

**Multiphase Flow**  
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**Lecture No. # 22**

**Separated Flow Model - Estimation of Frictional Pressure Drop and Void Fraction**

Well. So, today will be continuing with the discussions, on the separated flow model the we have almost come to a last end. Till now, what we have done, we have written down the mass momentum as well as, energy equations for the two phases taken separately. And then we have try to combine them in different ways in order to obtain the mixture momentum equation. Then we have discussed regarding the condition of choking for the two phase two fluid model rather, when the two phases are taken together; that means the condition of choking from the mixture momentum equation.

And also the condition of choking, when the two phases are considered separately; that means, we are considered the two phases separately in this case of course, to keep matters general what we are try do is we had also incorporated the effects of change of phase. So, we have taken the two phases separately and then we try to derive the momentum equations. And the condition of choking for the two individual phases and then we found that the choking of phase one or the choking of the phase two, it does not guarantee the compound choking of the two phases. Because when two phases are flowing together there is an additional parameter alpha the void fraction, which can adjust itself and prevent compound choking.


Even though, the two phases can be separately under choked flow conditions. So, therefore, we found that in order to estimate the choking the compound choking from the two fluid model. As it is said we would like, we would have to eliminate alpha from the two equations in accordingly we found out an expression of the condition of choking, where we found out that first choking to occur one of the fluid. As to be in the subsonic condition, while other is the supersonic condition and of course, after that we try to incorporate the effects of flashing and things like that.

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Combined Momentum Equation for Simplified one dimensional approach

$$-\frac{dp}{dz} = F_w + g \cos \theta \rho_{TP} + \cos \theta + \frac{1}{A} \frac{d}{dz} [W_1 u_1 + W_2 u_2]$$

where  $F_w = F_{w1} + F_{w2} = \tau_{wTP} \frac{S}{A}$



So, the more or less we have completed the two fluid model the only thing, which is remaining at present is if you remember the basic equation, which we had derived for the two fluid model the equation, which is their in sorry which is there in this particular p t t you can see. This was the more or less the general equation and we found out that, if you consider this equation than in this equation in order to find out rho T P you need to have an estimate of the void fraction. And the other thing was the frictional shear at the frictional pressure gradient, which has been represented as F W.

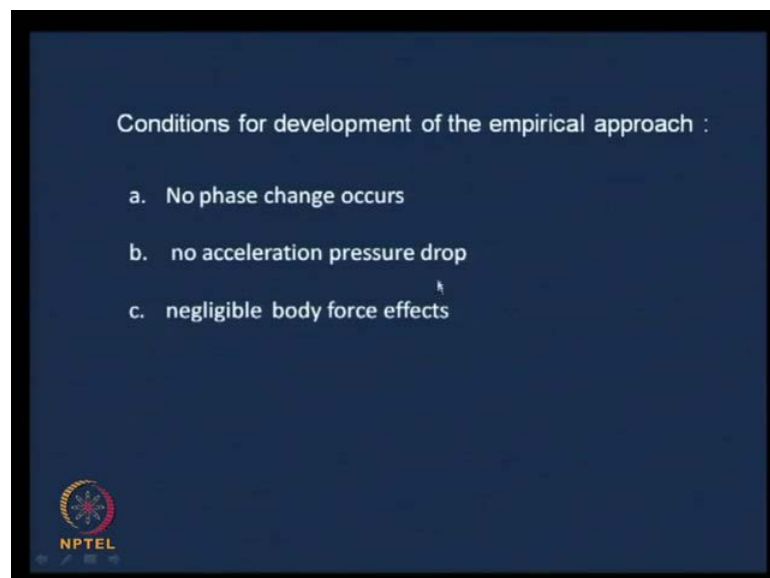
It comprises of the frictional pressure gradients, which phase one as with the wall as well as with phase two as with the wall. So, it can be written down in something of this or the two phase wall shear stress S by A, it can be written down in this particular form. So, that we can find out that finally, the combined momentum equation for the one dimensional approach can be written down. In this particular form and define that in order to evaluate pressure gradient we need to have an estimate of the frictional pressure gradient as well as alpha.

So, therefore, if you observe this particular equation very closely we find that it resembles the one dimensional homogenous flow model, which we have already done. The only variable be that in I mean allow with the frictional pressure gradient which as to be considered the alpha is also variable in this particular case. So, therefore, we find that just by relaxing, what is overly different from this equation as compare to the

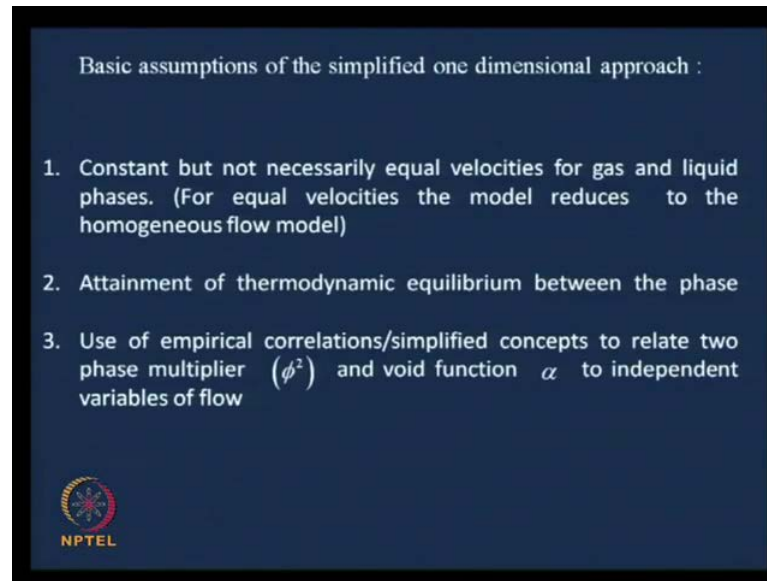
homogenous flow model expression that we had derived, if you remember the homogenous flow expression will find that.

We had one frictional pressure gradient term we had a gravitational pressure **sorry** this part is wrong the this part definitely it is wrong. So, I will try to let me see, if I can do it otherwise, just keep in mind that like this, this is a mistake this  $\cos \theta$  should not have been there anyhow will be correcting it. So, therefore, we find that there is a frictional pressure gradient we find that there is a gravitational pressure gradient. And if you remember the homogenous flow model you will find that in this expression we had  $d d z$  of  $G u$  or  $1$  by  $A d d z$  of  $W u$ , because in that case, we did not differentiate between velocities of the two phases.

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We assume that the two phases and there is no slip conditions they are flowing as a soda mixture with uniform velocity due to which we had written down  $u_1 = u_2 = \dots = u_j$ . So, what is the only assumption of the homogenous flow model that we have relaxed in this particular case, we assume the two phases to flow at two different velocities. Otherwise they cannot flow in the separated flow condition, because the densities are naturally different. So, therefore, in order to find out the frictional pressure gradient, what we basically did or the usually the approach is usually try to develop different types of empirical approach based on the condition the by the basic assumption if we can mention.

The basic assumptions if you find that most of the assumption they will be similar to the homogenous flow model except the first assumption. There we are set constant and equal velocities for the two phases. Now, remember one thing these empirical approaches, where initially developed for the gas and liquid or vapour liquid phases. Therefore, most of the terminologies will be dealing with vapour liquid or gas liquid the subscribes instead of one and two usually, will be putting the subscribe that is capital G and capital L, capital V and capital L. But in your whenever, you are using them you can use either one, two phase one is naturally, the heavier phase since it is a separated flow model phase two is the lighter phase.

In the beginning, when we were discussing about the normal cultures at already mention phase two is the depressed phase, phase one is a continues phase. When both of phases are continuous, then phase two is the lighter phase, phase one is the heavier phase. So, therefore, in order to keep priority with whatever, we have being doing till now you can use subscribe one and two to denote the heavier and the lighter phase or in other words. Since, most of them have been derived for gas liquid as well as, vapour liquid cases. You can use subscribes L and G for this case As well.

So, we find that the first assumption for the homogenous flow model the first assumption was equal velocities constant and equal velocities for the gas and liquid phases. So, this has been modified in this particular case it is constant, but not necessary definitely, not equal velocity. If it is equal then it is reduces to the homogenous flow model. So, therefore, constant but not necessarily equal velocities for the two phases. The other thing was there as well attainment of thermodynamic equilibrium between the phases; if you remember then I had said attainment of higher dynamic and thermodynamic equilibrium.

So, in this case it is attainment of thermodynamic equilibrium between the two phases. And based on this what we usually try to do usually, what happens, when we do not know anything we go for some empirical approach, how we take up we suggest some relationships, which have been obtain by curve setting a large data bang obtain from experiments. So, we try to conflict those particular data bang and then we try to obtain and empirical relationship and nothing is available. Now, how this empirical relationship is developed naturally we will try to develop it in terms of something, which we already know.

Now, the convectional practice, which I had already mention, when I have I was teaching you the homogenous flow model usually, we try to express the empirical correlations in two ways. Either, it can be in terms of friction factor or in terms of two phase multiplier. That I had already mention we can either in homogenous flow model, we had try to express in terms of  $f_{TP}$ , if you remember when we have said that  $f_{TP}$  is a function of two phase generals number are  $E_{TP}$ . There also it was not very convenient, because we have to incorporate new  $T P$ . New  $T P$  was logical any two phases by intimately makes, but when the two phases are flowing separately naturally in that case, incorporating a two phase viscosity is not all a logical thing.

So, what we have to do in this particular case, so the only option is to suggest empirical correlations in terms of two phases multipliers. Now, what are these multipliers that we have already discuss, before the two phase multiplier they expressed the frictional pressure gradient for the two phase flow. As a ratio of the single phase pressure gradient which would occur if either the gas or the liquid would be flowing alone in the pipe. So, as I had already discussed, they where four different definitions of the two phase multipliers that can be suggested.

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$$\phi_{L0}^2 = \frac{(-dp/dz)_{TP}}{(-dp/dz)_{L0}}$$

$$\phi_{G0}^2 = \frac{(-dp/dz)_{TP}}{(-dp/dz)_{G0}}$$

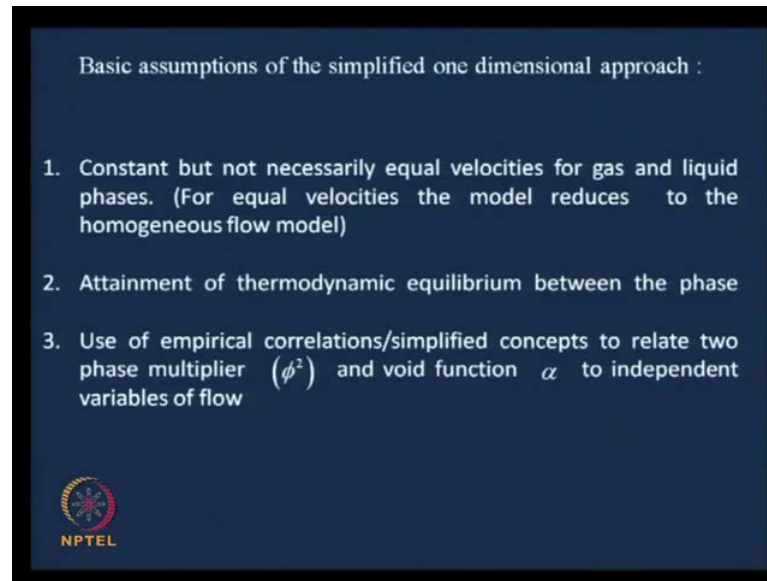
$$\phi_L^2 = \frac{(-dp/dz)_{TP}}{(-dp/dz)_L}$$

$$\phi_G^2 = \frac{(-dp/dz)_{TP}}{(-dp/dz)_G}$$

When it is either the liquid of the pipe? Which will be flowing alone now, the gas which will be flowing alone now, the entire mixture flowing as liquid, the entire mixture flowing as gas. Accordingly, we had define four frictional multipliers, you must be remember in this things. The first one was phi square L o remember these, substitute usually we use them as capital then there was phi square G o usually, we use them in terms of liquid.

So, this is minus d p d z divided by minus d p d z, when the liquid the entire mixture flows as liquid, then this was just from analogy this was as the ratio of the entire mixture flows in flowing as gas. And then we had phi L square, where the liquid portion of the mixture it is expressed as a ratio of the liquid portion of the mixture flowing alone in the pipe and phi G square, which is the gas portion of the mixture flowing alone in the pipe. So, these, where the four definitions that we had already, given.

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Now, at this particular junction I would like to mention that usually, these correlations if you see the my slide it is already written there usually, in this particular correlations they are expressed in terms of  $\phi L$  square and  $\phi G$  square. So, it can be the basic assumptions on the basis of which the empirical relationships to find out the frictional pressure gradient are based on the three assumptions, which you have given here constant. But not necessarily equal velocities for gas and liquid phases attainment of thermodynamic equilibrium between the phases.


Use of empirical correlations of simplified concepts to relate to phase multiplier as well as, void fraction  $\alpha$  to independent variables of flow. The first we would like to relate the two phase multiplier and then naturally, if the two phase multiplier is related with that particular parameter  $\alpha$  should also be related. So, accordingly, will find out empirical correlations to relate two phases multiplier as well as, the void fraction to independent variables of flow.

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Combined Momentum Equation for Simplified one dimensional approach

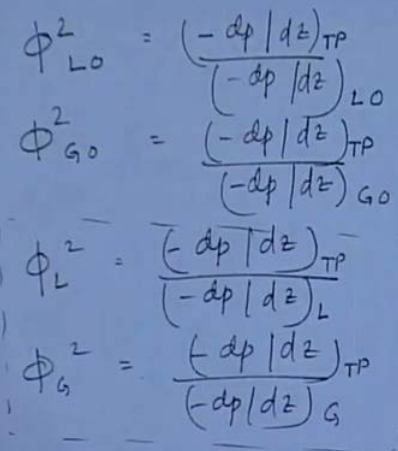
$$-\frac{dp}{dz} = F_w + g \cos \theta \rho_{TP} + \cos \theta + \frac{1}{A} \frac{d}{dz} [W_1 u_1 + W_2 u_2]$$

where  $F_w = F_{w1} + F_{w2} = \tau_{wTP} \frac{S}{A}$



Once this can we done we find out, that F W can also are the frictional pressure gradient can also be calculated and your once alpha is known your rho T P can also be calculated u 1 u 2 can also be calculated. So, therefore, the frictional pressure gradient can be obtain from this particular equation. Now, here I would like to mention that out of the four definitions of phi square, which I have already written down just now the four definition we have which we have written down.

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$\phi_{L0}^2 = \frac{(-dp/dz)_{TP}}{(-dp/dz)_{L0}}$

$\phi_{G0}^2 = \frac{(-dp/dz)_{TP}}{(-dp/dz)_{G0}}$

$\phi_L^2 = \frac{(-dp/dz)_{TP}}{(-dp/dz)_L}$

$\phi_G^2 = \frac{(-dp/dz)_{TP}}{(-dp/dz)_G}$





Just now, out of that usually, we adopt phi L square and phi G square, rather we correlate phi L square and phi G square for our purpose. We do not use phi usually, this phi L square and phi G square they are used for, when we encountered change of phases or when there is boiling condensation etcetera. When a single phase the liquid a saturated liquid they are saturated vapour is entering and change of phase occurs. Under that condition if you take phi L square phi G square, then it is much more convenient because liquid or the vapour was flowing alone in the tube at the n t condition.

But under normal circumstances, when we have developing the two phase multiplier correlations from the separated flow model we assume from the very beginning both the phases are present. In that case, we have found that it is much more convenient to use phi L square and phi G square, why is it more convenient? Because we means two phi that these two multipliers they confirm to certain limiting conditions, which are not obeyed by phi square L o and phi square G o.

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$$\Phi_L^2 = \frac{(-dp/dz)_{TP}}{(-dp/dz)_L} \quad \Phi_G^2 = \frac{(-dp/dz)_{TP}}{(-dp/dz)_G}$$

For no gas flow

$$\frac{1}{\Phi_L^2} = 1.0 \quad \frac{1}{\Phi_G^2} = 0$$

For no liquid flow

$$\frac{1}{\Phi_L^2} = 0 \quad \frac{1}{\Phi_G^2} = 1$$

Just, because of that, we would like to prefer or we would like to correlate phi L square and phi G square. Now, what are those limiting conditions let us see, what are those limiting conditions? Now, for no gas flow what will you get, let me write down the definitions once more phi L square minus d p d z by minus d p d z liquid phi G square is minus d p d z by minus d p d z gas. Now, tell me for no gas flow what will you get, what should be the value of phi L square and phi G square for no gas flow.

What is one by phi L square equals to for that particular case, only liquid for no gas promise only liquid is flowing. So, one by phi L square it will be equal to since only liquid is flowing. So, therefore, the two phase sorry these, are all two phase pressure gradient. Here also I would like to make the changes they are all two phase pressure gradient I am **sorry** in case, the I missed it just remember that they are all two phase pressure gradient. Now, you tell me, what is the when there is no gas flow then naturally it is only liquid, which is flowing through the pipe. Is it not?

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Limiting conditions of correlation :

For no gas flow :

$$\frac{1}{\phi_l^2} = 1.0 \quad \text{and} \quad \frac{1}{\phi_g^2} = 0$$

For no liquid flow :

$$\frac{1}{\phi_g^2} = 1, \quad \frac{1}{\phi_l^2} = 0$$

NPTEL

So, therefore, under that condition phi L square becomes or 1 by phi L square phi I am using 1 by phi L square you will understand. 1 by phi L square becomes 1.0, and what about 1 by phi G square 0? Just for this I have define, them not as phi L square phi G square, but as 1 by phi L square 1 phi L square. Similarly, for no liquid flow I also have them in my in may presentation you can see the p p t, which I have prepared, here also for no liquid flow no. So, naturally what do you have it is 1 by phi G square equals to 1 and 1 by phi L square equals to 1.

Remember one thing, these are the limiting conditions, which phi L o square and phi G o square would not have confirmed two. Is it not? So, therefore, when there was no liquid flow also we would find that there would be some value of phi square L o. Is it not? Because we what do we assume that the entire mixture is flowing as liquid. So, when there is no gas flow also the liquid flow would there do you understand? So, such

limiting conditions would not have been or they would not have been subscribe by your phi square L o and phi square G o.

Now, another limiting condition that is also obeyed by this, that limiting condition is relationships between phi L square and phi G square at the critical point. All of you know, what is the critical point, what happens at the critical point? Louder the two phases cannot be distinguished at the critical point that is the definition. Why it as that happen, rho one becomes equal to rho two mu 1 becomes equal to mu two and therefore, all the properties of the two phase is become identical. Now, let us see how the relationship between phi L square and phi G square they change at the critical point.

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SECRET  
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$$\phi_L^2 = \frac{(-dp/dz)_{fTP}}{(-dp/dz)_{fL}} = \frac{\rho z f_{TP} G_{TP}^2 U_{TP}}{\rho z f_L G_{TP}^2 (1-x)^2 U_L}$$

$$(-dp/dz)_{TP} = (-dp/dz)_{fTP} \frac{\rho z f_L G_{TP}^2 (1-x)^2 U_L}{\rho z f_L G_{TP}^2 (1-x)^2 U_L}$$

$$\phi_L^2 = \frac{f_{TP} U_{TP}}{f_L (1-x)^2 U_L} \quad \text{At critical point}$$

$$U_L = U_G = U_{TP}$$

$$f_{TP} = K \left[ \frac{M_{TP}}{D G} \right]^{-n}$$

$$f_L = K \left[ \frac{M_L}{D G (1-x)} \right]^{-n}$$

NPTEL

Now, what is the basic definition of these two, which here I just completed this was one by phi L square equals to zero and this was 1 by phi G square equals to 1, anyhow. Now, at the critical point what do you get? We find that at the critical point or rather let us, first find out the expression and then will the substituting the expressions for the critical point. So, what is the definition of phi L square? It is as I have just now written down it is the two phase pressure drop divided by the pressure drop, when liquid flows alone in the phi. Now, if you substitute the basic definition thus, at there is one more thing sorry, which I had forgotten to mention now, these things or these empirical approach this has been derived under certain conditions.

These, conditions if you observe these conditions are these particular derivations or empirical approaches have been derived under conditions of no phase change no acceleration pressure drop and negligible body force happens; that means, for a horizontal pipe under isothermal two components flows. Do you agree with me, it as to be horizontal pipe under two component isothermal flow, yes or no? So, if that is the case, then what do we find your  $G \cos \theta$  this term it cancels out there is no acceleration pressure drop this term it cancels out. And therefore, under that condition your pressure gradient is nothing it comprises only herb of the frictional pressure drop. Is it not?

So, therefore, we find that this particular correlations they had been most of the empirical approaches they have been developed under conditions of no phase change, no acceleration pressure drop and negligible body force separates. Under which we can write under these condition, we can write it down as minus  $d p / d z$  two phase this comprises of minus  $d p / d z$  frictional component of the two phases. So, this can be written and later on some modifications, where they in alternative incorporate effects, when body force effects an acceleration pressure gradient become important. But naturally, the more important they become the less accurate this correlations become. So, under this particular condition the correlations where developed.

So, if this is the applicable, then in that case your  $\phi L^2$  it becomes here only I will write down, this is frictional pressure gradient of two phase and this also become frictional pressure gradient of the liquid phase. Is it not? So, therefore, by definition what is this? By definition this becomes  $2 F T P$  correct me if I am writing anything wrong,  $G^2 T P v T P$  by  $D$  yes or no divided by  $2 F L G^2 T P$  into  $1 - X^2$ . Is it not? This is the amount of liquid, which Is there it gen to  $1 - X$  is the amount of liquid, which is contained in the two phase mixture right this will be  $v L$  or  $v 1$ . Whatever, you want and this will be divided by  $d$ , you agree with what I have written yes or no fine. Same on other words see in this same way your minus  $d p / d z$  frictional for the gas phase that also can be expressed in a similar way.

So, therefore, we find what is the expression of  $\phi L^2$  when these particular the derivarti the assumptions which I have written down in my transparency under this particular assumptions. What is the expression of  $\phi L$  than, we find that these, cancels out,  $D$  cancels out  $D$  cancels out. So, therefore, this reduces to  $F T P v T P$  by  $F L 1 - X^2 v L$  yes or no. Is this portion clear to all of you, fine [FT]. Now, at the

critical point what do you get?  $\rho_1$  is equals to  $\rho_2$  or  $v_1$  equals to  $v_2$ . Is it not? So, therefore,  $v_L$  equals to  $v_G$  equals to  $v_{TP}$  yes or no. So, therefore, these two also they cancel out do you get this. Right, so therefore, this becomes  $F_{TP}$  by  $F_L$  into  $1 - X$  whole square. Now, what is the expression of  $F_{TP}$ , can you tell me? This  $F_{TP}$  this is a function of Ronaldo's number.

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The whiteboard contains the following handwritten notes and equations:

$$\phi_L^2 = \left[ \frac{\mu_{TP}}{D_G} \right]^{+n}$$

$$\left[ \frac{\mu_L}{D_G(1-X)} \right]^{+n} (1-X)^L$$

At critical point  $\frac{(1-X)^n}{(1-X)^2} = \phi_L^2$

$$\phi_G^2 = \frac{X^{+n}}{X^2} = X^{n-2}$$

$$\phi_L^2 = \frac{1}{(1-X)^{n-2}}$$

For laminar flow  $n = 1$

$$\phi_L^2 = (1-X)^{-1} \quad \phi_G^2 = \frac{1}{(X)^{n-2}}$$

So, therefore, this becomes a function or this is  $K \mu_{TP}$  by  $D_G$  whole to the power minus  $n$ , you agree  $[FT]$  same way  $F_L$  this is equal to  $K \mu_L$  by or in fine we can write it in this  $d_G$  into  $1 - X$  whole to the power minus  $n$  yes or no. So, therefore, here we can write it down as this is equal to I have to shift to the next page this is equal to five L square, this is equal to  $\mu_{TP}$  by  $d_G$  whole to the power minus  $n$  divided by  $\mu_L$  by  $d_G$  into  $1 - X$  whole to the power minus  $n$   $1 - X$  square this is the relationship, which we get again, at the critical point for we know  $\mu_1$  equals to  $\mu_2$  equals to  $\mu_{TP}$ .

So, therefore, at critical point what do we get  $\mu_{TP}$  is at  $\mu_2$   $\mu$  cancels out  $d_G$   $d_G$  they also cancel out, fine. So, what is the final thing that you get for this particular case, what is the final expression that you get? It is  $1 - X$  whole to the power  $n$  by  $1 - X$  square this is equal to  $\phi_L$  square yes or no, just check up whether I am doing things correctly or not. Similarly,  $\phi_G$  square what will you get for  $\phi_G$  square it is going to be  $X$  to the power  $n$  by  $X$  square will be equal to  $\phi_G$  square.

So, we find that phi L the relationship between minus, what happen it is allright only is it not **sorry sorry** here also I made this mistake is it not minus n **sorry sorry very sorry**, fine. In other words you can just write it down as minus n plus two. So, therefore, what do we find that the relationship between phi L square and phi G square they at the critical point they will depend on whether the flow is laminar or turbulent yes or no. Because we find that both at the critical point the phi L square it is a function of X and n is it not and in this particular case also it is a function of X and n.

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$$\phi_L^2 = \frac{(-dp/dz)_{TP}}{(-dp/dz)_L} = \frac{f_{TP} U_{TP}^2}{f_L U_L^2}$$

$$(-dp/dz)_{TP} = (-dp/dz)_L \frac{f_{TP} U_{TP}^2}{f_L U_L^2}$$

$$\phi_L^2 = \frac{f_{TP} U_{TP}^2}{f_L (1-x)^2 U_L^2}$$

At critical point  $U_L = U_G = U_{TP}$

$$f_{TP} = K \left[ \frac{M_{TP}}{D G} \right]^{1+n}$$

$$f_L = K \left[ \frac{M_L}{D G (1-x)} \right]^{1+n}$$

$$\frac{f_L}{f_{TP}} = \left( \frac{1}{1-x} \right)^{1+n}$$

So, therefore, the relationship between phi L square and phi G square at the critical point, that will depend upon whether, the flow is laminar or whether it is turbulent. If it is laminar then what we get for laminar flow n equals to 1 very correct. So, therefore, phi L square will be equal to come on phi L square it is going to be 1 minus X only whole to the power of minus 1 check it up how can it be minus 3 just check it n is 1. So, therefore, this is 1 by phi L square, if I meted then in that case, let me make it 1 by phi L square, then this is going to be 2. Just then provably I had made a just a minute let me just check up may be somewhere, there is a mistake, which I have made f L by f T P for did I get there preferably I have made a mistake.

So, suppose I start from the very beginning for the laminar flow then this becomes minus 1. Suppose, for laminar flow if I start from the very beginning. So, phi L square becomes this is minus 1, this is minus 1 and then in that case, we get F by f T P in this particular

case, what do we get?  $f_L$  by  $f_{TP}$  this is equal to  $1 - X$  very correct. And in the same way we will get just a minute, where  $f_L$  by  $f_{TP}$  next page. Here only I made mistake sorry here only very sorry here only I made a mistake just a minute if  $f_{TP}$  equals to  $K R e^{-n}$ . So, I have written  $1 - R e^{-n}$  there only I had a mistake this is  $1 - R e^{-n}$  I have already written. So, therefore, it as to be plus here only I had none of you pointed out the mistake that is thing from here itself I had started making the mistakes.

So, therefore, it is as to be plus and this as to be plus. Is it correct? Because the relationship is  $F$  equals to  $K R e^{-n}$  the power I had already written by  $1 - R e^{-n}$  in this particular case there actually I made a mistake I am very sorry about it. So, very sorry now it is alright is it not very sorry. So, therefore,  $\phi_L^2$  this is equal to  $1 - X^n$  minus 2 fine or in other words  $1 - \phi_L^2$  this is going to be  $1 - (1 - X^n)^2$  whole to the power  $n - 2$  correct. And  $\phi_G^2$  will be  $X^n$  to the power  $n - 2$  very true and I had just made it the denominator **[FT]** and this one is going to be  $1 - \phi_L^2$  acting at 1, because these two relationships will come out in a very straight forward manner fine.

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For laminar flow  $n = 1$

$$\frac{1}{\phi_L^2} + \frac{1}{\phi_G^2} = (1-x) + x = 1$$

For turbulent flow  $n = 1/4$   
(Blasius type of eqn)

$$\left(\frac{1}{\phi_L^2}\right)^{4/3} + \left(\frac{1}{\phi_G^2}\right)^{4/3} = 1$$

$$= \frac{1}{(1-x)^{1/4-2}} + \frac{1}{x^{1/4-2}}$$

Now, for laminar flow what do you get? For laminar flow your  $n$  equal to 1, is it not. So, when you get  $n$  equal to 1, then in that case what do you get  $1 - \phi_L^2$ ? So, therefore, you just do it and then will see how do you get in plus you added  $1 - \phi_L^2$

square plus 1 by phi G square this what you are getting this will become 1 minus X plus X equal to 1 for laminar flow correct, for turbulent flow suppose, you this is for laminar flow n equals to 1. Now, for turbulent flow what you will get turbulent flow n equals to generally, for Blasius type of equation, if we use Blasius type of equation, if we use then this n equals to mutually one forth is it not.

So, if you substitute that then in that case, we find that in this particular case, if you just do it just calculate 1 by phi L square whole to the power 4 by 7 plus 1 by phi G square whole to the power 4 by 7. So, that we can have one at the end just for that reason, if you just calculated and you can find out what is the thing that your getting this will become 1 by suppose, in this particular case we know 1 by phi phi L square was 1 by 1 minus X whole to the power n one forth minus two is it not plus 1 by X to the power one forth minus two.


So, in that case you do it is going to be 7 by 4, 7 by 4. So, therefore, this becomes equal to 1. So, now is it clear what are the limiting conditions to which the relation between phi L square and phi G square they comfort to the limiting conditions are in actually if I write down the limiting conditions for no gas flow 1 by phi L square equals to 1 and 1 by phi G square equals to 0. Confusions remaining now, also, just do the derivation. See these are small derivation this have come just from single phase fluid flow there is nothing to fees in this.

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At critical point :

for laminar flow and  $\frac{1}{\phi_l^2} + \frac{1}{\phi_g^2} = 1.0$

for turbulent flow  $\left(\frac{1}{\phi_l^2}\right)^{4/7} + \left(\frac{1}{\phi_g^2}\right)^{4/7} = 1.0$





But please remember whenever, you are doing any derivation for your exams are something please remember that start from the very basic start from the definitions and then we are going to proceed this remember. So, in a actual what are the limiting conditions of the correlation for no gas flow  $1 \text{ by } \phi L^2 \text{ equals to } 1$ ,  $1 \text{ by } \phi G^2 \text{ equals to } 0$  for no liquid flow  $1 \text{ by } \phi G^2 \text{ equals to } 1$  and  $1 \text{ by } \phi L^2 \text{ equals to } 0$ . And at the critical point we have two different relationships depending upon whether the two phases are in laminar flow or in turbulent flow for laminar flow as it is very evident it is  $1 \text{ by } \phi L^2 \text{ plus } 1 \text{ by } \phi G^2 \text{ is equals to } 1$ .

For turbulent flow assuming a Blasius type of relationships we get  $1 \text{ by } \phi L^2 \text{ to the power } 4 \text{ by } 7 \text{ plus } 1 \text{ by } \phi G^2 \text{ to the power } 4 \text{ by } 7 \text{ equals to } 1$ . So, remember one thing we have started deriving the relationships between the two phases yeah this things more less as been covered. So, therefore, we have started deriving the relationships between the two phases between  $\phi L^2$  and  $\phi G^2$ . Just because we known that from the very beginning both the phases are going to be present and  $\phi L^2$  and  $\phi G^2$  they confirm to certain limiting condition to which  $\phi L^2$  or  $\phi G^2$  do not confirm.

And the limiting conditions are as we have derive. Now, so therefore, based on these fact what have been done in empirical certain, empirical correlations have been purposed. Now, these empirical correlations they have related  $\phi L^2$  and  $\phi G^2$  based on certain assumptions, what are the assumptions that I already mention. The first assumption is that the flow occurs or the in empirical correlations have been derived for circumstances of isothermal two components flows. Where in horizontal pipes or in other words to be more explicit the correlations have been derived for conditions were negligible body forces are there, there are no acceleration pressure gradient or no phase change.

And more or less the total pressure gradient comprises of the frictional component only under that circumstances, these emissions as been derived. Now, the [FT] derivation are the mostly widely used derivations, which was derived as it was the first almost the first derivation, which was derived in this regard was made by a group of workers. Commonly known as the Martinelli and Nelson coworkers or the Lockhart and Martinelli correlation these two persons they first derive the empirical correlations. And a

subsequently they were several other modifications of this correlations, but the basic form as remain.

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Assumptions of Lockhart and Martinelli correlation :


(1) They flow in two separate cylinders such that the cross-sectional area of the two cylinders is equal to the pipe cross section

$$A_1 + A_2 = A$$

(2) The two phase do not interact with each another

(This is the most severe assumption and is the primary cause for the mismatch between the experimental and predicted values.)

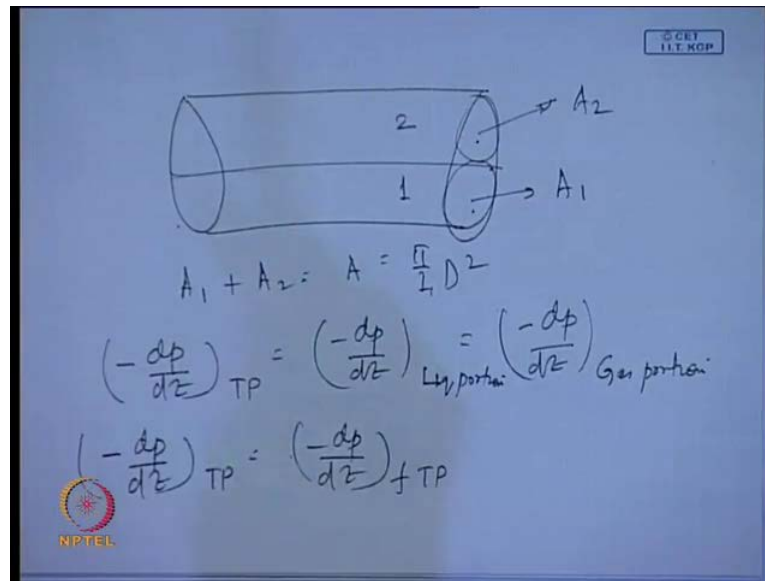
(3) The pressure drop in each imagined cylinder is same as in actual flow or

$$\left( -\frac{dp}{dz} \right)_{TP} = \left( -\frac{dp}{dz} \right)_l = \left( -\frac{dp}{dz} \right)_g$$


So, will be discussing the Lockhart and Martinelli correlation first and then will be discussing the modifications, which has been suggested to expand the range of applicability of this correlations. Now, this Lockhart and Martinelli correlation it was derived on the basis of certain assumptions. What are the assumptions? The assumptions where some assumptions are justified some are not the first assumption, is that the two phases they flow in two separate cylinders that is why justified that they do not include into one another. And therefore, the cross sectional area of the two cylinders is equal to the pipe cross section.

This was the first assumption, it was first assume that both the phases they flow under in separate cylinders. Such that, the cross sectional area of cylinder 1  $A_1$  plus, the cross section of area of cylinder b if the cylinder two or cylinder b a two they had a to give you the total cross sectional area of the pipe.

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So, mathematically the first assumption is  $A_1 + A_2 = A$ . What is the next assumption? Come to assumption three, the pressure drop in each imagine cylinder is same as in actual friction quite natural. So, the thing is it is sort of if you imagine it is something of this sort we have pipe say more or less fluid 2 is flowing here fluid 1 is flowing here. We assume that more or less they are flowing in two separate cylinders such that, the cross sectional area of this and cross sectional area of this they had two form the total cross sectional how area of the pipe, which is nothing but  $\frac{\pi}{4} d^2$  square.

The next assumption is they do not interact at the inter phase, if they will not interact at the inter phase; that means, they are flowing irrespective of the presence of the they are do not know that the other fluid is present. Under that condition the pressure drop across the cylinder 2 will be equal to the pressure drop across the cylinder 1, which will be equal to the pressure drop across the total contact yes or no it is just like, a parallel connection the voltages are seen this is just a same thing. So, therefore, we find that if we assume if it consider this second assumption the two phases do not interact with each other. Then we find under that condition the pressure drop or the pressure gradient across the entire cylinder this will be equal to the pressure drop across the cylinder.

The liquid cylinder, if we say or the cylinder 1 this will be equal to the pressure drop across the gas portion. So, let us see remember one thing we have already define,  $\phi L$

square and phi G square lot of definitions are coming into picture. If you remember this L was when liquid was flowing alone in the pipe. This was when gas alone was flowing in the pipe. Now, when we are we are defining this, this is the liquid portion when both the phases are flowing together. So, definitely the cross sectional area occupied in this case will not be equal as the cross sectional occupied from the liquid portion is flowing alone in the pipe do you get it.

So, therefore, this  $dp/dz$  and this  $dp/dz$  this cannot be the same. So, therefore, we have to remember it and we have to denote this with a different normal culture we just put it liquid portion gas portion whatever, you want you can use your own normal culture it does not matter. But we have to remember one thing that here for what is the assumption that I have written assumption is pressure drop in each imagine cylinder is the same as in actual flow. So, therefore, here the pressure drop refers to the pressure drop across the liquid cylinder in this case, it is across the gas cylinder. And the pressure gradient which I had define for the two phase multiplier they refer to the pressure drop, which would be encountered when the liquid, which is flowing here only in this portion.

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(4) The pressure drop is mainly due to the frictional component or

$$\left( -\frac{dp}{dz} \right)_{TP} = \left( -\frac{dp}{dz} \right)_{fTP}$$

Where  $\left( -\frac{dp}{dz} \right)_{fTP}$  can be calculated from single phase theory since

$$\left( -\frac{dp}{dz} \right)_{fTP} = \left( -\frac{dp}{dz} \right)_{fl} = \left( -\frac{dp}{dz} \right)_{fg}$$

5 They have defined four flow patterns on the basis of flow behavior (laminar /turbulent ) when the respective phases flow alone in the pipe.



Now, flow to the whole pipe is it clear. Since, they have occupying different cross sectional area naturally the frictional factor everything the total everything is going to change. And therefore, this particular pressure drop and this particular pressure are going to be completely different this part you suppose to remember. So, therefore, there is a

need so since will be dealing with this much more. So, when liquid or gas flows alone in the pipe they will be denoted by a subscript capital L or capital G. And first it is the liquid portion, which is flowing then we are going to protect either liquid portion or liquid cylinder whatever, it is. So, this is the third assumption, which we have going to gain and the fourth assumption, which you already know, we assume that the pressure drop is mainly due to frictional component which are also put down in my ppt.

In the ppt also this has been put down so therefore, the fourth assumption, which has been put down in my transparency or in my slide, which I have prepared. So, this is minus  $\frac{dp}{dz}$  this primarily compute comprises of the frictional pressure gradient is it clear are these two assumptions clear. The first assumption was fine it assumes to flow in two separate cylinders, where the two cross sectional areas at the to form the total cross section area of the pipe. Second assumption is the two phases do not interacted one another remember, one thing this is the most severe assumption and we will see at one point why that it does not give you completely accurate results. The primary result is the assumption two which I have put down in my transparency.

In the transparency or in my projector second assumption, which I have put down which tells you that the frictional pressure sorry the two phases do not interacted with one another. If it will not interact then naturally it is not two phase flow at all, because the definition, which I had given about two phase flow is it is the interacting flow of two phases. So, therefore, remember one thing if that assumption is not valid then there is no point. So, therefore, this is the most severe assumption and we will see that for this particular assumption it does not give you completely accurate results. But for the time being since the relationships have been will be derived based on certain curve fitting of experimental data large number of data.

So, more or less the relationships, which we get are accurate. So, the next assumption is the two phases do not interact with one and another. Now, movement this particular assumption comes into picture. So, naturally what happens? The pressure gradient across the liquid portion or across the liquid cylinder will be equal to pressure gradient, across the gas cylinder and that will be equal to the pressure gradient, across the two phase flow or across the mixture, across the composite gas liquid cylinders is it not. So, therefore, this second assumption, which I have written down this, is nothing but if you take the

transparency you are going to see. The second assumption is that the total pressure yeah the total pressure gradient, it is equal to the pressure gradient in each imagine cylinder.

So, the third assumption is basically nothing but it is a extinction of this second assumption. So, the next assumption is the pressure drop arises mainly due to frictional component how it assumption did this assumption come up, because this correlations where derived for two component flows. They will derive for air water mixtures in horizontal pipes. So, therefore, in horizontal pipes means, there was no gravitational component two component air water mixtures means, there was no face change. So, there was no acceleration pressure gradient and they were in circular pipes.

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The image shows a handwritten derivation on a blue background. It consists of three numbered steps:

- $$\textcircled{1} \left( -\frac{dp}{dz} \right)_{TP} = \left( -\frac{dp}{dz} \right)_{f, TP} = \left( -\frac{dp}{dz} \right)_{f, \text{liquid portion}}$$

$$= \left( -\frac{dp}{dz} \right)_{f, \text{gas portion}}$$
- $$\textcircled{2} A_L + A_G = A$$
- $$\textcircled{3} \phi_L^2 = \frac{\left( -\frac{dp}{dz} \right)_{TP}}{\left( -\frac{dp}{dz} \right)_L}$$

$$= \frac{\left( -\frac{dp}{dz} \right)_{f, TP}}{\left( -\frac{dp}{dz} \right)_{f, L}}$$

$$= \frac{\left( -\frac{dp}{dz} \right)_{f, \text{liquid portion}}}{\left( -\frac{dp}{dz} \right)_{f, L}}$$

There are logos for 'CET IIT KGP' in the top right and 'NPTEL' in the bottom left of the slide.

So, there was no area change as well. So, therefore, the pressure drop is mainly due to frictional component in other words minus d p d z T P as I have written in down. This will be equal to minus d p d z the frictional component of two of the two phase mixture, fine. Now, from this particular assumption and this particular assumption what do we get from this two assumptions we get. So, therefore, minus d p d z two phase this is nothing but the frictional component of the two phase pressure gradient, which is nothing but the fractional component in the liquid portion, which will be equal to the frictional component in the gas portion.

So, this is the finally, this is the final expression, if you couple up all the assumptions which we have. So, finally, the assumption one is this assumption two is A 1 or A L plus

A G equals to A and what is the next assumption. Now, so therefore, in order to find out the or rather, how to find out your fictional pressure gradient? We assume that this frictional pressure gradient can be obtained, if we know the frictional pressure gradient in the liquid portion. Now, in the liquid portion it is simply single phase liquid flowing through the pipe. So, therefore, this can be obtained from single phase flow equations this can also be obtained from single phase flow equations. Is it not?

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$$\phi_L^2 = \frac{(-dp/dz)_{f \text{ liquid portion}}}{(-dp/dz)_{fL}}$$

$$\phi_G^2 = \frac{(-dp/dz)_{f \text{ gas portion}}}{(-dp/dz)_{fG}}$$

Lockhart & Martinelli correlation  
or separate cylinder Model

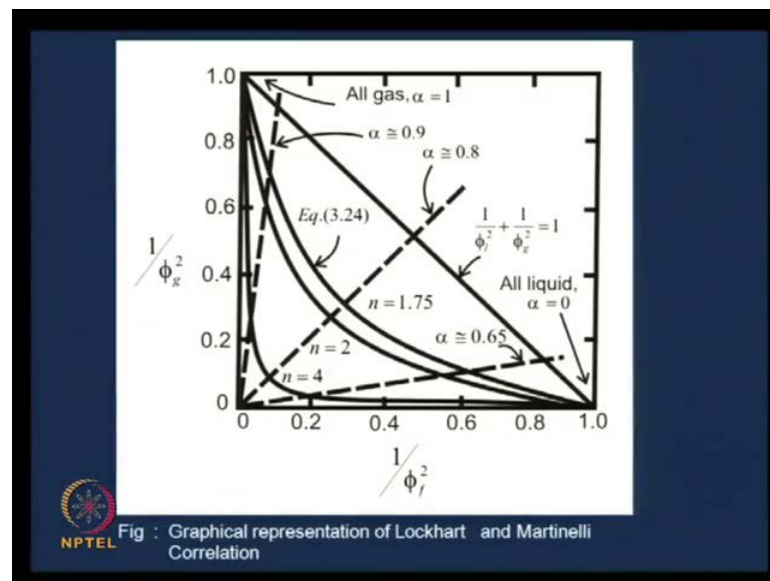
So, therefore, under these two conditions what happens my phi L square this was originally this was minus d p d z T P, if you remember by minus d p d z liquid right. Now, this becomes minus d p d z frictional T P fine or let me write it sorry f T P by minus d p d z your frictional part of the liquid, because for liquid there is no acceleration it is horizontal pipe. So, all this assumptions were applicable or in other words this is nothing but equal to minus d p d z frictional of the liquid portion divided by minus d p d z frictional, when the same liquid flows alone in the pipe. Could you get the thing the simplification, which has been reduced in this particular portion?

Now, we will see that see same liquid is flowing here it is flowing through some portion of the pipe here it is flowing through the entire portion of the pipe yet, these two pressure gradients are not equal. Do you understand? As a result of the assumptions, which we have I had written down in my transparency as well as, here as a result of these assumptions what I have arrived at? I have arrived at finally, as a result of this from the

Lockhart and Martinelli correlation just remember the first group, which I had given the correlation it is the Lockhart and Martinelli correlation or this is usually told as the separate cylinder model.

This is also known as the separate cylinder model. So, therefore, from this model we get  $\phi_L^2$  this nothing but minus  $d p d z$  frictional liquid portion of the pipe divided by minus  $d p d z$  frictional, when that liquid flows alone in the pipe. Similarly,  $\phi_G^2$  square this will be minus  $d p d z$  you can take frictional gas flowing L see gas portion and liquid portion they are the equal is it not it F gas portion and F liquid portion is equal, which I have already shown you. Here minus  $d p d z$  two phase minus  $d p d z$  frictional component is the frictional component in the liquid portion frictional component in the gas portion.

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So, therefore, this therefore becomes by minus  $d p d z$  frictional and the gas flows alone in the pipe agree. So, therefore, what basically they did the Lockhart and Martinelli they purposed a correlation based on the separate cylinder model, where they try to correlate these two parameters  $\phi_L^2$  and  $\phi_G^2$ . More or less the graphical correlation is if you see here the graphical correlation is something of this chat. They have try to correlate  $1/\phi_G^2$  one by  $\phi_L^2$  will be discussing this graphical correlation. And then will find that just very simply that seeing very simply



analysis we can find out, the analytical expression for the family of curve which I have shown here.

So, basically Lockhart and Martinelli they stopped by purposing this graphical correlation and then subsequently the coworkers Martinelli etcetera. They put purpose graphical correlation simple by substituting minus  $d p d z$  F liquid portion minus  $d p d z$  F liquid flows alone in the pipe by simple substituting these, expressions correctly from single phase flow equations. They could arrive at analytical expression relating  $\phi L$  square and  $\phi G$  square and as it evident you can very well understand. That this relation will again depend upon whether, that two fluids are flowing through laminar flow turbulent flow or whatever it is tomorrow will be continuing our discussion regarding the Lockhart and Martinelli correlation. And then if time permits will we going to the other modifications of the correlation thank you very much.