Multiphase flow

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Model No. # 01
Lecture No. # 23
Separated Flow Model- Estimation of Frictional Pressure Drop and void fraction (Condt.)

Well, so, good morning to all of you so, in the last class what we were doing we had come almost to the end of this separated flow model. The only thing, which was remaining was the estimation of the frictional pressure gradient as I have written it down the only thing, which is remaining it is the estimation of the frictional pressure gradient and as well as the void fraction from the separated flow model. So, we discussing it we found out rather, we were discussing that mostly large number of empirical correlations have been proposed for this particular purpose. And, the basic thing of this empirical correlation is that first.

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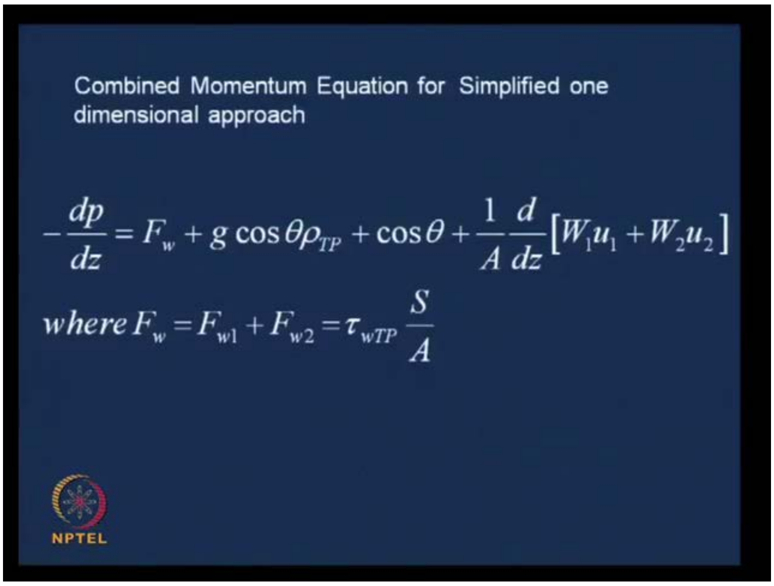

We assume that we have just relaxed one particular assumption of the homogeneous flow model and we assume that the two phases are flowing at different velocities. So, we are in order to develop the empirical approach the first thing which we do is, we assume that the effective frictional pressure gradient can be expressed in terms of something like the two phasive wall shear stress into the S by A in this particular form and we would like to find out either tau W T P S by A or in other words F W. So, if you observe this particular form of the equation and the homogeneous flow model.

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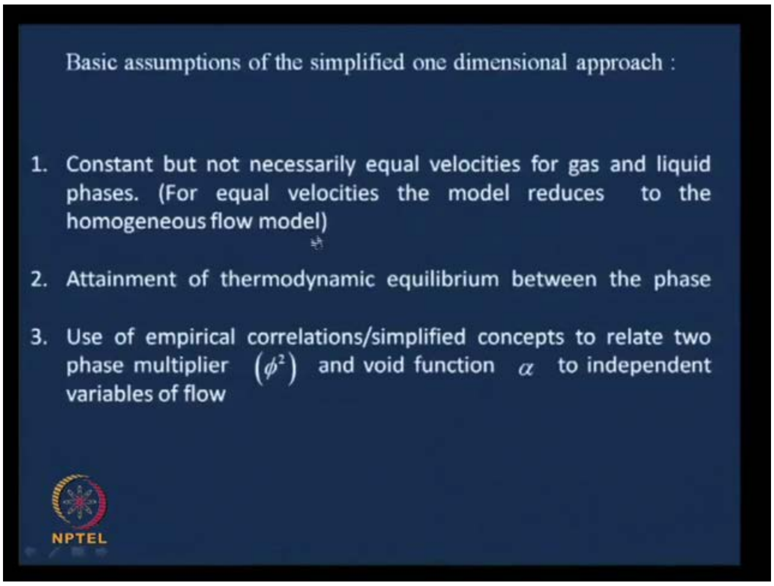

We had developed we find that the only relay only assumptions. Which we had relaxed in this particular case was that, the two phases were allowed to have different velocities. Other than that this expression is identical to the expression that we are derived for the homogeneous flow model. So, naturally accordingly we had laid down the basic assumptions of the simplified one dimensional approach. And then in order to develop certain correlations for finding out the frictional pressure gradient as well as the void fraction, if they were developed on the basis of no phase change no acceleration pressure drop negligible body force.

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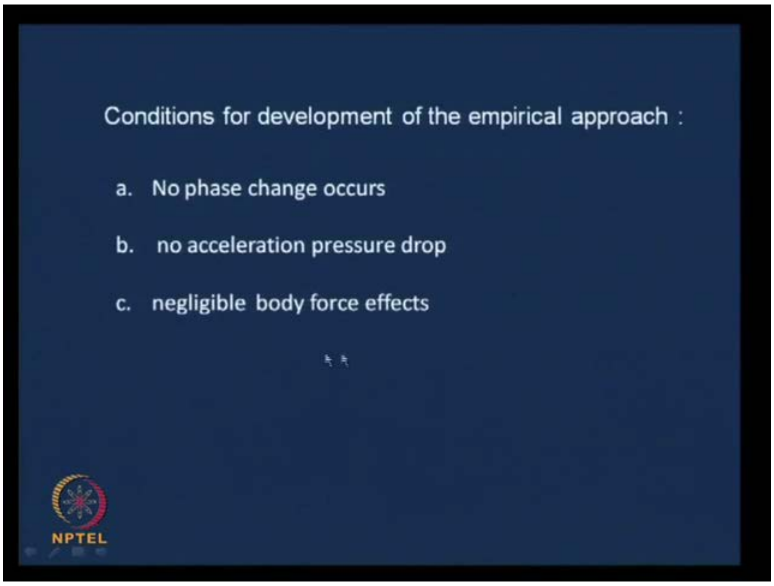

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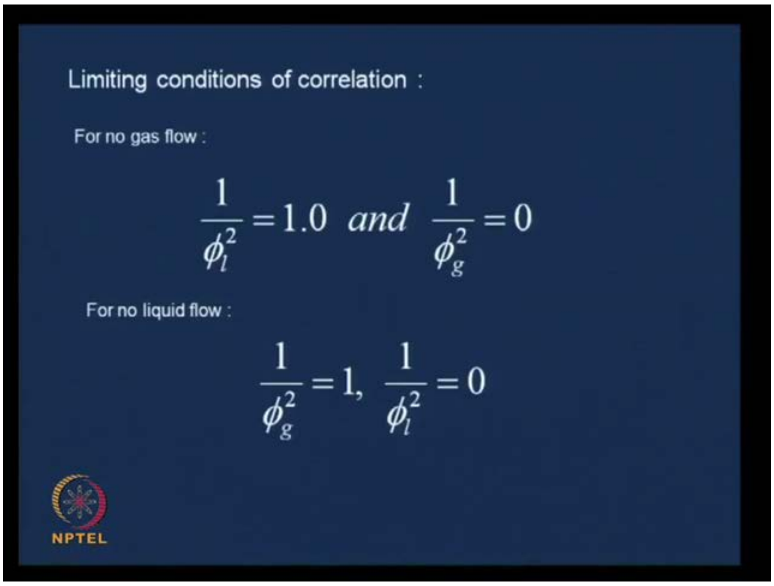

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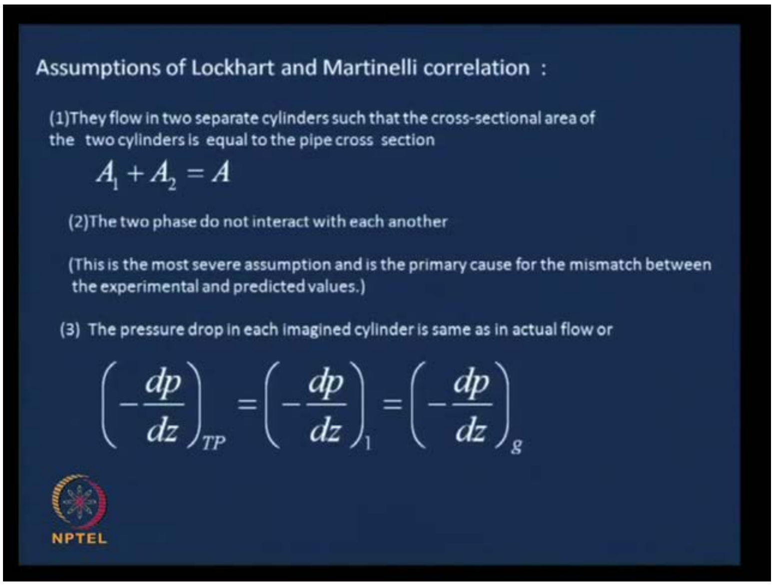

Such that the effective that total pressure gradient can be made equal to the frictional pressure gradient. And after that what we did? We found out or other idea discussed phi we have tried to develop a correlation between phi L square phi g square and not phi L square and phi g square. That means, we had tried to develop a correlation between two phase multipliers, which were based on the liquid portion flowing alone in the pipe and the gas portion flowing alone in the pipe. Instead of the entire mixture flowing as liquid or the entire mixture flowing as vapor the reason we had discussed in the last class it was just, because this conforms to certain limiting conditions that we had already done.

So, after that we have discussed that the oldest correlations, which is also the most widely used correlation that is due to Lockhart and Martinelli. And, they had developed the correlations. So, and the basis of certain assumptions we had already discussed the assumptions in the last class, the assumptions I just tell you very briefly. The first one was the two liquids they are assume to flow in two separate cylinders, such that the two the cross sectional area of the two cylinders add up to form the cross sectional area of the pipe this was number one.

The other thing is the two phases do not interact with each other this was the severe assumption. And I will just show you the anomaly in the results, which we get just, because of this particular assumption before we go into deriving the analytical expression and discussing the graphical correlation between phi L square and phi g square I would just like to show you that. So, that you can understand exactly this assumption is severe never the less we take it just, because it guarantees simplicity. And an under the within practical limits this gives as a more or less accurate estimation of the frictional pressure gradient, particular when the frictional pressure gradient is more important than the gravitational or the acceleration component.

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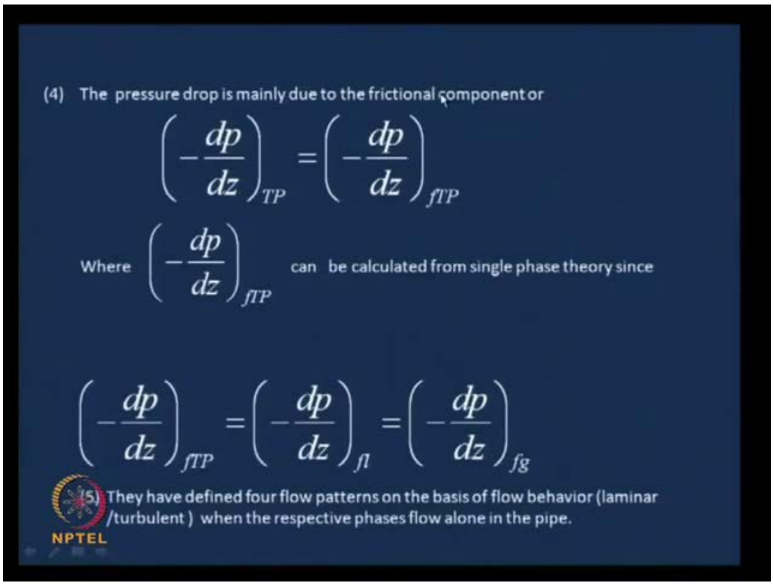

So, this was the second assumption and as a result that since the two phases do not interact at one another, we can always say that the pressure drop in each of the imagine cylinder is same as the pressure drop in actual flow on other words as have written down that the two phase pressure drop is equal to the pressure drop in phase one. And it is equal to the pressure drop in the liquid cylinder it is equal to the pressure drop in the gas cylinder. And the other thing is, the pressure drop mainly arises due to the frictional component.

So, due to which we can write that the two phase pressure gradient is primarily the frictional pressure gradient. And the two phase pressure gradient is equal to the pressure gradient in the liquid cylinder is equal to the pressure gradient in the gas cylinder. And combining everything we can write that the two phase pressure gradient is the frictional pressure gradient. And, this is equal to the frictional pressure drop in the liquid cylinder, which is equal to the frictional pressure gradient in the gas cylinder. So, this is finally, what we come now.

So, if we find out that based on this assumptions if you can find out phi L square and phi g square, then we will be in a position to find out the two phase pressure gradient. For that what we have to know? We have to know or we have to have methods in order to find, if find out the single phase pressure gradients for liquid flowing alone in the pipe for gas flowing alone in the pipe. Now, they can be obtained when an liquid or an gas is flowing alone in a pipe. Then naturally, it is just single phase flow and therefore, these pressure gradients can be obtained simply from single phase fluid dynamics.

The only thing we need to know is whether when the single, the single phase flow is occurring within the pipe whether that is occurring under laminar flow conditions or under turbulent flow conditions. If we know that we can find out the pressure gradient when either liquid or gas is flowing once we can find that out, we can find out phi L square phi g square. Once we have found out phi L square and phi g square then we can find out the two phase frictional pressure gradient right. So therefore, the fifth assumption is that four flow patterns have been defined. As I have written down in my transparency that four flow patterns have been defined on the basis of flow behavior.

When the phases are flowing alone in the pipe; that means, it can be gas laminar liquid laminar gas turbulent liquid turbulent gas laminar liquid turbulent. And gas turbulent liquid laminar last one of course, is something very real and so therefore, it is not widely used. So therefore, what we do? We first find out the frictional pressure gradient in the liquid portion or in the gas portion that gives us the two phase pressure gradient frictional pressure gradient then what we do find out? The pressure gradient when this liquid or this gas flows alone in the pipe form an idea of whether the liquid or gas is flowing in laminar flow or turbulent flow.

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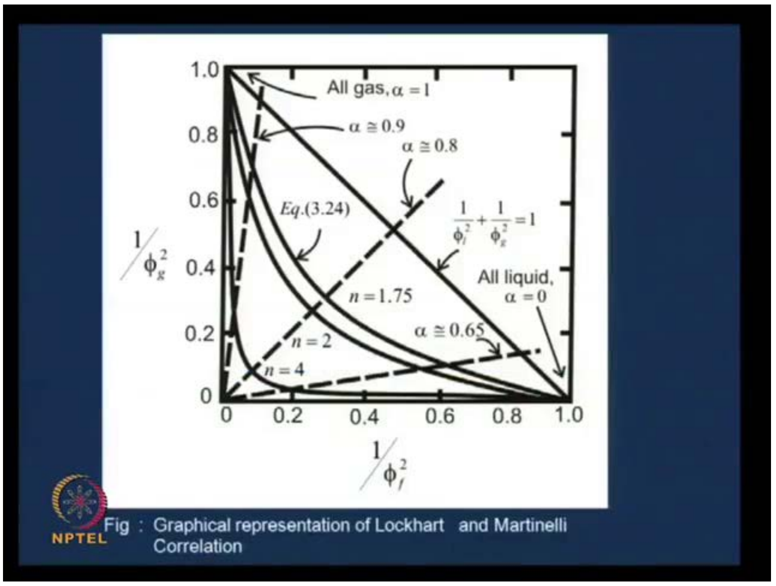

So, accordingly we know the pressure gradient in the liquid portion or the gas portions when both are flowing together in a pipe we know the pressure gradient when either of them are flowing alone in the pipe, if we divide one by the other we can find out phi L square or phi g square as the case meaning. So, accordingly phi L square and phi g square has been evaluated. And Lockhart and Martinelli had basically given a graphical correlation between the two. So, one by phi g square and one by phi L square they that they have been plotted and we obtain a graph something of this sort where we find that more or less it gives you for different values of n, where n is the Blotius or rather the coefficient of the equation F is equal to K r e to the power minus n.

So, n equals to one means it is naturally the laminar flow n if other values of n naturally it refers to turbulent flow. And we find that we have obtained a family of curves in this particular situation. So, this equation this is for n is equal to one this is for laminar flow and accordingly we have for turbulent flow. Now, this graphical correlation can be expressed than analytical expression simply, if we start from the basic derivation of phi L square and phi g square from the basic single phase hydrodynamics. And we try to correlate them then we can find out an analytical expression between the two.

So, we will be basically doing two things in the class first we will try to find out the analytical expression. And secondly, we will take up the annular flow pattern we will find out phi L square from the separated flow model we will find out the actual phi L squares annular flow model. And we will find how well they two agree from that? We can an idea regarding how accurate this model is correct. So, these are the basic two things which we are planning to do in this class once we finish off these things we obtain an analytical expression after that we will be discussing of about further refinements of this particular model. And what are the other correlations that follow and so on and so, far.

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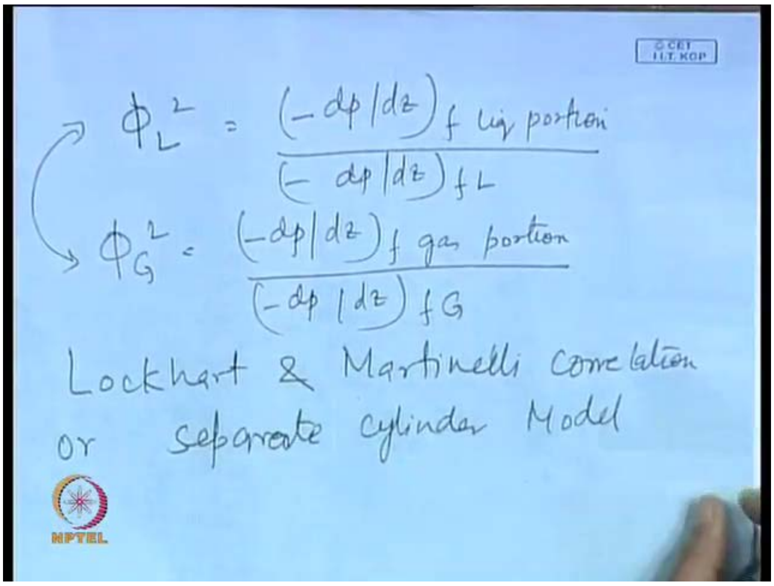

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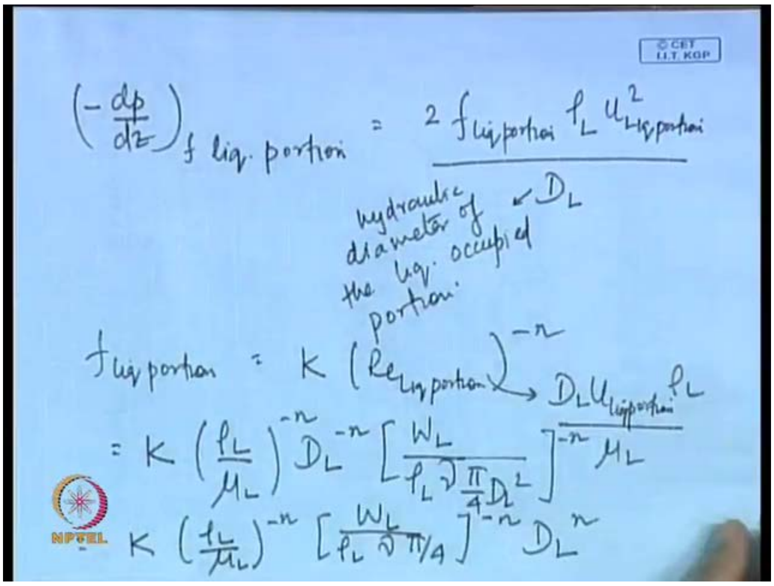

So, this is the next thing that we will be doing. Now, let us start from the basic definitions of phi L square and phi g square before. And then let us see what is the analytical expression the two now, in the last class if you see in the last slide had already written down phi L square it is minus d p d z frictional for two phase, which is nothing, but minus d p d z frictional liquid portion divided by minus d p d z frictional when liquid flows alone in the pipe similarly, phi G square was defined. Now, if we start from the basic definition what do we get? What is this minus d p d z F liquid portion? Wet us do the derivation and let us see what we arrive at this is F liquid portion.

So, therefore, this is nothing, but 2 F liquid portion row L u L square this is again liquid portion, liquid portion means the initiative velocity of the liquid phase when it is flowing under separated flow in the conduit remember one thing we are deriving everything for circular pipes of diameter d this divided by D L what is D L? It is the hydraulic diameter of the liquid occupied portion. So, therefore, this is the exact definition. Now, what is this F liquid portion? This is nothing, but K r e liquid portion, if I write it down whole to the power minus n yes or no. And what is the r e liquid portion? This will be d L hydraulic diameter of the liquid occupied portion u liquid portion this writing is very big botheration row L by u L yes or no.

So, therefore, if you substitute this here then we get for K row L by mu L to the whole to the power minus n just see whether I am writing it correctly or not. And instead of this u liquid portion we can write it down as W L by row L a L we can write it down what is this a L? This is nothing, but gamma phi by 4 d L square where gamma is the shape factor that cylinder might not be completely circular is it not. So, this just accounts for the shape factor, which at the end we may take it as one. So, for the time being you would like to keep it. So, this is what? Is the final expression of the frictional factor of the liquid, in the liquid occupied portion during two phase flow clear to all.

So, therefore, or in other words this can be written down as it is just some simplifications that I am doing for W L by row L comma phi by four whole to the power minus n D L to the power minus plus two n minus n. So, it becomes d L to the power n correct. So, this is for the friction factor. So, therefore, if you substitute F here we get the minus d p d z for the minus frictional d p d z for the liquid portion just do it and see what you are going to get as a same way we can write down minus d p d z frictional for the gas portions that we will do later.

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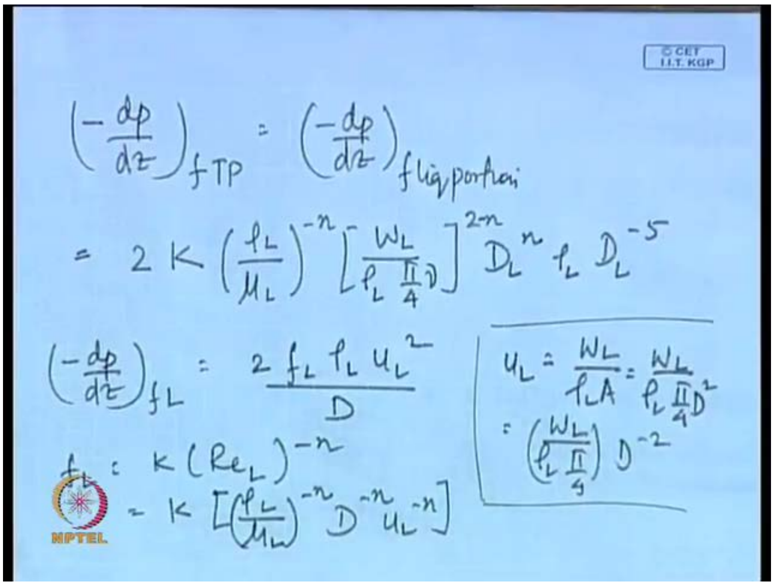

So, minus d p d z frictional two phase equals to minus d p d z frictional liquid portion what we will do? We will just substitute this F portion here and instead of u L square we are going to substitute the expression of u L. So, that we have obtained everything in terms of known input parameters d L is of course, not known, but can be correlated with alpha fine. So, if we substitute that then what do we get? This is nothing, but equal to you just substitute and then you make all the necessary simplifications. If final things, which you are going to arise at is something of this sort please do this derivations in your hostels. And in case you have any problems you can come back to me and you can ask me whether you have any doubts or something this is equal to two n.

Because we will be having one is this and other is for to liquid portion part it is for it is 2 minus n probably sorry 2 minus n this is d L to the power n row L d L to the power minus five correct. So, this was for frictional pressure gradient expression in the liquid portion which is nothing but the frictional pressure gradient in the two phases fine. Now, suppose this liquid flows alone in the pipe once we can find out these two expressions F liquid portion we found out. Now, when this liquid flows alone in the pipe if you can find out and if can divide the two we can get phi L square. So, now, let us find out minus d p d z frictional liquid flowing alone in the pipe.

What will we get? If you compare with this particular equation it will be 2 F L yes or no row L correct u L square the liquid is occupying the entire cross section and flowing.

And this D L is going to be D now the liquid is occupied the entire pipe is it not? So, therefore, this is going to be 2 f L row L u L square divided by D fine [FL] here what was u L u liquid portion it was W L by row L gamma phi by 4 D L square agreed. Now, it is going to be W L by row L phi by 4 D square when the liquid occupies the entire cross section the liquid flows alone in the pipe clear to all of you. Anybody who wants to be repeat this initially we have derived the frictional pressure gradient.

When liquid was occupying a portion of the pipe so, for that we had derived this expression fine. Now, the same liquid has occupied the entire pipe cross section agree. So, for that case what you are going to get instead? So, for that case this F will be F L liquid flowing alone in the pipe row L by d fine it is occupying the entire pipe. And this u L square will be equal to W L by row L into a. Now, it has occupied the entire pipe cross section. So, therefore, this is going to be in this particular case. So, where u L equals to W L by row L a which is nothing, but W L by row L phi by 4 D square or since we have taken out these separately here. So, in the same way, we will be taking this D out this will be W L by row L phi by 4 into D to the power minus 2.

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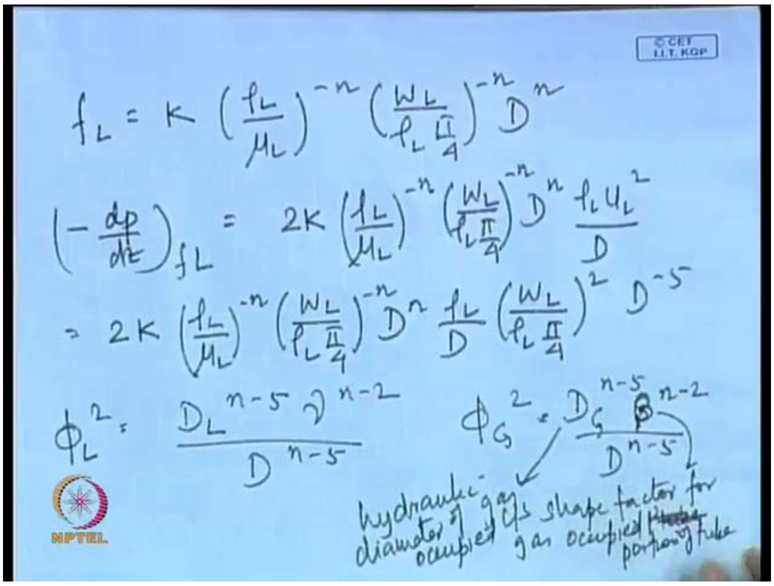

We can write it down in this particular way. And similarly, what is F L? Is equal to this will be K r e L whole to the power minus n right what is this r L in this particular case? R e L was d L u liquid portion row L by mu L in for our case it is going to be d u L row L by mu L correct. So, K this is going to be row L by mu L whole to the power minus n d to the power minus n u L to the power minus n I have just broken it down. So, that I can substitute I can make the necessary substitutions fine or this can be written down as F L equals to K just things, which I had written down row L by mu L whole to the power minus n instead of u L I write W L by row L phi by 4 whole to the power minus n.

Then D to the power D square to the power minus n and there was a D to the power minus n. So, finally, it will answer up to D to the power n. So, now, if I substitute this F L in the expression for the frictional pressure gradient then what do I get? I get minus d p d z frictional liquid this will be equal to two K just I will write down the expressions one by one row L by mu L whole to the power minus n W L by row L phi by 4 minus n d to the power n or in other words, which we can substitute it a little this is going to be 2 K this u L square also we can substitute row L u L square divided by phi this row L u L square is coming from here row L u L square by D.

And instead of this F L we have written it in this particular form this form then this mu L has come here from that Renaults number these derivations they need a lot of practice please do this, because either your thoroughness or your fun less something will be tested in the mid sem. So, that is your option which one you want to be tested you are going to tell that to me. And accordingly either your thoroughness or your concept something will be tested. So, this is the final form of the frictional pressure gradient. So, this is the final form for the frictional pressure gradient and this was the final form for the two phase frictional pressure gradient or the frictional pressure gradient in the liquid portions.

So, if we divide one by the other this by this we are going to get phi L square. So, just divided it and see if what to get? When you are dividing it, if you compare the too you find your 2 K from here cancels out 2 K from here cancels out row L by mu L row L by mu L they cancel out just, because of that I have written it in this form your this part cancels out this part when the gamma part remains. And then this row L cancels out with this row L fine only D L parts remain D parts remain and gamma part remains nothing else. So, if you make all the cancelations finally, probably you will land up into something like D to the power n minus 5.

So, from this what do we come to know we find out that the phi L square the two phase multiplier it is the ratio of the two phase frictional pressure gradient, if it exists if liquid flows alone in the pipe. And if it exists for the two phase case they depend on just two things they do not depend on flow rate they do not depend on anything they just depend on two things what are the two things? It is the fraction of the flow area occupied by the liquid phase and the geometry of the flow area occupied by the liquid phase it just depends upon the two things. And this is the final phi L square which I have obtained similarly, we can proceed and we can come we can arrive at a phi G square as well yes or no.

Fine this will again be d g o the power n minus five where d g is the hydraulic diameter of the gas occupied portion any doubts? You can raise your hand we can put as delta or something sort of a thing which is the shape factor for the gas occupied portion. This can be n minus 2 by D to the power n minus 5 delta and gamma they can be the same thing I can put it something else may be beta I put it this is the shape factor for gas occupied tube his gas occupied sorry gas occupied portion of tube. And D G is the hydraulic diameter of gas occupied cross section or gas occupied portion fine.

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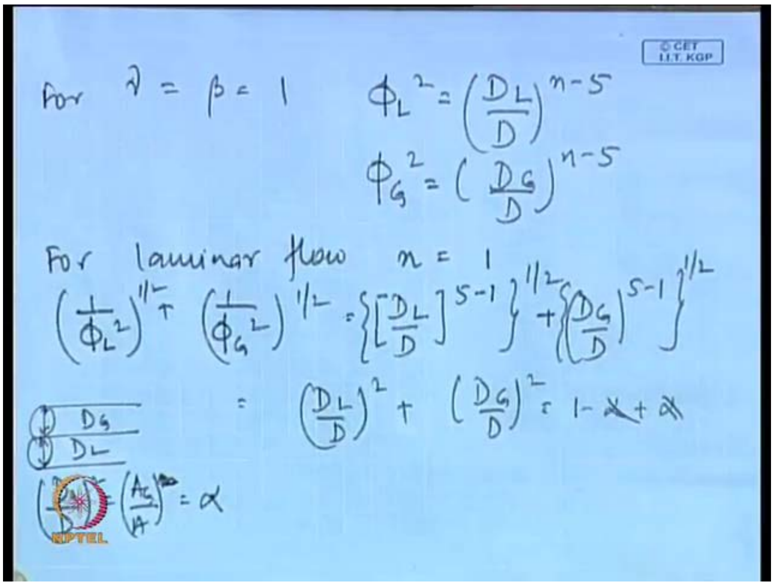

So, in the same way we can get a phi L square and we can get a phi g square. Now, usually we can take these two as equal or else we can assume that more or less since from the separated cylinder model what was the first assumption that I made? First assumption that was written down in my transparency slide they flow in two separate cylinders such that the cross sectional area of the two cylinder is equal to the pipe cross section. So, there is in the first assumption itself we had assumed gamma equals to beta equals to 1. So, therefore, we find that for the situation where gamma equals to beta equals to 1 for that particular case phi L square is nothing, but D L by D whole to the power n minus 5 phi G square will be equal to D G by D whole to the power n minus 5.

Where what is n? This n we had we have arrived at this n from the basic equation relating friction factor and Renaults number by a Blotius type of relation for laminar flow what is the value of this n? And for turbulent flow, what is the value of this n? If we assume Blotius equation 0.25 or one by fourth fine. So, now, for n equals to one so, therefore, for laminar flow n equals to one in that case what is the relationship of? What is 1 by phi L square plus 1 by phi G square can you tell me? Sorry 1 by phi L square whole to the power half 1 by phi G 1 square whole to the power half what is this equal to can you tell me? This is D L by D 5 minus D L by D I have made it yeah 5 minus 1 whole to the power half it should be 1 minus 5 I believe.

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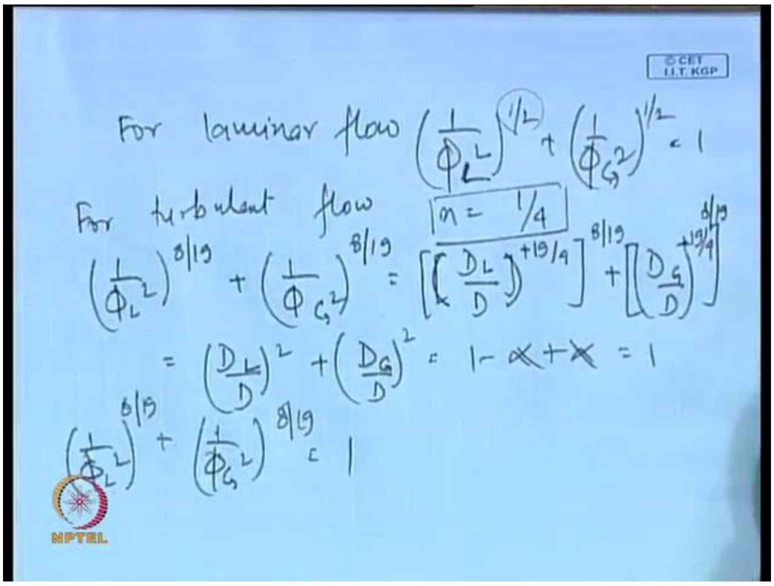

Now, it is alright 5 minus 1 fine plus D G by D 5 minus 1 whole to the power half. So, therefore, this because 4 by 2 square is it not. So, therefore, D L by D whole square plus D G by D whole square. Now, tell me one thing? Suppose it is this type of a thing and this is D G hydraulic diameter and this is D L then D G by D L or rather D G by D whole square this is equal to A G by A whole square yes or no, yes. And this is equal to A G by A whole sorry very sorry A G by A equal to alpha what fraction. So, therefore, this can be written down as 1 minus alpha this can be written down as alpha yes or know. So, therefore, this is equal to one agreed. So, for laminar flow what have we arrived at we are found that for laminar flow 1 by phi L square whole to the power half plus 1 by phi G square this all capital this is equal to 1 and for turbulent flow.

We have to find out n equals to one fourth is it not? So, there we have to find out that what will be this particular coefficient? Such that 1 by phi L square whole to the power something plus 1 by phi G square whole to the power something that comes to be equal to 1. Now, if you try out and if you see you will find out that this particular coefficient if it is made 19 by 8 by 19 then probably you are going to get 1 by phi L square whole to the power 8 by 19 plus 1 by phi G square whole to the power 8 by 19 equals to 1 you can try it for yourself and you can see it. When n equals to one-fourth then in that case your phi L square if you are suppose to do put one-fourth is it not?

So, if you if you put one-fourth here then what do you get it is 19 by 4 here also you get 19 by 4 is it not. Sorry minus 19 by 4 rather. So, therefore, if you have to write it in this particular form then in that case it has to be multiplied by 8 by 19 fine then in that case you get a form of D L by D whole square plus D G by D whole square this form and finally, it becomes equal to 1. So, far turbulent flow assuming n equals to one-fourth we find that 1 by phi L square whole to the power 9 by 19 plus 1 by phi G square whole to the power 8 by 19 this is nothing, but equal to D L by d minus 19 by 4, we put it other bracket into 8 by 19 plus D G by D again minus 19 by 4 8 by 19.

So, this reduces to d L by d whole square plus D G by D whole Square which is nothing, but 1 minus alpha plus alpha equals to 1. So, therefore, for turbulent flow assuming n equals to one-fourth under this condition, we find the relationship is going to be 1 by phi L square whole to the power 8 by 19 plus 1 by phi G square whole to the power 8 by 19 this is equal to 1. Which 1 D L by D this is going to be just a minute me check this up D L by phi yeah very sorry both of them are going to be plus agreed small mistakes with. So, many calculations it is comes up, but if you make the mistakes the mid sem then I am not going to part on you that is this at what of the story.

So, therefore, we find that in this particular way starting from the basic derivation basic derivation means the basic definition of phi L square phi G square the basic definition of the pressure gradient expression for the liquid occupied portion of the pipe. And find that liquid occupies the entire cross section of the pipe for gas occupied portion of the pipe and that gas flows alone in the pipe just from the basic derivations basic definitions, if we start we arrive at some sort of an analytical expression between phi L square and phi G square the initial attempt as I have already told you the initial attempt was this particular graphical correlation which I have showed in this particular graph. And this probably explains why I have express this graph in the form of 1 by phi G square and 1 by phi L this should have been phi L square this has been expressed in this form.

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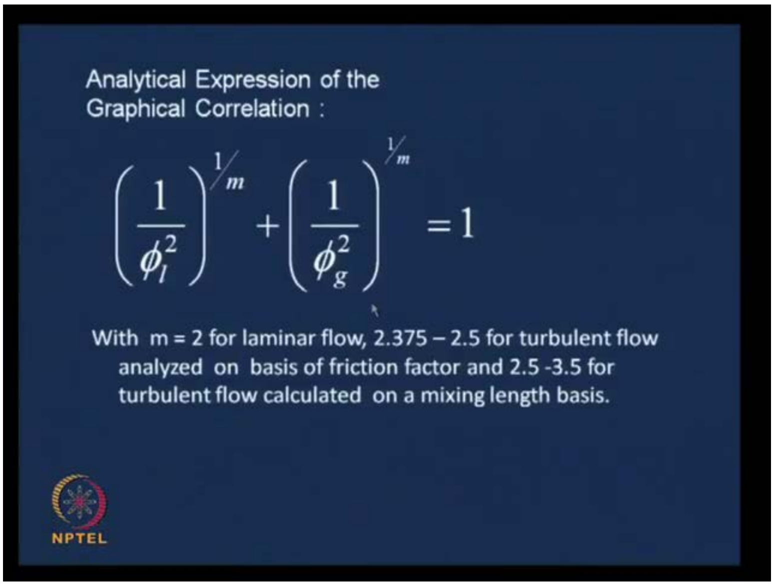

Because from the analytical expressions also we find that the relationship between these two exists the straight forward relationship exists not between phi G square and phi L square, but between 1 by phi G square and 1 by phi L square. So, this was the particular expression that we had got and the analytic analytical from can be expressed as it is written down here it is 1 phi L square whole to the power 1 by m. Remember this m is different from the n, which was the Blotius which was the coefficient of Renaults number in the friction factor Renaults number equation when the equation is in the form of a Blotius type expression please remember the main problem is when.

We have increased the number of phases the number of properties, the number of constants the number of exponents have increased, if you do not confuse them then to phase flow is not going to be a very difficult subject for you. So, therefore, remember the analytical expression that has been expressed as 1 by phi L square whole to the power 1 by m plus 1 by phi G square whole to the power 1 by m equal to 1 where as we have already derived m equals to 2 for laminar flow is it not. We have already derived that if it is on the basis of a Blotius type expression then it comes to m will be equal to 19 by 8 which is nothing but 2.375e and this is your home assignment you try to calculate the turbulent flow case on a mixing length basis.

And find out what is going to be the value of m in that particular case, you will finds that the value of m under that particular condition is going to be 2.5. And more or less with experimental data they are found it varies from 2.5 to 3.5. So, therefore, this is the analytical expression of the graphical correlation which was originally proposed, but now of course, this analytical expression that is much more used fine. Now, before I proceed I would just like t the things which I had told. So, is if when you using a correlation you must also know how accurate it is that is also very important. Now, let us see intake of the case of the annular flow pattern this is the most or rather the most common example for a separated flow situation is it not. So, horizontal annular flow pattern.

We make a few assumptions, what are the assumptions that we made for the derivation? We would like to find out phi L square, what the annular flow pattern in the horizontal phi? So, we will find out the phi L square from the separated flow model and we will consider the actual annular flow situation. And try to find out the phi L square there then we will compare the two values and then we will see show accurate this separated flow model is. Now, whenever we take up any particular analysis of any particular flow situation we try to take up the idealized form even in single phase flow you have found out what we try to do frictionless flow in visit flow? Because the derivations are easier here when we take up the annular flow pattern what we assume?

We assume that the liquid flows only as a film at the wall there is no entrainment of the liquid by the gas core again an extremely idealized situation remember it there you will hardly find annular flow in with a case where there is no entrainment of liquid droplets in the gas core under most circumstances there is a good amount of entrainment. And sometimes the entrainment is. So, very high that we can hardly differentiate between bubbly flow and annular flow it happens at times, but for the time being for assumption meet rather for or derivation we assume two things one is the liquid flows entirely as a film, which is a annular film between the gas core and the pipe valve.

The second thing which we assume is the inter phases smooth again it is quite drastic assumption. Such an annular flow occurs only for vey high gas and very low liquid flow rates for most of the cases this does not happen. But we assume these two conditions so, the things, which we are we assume is inter phases smooth entire liquid flowing as a film are as flowing as annular film between the gas core and the valve it is going to be a horizontal pipe. So that, there is a no gravitational pressure gradient it is going to be an air liquid and air water system. So, that there is no phase change and there is if and the acceleration pressure gradients can be minimized it is a circular pipe of conduit diameter d so that the acceleration pressure gradient due to area changes and negligible.

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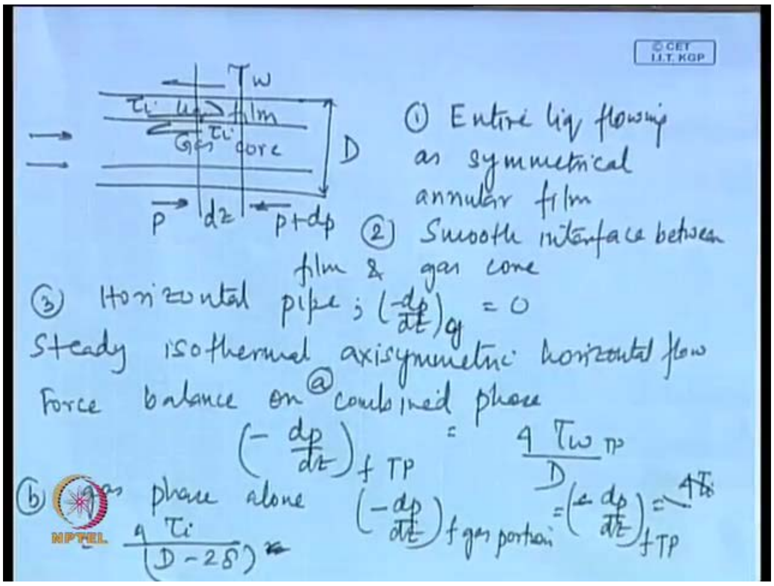

So, with all this assumptions we can we find out that we are in a position to apply the separate cylinder model to this particular type of annular flow situation. So, for such an annular situation, if we can represent it is sort of this very straight lines I have drawn this is a gas core this is a liquid film, if flow occurs in this particular direction then wall shear stress will be in this particular direction. And they will be a tau I in the interfacial shear I what will be the direction of tau I can you tell me? Very correct it will be in the direction of flow for the gas phase, know good it will be in the direction of flow for the liquid phase and opposite to the direction of flow in the gas phase.

Because the gas always flows faster than the liquid phase very correct. So, therefore, this is going to be fine this is tau I and suppose, just like I do in the single phase flow case here it is p here it is going to be p plus d p and this is at d z portion that is the entire pipe diameter is d agreed. So, in this particular case the assumptions, which we have is and I have already discussed them entire liquid flowing as symmetrical and see this is also a drastic assumption as far as horizontal flow is concerned. Because whenever we can say horizontal flow due to gravity most of this times it happens that the upper film will be thinner than the bottom film, but when the gas core it is quite high liquid when it quite low under that circumstance the distribution tends to a symmetric. So, since we have assumed.

Considering the other assumption this symmetrical is quite a justified flow the first thing entire liquid flowing as symmetrical annular film between the channel valve. And the gas core this second assumption is smooth interphase between film and gas core that is not an assumption, it is a consideration it is a horizontal pipe. So, therefore, gravitational pressure gradient minus d p d z is small g this is equal to zero. And this is so, therefore, this is steady isothermal axis symmetric horizontal flow in short we can write it in this particular way. Now, suppose we do the force balance on the combine two phase system. So, from force balance just like, if you remember we used to do for this single phase flow we take a control volume the pressure gradient and the wall shear stress.

And then we use to balance them and we use to get an expression for tau W in terms of d p same way force balance on combined phase what will we get here? We will get minus d p d z frictional two phase this is simply four tau W two phase or tau W here divided by D yes or no agreed. We can put it two phase, if you want we have always be do doing it now force balance on gas phase force balance on a combined phase. Now, if we just take the gas phase alone then fourth we will get we will get minus d p d z yes the gas portion this is equal to minus d p d z F two phase, which is will be equal to can you see what I am writing this is 4 tau I will write it here equals to 4 tau I divided by D minus 2 delta whole square do you understand what I have written sorry, there is no whole squares here.

Is this portion correct the portions, which I have written down here this portion is fine. Now, again what do I know just see the thing is, if you have to understand these things very clearly you have to keep the assumptions of separated cylinder model very clearly you have to remember when you are using or when you are finding out the friction for liquid occupied or the gas occupied portions and both the phases are flowing together in the pipe. And when you are considering when only that liquid is flowing alone in the pipe and only the gas is flowing alone in the pipe, if you do not confuse between these things then the derivations are very simple, but moment you confuse these things you are gone. So, you have to remember certain situations what are the situations that you are considering?

One is the two phase flow situation even, if it is a two phase flow situation the liquid phase is flowing separately the gas phase is flowing separately, if it is a separate cylinder model then the liquid phase and the gas phase they do not interact with one another fine. So, they therefore, under that circumstance the frictional pressure gradient in the liquid occupied portion will be equal to the frictional pressure gradient in the gas occupied portion. And this will be equal to the frictional pressure gradient in the two phase separated flow situation please remember these things I am repeating it repeatedly. So, that you do not have any confusions regarding the other thing, which I am considering is this particular liquid is flowing alone in the pipe this particular gas is flowing alone in the pipe.

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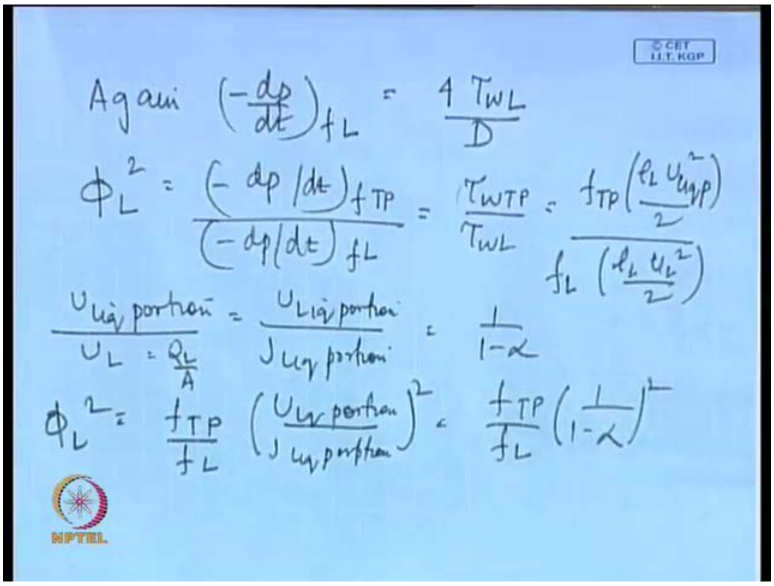

So, when this flowing alone how do we denote it is just a capital L or a capital G as a subscript. And when it is flowing to get there in the two phase mixture it is liquid portion or the gas portion in the subscript, if you want you can use some other subscripts I do not have a problem, but use some consistent nomenclature. So, that you do not get confused at the end agreed. So, therefore, from what do we get from, this particular horizontal flow if you see. So, we find out that from force balance on the combined phase and from force balance on the gas phase we have obtained two expressions. Again we know minus d p d z frictional when this particular liquid is flowing alone in the pipe, once we have liquid flowing alone in the pipe and the liquid flowing as the film moment we get the two expressions of pressure gradient.

We can divide one by the other and obtain phi L square. So, therefore, this d p d z frictional for liquid flowing alone in the pipe this would be equal to four tau W L by d same way first we obtain two phase flowing then the gas phase for that particular situation. And now we are we are found it out minus d p d z f l. So, if this f T P is divided by this particular f L then we can get phi L square agree. So, therefore, if you divide 1 by the other what do we get we get phi L square this is minus d p d z frictional two phase by minus d p d z f l. So, this is nothing, but equal to tau w T P by tau w L yes or no agree. Now, what is this equal to? This is f T P row L u liquid portion liquid portion square by 2 divided by f L row L u L square by 2 yes or no.

That this tau w T P it is the wall shear stress when the two phases are flowing as separated flow in the pipe and this is the wall shear stress, when the liquid is flowing alone in the pipe agreed fine. So, now, you tell me a few things? This u liquid portion by when this u is flowing alone in the pipe; that means, this u L is nothing but q L by a yes or no fine or in other words this can be taken as j L volumetric flux is it not? So, this can be written down as u liquid portion by j liquid portion then I write down this or not tell me and what is the relationship between these two is it not equal to 1 by 1 minus alpha you remember this fine.

So, therefore, from here phi L square if I substitute it here what do I get phi L square this is equal to f T P by f L into u liquid portion by j liquid portion whole square or in other word this is equal to f t p by f L 1 by 1 minus alpha whole square yes or no. So, we have a small portion of this left, which we can continue in the next class very small portion of this left. So, we will be doing this in the next class we will be deriving the phi L square for this actual annular flow situation. And we will find out the phi L square, which we can obtain from the separated flow model. And we will compare that to this we will be doing in the next class and then we will be proceeding for the for certain other things. So, thank you very much.