and then we will be seeing how the analysis takes into account the different shapes and sizes of the bubbles in the different extreme cases. Now, suppose in this particular bubble flow, now what we know when the bubbles are uniformly depressed as I have shown in this particular picture, under such condition we very well know that the homogenous flow theory is applicable. When the homogenous flow theory is applicable we know that alpha equals to beta, or in other words u 1 equals to u 2 equals to J, isn't it? Under such circumstances what do we know? We know that your tau w it is given this can be obtained, or rather we can write it in terms of f 2 phase, the two phase friction factor. That can be written in terms of tau w by G J by 2. Under such circumstances we can find it out.

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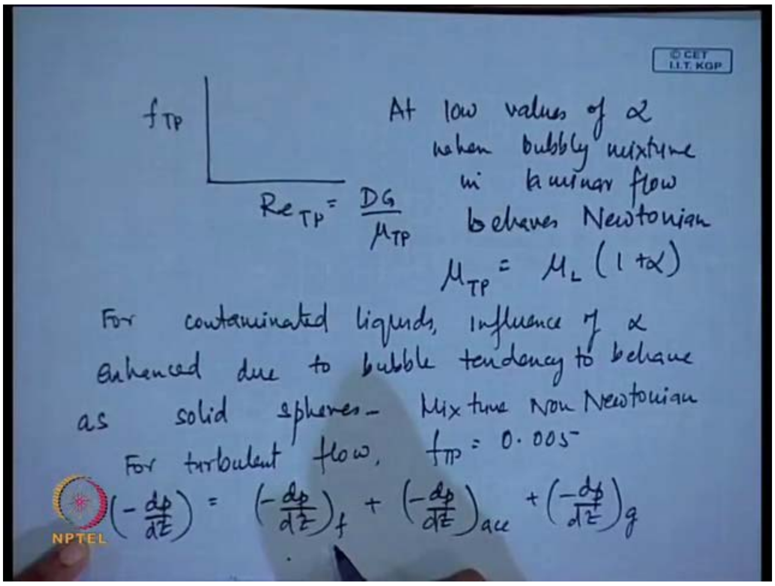

Then if we plot this f T p as a function of R e T p, or rather f T p if we plot as a function of R e T p, than in that case we can find out f T p and accordingly we can find out the frictional pressure drop under these conditions.

Now, one thing we know that at low values of alpha for low values of alpha we know that the only hitch of finding out R e T p f is to find out mu T p. Now, for low values of alpha we know when the flow this laminar, when bubble mixture in laminar flow behave Newtonian. So, at low values of alpha when bubble mixture in laminar flow behaves in Newtonian fashion, under that condition we know mu T p it can be written down as mu l or mu 1 into 1 plus alpha. And for contaminated liquids of course, for contaminated liquids we find out that the influence of alpha, it enhances the tendency of the bubbles to behave as solid spheres.

So, therefore, when the value of alpha is very low, we find that the bubble mixture it is in it is in laminar flow, it behaves Newtonian fluid, under such conditions mu T p can be written in this particular expression. And when the liquid is contaminated, influence of for contaminated liquid, influence of alpha enhanced due to bubble tendency to behave as solid spheres. Why does it happen? It happens, because whenever there is some contamination, the contaminations tend to deposited on the interphase between the gas and the liquid, interphase is nothing but the bubble surface.

So therefore, when the bubbles surface becomes contaminated its start behaving as solid spheres. So, under that condition of course, we cannot say that the bubble mixture is in laminar flow, and under that condition the mixture it becomes, it exhibits non Newtonian behavior, it can exhibit or yield stress or it show different non Newtonian behaviors, it can exhibit in stress, it can exhibit a decreasing apparent viscosity with increasing shear rate, it can also exhibit certain other electric effects and so on so forth.

So, therefore, under such circumstances we cannot use mu T p, for turbulent flow usually when happens when the liquid is contaminated the bubble concentration is very high. So, for turbulent flow usually f T p can be taken as a constant value of 0.005, this is a good approximation. Now, we find that whenever we have find out the pressure gradient for this particular bubble flow case, the pressure gradient as you already know this is a summation of three components. This we have discussed several times, you already know it is a summation of the frictional component, the acceleration component as well as the gravitational component. For horizontal flow we know that the friction the gravitational component becomes equal to zero.

Now, for the frictional and the acceleration components, now we find that usually the frictional and the acceleration components they are important at high phase velocities, whenever the relative velocities higher the wall shear stress becomes more important therefore, the momentum flux effect as well as your wall stress effects, they become much more important for high flow velocities. Fortunately, under that condition the flow exhibits homogenous flow characteristics. So, when your wall shear stress becomes important, when the momentum flux becomes important, under that circumstance we find that homogenous flow theory is applicable.

Now, at low phase velocities when the relative motion of the two phases become important, or the gas or its important to consider the relative raise of the gas bubbles with respect to the surrounding liquid phases or in other words when the drift flux model has to be applied, we find that the wall shear stress as well as the momentum flux influences, they are not so very important. So therefore, the bubble flow pattern the analysis can be divided into two parts; one is at high phase velocities where we can use the homogenous flow model, this is applicable when the liquid flows at a high velocity, and the bubble is depressed as very fine depression in the liquid phase. Just as I have shown in the first figure here in this particular case. Now, as if we come down to lower and lower phase velocities naturally the bubble sizes increases, as the bubble size increases, they get more deformed, and they take a variety of shapes starting from spherical, spheroidal, oblate spheroids, cap shapes and so on and so forth. Now, when this happens the bubble definitely has a relative motion with respect to the liquid, and under such circumstances homogenous flow model is going to be erroneous. Under such circumstances we apply the drift flux model, what does the drift flux model do?

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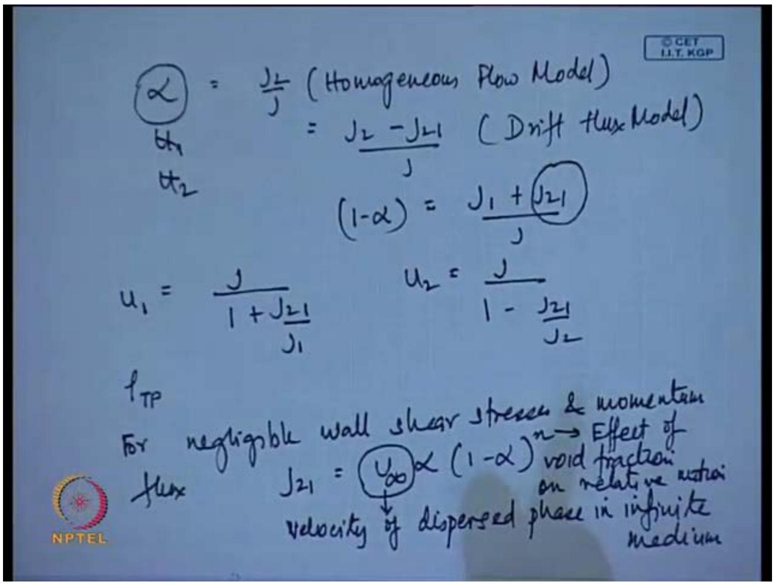

We have already discuss the drift flux model it modifies the expression of all the parameters starting from alpha, u 1, u 2 and momentum flux kinetic energy etcetera, by incorporating the relative motion between the phases, or by incorporating the drift flux between the phases.

Now, we have already reduce that alpha from homogenous flow model it is J 2 by J. This is applicable for the at high phase velocities, now movement the phase velocities become lefts, this becomes J 2 minus J 2 1 by J, is it not? This is this we get from the drift flux model. So, moment relative velocity becomes important, this is incorporated as J 2 one in the model, this we have already done. Similarly, if we take a 1 minus alpha, this is nothing but equal to J 1plus J 2 1 by J. Same way we had already discuss, I will just write down the expressions, u 1 it was obtained as J by 1 plus J 2 1 by J 1, this derivations I believe we have already done. u 2 was 1 minus J 2 one by J 2 and accordingly we had already derived momentum fluxes and things like that.

Now, we find that whenever we are operating at not very high velocities alpha as to be modified, moment we have modify alpha we can modify rho T p, we can modify u 1 u 2 and so on and so forth. Now, for alpha, in order to find out alpha when the relative motion is important, what are the… What is the parameter that we have to find out. The parameter which is not a input but as to be determined is J 2 1, J 1 J all those things we can already find out from input parameters. So, therefore, if relative motion is important then naturally J 2 1 becomes important, and we need to find out J 2 1.

I had already we are already find out that when wall shear stress for negligible wall shear stresses, wall shear stresses and momentum flux. This we had already derived, we found out that J 2 1 can be expressed as a unique function of alpha, as well as the two other parameters; one was u infinity which is the bubble raise in a infinite medium, and the other was end which in incorporates the effect of void fraction on relative motion. Remember, this we have I already told you while studying the delft flux model, that more or less J 2 1, it can be expressed or the call of J 2 1 can be approximated by this particular equation, or void verity of fluid pairs fluid property etcetera, where for the different flow situation u infinity and n take different values.

So, this was the velocity of dispersed phase in infinite medium, and this incorporated the effect of void fraction on relative motion, is it not? So, this we have already done. So, therefore, we find that in order to find out J 2 1, we need to know u infinity and n, and at this point I would like to mention that we cannot adopt as unique value of u infinity and n for the entire range of bubble flow, even then we said bubble flow which is characterized by a more or less uniform depression of bubbles in a liquid medium for all the entire range of bubble size, entire range of bubble flow distribution, we cannot adopt a single u infinity and the single value of n. It is phi, because again if you see this picture it is quite evident, that derive velocity of this particular single bubble in an infinite medium, cannot be same as raise velocity of this particular single bubble in infinity medium, cannot be equal to this particular single bubble in an infinity medium, is it not?

So, therefore, we find that u infinity and n have to have different values in order to incorporate different ranges of the bubble flow pattern.

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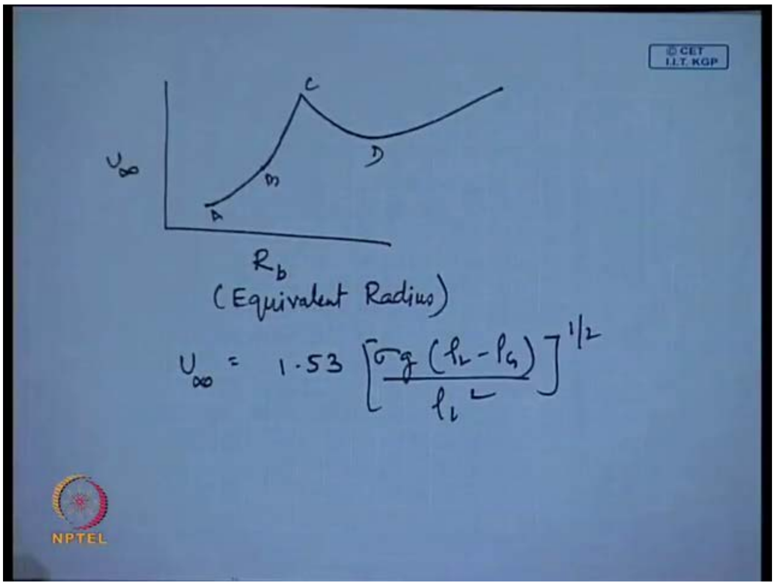

Now, usually we find that if suppose you plot u infinity with n, sorry u infinity with say the bubble radius R b, than in that case you do not get a smooth curve.

The curve is something of this sort, you get something on this sort, and then it is increase raises deeper, and then it falls, and then it something of this sort, more or less this can be distributed into say 4 or 5 sort of distributions. At this R b it is the equivalent radius, where it is the radius of the sphere which as the same volume as the bubble. So therefore, we find out that in order to find out the pressure drop or any particular property of the bubble flow pattern we need to know J 2 1, now J 2 1 is a function of u infinity and alpha. We need that u infinity… We will also find n that is also function of the bubble radius.

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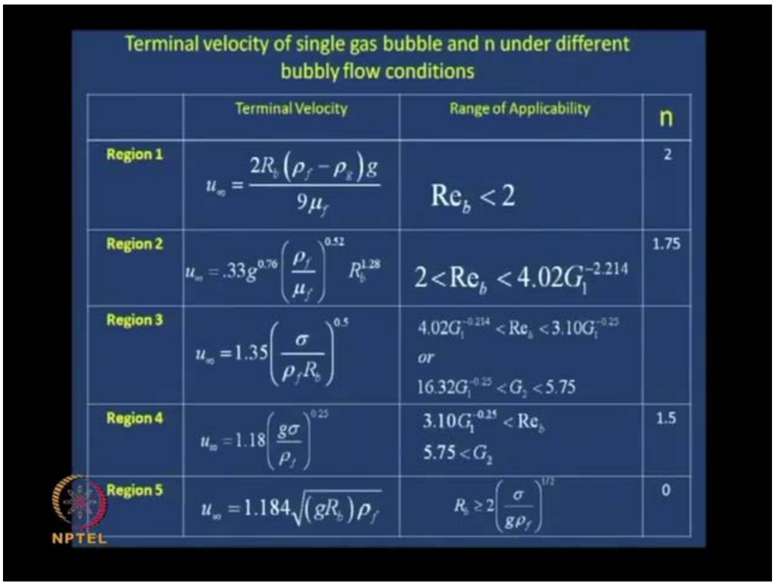

Terminal velocity of single gas bubble and n under different bubbly flow conditions

	Terminal Velocity	Range of Applicability	n
Region 1	$u_\infty = \frac{2R_b(\rho_f - \rho_g)g}{9\mu_f}$	$Re_b < 2$	2
Region 2	$u_\infty = .33 g^{0.76} \left(\frac{\rho_f}{\mu_f}\right)^{0.52} R_b^{1.28}$	$2 < Re_b < 4.02 G_1^{-2.214}$	1.75
Region 3	$u_\infty = 1.35 \left(\frac{\sigma}{\rho_f R_b}\right)^{0.5}$	$4.02 G_1^{-0.214} < Re_b < 3.10 G_1^{-0.25}$ or $16.32 G_1^{-0.25} < G_2 < 5.75$	
Region 4	$u_\infty = 1.18 \left(\frac{g\sigma}{\rho_f}\right)^{0.25}$	$3.10 G_1^{-0.25} < Re_b$ $5.75 < G_2$	1.5
Region 5	$u_\infty = 1.184 \sqrt{(gR_b)\rho_f}$	$R_b \geq 2\left(\frac{\sigma}{g\rho_f}\right)^{1/2}$	0

So, therefore we need to know either graphical or analytical expression, in order to express u infinity in terms of some sort of a bubble parameter, it can be bubble be equivalent bubble radius or something of that. Now, several different correlations have been purposed for this particular condition, and we find that the correlations which have been purposed by pebbles and grabber, and this has been given in a table of form in values, this suggest the value of u infinity and n for different particular flow regimes. What they have done, they are derived the entire bubble flow condition into five flow regimes, what is the first regime? In the first regime we have very fine depressed well depressed bubbles, if you find we will find that this the raise velocity of a single spherical bubble in a infinity medium, which we can all obtain simple for stokes law. Now the first regime it comprises to thus strop loss out of a regime, and that is applicable for a bubble in a very low bubble Reynolds number.

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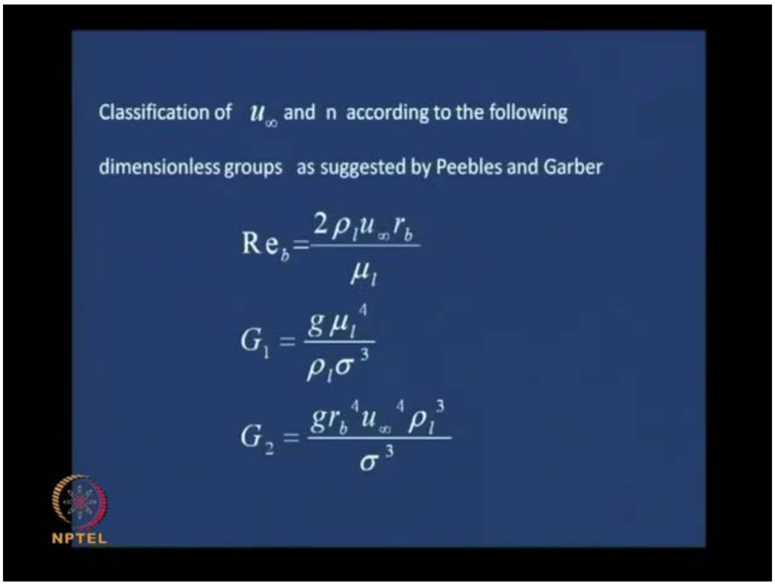

Now, we find that the entire bubble flow has been divided into five flow regimes on the basis of three dimensional groups. First group is the bubble Reynolds number, and the other two groups they are some sort of property groups, there are does been designated as G 1 and G 2, where G 1 is nothing but G mu l to the power four by rho l s sigma q, G 2 equals to it is also a property group which also incorporates the equivalent bubble radius, as well as the bubble raise velocity in a infinity medium. This all this things available in the book of values you can refer that if you can see it.

Now, we find that based on the values of these dimensional list groups, the entire bubble flow and condition can be divided into five flow regimes. The first flow regime is naturally the sloop law regime, regime two is we spends lightly larger bubbles are there. And then gradually we find that as we go into regime four and regime five, we find that usually the regime five expression to the bubble is not correct here, I do not need remember the regime five expression, but most probably the regime five expression we have to check it. Probably there where was no R b in the regime five expression that as to be checked, this particular regime five expression please do not take it from here, this regime five expression is not correct. As far as I remember this regime five expression this is 1.53 sigma G rho l minus rho G 5 rho l square whole to the power half, it is something of this sort as far as I remember I am not very sure about it.

But, regime five expression is not correct here, regime four and regime five we find that the raise velocity expressions they are independent of equivalent bubble radius, and they are a function of liquid property only. So therefore, here we have the terminal velocity u infinity values, and for each regime the corresponding n values are also given here. So, once what you suppose to do? Suppose, your suppose to find out the bubble, suppose you suppose we require to find out the void fraction or the pressure drop then bubble flow pattern. First for that you have to write down the expression of the pressure drop, there we find that alpha is important. Moment the alpha becomes important than that alpha it as to be found out from the delft flux model, for finding out alpha we need to find out J 2 1, for finding out J 2 1 we need u infinity and n, for finding out the u infinity and n you have to refer to this particular table.

Now, for determining in which particular range your particular bubble lies, you have to find out the values of R e b G 1 G 2. Accordingly, you select the range, once the range is selected you find out u infinity and n corresponding to that particular regime, and then we can find out J 2 1, we can find out alpha, we can find out the pressure gradient. Now, if you observe this closely you find that for regimes 1, 2 and 3, in order to find out u infinity you need to know R b, what is the R b? R b is the equivalent bubble radius or T wages of the volume of a sphere which as the same volume as the bubble.

So therefore, next what we would have to do is? We will have to find out different methods of determining the equivalent bubble radius. How to find out equivalent bubble radius?

Now, we know one thing that we wave a bubble is formed that depends, or rather the size of the bubble depends on the way it is formed. Bubbles can be generated by different ways for examples, we can have orifice phase with its tip which is directed inside the liquid, we can introduce some gas to the orifice and the gas comes out as individual bubbles, or in other words we can have a poorest plate, and may be gas is introduced to the poorest plate, so its forms the blanket over the poorest plane, and from this blanket the gas distiches and it comes out. We can have an agitator, where liquid gas mixture is agitated and some amount of bubbles comes out. Now, the size of the bubble depends upon how the bubbles have been generated.

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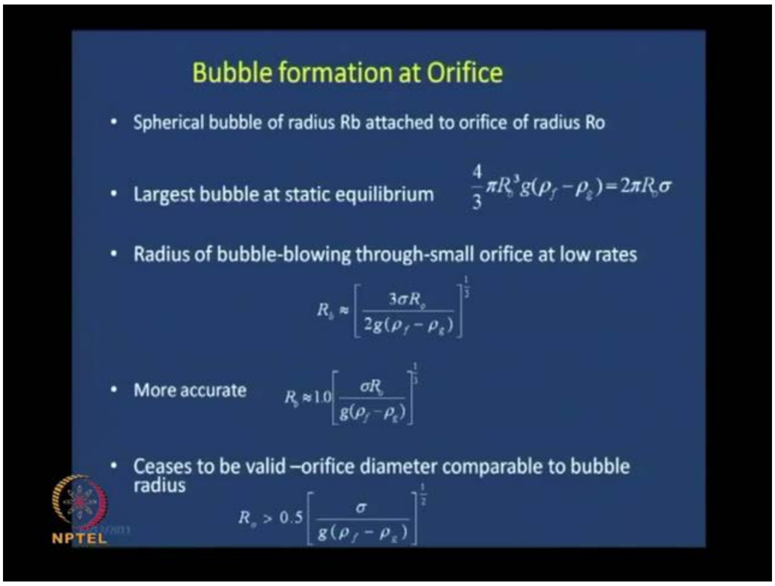

The different ways of bubble generation, and the size which you get as a result of this, I have also noted it down because they simple comprise of some particular equations, equations come from force balances, some of the equations they are empirical equations but all of them are available in standard text book, and so we need did not take them down. But I have just I will just be discussing the different methods of bubble formation, and how the bubble size is determined for the different methods. For example, the most common technique is bubble formation at the orifice. Now, for this particular case we find that assuming that the spherical bubbles of ranges R b, they are attached to the orifice of radius R 0. So, it will be attached for a finite interval of time till the neck it distiches at the neck and the bubble is declutched, it raises up.

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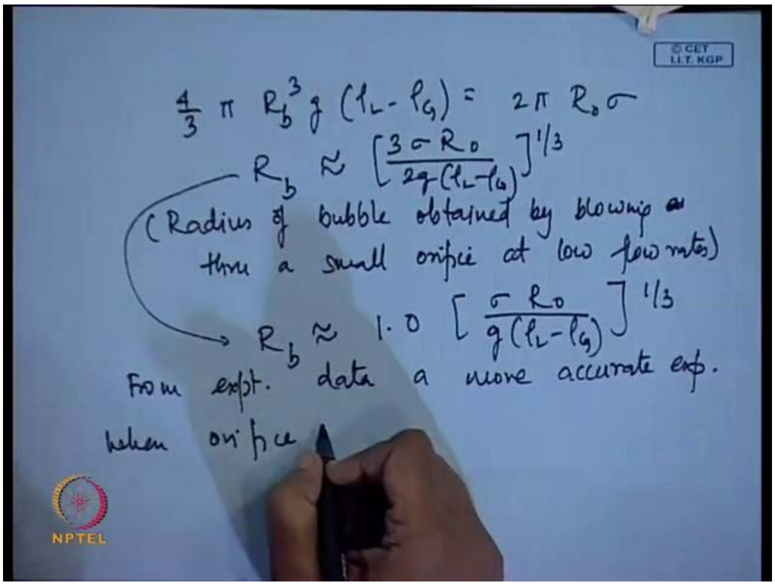

Now, from force balance what do we get, if you perform of force balance in this particular case we find that four third phi R b cube into G rho l minus rho G, this will be equal to two phi R 0 sigma isn't? The surface tension forces that basically helps the bubbles to remain attached at the orifice, and this is equal to the buoyancy force due to which the bubble will detach and it will raise.

Now, from this particular situation we find that the wages of the bubble, which we obtained by blowing through this small orifice at low flow rates, this is the radius of bubble obtained by blowing through a small orifice at low flow rates. So, from this particular mass balance if you work it out, you find this will be more or less equal to 3 sigma R 0 by 2 G into rho l minus rho G whole to the power one third, it simple comes from this particular equation.

Now, we have observe or rather people have reported that instead of this particular expression, a more accurate expression is 1.0 sigma R 0 by G into rho l minus rho G. People have observed that from experimental data, a more accurate expression is this particular expression. So therefore, we can find out R b either from this expression or from this expression, and we also find out that when orifice diameter, probably this things already written down here, orifice diameter if you can see, if in the orifice diameter is comparable to the bubble radius, then under that condition these particular equations this is to be valid.

When will that happen? When your but d b… The d zero in other words two R 0 becomes comparable to R b, under that condition we find that the condition can be represented by this particular expression, under that situation the expression or the equation is no longer valid in this particular case. Now, we further observe that when the bubbles which are formed at a finite rate, under that circumstances the bubble radius or the bubble dimension it depends upon a large number of others things also for example, gas and liquid properties, details of the orifice design, the nozzle and so on and so forth. It depends upon a number of things for such particular situation. But more or less under normal circumstances we find that this spherical bubble, the radius can be obtained from this particular expression, and it is more or less accurate under normal circumstances. So, this was for bubble formation at the orifice.

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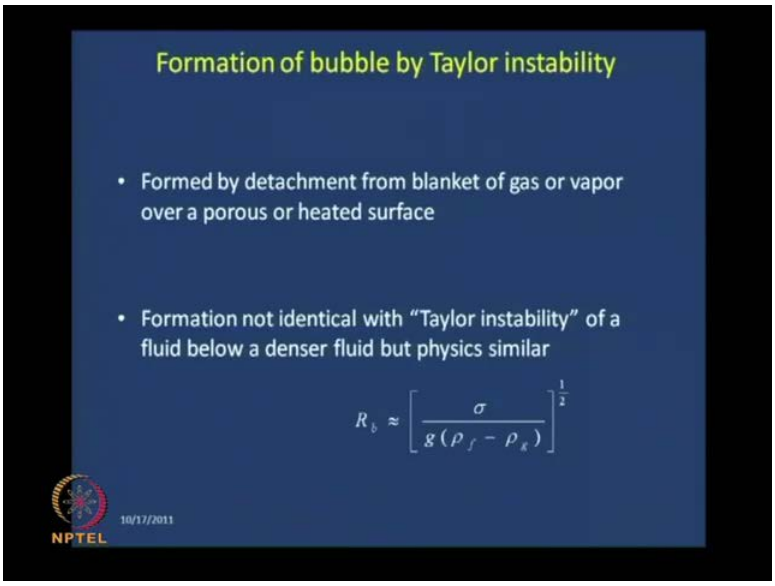

Now, what is the next way of forming the bubble? It is which tailor instability, what is the tailor instability? This is that when there is a…. Just as I have telling suppose, there is a porous a cylinder porous medium or may be the heated surface, so where the heated surface what happens? There is a blanket of gas or a vapor which is deposited on that particular surface, and from this particular blanket gradually the bubbles they get detach, and they start raising in the tube or in the in that particular conduct.

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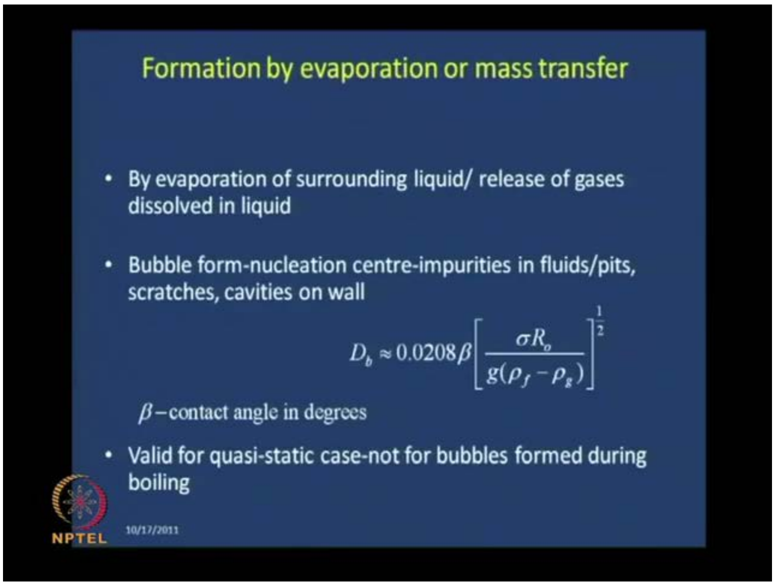

So, for such particular circumstances we find that R b can be obtained from this particular expression under this case, so we get this is particularly important for film boiling. So, for such situations, we get or rather we can obtain R b from this expression. What is the next way of deciding? The next way of deciding is suppose the mass transfer or the evaporation of this surrounding liquid, or may be some amount of gas is dissolved in the liquid, due to mass transfer this amount of gas they get released from the liquid, may be bear, soda etcetera, and this particular things some amount of gas is dissolved, due to release of pressure, this gas they get dissociated and they start raising as bubbles. Or maybe, due to evaporation of the surrounding liquid some amount of bubbles are generated and they start rising.

So, under such circumstances what we have? Under such circumstances we find that in this particular case the expression is in terms of equivalent diameter. So, equivalent diameter which is just large enough to break away from a horizontal surface, this is obtained from the expression here, where since the surface is important here, the beta be contact angle in the gears is also becomes important and it is included in the expression, right?

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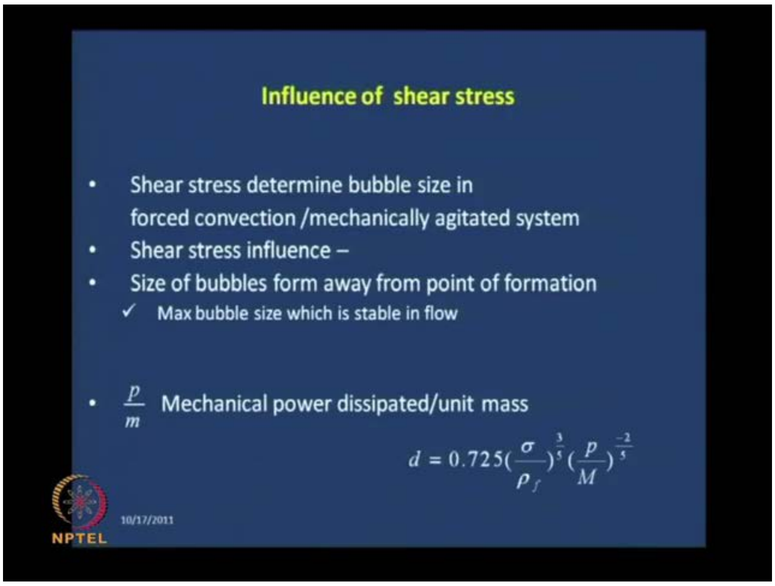

Well, the other way in which the bubbles can be formed, this is by the influence of Shear stress. As I was telling in force convection or in mechanically agitated systems, then under such circumstances the Shear stress it influences the size of bubbles away from the point of formation.

And the size of bubbles, they are as well as the maximum bubbles size which is stable in the flow, both of them they are decided by the shear stress in this particular case, and under this particular circumstances this should have been R b Sorry d b here, the equivalent bubble diameter can be obtained from this expression, where p by m is nothing but the mechanical power dissipated per unit mass. So therefore, depending upon the way bubbles have been formed we can find out d b or R b as the case may be. Usually they are form by when any particular gas is ensuring through an orifice or some sort of special nozzle, they can also ensure by shear stresses or may be when there is evaporation of the surrounding liquid, or may be dissolution of the gas dissolved in the liquid, or may be when the gas blanket is raising through a poorest plate, or during film volume.

So, whatever the case may be depending upon the situation, we are going to find out or rather we are going to select the accurate expression of d b. So therefore, using that expression we can find out either the equivalent bubble diameter or the equivalent bubble radius. Please remember, the equivalent diameter is nothing but the diameter of a sphere which as same volume as the bubble.

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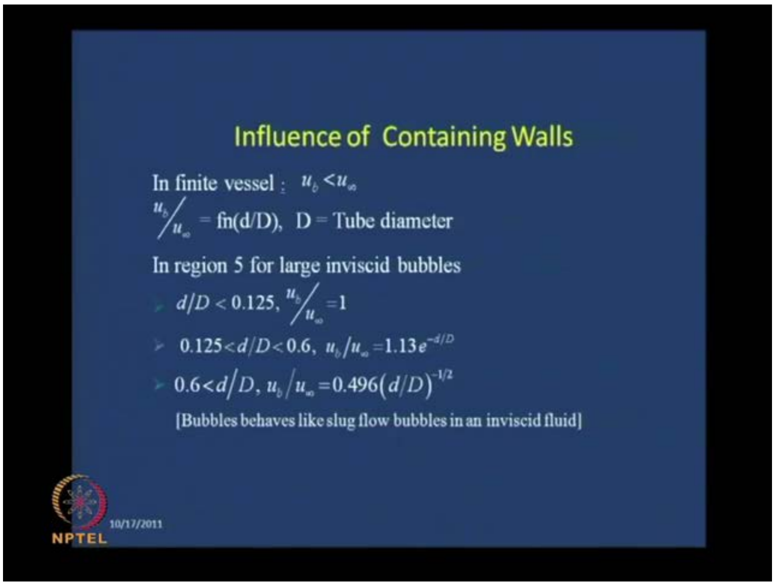

So therefore, once you find out that then what we do we go to this particular table we use the value of R b that we obtain, find out R e b G 1 G 2 from these expressions, and then decide in which particular regime you are going to lay, accordingly select terminal velocities and n. Now, at this particular point I would like to tell you 1 particular fact, in order to find out n sorry in order to find out u infinity, what is your infinity? Velocity of a bubble in an infinity medium, but can in reality can you generate an infinite medium? For example; say tailor bubble, it will not be formed in an infinite medium, so we have to perform that the experiments in a confined tube.

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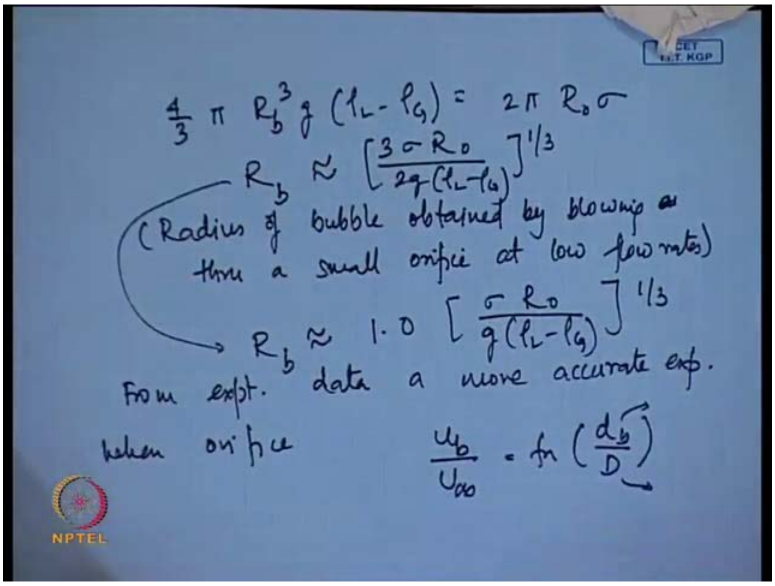

Now, whenever we are performing in a tube due to the wall effect, the bubble will rise at a slower rate as compared to the velocity at which it would rise, had it been in an infinite medium? Because naturally, the confining walls will try to restricted its motion is it not? And it is observed that the rise velocity of the bubbles u b by u infinity, this is a function of the d b by d where this is equivalent bubble diameter, and this is diameter of the tube. If you observe this particular transfer slide which I have got, we find that in regime five, the last regime which we had for large invested bubble, it is found that when d by D is less than 0.125, than in that case we can very well assume that the u b that we have we have found out that can be used as u infinity. But movement the ratio d by D it keeps on increasing, the relationship between u b and u infinity they also change, and depending upon the ratio of… these are all d b the equivalent bubble diameter. Depending upon the ratio d b by D, we will be using the appropriate expression of u b by u infinity, and then we can find out u b from the experiments and accordingly we can find out u infinity in these particular cases. Now, this was what in visit bubbles.

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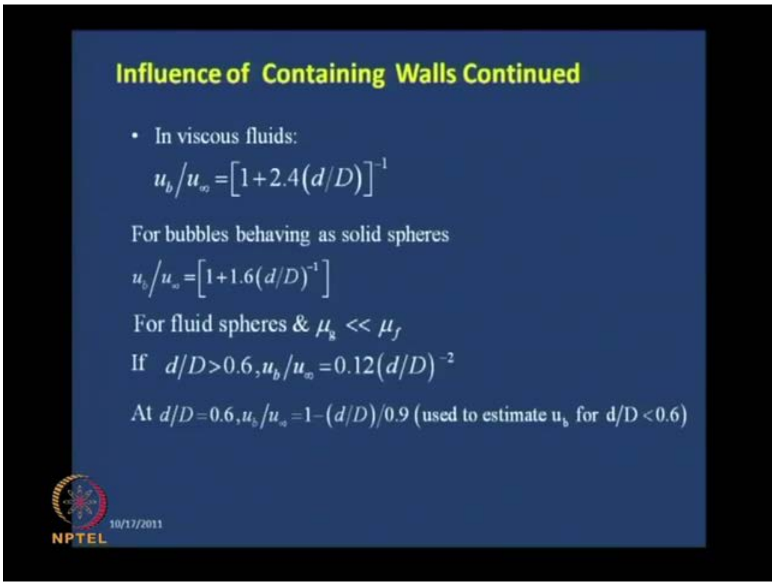

Now, for Viscous flow again there are different expression, for bubble behaving a solid sphere, for flow for flow bubbles behaving as fluid sphere; that means, liquid raising through the liquid for each and every case there are different expressions, which we can use under this particular cases.

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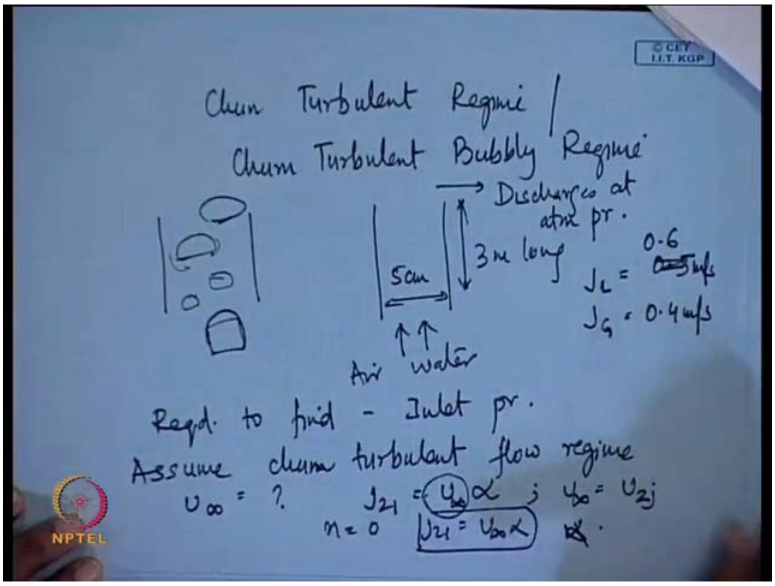

So therefore, this is how we find out the or rather we use the delft flux model for the bubble flow pattern. Now, remember whenever we using the delft flux model for bubble flow pattern, under that circumstances usually the acceleration pressure drop, and the frictional pressure gradient they are usually much less as compare to the gravitational pressure drop, in order particular in a vertical pipe. And for finding out the vertical, the gravitational pressure drop we need to know alpha, for finding out alpha we need to know J 2 1, and particular for bubble flow how we have found out J 2 1, we have already discussed.

Now, there are certain other important things which I would like to mention at this particular stage. Now, as I have told you regime one, this is the stokes regime where the bubbles are prefect sphere and the raise in almost straight lines. Now, as we keep on going for higher and higher gas velocities, the bubble straight getting larger and larger. Finally, we find regime four, this particular regime the bubbles are quite large, since that quite large there is a weight effect of this particular bubbles, they are very much entrapped into each other's weight, and the particular situation it appears as a, more or less as a very random sort of a distribution, and therefore the u infinity under this case it is not a function of bubble equivalent bubble radius.

And regime five it is not actually the bubble flow pattern, it is the transition between the bubble and the slug flow pattern. So, this your suppose to remember, it is known as the Churn turbulent bubble flow pattern. Why it is given this particular name? It is given this particular name because the distribution resembles the Churn flow regime. So, it is either the Churn turbulent regime, or it is named as the Churn turbulent bubble regime. This particular situation we find the situation, which I have try to draw in the previous slide the situation is completely erratic, and we have a large number of cap shape bubbles sorry the cap shape bubbles and several bubbles, and what happens the bubble since they are quite large, they have good amount of weight effect and the bubbles get entrapped into each other weight.

And therefore, there is no influence of void fraction on G 2 1, and that is why the value of n it is zero in this particular case, please remember this. So therefore, regimes one to four, they are the different divisions of the bubble flow pattern, while regime five it is commonly termed as the Churn turbulent bubble regime, which is the transition between the bubble this slug flow pattern, where the bubbles they fall into each other wakes, there is a wide range wide variety of bubbles sizes and shapes, and more or less the influence of void fraction then it starts becoming less than one, and in the limit it becomes zero.

There is one more thing also which I would like to tell you, suppose you do not make any additional effects to control the bubble size, under that condition suppose you are just introducing air, there is water flowing and you just introducing air, you make no additional effects to control the bubble size, the bubble flow or anything. Then in that case the bubble will do whatever is best situated for it, under that condition depending on the fluid rates the bubble will just simple break down into a range of sizes, into range of shapes and there will be randomly distributed in the fluid mixture, so and it will lie in regime five, the Churn turbulent flow regime. So therefore, in any problem, if it is not mention that any special effect is being made in order to control the bubble size the gas flow rate or something on that sort, then you can very safely assume that the distribution lies in the Churn turbulent regime, and accordingly you have to use the u infinity and the n which are specifically applicable for such flow conditions. This is the thing that you suppose to (( )). So, nothing is given, then you can straight away assume that the bubble or the flow pattern lies in the Churn turbulent bubble flow pattern.

For other situations we have to make some special effects to control the bubble size, the bubble gas flow rate and so on and so forth. For example, suppose if you do a problem and I will just discuss the problem here, I will be giving it to you as a home assignment, so that you can do it and we can discuss it later. Say simple in one particular vertical pipe, say it is 3 meters long and it is 5 centimeters wide say, it is 5 centimeters wide say it is 3 meters long, and air and water are being introduced here in this particular pipe, and we know that it discharged at atmospheric pressure, this discharges at atmosphere pressure.

Now, we can assume the inlet flow rates they are given your J l, this is given as 0.3 or 0.5 meters per second, and J G is given as 0.4 meters per second, or in other word this can be 0.6 meters per second, J G can be 0.4 meters per second. So therefore, we know that through this particular 5 centimeter diameter, 3 meters long pipe air water is introduced, and they discharge at atmosphere pressure, you are required to find the inlet pressure? required to find the inlet pressure Or in other words we required to find out the pressure drop across this, once you know the pressure drop you know the exact pressure you can find out the inlet pressure.

Now, for this particular situation how do you purpose to proceed? We see one thing that both J l and J G, and one thing is told that you can assume bubble flow to occur; this has been told to you. Now, how to proceed in this particular case, what do you suggest that you should do for this particular case? First thing is, moment it is written assume bubble flow and nothing is given, we straight away assume the Churn turbulent flow regime, this is number one that we can do. Once you assume the… So therefore, since nothing is specified first thing we do, because more when you do not try to control the bubble size, they will be distributed in a wide range of sizes, and whenever there is a wide range sizes it will be a Churn turbulent flow regime.

So, the first thing that you can do is, you can assume a Churn turbulent flow regime. The movement you assume a Churn turbulent flow regime, you find that u infinity or rather u infinity this expression is not correct which is their, u infinity can be obtained for the Churn turbulent flow regime. Now, next we can find out J 2 1, we know n equals to zero, so therefore u infinity into alpha is going to be your J 2 1, where your u infinity is nothing but equal to u 2 J in this particular case. So, what we do? First thing we do is we assume Churn turbulent flow regime, movement we assume Churn turbulent flow regime, we can find out u infinity from the table or rather from the table which is given, and we know n equals to zero. So, J 2 1 equals to u infinity into alpha, this we already know.

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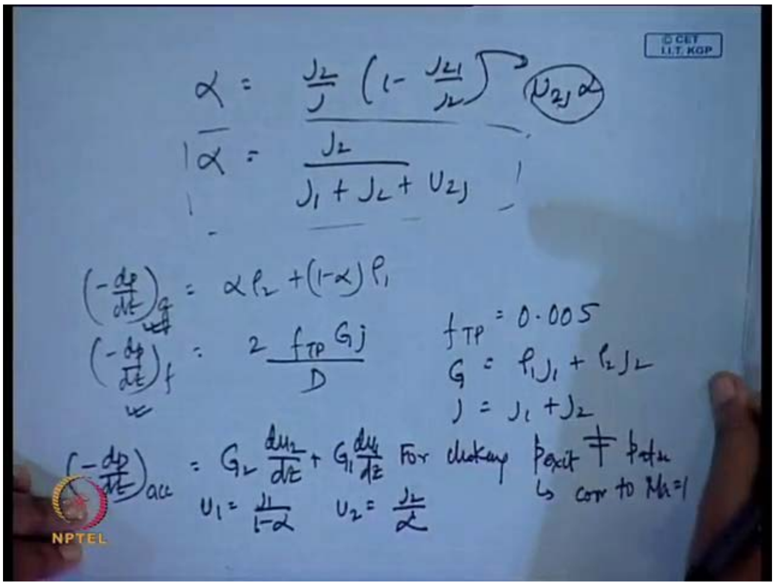

And again from drift flux model what do we know? We know alpha, I will write it in the next slide. We know alpha this is equal to J 2 by J into one minus J 2 1 by J 2, this also we know. Where this J 2 1 it is nothing but u 2 J into alpha. So, if we substitute this particular value of J 2 1 here, then we find that alpha can be obtained as from this particular expression, right? So therefore, alpha can be obtained from the expression which we have provided, u infinity equals to u 2 J this is known, J 1 j 2 is known, so alpha can find found out very easily. Now, what are the pressure gradients here? There is an acceleration pressure gradient, there is a frictional pressure gradient, there is a gravitational pressure gradient.

Gravitational pressure gradient is very simple, movement you know alpha you can find it out. Your frictional pressure gradient that also we have discussed out, the frictional pressure gradient this is nothing but 2 f T p G J by d isn't it? You can find out the whether it lies in the laminar or the turbulent flow regime. Usually, under normal circumstances for Churn turbulent bubble regime, f T p it rather it lies in the Churn turbulent or in the turbulent flow regime, so f T p can be taken in this particular form.

So therefore, and what is G equals to, this is nothing but rho 1 J 1 plus rho 2 J 2 agreed? What is J equals to J 1plus J 2? So therefore, this can also be found out, this can also be found out, agreed? Now, the next thing is finding out the acceleration pressure gradient. Now, one small catch I would give here, what you have to check up? It is given that its exit pressure is atmosphere pressure, you are required to find out the (( )) pressure. Now, you have to check whether the flow is under sonic conditions or not, if the flow is under sonic conditions then naturally, the outlet pressure of the pipe will not the atmospheric pressure, it will be the pressure corresponding to mach number equal to 1, please remember this thing. These are small catches in the mid sem also I had given you I had given you a problems by the catch, most of you could not get of the catch, but remember these things are very important.

So therefore, this we have to keep it in mind when you proceed, that the just check whether the flow corresponds to sonic flow or not, whether there is choking or not. If choking is there, for choking I will write down p exit will not be equal to p atmospheric. So, it will be p exit will correspond to m a equals to 1, so these small things you are supposed to remember. So, what about the acceleration pressure drop? It can be obtained as G 2 d u 2 d z plus G 1 d u 1 d z isn't it? This can be obtained, u 1 and u 2 we know very well, u 1 equals to J 1 by 1 minus alpha u 2 equals to J 2 by alpha alpha and 1 minus alpha we can obtained from this particular expression. I will just discuss the problem, and then you are suppose to do it and your suppose to submit this problem.

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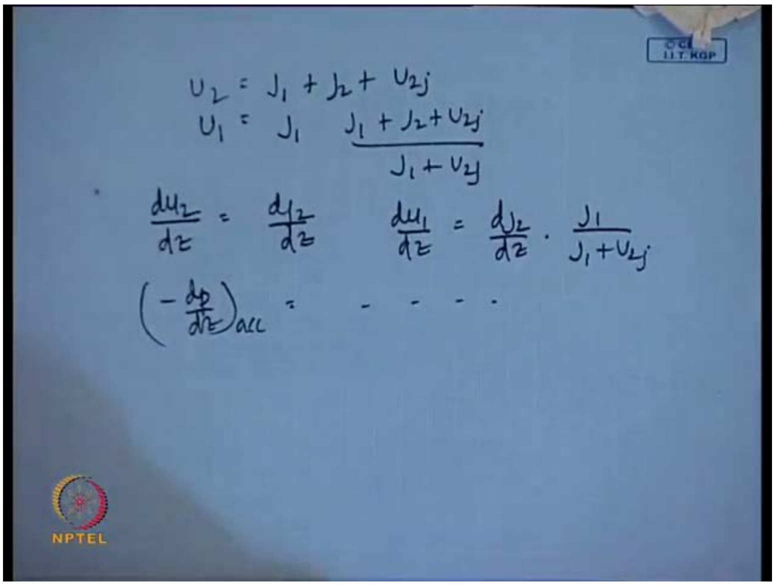

So, then accordingly we find u 2 form the substitutions, this u 2 equals to J 2 by alpha and this alpha we substitute this particular expression, so therefore we find out that u 2 it is nothing but J 1plus J 2 plus u 2 J. Similarly, u 1 this is J 1intoJ 1plus J 2 plus u 2 J by J 1plus u 2 J. Now, we find out that since the pressure is changing along the pipe, what will change? J 1is not going to change only J 2 is going to change is it not?

So therefore, just because of J 2 change, there is a going to be a acceleration pressure gradient in this particular case. Now, since there is no temperature change, I do not know whether I had given the temperature, write down the temperature condition it is around say 27 degree centigrade, T it can be taken 27 degree centigrade. So therefore, we find that…. Under such circumstances we find that only J 2 changes down the duct, nothing else changes. And what is the change of J 2 in the duct? The change of J 2 it occurs just because there is a pressure change in this particular case.

Now, so therefore, due to J 2 change what do we get? G u 2 d z its going to be d J 2 d z and d u 1 d z, this is going to be d J 2 d z J 1 by J 1 plus u 2 J. Accordingly, minus d p d z acceleration this can be, this particular expression can be written down here. Then we can add up this acceleration expression, frictional expression, gravitational expression and get the expression of the total pressure gradient. Once we get the expression of the total pressure gradient then from there we can find out the condition of choking, we can find out what is m a, the value of m a. Verify that m a is not equal to 1, it is less than 1? And when it is less than 1, we can assume the exit pressure is equal to the atmospheric pressure, and accordingly we can find out the d p d z, and for three meters length we can find out p entry when the p exit is atmospheric pressure.

So, this is the way you are suppose to proceed for such type of problems, when we applied drift flux model to the Churn turbulent bubble flow regime. So, tomorrow we will be taking up this slug flow regime, and will find how the slug flow regime is going to be, rather how we applied the drift flux model to the slug flow regime. And if I find time we will be discussing the annular flow pattern or else annular flow pattern in will be left as a self study for you. Thank you very much.