Multiphase Flow
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Lecture No. #25
Separated Flow Model Estimation of Frictional Pressure Drop and.
Void Fraction (Contd,)

Well to continue with our discussions on the improvements of the Lockhart Martinelli correlation which I should say. So, initially as I have already mentioned the Lockhart and Martinelli correlation, this was applied for your adiabatic two phase two component flow systems in horizontal pipes.

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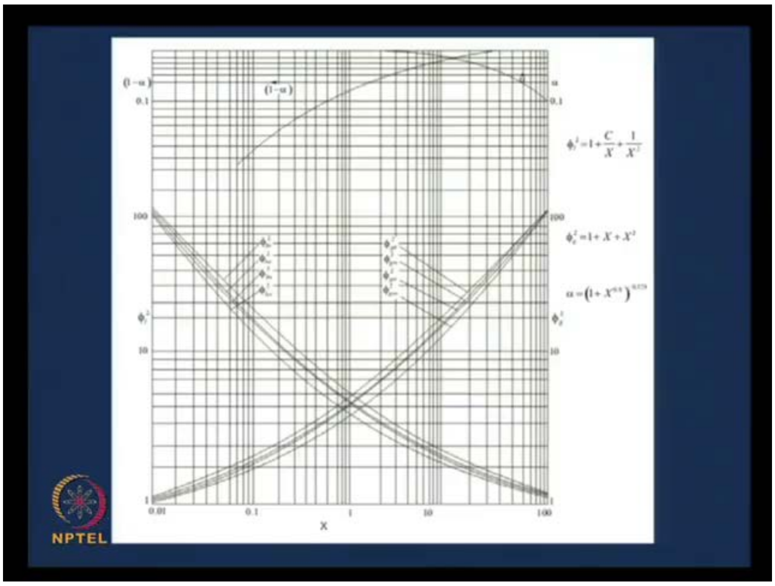

So, after that the next modification which people suggested were, firstly they noticed two things, if you notice the graph which has been plotted here you find the people they noticed that this the curves are not continuous throughout, they show certain they are not smooth actually they show certain discontinuity of sloop and people thought that this must be associated with a change of flow pattern.

Because with flow pattern naturally the pressure drop characteristic are bound to change since the interfacial distributions are different for the different flow patterns, this was the first observation which people found. The next thing which people observed were that these particular curves they are dependent they are shown as if they are completely independent of all input parameters, but they have a dependence on flow velocity, or they are dependent on mass flux, and mostly these particular correlations they are applicable for the mass flux g between 500 to 1000 kg per meters square second, this was what they had notice people have plotted this for different mass flux situations and they found that there is a definite influence of mass flux on x.

I do not have the flux but any particular text book will give you the dependence of x on mass flux, you find that they are definite different curves which you obtained when you plot these particular this phi this x with g, and they found the ideal curve which was given that was applicable for a mass flux between 500 to 1000 kg per meter square second. So, this was the first thing. So, so these two were the first things which were noticed in this particular case.

Now, the first modification in for this particular situation was purposed by Martinelli and Nelson, for they observed that the correlation has been developed for two phase two component system as I have already told you, and that too at low pressure close to atmospheric, and it is application to the range of condition outside the range for which this empirical correlation was developed is definitely not recommended. This is something very understandable for any empirical correlation that you develop, you cannot use moody’s plot f is equal to 16 by r e if the flow is not laminar is not it.

So, that is common for all empirical correlations. So, subsequently what marten they are Martinelli and Nelson these coworkers what they did they tried to generalize it, they found out that this was also applicable for single component two phase flow situations as well, and they could be used for steam water mixture under low pressure conditions. But for prediction of pressure drop during forced circulation boiling, they found that Martinelli and Nelson these workers they found that the correlation is not always applicable. So, they tried to improve it they assumed that for force convection boiling both the phases would be in turbulent flow.

So, naturally the graph here which is applicable for phi L t t that particular graph will be applicable, so the correlation of frictional pressure gradient it is expressed by Martinelli and Nelson as phi L o square, because that is much more convenient in boiling and condensation as I have told you.

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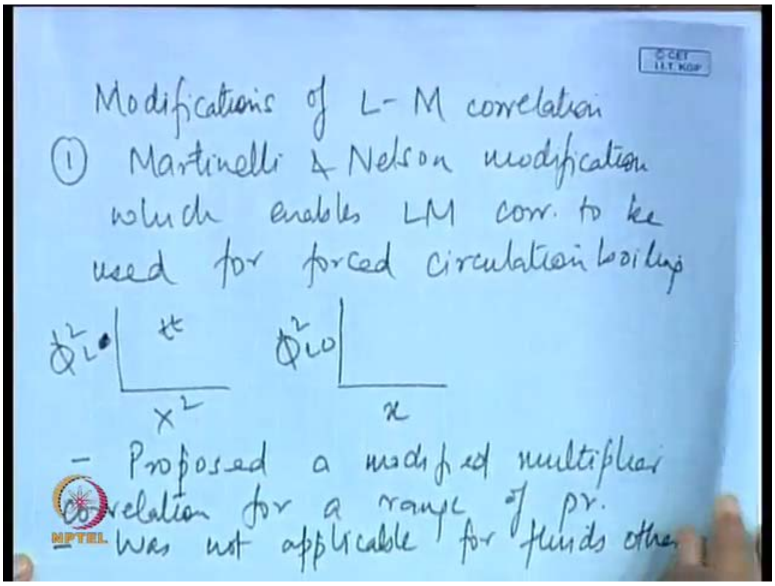

So, if I consider the modifications of Lockhart Martinelli correlation or the separate cylinder model the first modification was given by Martinelli and Nelson, Nelson modification, which enables Lockhart Martinelli correlation to be used for forced circulation boiling. So, for this what they did instead of phi L square they plotted phi square L o, and they used this particular situation for the case where both the phases are turbulent turbulent they plotted phi square L o verses X square. And then what they did they assumed thermodynamic equilibrium at all conditions and then applied this phi L t t correlation, to atmospheric pressure steam water flow and then they also tried to find out the relationship under critical pressure conditions. And then for intermediate pressure conditions they tried to interpolate the graph between the atmospheric pressure and the critical pressure.

So, accordingly they generated the family of curves for different pressure conditions for steam water flow, and they found out that more or less with a value of c equals to 1.36 the different values of c, which I had give here, so they found out that with a value of c equals to 1.36 more or less the relationship can be expressed. So, from phi L square sorry this was phi l square verses X square for the turbulent flow regime they could obtain the graph for atmospheric pressure and under critical pressure conditions, and after that they generated a curve of phi L o square with mass quality and from this particular curve they found that is was easier to find out the two phase pressure gradient here.

So, accordingly they purposed a reverse multiplier correlation to fits steam water data over a range of pressures, so there thing is there modification according to their modification they purposed a modified or reversed multiplier correlation reverse multiplier correlation which could be applicable for a range of pressure, which could be could because your whenever it is a change of phase then under that condition your pressure becomes very important. So, in this particular case they plotted the curves and the for low pressure situation and they found that the curves did not become (( )) under the critical conditions where the two phases become in distinguishable.

So, accordingly they modified the data and they found out reverse multiplier correlation to fit data for steam water mixtures over a range of pressure, and they found out that this relationship it was good for steam water, but it was not accurate for other fluids at the same reduce pressure, what do we except? That for the same reduce pressure means the ratio of pressure to critical pressure. So, under these condition from the law of corresponding states which we have already studied in thermodynamics it is excepted where the same correlation should be applicable for all fluids under the same reduced condition that was not that did not happen, although your they found that this particular correlation this is applicable for your steam water mixture they could not use it or are the correlation was not accurate for other fluids under the same conditions of reduced pressure.

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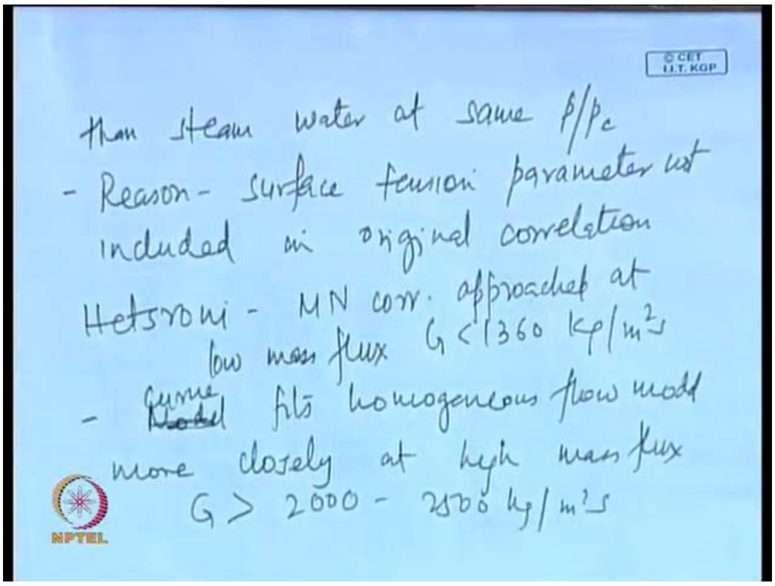

For they found out they thought that this arises because the correlation does not have a surface tension parameter, so therefore, what they found was that this particular correlation this Lockhart Martinelli correlation did not contain a surface tension as a parameter, and therefore, the part of the pressure effect, which was observed by the researchers this was due to the variation of surface intension of water with pressure, this is the thing which people found out, that it they purposed this; this was not applicable for fluids for fluids or other than steam water at same p by p c reason surface intension parameter not included in original correlation.

So, this was there observation that all although it was very good for steam water cases, but it was not good for other liquids and that is the reason the attributed that the surface tension parameter as not been included, and a part of the pressure effect which has been observed by researcher that was due to the variation of surface tension of water with pressure. So, in this particular way they where they observe this and this mass flux effect this was observed by a large number of researchers, if you take up any text book say for example, the hand book of multiphase system, which is there by Hetsroni you will find that they have given you a curve or a family of curves where this Lockhart Martinelli parameter it varies as a function of mass flux.

And they found out that the curves they correspond to the Lockhart Martinelli correlation for lower mass flux, and they convert with the homogenous flow model for higher mass flux, quite in expected situation because for lower flow conditions only the two fluids will be under separated flow, and as you go for higher and higher mass fluxes naturally they will tend to depressed into one another.

So, therefore, it was observed by Hetsroni and he also shown it by a family of curves, I did not reproduce this curves here, they found out that the Martinelli Nelson correlation approach at low mass flux g less than 1360 kg per meter square second, and model which or rather curve fits homogenous flow model more closely at high mass flux for which G is greater than 2000 to 2500 kg per meter square second.

So, this was also not a counted by the Martinelli correlation, so with the two problems one was in the horizontal correlation the pressure effect was not very well taken care of, and the mass flux effect was not very well taken care of, the other thing is the curves they show at definite discontinuity in sloop which shows that this particular discontinuity probably is related with the flow pattern transitions. So, if this pressure effect is not taken into account then naturally what happens for two phase two component it is fine, but for single component two phase flows naturally pressure effect becomes very important, Martinelli and Nelson, they tried to derived correlations taking the special effect into account.

So, they obtained curves for atmospheric pressure and they all also obtained curves for critical pressure and then they interpolated between the two extreme curves to obtain the curves for other pressure conditions as well. And based on the curves of phi L turbulent turbulent as a function of x square, they tried to generate curves for phi square Ll o as a function of mass quality, since they consider change of phase. So, when we start from saturated conditions and we undergo change of phase with heat flux inside the pipe, naturally we are seen that phi l square L o is much more appropriate then phi square L.

So, they tried to generate those particular curves, but yet they found that although it was fine for steam water cases, but it could not be applied for other fluids, now this again they counted for the fact that this surface tension is also function of pressure, and since surface tension parameter is not included in the correlation that is why although it is good for steam water it is not good for other fluids, the other fact, which is was not accounted for was there was a definite effect of mass flux, so these this Martinelli Nelson correlation the Lockhart Martinelli correlation while they were appropriate for lower mass fluxes between 500 to 1000 kg per meter square, we found that the mass flux curves or the curves with mass flux as parameter they corresponded more closely to the homogenous flow model for higher mass fluxes.

When the mass fluxes they exit 2000 to 2500kg per meter square in definitely the homogenous flow model is better, when there is a lower mass flux say from 500 to 100, or less than 1360 kg per meter square second then this Martinelli Nelson correlation was appropriate. So, the first thing which they could do is to modify the correlation for other pressure conditions for steam water fluids, but yet they could not use it for other fluids.

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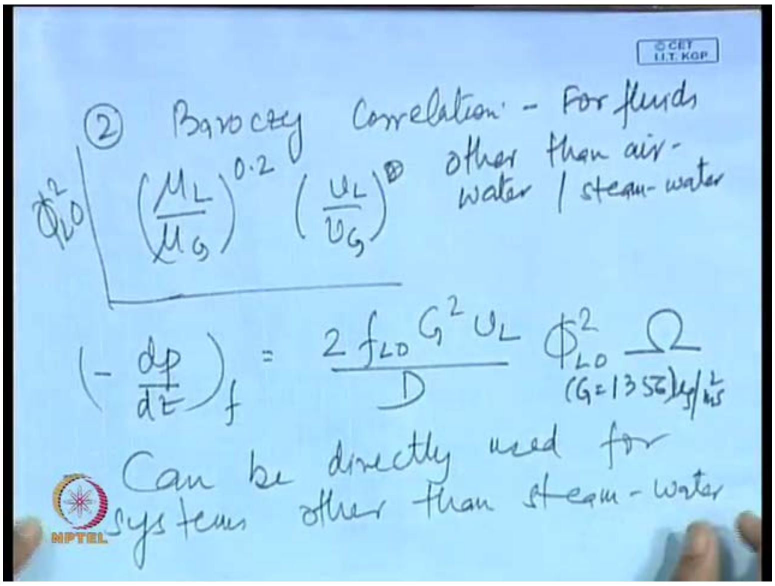

So, for other fluids the next correlation came up, the next modification this correlation was purposed by Barocty this is the most widely used correlation for fluids other than steam water flows.

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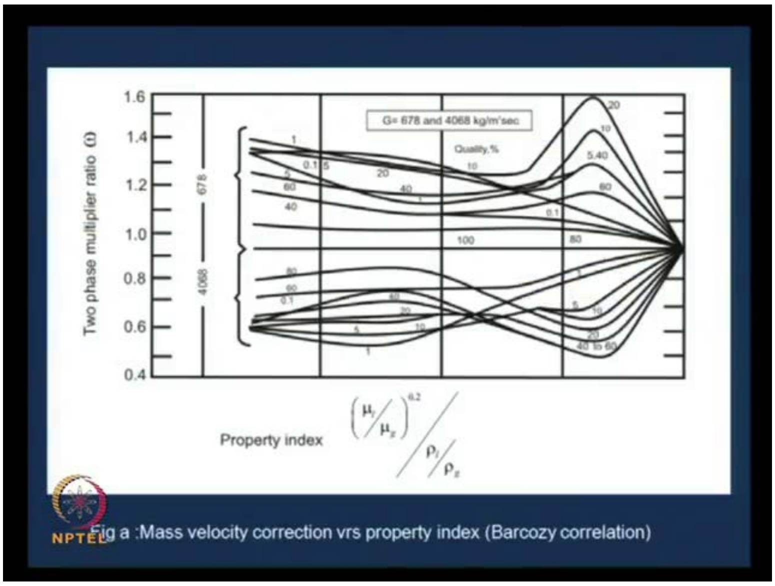

Fig a :Mass velocity correction vrs property index (Barcozy correlation)

Now, this particular correlation the only problem of this correlation is it is a graphical correlation, the graphical correlation if you find that they relate the two phase multiplier the phi l square with property index, if you see the property index it contains the ratio of viscosity of the gas phase with the viscosity of air, and the ratio of the the sorry the ratio viscosity of the liquid phase with the viscosity of water, and the ratio of density of the liquid with the density of water, this is not very well given the ratio basically if you write it down the ratio is something of this sort, it is mu l by mu g whole to the power 0.2 v L by l g to the power nothing no problem. So, so they basically tried to correlate phi l o square, it is not very clearly written this was the x axis and the y axis was phi square l o.

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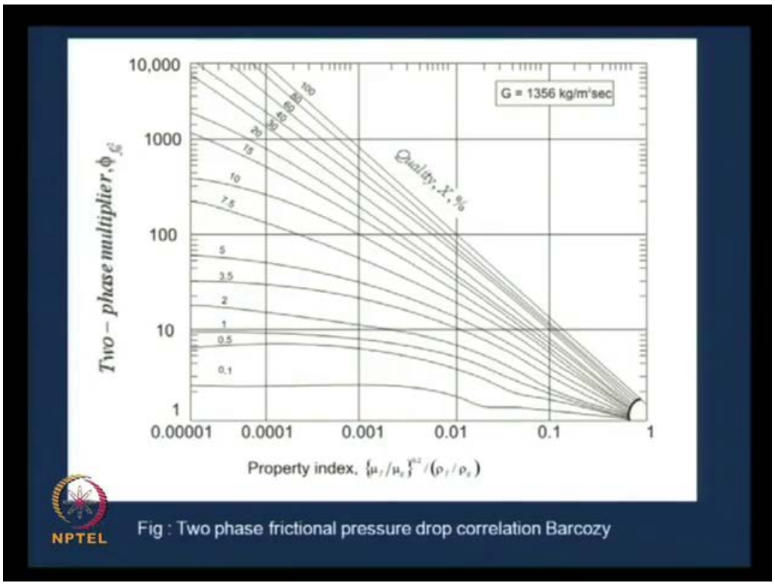

Fig : Two phase frictional pressure drop correlation Barcozy

So, they obtain two separate set of curves for if you observe this particular graph this is for two different mass velocities, they basically they had generated four set of curves, for four mass velocity conditions, or four mass flux conditions, if you have to deal with any other mass flux conditions then it can be done by interpolation between the mass fluxes which are giving here, so therefore, they generated a set of four set of curves or four family of curves for four different mass flux condition, then for each particular set they tried to relate omega square sorry phi square L o with a property index term. So, depending upon the property index they could use it, and the basic curves which were there the first curve which they have obtained this correlated your phi square l o with your property index which has been given.

This particular curve it had been generated for g equals to 1356 kg per meter square second. So, once you have this particular curve you are in a position to find out phi square l o, there was a certain mistake which I had made this is the original curve which he purposed Baroczy, this curve it correlates phi square l o with the property index which have already mention. So, from this particular curve if you know the quality or the mass fraction of the two phase mixture then using the property index you can find out the two phase multiplier phi square l o from where you can find out the frictional pressure gradient, now this is for a particular mass flux, if you have to deal with other mass fluxes what you need you need a correction factor.

Now, this correction factor is purposed by this set of curves, this curves they seem to be very erratic and it is very umbilical sort of thing, but they are very effect and they give more or less accurate results. So, once you sorry once you find out the two phase multiplier from this you can get it for a particular mass flux, for other mass fluxes what you have do you have to refer to these particular curves, they give you a sort of a correction factor as a function of property index, this correction factor this is represented as omega, it I do not know whether it is evident here this particular correction factor is omega.

So therefore, the second set of plots these plots they solve to correct the phi square l o which we obtained from here, from here we want phi we have obtained a phi square l o this is corrected by the omega which can be obtained from this particular curves, and finally from this the expression of the frictional pressure gradient which we get it is obtained as minus d p d z f this is 2 f l o g square v l by D, this is the frictional pressure gradient for liquid flowing or the entire mixture flowing as liquid, this into phi square l o this is the correction factor, this gives you the frictional pressure gradient for a mass flux of g as something of this sort.

And after that this is for a particular mass flux, after that what we can do we can simply multiply this with an omega, this phi square l o has been obtained for g equals to 1356 kg per meter square second, then you multiply this with omega this finally this whole expression finally gives you the frictional pressure gradient for the two phase separated flow for any particular fluid other than air, water, mixtures, now tell me if this portion is clear to you how to use the Baroczy correlation, this correlation is specifically for fluids other than air, water clear, for steam water air water we had already discussed air water or steam water for other fluids other than air water steam water.

We found out that Martinelli nelson was not appropriate for this particular case. So, in this particular case what we can do is if we have to do deal with other fluids then we have to go for the Baroczy correlation, this correlation is entirely graphical what he does is it generates a set of two sets of curves. What are these curves? First curve it gives you phi square l o as a function of property index with quality as parameter, from here you can find out phi square l o for g equals to 1356 kg per meter square second.

Now, naturally you have to deal with other G values as well, when you have deal with other g values as well then you need a correction factor, this correction factor is again plotted as a function of property index in this set of curves, from here you can find out omega and then finally the frictional pressure gradient can be obtained from the expression which I have written down, where the first portion it contains the frictional pressure gradient when the entire mixture is flowing alone as water fine, or alone as a liquid fine, this is multiplied with the two phase multiplier which gives us the equivalent frictional pressure gradient when under the two phase flow conditions separated flow.

This gives you the two phase flow pressure drop for g equals to 1356 kg per meter square second, for other g you have to include this omega you to multiply this with omega. So, this finally gives you the frictional pressure gradient expression, in order to calculate frictional pressure gradient when a two phase mixture flows as a separated flow situation, this is the most widely use correlation for fluids other than air water or steam water, it can be used directly with systems other than the important part is it can be used this is the important part of this correlation, it can be directly used for systems other than steam water.

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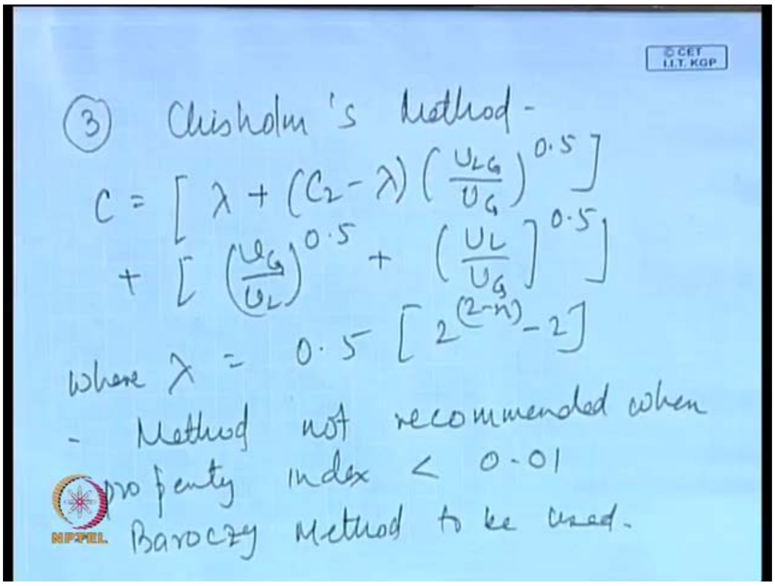

Now, what Chisholm did was there was a third these are just certain suggestions we need not to mug up these correlations or anything, these as certain things to show you that the extent of effects which people have given for developing the different thing, what Chisholm's did they purposed a simple method to incorporate the effect of mass velocity on phi l square.

What they did if you remember we had a C expression here, so this particular C they defined it as a very elaborate function of mass flux and certain other quantities.

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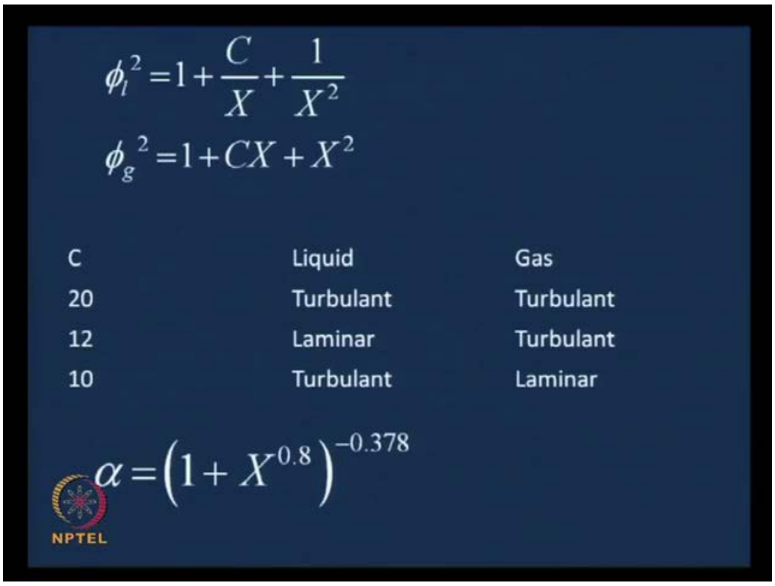

So, there correlation basically there is no point in just you can see the form and you can see that the extent of complexity which is there here, plus v g by v l whole to the power 0.5 plus v l by v g whole to the power 0.5. So, this is the correlation where lambda equals to 0.5 2 to the power 2 minus n minus 2, so this was the type of correlation which Chisholm purposed, and this particular correlation this incorporates the effect of mass velocity on phi l square by assuming that C is not a constant as was purposed by Lockhart Martinelli correlation, but it is a very complex function, and this n it as different value for smooth tubes, rough tubes, for different pressure, for different mass flux this n as got different values.

So therefore, big chats which you can see and you can find out the value of n for your particular situation, for your particular tube, for your particular mass flux, your particular pressure conditions, accordingly you can find out the value of n substituted find out lambda and then you can find C from here and you can do it, now this method it can be adopted for systems other than steam water flows, simply by calculating the property index value for the system considered, but remember one thing this Mathnasium method it is not recommended when property index value is less than 0.01.

For under such circumstances the Baroczy correlation is recommended. So, remember this method not recommended property index less than 0.01, Baroczy method to be used for such circumstances this Baroczy method has to as to be used, now there are certain other set of correlations also, I would not like to go into all the details we there is feudal correlation as well.

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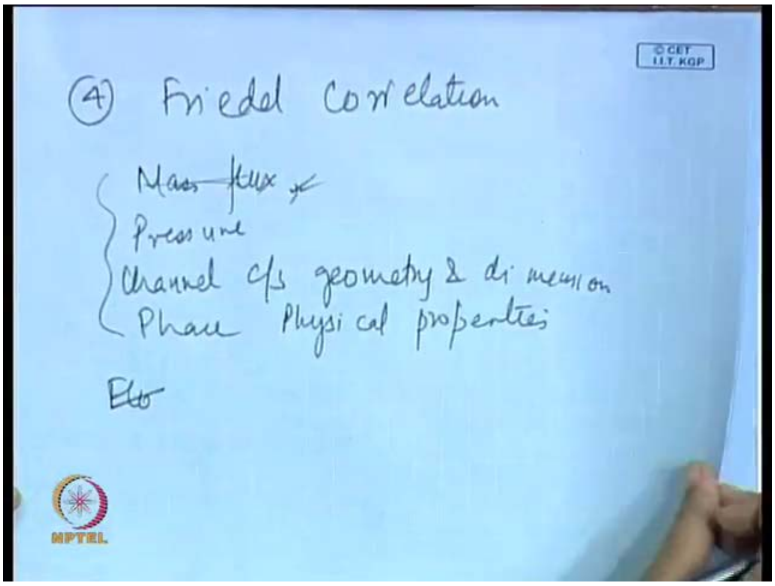

So other than this Baroczy correlation, there is a feudal correlation you just remember I will not go into the details of everything, but more or less the most common things which we use is Lockhart Martinelli or the Martinelli Nelson curves, if you have to go for other fluids we used a Baroczy correlation.

But we have a rather than I have been certain improvements of other correlations as well, now in this particular case there just one thing which I would like to mention that over the past say half a century at a over the past 50 years also lot of correlation have been purposed, there has been improvements over the exiting correlation and this purpose is still now going, still now we cannot say that this method is definitely very appropriate to find out the frictional pressure gradient for two fluid model or for separated flow conditions, now this arises because two phase flow is a completely random phenomena.

Now, when we tried to develop correlations what are the parameters that we consider, we consider say one is there flow rates or we consider say mass flux as one, then we consider say the pressure, and then we consider the main things is may be the channel cross sectional geometric, channel cross sectional geometric and dimension, then phase physical properties, we always thing that whenever two phase flow occurs these are the things which are important, mass flux is a function of these things. So, generally sorry sorry sorry this is all right. So, generally we consider that the empirical correlations they should be based on the channel cross sectional geometric and dimension, because it is the contact of the channel wall with the fluid which gives you the wall shear stresses.

The phase physical property definitely must be important, the pressure the mass flux etcetera must be important, but remember one thing when even two phase flow occurs then in that case the way the two phases are introduced that is very important, and we do not know what is the developing length for two phase flows, under certain circumstances there cannot be any development for example, when there is change of phase occurring when we give some heat flux and the change of phase which occurs then the entire pipe is developing length at it consist of developing flow.

So therefore, unless these things can be considered a proper correlation a proper relationship to find out the frictional pressure gradient is not possible, because we just take the core things, may be the channel dimension, the phase physical properties or almost whether the pipe is rough or smooth, just things which we use for single phase flow situations, but apart from this two phase flow being a much more complex and much more random phenomena, there are other things also which we cannot account for, if the flow is over the entire thing the flow is unsteady state, if over the entire range the flow is a developing type of flow, say counter current flow, in counter current flow what is the developing length you tell me?

For the entire range your liquid is entering the gas is going up for the a throughout the entire pipe the flow is developing, when there is change of phase until the entire liquid converts gets converted to the vapor phase there is simple change of flow, so it is a totally developing flow type of a thing. So, under such circumstance how can we think that a single correlation will be able to predict your frictional pressure drop for all the flow distribution which are encountered, when saturated liquid is entering it is changing phase and as the it flows up and down more and more amount of vapor is being produced, when more amount of vapor is produced distribution between like liquid and vapor changes.

So, naturally frictional pressure gradient cannot be predicted by one particular method in this particular case is not it. So, due to all these uncertainty due to all this anomalies we find that there is no unique method for finding out the frictional pressure gradient, at the same time there is no point in going for more and more complex situation, because as you go for more and more complex correlations the amount of uncertainty associated with finding of the constants become much more. So therefore, the balance as to be stack and depending upon your particular flow situation you have to assume or you have to adopt a proper correlation, so more or less for your purpose your Lockhart Martinelli correlation and the Baroczy correlation Martinelli Nelson curves these are sufficient.

Now, before I end I would like to discuss one particular situation which is which since because of it is industrial application it is very well means it is used so frequently that a set of graphical correlations are available, and computing that particular situation is very easy, that particular situation is flow of boiling water in horizontal circular pipes, now for this particular case under low pressure conditions for a circular pipe and n under horizontal condition we find that each and every pressure gradient term that has been expressed as a function of your mass fraction and other input parameters, graphically in such a good way that just by referring to those curves or just by referring to those curves you will be in a position to find out the pressure drop or the pressure gradient when boiling water flows through a pipe.

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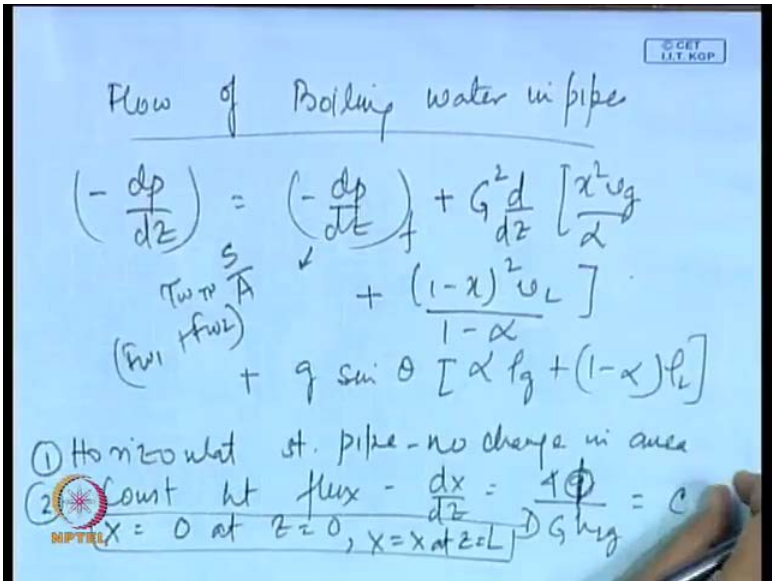

So, will just discuss those particular graphical techniques before we end the class, so next the thing we are going to do is well take a different pipe now in this particular case if you find out you see that what was the original derivations which we did, when we deriving the two fluid model the mixture momentum equation which we obtained what was that this was equal to minus d p d z frictional, which you can express it as tau w two phase your weighted parameter by a, or you can express it as f w 1 plus f w 2 in whatever way you can express it.

So, the frictional pressure gradient plus the acceleration pressure gradient if you remember the expression we at finally obtained a expression something of this sort, I do not know you must have this derivation in your plus your g sine theta into rho t p which is nothing but alpha rho g plus 1 minus alpha rho l, this was the basic definition which rather this was the basic expression which we have derive for the mixed from the mixture momentum equation for two fluid model, I presume you remember this particular expression is not it.

Now, in this particular expression if we assume that the tube is horizontal and liquid enters under saturated flow conditions, and one horizontal straight pipe no change in area, so therefore, the acceleration pressure gradient due to change in area that can cancels out, horizontal so therefore, your g sine theta path cancels out, so under that condition and there is a constant heat flux, what does this constant heat flux term imply can you tell me? When I tell you that there is a constant heat flux what does it imply?

Louder.

d x d z equal to.

It is constant ok.

So, constant heat flux it automatically implies d x d z this is nothing but four heat flux or rather yeah we can put it I think we put it a phi is not it, d g this is there so movement d g h the enthalpy of evaporation all these are constant, so constant heat flux automatically implies that your the change of quality is linear with distance is not it, or d x d z this is equal to c or this is equal to constant, and the boundary conditions are x equal to 0 at z equals to 0, and say x equals to x at z equal to l is not it, so this is the other assumption that we took up, now based on these assumptions if we integrate this particular equation in order to obtain the pressure gradient under low pressure conditions, when I say low pressure condition what does it mean? It means that the variation of specific volume with pressure is not there, or in other words we can assume v g v l to be constant it does not value with pressure.

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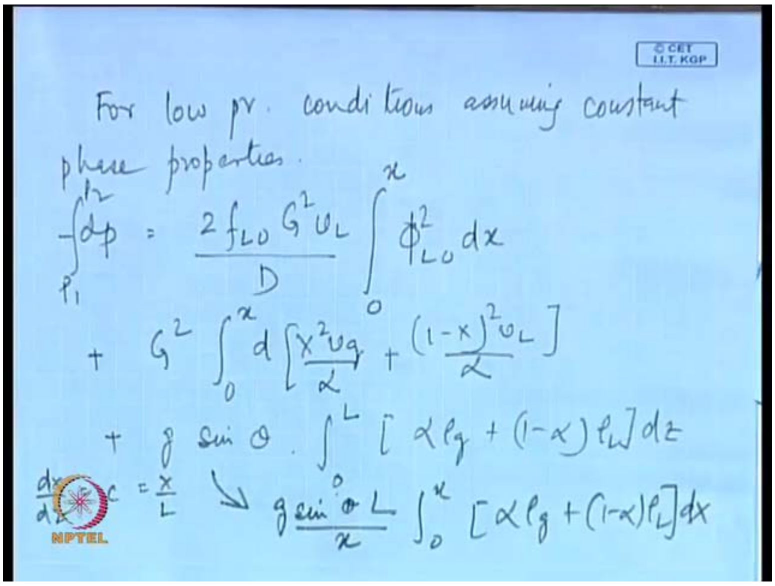

So, under such circumstances if we integrated to obtain the pressure drop for a length l of the pipe conduit what we will be get? For low pressure conditions assuming constant phase properties what can we do? We can simply integrated this the expression minus d p from p 1 to p 2, and this frictional pressure gradient since it is boiling water naturally it will be expressed in terms of phi square l o entire mixture flowing as liquid, because that is the entire condition agree, so therefore, this can be expressed as 2 f L o G square v l by D into phi square L o, yes or no you tell me the two phase frictional pressure gradient express as a function of the single phase pressure gradient when the entire mixture flow as liquid.

So, that is the entire condition under this case, now this phi square l o this should be or this should be varying with your quality is not it, this should vary with quality, so therefore, this should be 0 to x d x or in other words 0 to l d l it does sorry 0 to l d z, because quality varies with length in a linear fashion, any place you do not know understand you just tell me to repeat, plus g square your 0 to x your d of x square v g by alpha plus 1 minus x whole square v l by alpha, plus your g sine theta integral 0 to l, alpha rho g plus 1 minus alpha rho l into d z, tell me whether you have understood the integration which I have done in order to find out the pressure drop over a conduit of length l where the conduit diameter is d and the more or less the other terms they are self explanatory, this part I presume you have understood agree?

So therefore, if we integrated then what do we get, now remember one thing this g sine theta 0 to l d z this if it is converted to a variable x, then what does it become? This becomes we know that d x d z equals to constant, and this constant is nothing but equal to x by l, because that x equal to z equals to 0 x equal to 0 at z equals to l x equal to x. So therefore, if we have to make convention then this term it becomes just see whether you understand g sine theta l by x 0 to x, alpha rho g plus 1 minus alpha rho l into d x, can we write it in this particular form, why do I want to want to make this convention? Because I can find out how alpha varies with x.

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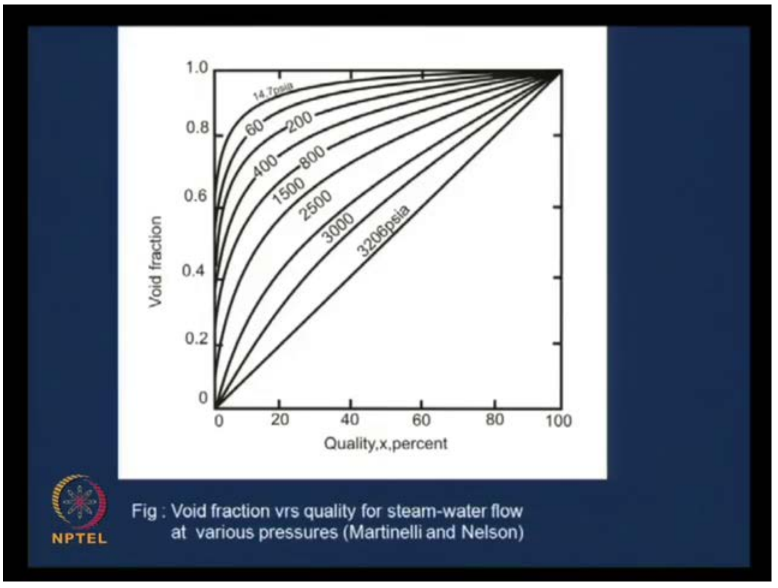

Fig : Void fraction vrs quality for steam-water flow at various pressures (Martinelli and Nelson)

So, whatever be the length they variation of alpha with x this is very evident, and graphs are available for this, I have got those particular graphs see this was one particular graph where we show how void fraction varies with x with pressure as a parameter, so if you want to find out how alpha varies with x what we do we simply go there we calculate the x calculating x is very easy, what is the pressure under which we operating go to that particular pressure parameter and you can find out void fraction, you know that the qualities changing from say 0 to 260 over this particular pipe length, so for or may be 10 to 60. So, accordingly you can calculate the void fractions for that particular pressure conditions and you can find out the void fraction you can substitute here.

So, thing is we do not know how your void fraction varies with length, it varies with length just because quality varies with length, how quality varies with length we know it is a linear variation, but how void fraction varies with length we do not know because we do not know how void fraction varies with quality, is it clear to you. So, what we do we tried to express the basic at equation it at the variation of void fraction with z the actual distance, we simple converted it to the variation of void fraction with quality, we simply converted the variable from z to x, and why? Because in order to find out alpha as a function of x and pressure standard graphs are available, remember only for water under fixed set of conditions which are horizontal pipe, no area change, more or less low to moderate pressure, constant heat flux.

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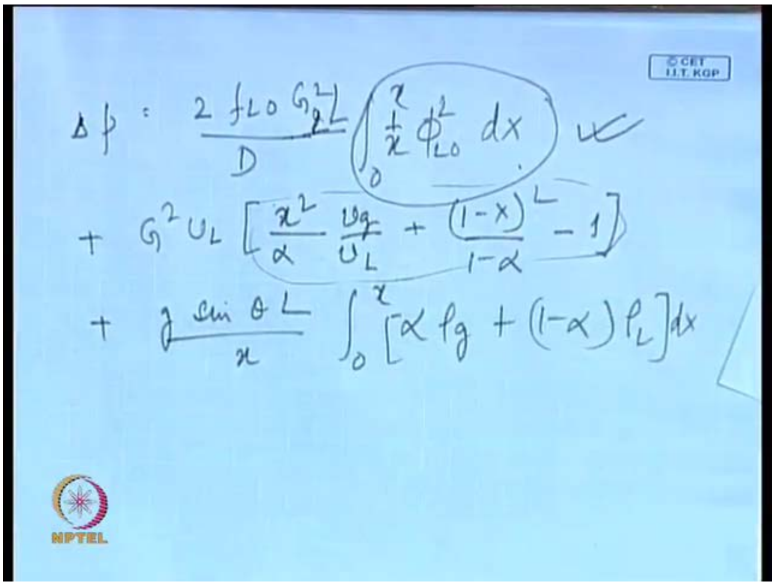

So, for all this situations curves are available. So therefore, what do we find the final expression in this particular case this reduces to it is something of this sort if you see your delta p this is nothing but equal to if you see this gives you 2 f L o G square by D integral 0 to x phi square L o d x, here also it has been converted from 0 to x from 0 to z phi, because a phi square l o how it varies with quality if that is given then we can use it for all pipe dimensions is not it, just if we know the change of quality, and this plus G square v l just see if you understand we are integrating this one term by term, now for this case you variable is x.

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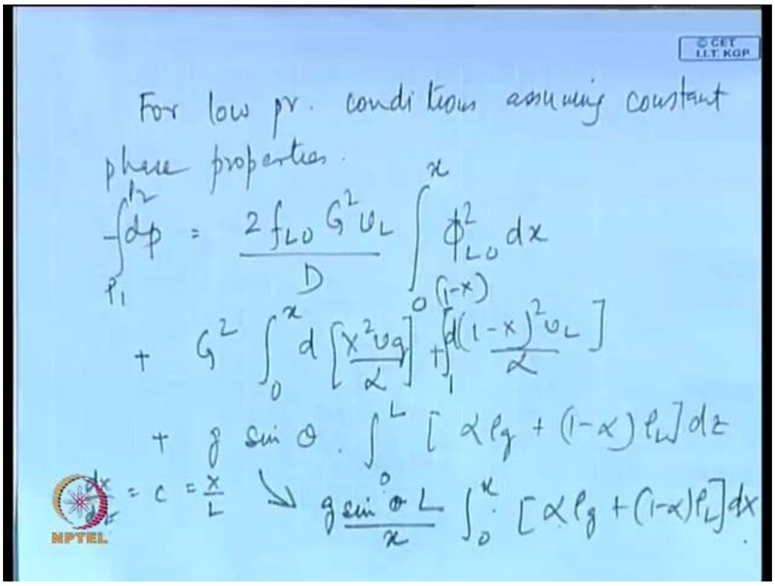

So, from here it is 0 to x, just tried to understand this part, and again for this part if you take this is going to be your when x equal to 0, 1 minus x equals to 1 is not it, so here it is d of this 1 to 1 minus x, see if you understand this part or not, so when you integrate this, this becomes x square v g by alpha minus 0 clear, and when you are integrating this part what it becomes? It becomes 1 minus x whole square v l by alpha minus 1, do you get the point just because the limits of integration are different. So therefore, on differentiating what you get in this particular case is G square just a minute here I made a mistake, g square sorry sorry there as to be a l here very sorry and 1 minus x here.

Because what I had what I have done this was actually 0 to z d z I have change the limits is not it, and so this plus g square v l x square by alpha v g by v l, plus 1 minus x whole square by one minus alpha minus 1, plus g sine theta l by x integral 0 to x, this is d x, this is the final expression, tell me if there is, but this has been derived on the first assumption that your quality variation with actual distances linear, now if you observe this equation we find in order to find out pressure drop what are the things that you required one is you require how this particular term varies with quality.

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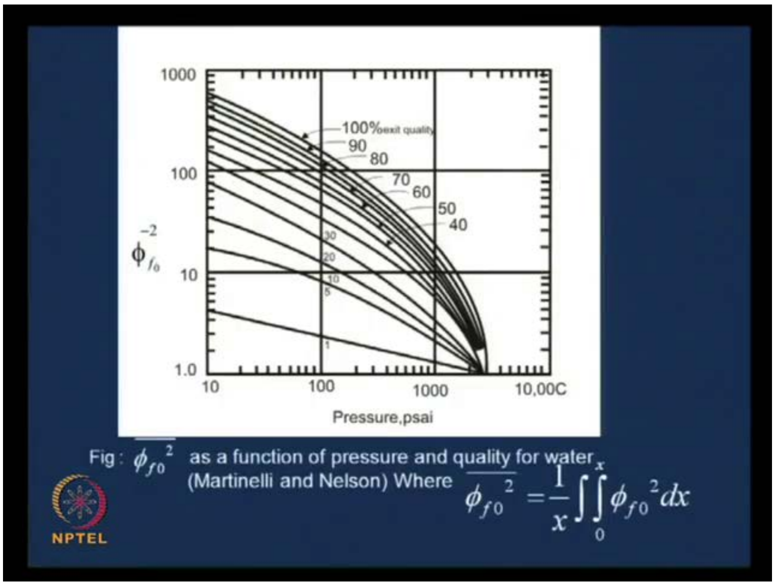

Now, all this graphs they are widely available for boiling water in straight horizontal pipes, the graph for this particular cases here, it is not this one, this is the graph, we find that we have got readymade curves for phi square l o 1 with this particular terms which we have already derived if you see, this particular term this gives you the this is the component of the frictional pressure gradient, there is by mistake another integral as come just ignore it, so this particular quantity if is plotted with pressure as a function of quality.

So, from here what you can get, if you see the expression which is have written down the expression which you have written down in my in just now which I have written down, so this is basically the quality which has been plotted as a function of quality as well as pressure, so from the curve which I have shown here we find out that from this particular curve we can find out the size 1 by x phi square l o this term as a function of quality and pressure, so movement I can do this if you see this particular expression and we find that f l o g square l d everything known for you, so the first term it can be calculated, then we also have standard curves for calculating this particular function as sorry this particular function if you see the transparency which I have prepared we find that this as a function of quality and pressure is also available for flow of boiling water in pipes, this is that particular function, if you see this is that particular function.

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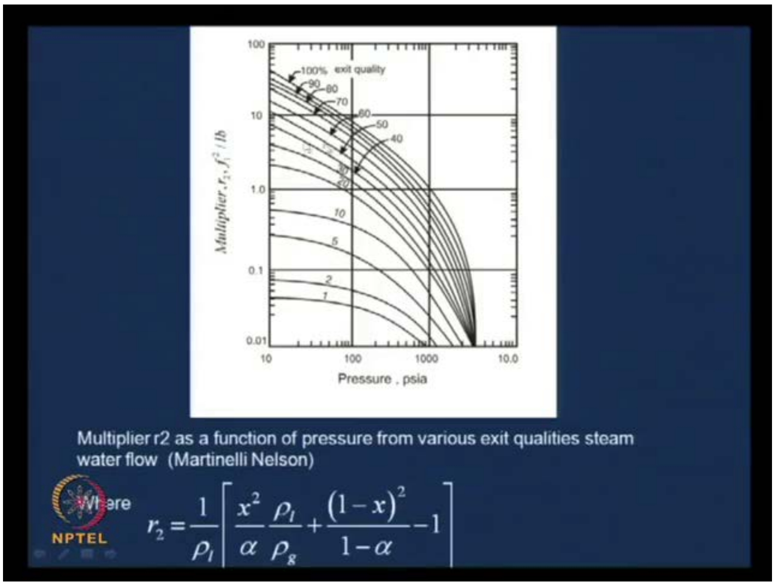

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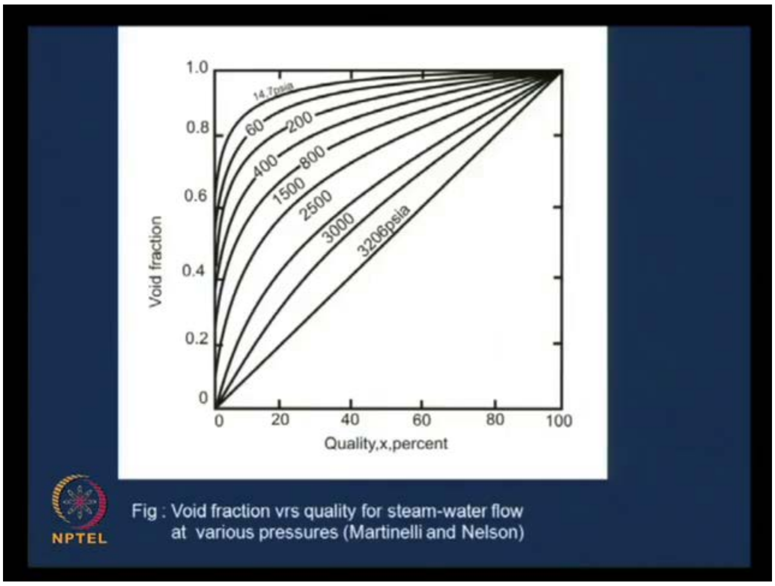

Where this r 2 this r 2 it has been in plotted this r 2 is nothing but, the term within the parenthesis which I had derive, so this r 2 also it is predicted as a function of pressure and as a function of exit quality. So, now we find that if it is the flow of boiling water in pipes, then in under for that particular situation we find that set of three curves are available, the curves they give you or they enable you to calculate the acceleration pressure gradient as a function of pressure and exit quality, from this graph the frictional pressure gradient as a function of pressure and quality, from this graph and alpha is a function of pressure and quality from this particular graph, once these graphs are there then by using this particular curves we can find out those particular portions of the gravitational frictional and acceleration pressure gradient which cannot be computed or which is difficult to compute.

So, from for those standard graphs are available and this can be used and they can be found out, the reason for purposing standard graph for these particular situations are they are very frequently encountered in industries, when you have to calculate pressure drops in industries is very difficult one you to do long integration and to find them out, but if you have some such readymade curves then very fast the technical people or the techniques they can just refer to the graph and they can find out the pressure drop at a much faster rate, is it clear to you, and since this boiling water in pipes it is very frequently encountered, so for this particular situation curves are available this is not available for others situation as well.

So, with this what we come to the end of the separated flow model, just in a naturally if I tell you what are the things that we have done we will find that the first thing we did we drive the two fluid model, we considered the two phases to flow separately and to occupy different cross sectional areas of the pipe, after that what we did we found out the mass momentum in energy equation for the two fluids separately, and then we tried to combine them to find out the mixture momentum equation, we tried to find out the condition of choking for two phase flow from the mixture momentum equation, then we took up the situation for phase change as well, we found out that when there is phase change than apart from momentum change due to the interaction between fluid one with the wall fluid to with the wall fluid one with fluid two there is an additional force which is involved due to the momentum change which is associated with the phase change of some fraction of the mixture from the liquid to the gas or vice versa.

So therefore, we incorporate it and we obtain the component momentum balances and the mixture momentum balance and the condition of choking from there, we found out that in order to use this equation for homogenous flow model we had to find out the frictional pressure gradient, we calculated the frictional pressure gradient by incorporating the two phase multipliers, here along with the frictional pressure gradient it was very important for us to calculate the void fraction as well. So, the empirical approaches again in terms of frictional two phase multipliers was adopted, phi l square and phi g square adopted and we found that there are large number empirical correlation for finding out the frictional pressure gradient as well as the void fraction, the oldest and the most widely use to Lockhart Martinelli correlation this has been modified for other fluids by Baroczy, and this has been modified for steam water flow using Martinelli Nelson.

And so accordingly we can use and find out the frictional pressure gradient as well as the void fraction, once we have done this we are in position of finding out that the pressure drop for a situation as well, now this finishes of our separated flow model. Now in the next class we will be applying the homogenous the drift flux and this a separated flow model for a different flow situations, maybe we will be taking up the bubble stag and the annular flow cases, now before I end there is one question which I would like to ask you, I had given up a problem in the class and I told you to solve the problem, have you found out the pressure drop for the homogenous flow case and for the separated flow case? Can you tell me which pressure drop was higher? If you have done it correctly than probably you have found out that the delta p as obtained from the homogenous flow model was much higher, and the pressure drop which you had calculated from the separated flow model was much lower.

So, remember one thing these are the two extreme which I had told you to consider, your actual pressure drop will laid within this particular range, and what does this suggest? Suppose you want a very high pressure drop, so that there is no leakage from the tank, then in that case what you have do? You have to keep our homogenous mixture of the two phases in the tank, and we have ensure that the homogenous two phase mixture flows out through the nozzle, if you want to low pressure drop from pumping power considerations than what you have do? You have to separate the two phase and you have arrange the nozzles such that the interface lies within the nozzle.

So, that the two phase flow as separated flow through the nozzle, under that condition you will get the minimum pressure drop, so depending upon your condition whether you do not want a leakage or whether you want a low pressure drop your flow distribution or the arrangement of the two fluids have to be decided accordingly, there was one more interesting thing about the problem if you notice it comprised of a very high gas under very low liquid flow rate, which shows that bubble flow is not possible in this particular case, what is the homogenous flow distribution that you can adopted here it as to be the droplet flow distribution.

So, whenever you are given two phase flow problems either in your exams or something remember that other than the final answer this type of analysis is much more important, so you have to analysis the situation and you have to write the answers accordingly, and we will be evaluated even if you are answer is wrong, but the logics are correct or they are wrong and strong than also you will get marks. So, I will be mainly looking for your listening power other than how well you can represent the thing for your for any particular evaluation or this particular subject. So, Thank you very much in the next class will be dealing with flow patterns, will be applying this simple analytical model to the different flow patterns, and then we will go for some classes on boiling and conversation, and after that some classes on the development of instrumentation for different two phase flow situations. Thank you very much.