Multiphase Flow
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Lecture No. # 21
Separated Flow Model - Condition of Chocking (Contd.)

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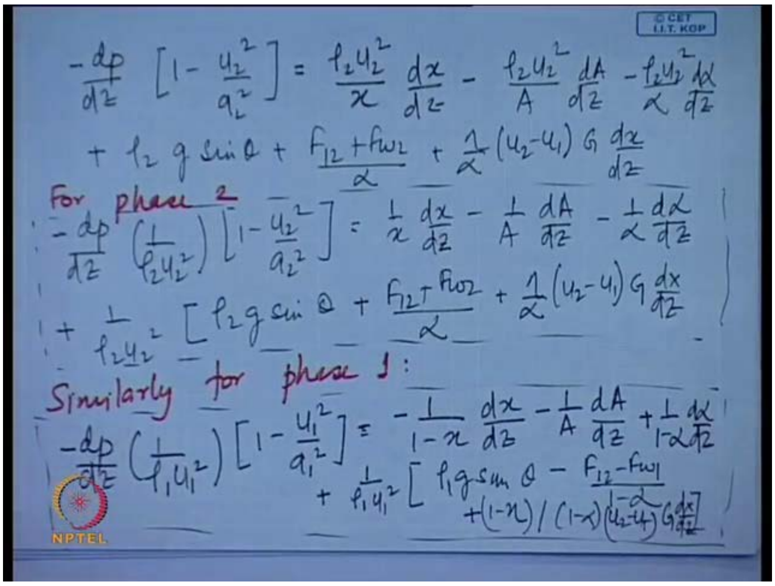

So, we will be continuing with our discussions, which we already had started in the last class. We were trying to derive the condition of chocking constrained with two phases separately and had also, assumed or we had also considered the change of phase, so that at one go I can make the whole derivation, and then you can make the appropriate simplifications as and when necessary. So, if you follow what we had done till the last class we will find that what we did we finally, arrived at a momentum equation for phase two. In the same way, we can write down a momentum equation for phase one as well just write it down let us see what you get in this particular case, same thing we get minus d p d z in this particular case, 1 by rho 1 u 1 square 1 minus u 1 square by a 1 and this particular case it was 1 by 1 minus x.

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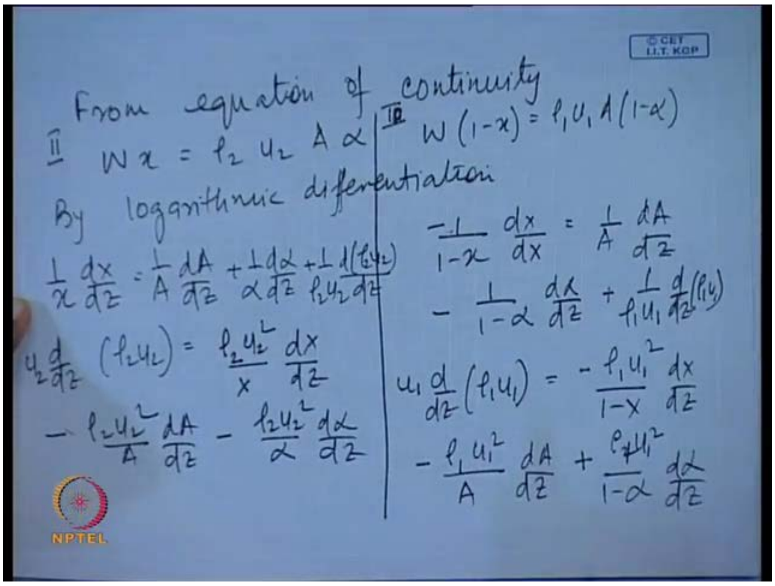

So, therefore, it is minus d x d z is it not here, if we write it down this is going to be 1 by 1 minus x minus d x d z this will be 1 by a d a d z again minus 1 by 1 minus alpha d alpha d z if you remember, again plus 1 by rho 1 u 1 d d z of rho 1 u 1. Remember these things just instead of this equation of continuity I have I am taking this particular equation of continuity. So, therefore, in this particular case also if you write u 1 d d z of rho 1 u 1, we would get minus rho 1 u 1 square by 1 minus x d x d z again here, minus rho 1 u 1 square by a d a d z in this case plus rho 2 sorry rho 1 u 1 square by 1 minus alpha d alpha d z.

So, see even if there are two phases and we doing the same thing for the two phases as well there is some minus plus sign here. So, there are plenty of scopes for you to do careless mistakes and plenty of scopes for me to cut marks. So, these things you are suppose to remember otherwise, I would not have elaborated the calculations here just to tell you that do not do things blindly even if you doing it blindly also you must use some of your common sense for doing it. So, therefore, in the same way if you write it down just look out whether I am doing any mistakes or not.

So, it becomes minus 1 by 1 minus x d x d z minus 1 by a d a d z then plus 1 by 1 minus alpha d alpha d z plus 1 by rho 1 u 1 square rho 1 g sine theta then this will be minus remember f 1 2 minus f w 1 by 1 minus alpha five. Where f 1 2 was positive for phase two and negative for phase one if you remember the direction of position as we had taken, plus that 1 minus eta by 1 minus alpha that particular term g d x u 2 minus u 1 and this term fine, I will write it down u 2 minus u 1 g d x d z agree. So, for phase two we have got one particular equation this is the equation for phase two, this equation we had already got in the last class.

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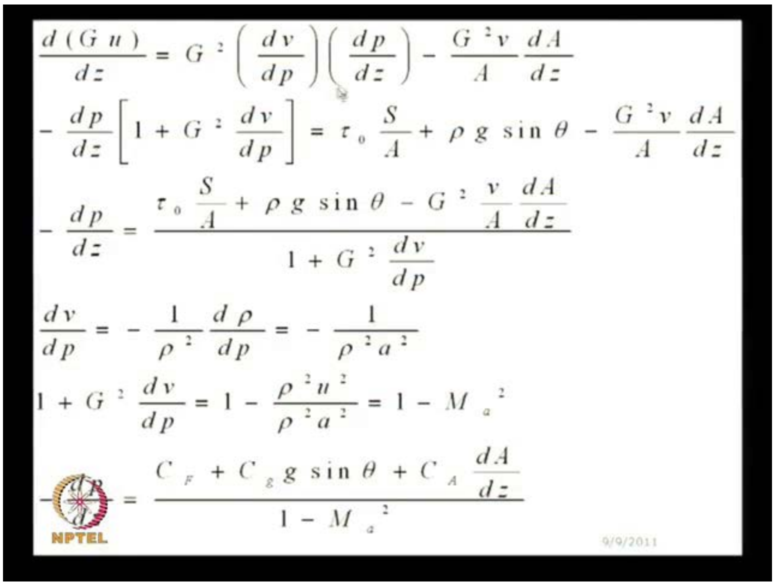

This particular equation for phase two and a similar equation we have obtained for phase one as well. So, if you compare these two equations what do you get you find that both this equations they resemble one dimensional steady flow single component compressible force is it not. If you just compare these equations with this particular type of equation you find that if you see the transparency you are going to see that, in this particular case also you have something of this sharp and there is a wall shear stress here, also if you find there is one f 1 2 if you see this f 1 2 plus f w 2 here you have got a rho g sine theta.

Here also you have got a rho 2 and rho 1 sine theta, then in this particular case you have got a d a d z term and in this particular case also you have got a d a d z term. What is extra here? Extra in this particular case, which you get is number one the effects of phase change d x d z terms and other one is d x d z then eta all this term and the additional degree of figure which is introduced by alpha. Other than that if you find if you compare this with single component compressible flows, where we had performed a 1 dimensional steady state analysis remember that we have done for all the cases. So, if we compare this we find that in that particular case also.

We had something like the frictional component where the frictional component compresses some only the wall shear stress in this particular case. You also have a frictional component which comprises of both the wall shear stress as well as your interaction between phase one and phase two. Then we also have some additional terms which came up, because phase change occurred here also in this particular transparency or in this presentation we have got some particular gravitational component we have an identical gravitational component here.

Here we have some area changed term in this particular case also we have a area change term what extra we have effects of phase change and effects of your the additional degree of freedom introduced by alpha. So, from this particular this expression and this expression if you see the left hand side, except for alpha this 1 by alpha d alpha d z. All the expecting for this alpha more or less all the other terms, they are free of alpha. On another words they will not change with any particular parameter of two phase flow. What they depend? They depend on area changed they depend on physical properties and so on and so what is it not. So, therefore, from here just as I was discussing in the last class that we can get the condition of chocking phase two we can get the condition of chocking for phase two we can get the condition of chocking for phasing.

Where the problem starts? The problem starts that this alpha keeps on adjusting itself. So, just now if we simply add up the two equations we were we will not get the condition of compound chocking, because that there is going to be an alpha variable and then this alpha variable it will be adjusting itself according to d alpha d z. And therefore, we unless we can eliminate this d alpha d z we cannot arrive at the condition of chocking, but just absorb these two equations and tell me how to eliminate d alpha d z from this equation and from this equation? If we eliminate that and then if you add the two equations then probably we can arrive at a condition of compound chocking.

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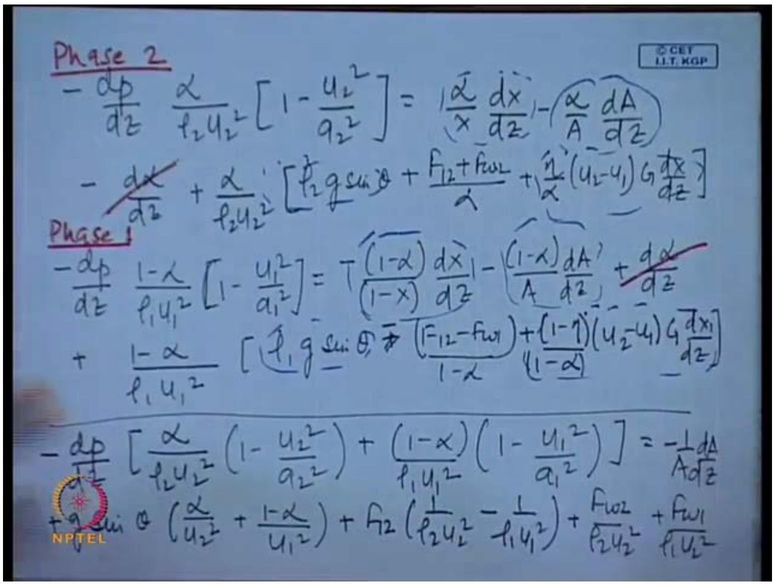

So, what we have to do? We have to multiply the first equation with alpha the second equation with 1 minus alpha and then we can add the two equations. Let us do that and let us see what we are going to get for that particular phase, once we can do it it is expected that we can arrive at the condition of compound chocking for that particular case. So, let us do this particular addition and then let us write it down and then let us see what are the things that we are going to get? So, for that particular case the things which you are going to get is minus d p d z alpha by just multiplying one equation by this, this will be equal to alpha by x d x d z minus alpha by a d a d z minus d alpha d z plus alpha by rho 2 u 2 square again rho 2 g sine theta plus whatever, was there in side that will remain intact u two minus u 1 g d x d z fine.

Same way the other equation this was for we are doing phase two first because it has alpha x so, we have to write less in that particular case. In this particular equation for phase one what do we get two 1 minus alpha by rho 1 u 1 square into 1 minus u 1 square by a 1 square this is equal to minus 1 minus alpha by 1 minus x. Do it very carefully, you can instead looking from me you can write down from the expression which you have derived in your note book that will be easier for me plus 1 minus alpha by rho 1 u 1 square rho 1 g sine theta just like had got in the previous sorry this is minus f 1 2 minus f w 1 plus 1 minus eta by 1 minus alpha u 2 minus u 1 g d x d z.

So, therefore, so this is for phase one which one sorry by 1 minus alpha very true this alpha this alpha will be cancelling out at the end very true thank you very much this is 1 minus alpha true. So, and finally, this 1 minus alpha also we will cancel here also alpha alpha cancels out. Now, if we add the two simply after this we can add the two equations this term will get cancelled out minus d alpha d z plus d alpha d z it cancels out is it not. And then what do you get in this particular case minus d p d z alpha by rho 2 u 2 square 1 minus u 2 square by a 2 square plus 1 minus alpha by rho 1 u 1 square 1 minus u 1 square by a 1 square this whole thing this will be equal to first, if we start adding this alpha d a d z and here also we have 1 minus d a d z.

So, therefore, alpha plus 1 minus alpha becomes d a d z so, by adding up this particular term and this particular term, we get minus 1 by a d a d z correct same thing if you take the rho 2 this particular term this g sine theta and if you take this particular term. So, then in that case what do you get plus g sine theta you can take as common and this is alpha by u 2 square plus 1 minus alpha by u 1 square correct, plus f 1 2 if you take 1 by rho 2 u 2 square minus 1 by rho 1 u 1 square agree plus f w 2 by rho 2 u 2 square plus f w 1 by rho 1 u 1 square plus whatever, d x d z terms we have plus one d x d z term here everything concerning d x d one and this is the other thing. And again this is one d x d z this is another d x d z.

So, we can come combine fine we have written the d x d z terms we have written the g sine theta terms we have written f 1 2 terms we have written f w 2 we have written f w 1 now, remaining is the d x d z terms.

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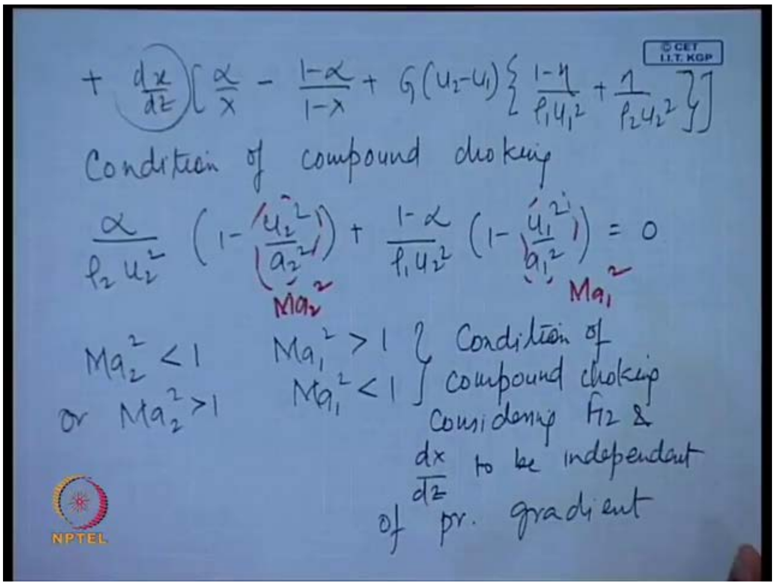

So, therefore, this plus d x d z into alpha by x minus 1 minus alpha by 1 minus x plus g u 2 minus u 1 1 minus eta rho 1 u 1 square plus eta u 2 u two square agreed. So, therefore, this has come to two pages, we have this particular equation this left hand side equals to this whole right hand side. Now, you tell me from this particular expression what will be your condition of chocking under the present case. What is the condition of chocking? Again in the same way if you consider we find that the term which is associated with d p d z that will go as the denominator and therefore, this terms when I make this equal to one, then it sorry this term becomes equal to 0 because this corresponds to 1 minus m e t p square.

So, this term equal to 0 corresponds to my case of condition of compound chocking agreed. So, therefore, if you can tell me what is the condition of compound chocking this is alpha by rho 2 u 2 square into 1 minus, if you just see you are the term attached to your pressure gradient you are going to get it minus u 1 square by a 1 square this was the term which was attached with the pressure gradient term. So, therefore, if this particular term this term this is this is nothing, but 1 minus m e t p square is it not and this 1 minus m e t p square equal to 0 gives you the condition of chocking. So, therefore, this term when it becomes equal to 0 it should give you the condition of compound chocking.

So, therefore, condition on compound chocking is this is equal to 0. Now, look at this particular expression which I have got now in this particular expression if you see alpha

1 minus alpha both are positive quantities yes or no, rho 1 rho 2 cant they ever be negative u 1 u 2 cannot be negative. So, therefore, something plus something if it has to be equal to zero, then how to adjust for that only the terms in the parenthesis they can only be all other factors since it is positive. So, only in the terms in the parenthesis they can be adjusted to give a the condition of compound chocking. So, from here you can very well understand when we will have compound chocking for the case of your from the two fluid model.

When one of them is less than 0 the other of them is greater than 0. So, that they cancel out one another and they give us the condition of compound chocking. So, therefore, now can you tell me when we are going to have the compound chocking as derived from the two fluid model or for the separated flow of two phases? When this particular term this is nothing but m a 2 square this is nothing but m a 1 square correct. So, therefore, we can have compound chocking m a 2 square less than 1m a 1 square greater than 1 or m a 2 square greater than one m a 1 square less than one only under that condition, under that condition we are going to have.

So, therefore, if both of them are either both of them have to be equal to 0 definitely under that condition we are going to have chocking. Therefore, one thing is for sure when both the phases are flowing under chocked flow conditions definitely we are going to have compound chocking this is for sure both the terms are individually equal to 0. Otherwise, if one is supersonic the other is subsonic or wise versa we can have the condition of chocking. But provided what are things that you have assumed here? Again neglecting flashing number one and the number two, if you notice this equation you are going to find out first thing is I have neglected the d x d z terms.

I have assumed d x d z to be constant for flashing again in the same way this x will be a function of h and p this is first assumption. The other thing is I have assumed f 1 2 also to be independent of your pressure gradient this is more or less resemble assumption, but I assume that in fraction between the two phases they are independent of the pressure. So, therefore, this gives you the condition of compound chocking assuming or considering f 1 2 and d x d z to be independent of pressure gradient, under this condition we have got the condition of compound chocking.

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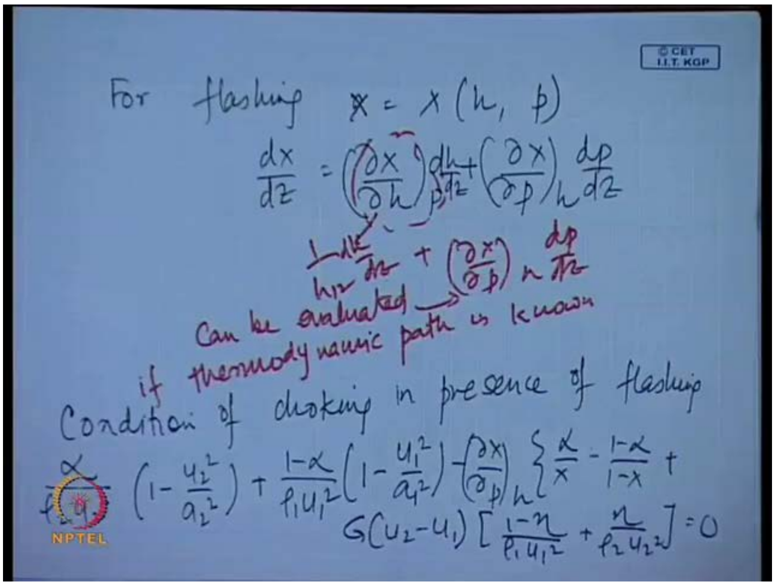

Now, tell me for flashing what is going to happen? For flashing suppose we consider flashing suppose, we consider flashing then in that case what happens, for that particular case we can write x is a function of enthalpy as well as pressure is it not or in other words d x d z just like I had written down in the previous class constant p plus del x del p a constant h sorry this is d h d z this will be d p d z same thing we had done. And here this what is this term del x del h a constant p or in other words del h del x at this is equal to 1 by lambda 1 by h 1 2, we call it very correct this is 1 by h 1 2. So, therefore, this reduces two 1 by h 1 2 d h d z plus del x del p at constant h d p d z agree.

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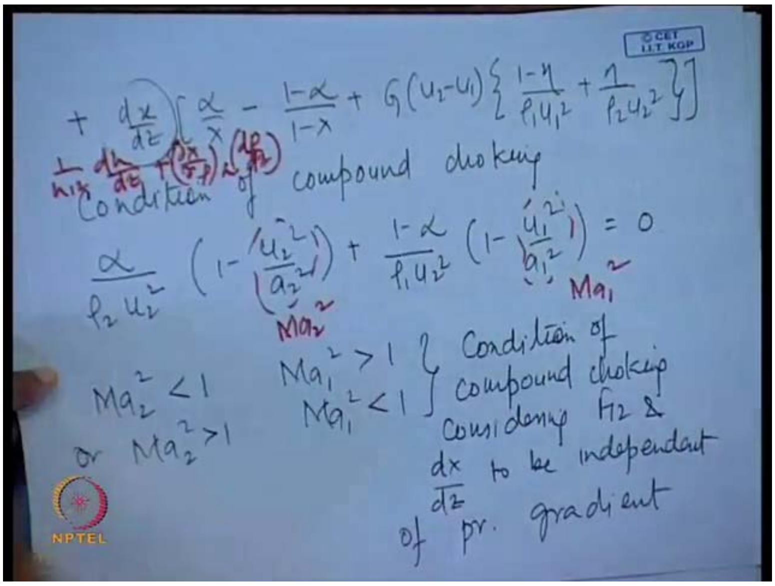

So therefore, instead of this particular d x d z we can write it down as 1 by h 1 2 1 by h 1 2 d h d z plus del x del p at constant h d p d z yes or no. So, for flashing what happens this term along with this it will go to the d p d it is going to attach with d p d z and that entire thing will come out as the condition of flashing do you understand. So, therefore, for flashing in presence of for flashing we find that, we are going to introduce two terms one is 1 by h 1 2 d h d z other is del x del p at constant h d p d z now, the thing is you can find out del x del p at constant h and in the thermo dynamic path is known is it not. Once we know that is what particular path we are going to effect the phase chance we can find out this particular term this can be evaluated, if thermo dynamics path is known is it not.

So, therefore, this particular expression finally, for the condition of flashing under the condition of flashing the condition of compound chocking what is going to be we will just attach this particular term. This del x del p at constant h into this whole term will come as the exponent I will come on the left hand side with minus d p d z and this whole thing will be has to be equal to 0 agreed. So, if we just do it and then you find it out we find we shall be mooting that this will give us something like. The condition of chocking in presence of flashing this will be for then alpha by rho 2 u 2 square 1 minus u 2 square by eighty square 1 minus alpha by rho 1 u 1 square 1 minus u 1 square by a 1 square minus del x del p constant h into alpha by x minus 1 minus alpha by 1 minus x plus g u 2 minus u 1 1 minus eta by rho 1 u 1 square plus eta by rho 2 u 2 square this has to be equal to 0. Do you agree with me yes or no yes or no do you agree with me?

So, in presence of in absence of flashing your condition of chocking is given by this particular expression and in presence of flashing your condition of chocking is given by, this particular expression being equal to 0. So, we find that in presence of flashing your chocking should depend on the value of eta this is the problem, because eta also we are not very confident how we are going to define it. Usually from thermodynamic analysis that is quite an elaborate analysis probably I will not be going through this, but from this particular analysis people have found out that eta equals to half.

So, usually it is shared half and half by the two phases, but that comes under a certain specific assumptions from thermo dynamic analysis for isentropic conditions only. For other conditions it is very difficult to derive eta, under isentropic conditions for change of phase we find eta equals to half on other words this particular force which is required to bring about a phase change is shared equally by the two phases. So, we find that in presence of flashing the condition of chocking is a function of eta, except just look at the expression and tell under what condition chocking will be independently of eta under flashing conditions is it clear to all of you what I want from you. You just look at this particular expression, the expression which I have written down and tell me that under what conditions chocking will be independent of eta for flashing conditions.

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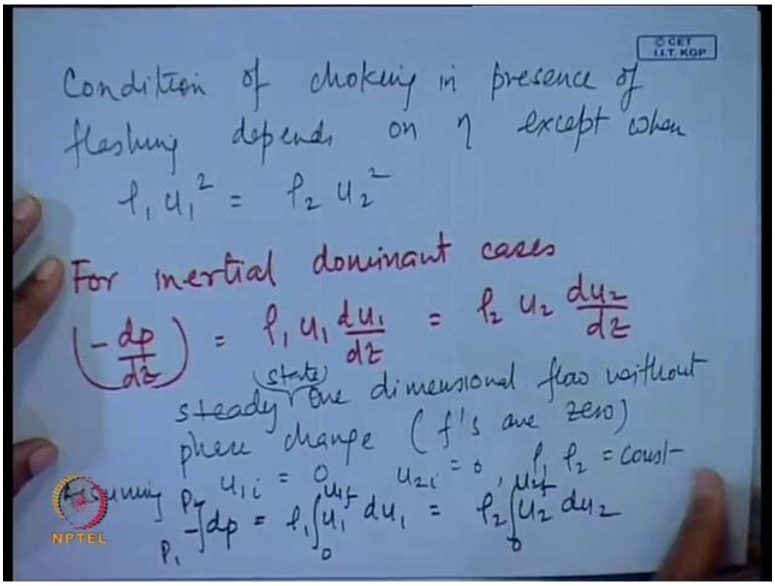

Can this particular term this disappears off is it not when this particular term disappears off it is going to happen now definitely g cannot be equal to 0 u 2 cannot be equal to u one. So, therefore, what can happen only your rho 1 u 1 square can be equal to rho 2 u 2 square under that condition 1 minus eta plus eta cancels out clear to all of you. So, therefore, condition just remember this thing condition of choking in presence of flashing depends on eta, except when rho 1 u 1 square equal to rho 2 u 2 square except for this condition it depends on your it depends on eta.

So, now I would like to deal with one another type of another type of very simple situation. See whenever, we find the two phases there accelerating through an nozzle even for a single phases accelerating through a nozzle. We have good amount of fluid in say any particular stationary tank and then we have a nozzle connected to this so, this fluid flows through the nozzle a converging type of nozzle. Now, for that particular situation switch are going to be the dominating forces normally, the forces we have a gravitational forces frictional forces and acceleration pressure gradient is it not. Now, for normal circumstances and it is a vertical pipe we know that for even the single phase loop we know that the gravitational pressure gradient is important.

When it is a horizontal type we know the frictional pressure gradient is important. Once when it is flowing through a nozzle say horizontal nozzle from a stationary tank it is flowing through horizontal nozzle. And that conditions which particular pressure gradient or component is going to be important and you tell me acceleration it is neither the gravitational nor the frictional pressure gradient.

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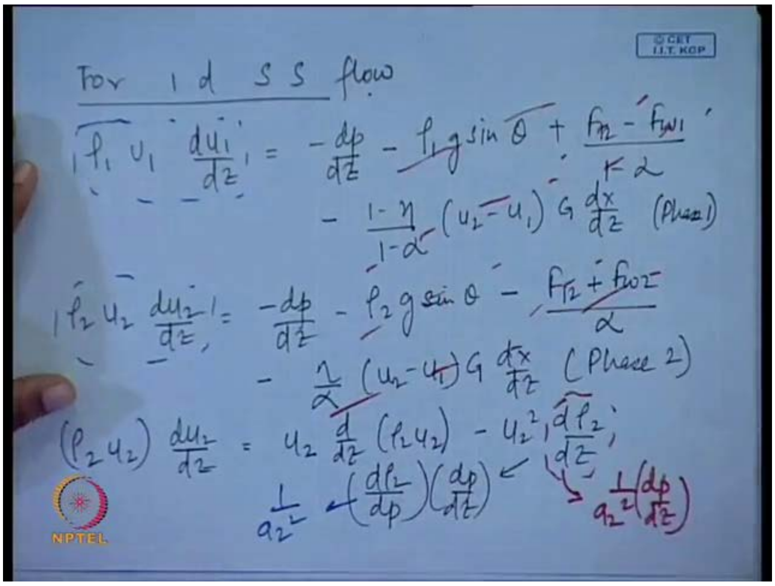

So, under such circumstances when in is in under such circumstances what happens, if we take up the basic equation of the movement basic momentum balance equation which I had written down on this, this is the equation. The basic momentum balance equation which the system balance equation so, if we look at the basic momentum balance equation under this condition what do we find there are just two phases which are flowing through a nozzle. So, under that particular condition there is no change of phase so, d x d z terms cancels out from both the cases yes or no. And since acceleration pressure under these particular circumstances more or less the phases they do not interact.

This interaction between the phases is not very important neither is the interaction with the walls very particularly important. Hence, it is a horizontal nozzle then naturally this gravitational component also disappears off so, under that condition what remains your minus d p d z equals to this minus d p d z equals to this. So, therefore, you find out that your equation of motion becomes very simple under this particular relation for what happens? So, this is inertial dominant cases for inertial dominant cases what do we get? So, for this case what did we get from here rho 1 u 1 d u and d z equals to minus d p d z similarly, rho 2 u 2 d u d z equal to minus d p d z.

Let us write down these two terms what do we get we get minus d p d z equals to rho 1 u 1 d u 1 d z this is equal to rho 2 u 2 d u 2 d z yes or no. And now in this particular case suppose see what I have said he start from a stationary tank what is the initial velocity then, the initial velocity of fluid one fluid two it is 0 is it not. And then under such circumstances if we assume that more or less rho 1 rho 2 they also remain constant. So, under such circumstances we find there is a particular relationship between the final velocities of the two fluids definitely even, if the start from the same initial velocity here final velocities are going to be the same.

Now, what is the relationship can you simply derive from the equations of the momentum equation that we have we have written down in this particular case now, this was again I will write down the assumptions that you do not forget same thing. Please write down these assumptions in your exam otherwise, steady state sorry steady state one dimensional flow without phase change. Under this particular condition if phase change would have been there then we could not have written such a simple equation. So, this is this equation has been written under the conditions of steady state one dimensional flow without phase change such that the f r 0. The g term g containing terms are also equal to 0 agree.

So, therefore, we have obtained this particular equation now assuming see u 1 i initial this is equal to very low velocity 0 u 2 i equals to 0. So, therefore, with all these and rho 1 rho 2 equal to constant so, therefore, under that circumstance it is very easy to integrate this particular equation is it not. So, under that particular situation what do we get? We simply get minus d p equals to rho 1 u 1 d u 1 this is equal to rho 2 u 2 d u 2 very correct this is say from p one to p two delta p agree this will be from 0 to u 1 final we can write it down, this will again be 0 to u 2 final we can write it down.

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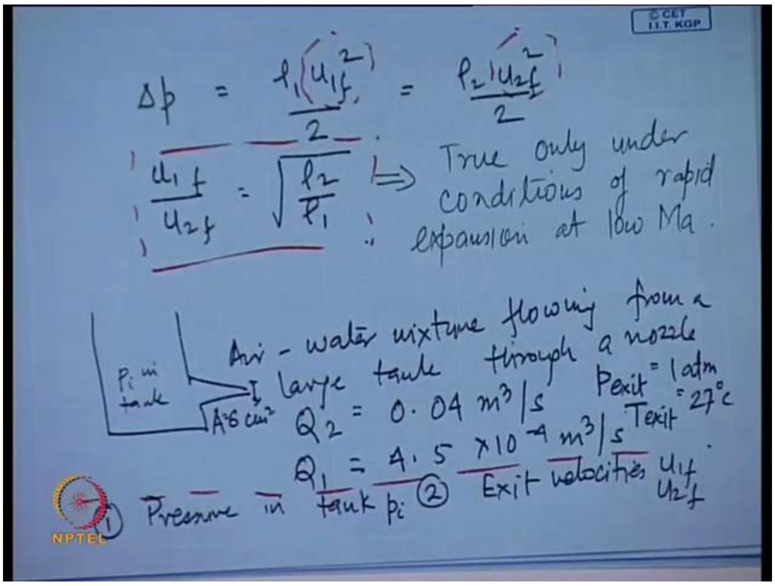

So, just if you perform the integration what do we get? We get delta p this is equal to rho 1 u 1 final square by two which is equal to rho to u 2 final square by two agree. So, how are these final velocities how are these the final velocities of the two fluids related after flowing through the nozzle assuming that they has started with a very low initial velocity. They were almost at rest and flow was offering under steady state one dimensional flow conditions without any phase change or without much interaction between normally, under this circumstances wall interactions we can neglect even for single phase flow situations as well.

So, therefore, under that particular situation what can we write down? We can write down so, under this situation u 1 final by u 2 final equals to can we write this term yes or no. So, therefore, we find that the final velocities after expansion they depend upon their densities this is a very simplified assumption the very simplified derivation we have got with a lot of assumption, but more or less under normal circumstances, they are they give us more or less appropriate presence. So, therefore, we find that under conditions of starting with very velocity and all the other assumptions that have already stated we find that, the final velocity of phase one and phase two they depend upon they are in the inverse ratio of the densities of the two phases.

Now, since we know that rho 1 is much much suppose, it is an air water mixture rho 1 will be much much greater than rho 2 and therefore, finally, what we find u 2 f will be much greater than u 1 f even if it is under separated flow conditions also. We find that the two velocities will be much different even shall we assume that the pressure gradient across them is the same just. Because of the densities this differences going to occur, but of course, remember one thing this expression this has really know universal validity and it is primarily true only under condition, this is true only under condition of rapid expansion at low mach number this is important.

Just because at low mach number only we can assume rho 1 rho 2 to be independent or rather rho 1 rho 2 to be constant and independent of pressure and under that condition we can assume this particular case. So, after this more or less what are the things we have completed? We have completed the we had first started with the two fluid model, we perform the equation or rather we wrote down the equation of continuity we wrote down the equation of momentum of two phases. Then we combine them in several ways from the combined equation what we did we found out the condition of chocking, after finding out the condition of chocking we found that the that the results can be misleading under certain conditions particularly, because del alpha del p this is usually derived for low to moderate pressures well friction dominates inertia.

So, therefore, after that what we did? We considered the two phases separately, but of course, we also considered the change of phase just to keep the matters much more generalized then from there we derived the equation of momentum for the two phases separately and then we combine them. Then we derive the condition of chocking under these conditions and we derive the condition both in presence of flashing and in absence of flashing. The only things which is left after this is the hitch which was there if we consider the basic equation, which we had already written down the basic momentum it combine momentum equation well we I do not have it at the moment.

Where we had found out that the only unknowns in that particular expression was the frictional pressure gradient and the dependence of alpha, with z or with pressure gradient so, these were the two things that had to be found out. Usually, there are several empirical approaches for finding this out in presence of or rather in absence of certain better techniques. So, we would be going through those particular whatever, empirical expressions are available and we will be doing then accordingly, but before we start that today I would like to do a problem for you I would just like to specify a problem based on the simplest thing that we had studied till.

Now, that is flow of an air water and mixture through a nozzle just based on that I would just like to discuss a problem, simply to show you how flexible or how much depends upon the design of two phase systems. Just to show you that I would like to do a problem and in the next class we will be discussing the different methods of evaluating the frictional pressure gradient and the what fraction for two phases would be in separated flow condition. Now, let us take up the problem now say we have a very large tank and in this particular tank from this particular tank air and water mixture they are flowing out among very large tank air water mixture is there in this particular tank and it is air water mixture flowing from a large tank through a nozzle.

Now, the flow rates there are more or less and this particular this final diameter that is 6 centimeter square, the nozzle value it is the diameter sorry cross sectional area is 6 centimeter square a equals to 6 centimeter square. And the flow rates I have the flow rates your q air or q 2 this is equal to 0.04 cube per second q 1 will be equal to it is 4.5 into 10 to the power minus 4 meter cube second. So, this is all of what you are required to find out? Where required to find out one pressure in the tank and two you are required to find out the exit velocity; that means, u 1 f and u 2 f.

These are the two things you are required to find out and pressure in the tank; that means, p initial [FL] one more thing is given that is exit pressure is atmospheric pressure. So, therefore, this discharges that atmospheric pressure so, therefore, q 2 q 1 p exit equal to one atmospheric pressure and t the temperature more or less it will remains constant at twenty seven degree centigrade. So, these are the data which has been provided to you q 2 is provided q 1 is provided and it discharges at a pressure of at an atmospheric pressure at a temperature of twenty seven degree centigrade this is also given.

So, what you are required to find out? You are required to find out the exit velocities u 1 f u 2 f and the pressure p i in the tank p s equal to one atmosphere it is given. Just this much is given nothing else is given how to proceed for what do you suggest how should we proceed to do this particular problem? See if you go o the problem very well you will find that in the present forms the problem cannot be solved just like that, because you ordered to solve the problem the first have to know how the two phases are mixed together and what is the flow pattern under which is flowing inside the nozzle? Just if it would have been only water flowing or only air flowing we could have done this problem, but since in this particular case it is an air water mixture flowing.

So, therefore, before we proceed what we are suppose to know? We are suppose to know that how the two phases have been mixed together and what is the flow pattern that is in exhibited in the nozzle, unless that is given we cannot solve the problem. So, what we can do under the present circumstance we can take two extreme conditions what are the two extreme conditions, one is the two phases are intimately mixed with one another and there are (( )) well mixed that it becomes homogeneous flow and a homogeneous mixture flows out through the nozzle. This is one extreme what will be the other extreme? The other extreme is going to be the two phases they do not interact at all at the interface and they flow in separate layers through this particular nozzle these two can be the two extreme cases.

So, what we can do we can take up this problem we can solve it first for two phases flowing separately or we can solve it first by assuming by two phases are flowing as horizontal flow. We can find out the pressure in the tank and the exit velocities then we can solve the same thing by considering that the two phases are under separated flow conditions with minimum interaction at the interface. Wall shear stress can be neglected probably in the problem also it will be specified wall shear stress can be neglected. So, therefore, before doing anything for suppose problem of this what is frame so, therefore, you have to first decide the conditions under which it can be solved and then taking up the individual conditions you have to solve the problem is it clear to you.

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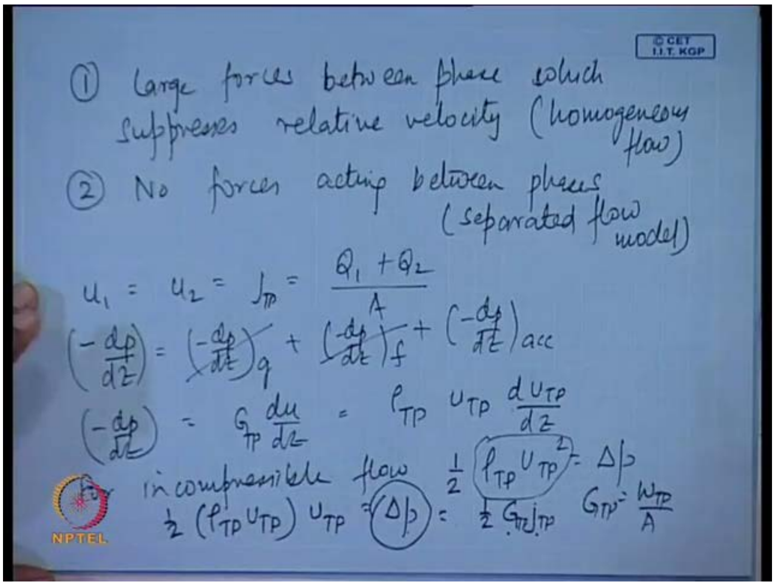

So, for this particular case what we can do? We can solve the for the problem for two cases one case is large forces between phases which suppress relative velocity what does it mean, that between the two phase phases such large forces are acting that the suppress the relative motion. What does it mean, what does this sentence mean? Homogeneous flow on other words this means that the two phases are under homogeneous flow condition. What is the next extreme that we can we can assume no forces acting between phases now actually, no forces will be acting between phases they will separate it out and they will be flowing separately is it not.

So, no forces acting between phases so, these are the two cases, this is the separated flow model so, these are the two cases for which you can solve it. Now, let us take the first case when there are large forces acting between the phases I will not be doing the whole thing I will just be giving you the hints of how to solve it so, that you do it come to the next class with the answers and then only I am going to discuss certain other things which I have to tell you relating this problem. So, when the large forces acting between the phases what can we assume we can very well say that u 1 u 2 they are the same no slip condition and this is equal to j the volumetric flux, which is equal to q 1 plus q 2 by a 1 q 2 are already given.

So, therefore, from this what do we know? We know that if you see the homogeneous flow model there you would find that minus d p d z was equal to a gravitational pressure gradient plus your frictional pressure gradient plus your acceleration pressure gradient and this particular case both these cancels out. So, therefore, minus d p d z would be equal to minus d p d z acceleration what was the expression of this particular case? This was nothing but g d u d z remember this was equal to minus d p d z in this particular case or in other words g was nothing, but equals to this was g two four this was g two phase rho t p u t p d u t p d z yes or no agree.

So, therefore, in this particular case we found that assuming since the density are more or less constant and it has been given and the pressure is not excepted to fluctuate much. So, you can see you might be given such problems in your exams well may be you have to make rational assumptions then you have to progress. Depending on what type of assumptions your made may be even if assumptions are wrong, but logic is correct also you will be evaluated. So, remember this it will be much more analytical as compared to anything else. So, therefore, under this circumstances we can assume that it is in compressible flow because what (()) sending compressible, here for such a small pressure drop it is going to be in compressible.

So, for in compressible flow what we can write? We can just integrate this equation and we can have it as half rho t p u t p square is equal to delta p, rho t p you can very well find it out u t p also you can find it out or else you can make this in a much more friendly form what is it? You can write it down as half rho t p u t p into u t p this is equal to delta p what is this rho u equal to g? So, therefore, this is nothing, but half g j or g t p j t p yes or no . So, therefore, g t p is nothing but equal to your w t p by a or w one plus w two by a j t p is nothing but q 1 plus q 2 by a. So, we can very well find out j t p g t p and we can find out delta p.

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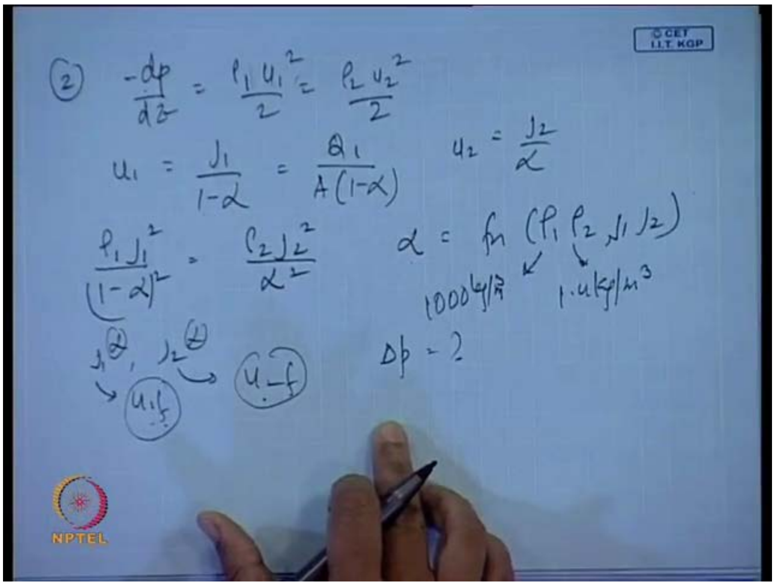

We know the final pressure as one atmosphere. So, we can find out the initial pressure very simple problem for this second case this was for the first case. For this second case what is suppose to do? Second case already we have derived the equation. So, the equation was minus d p d z was rho 1 u 1 square by two equals to rho 2 u 2 square by two is it not, this particular case we know what is u 1 equals to j 1 by 1 minus alpha agree this is nothing but q 1 by a into 1 minus alpha. Same thing u 2 this will be equal to j two by alpha fine. So, instead of u 1 square rho 1 u 1 square what can we write? We can write it as rho 1 j one square by 1 minus alpha equals to rho 2 j two square by Alpha Square sorry can we write down this particular equation yes or no.

So, once we can write down this equation we can find out alpha is a function of rho 1 rho 2 j one j two can we do this rho 1 rho 2 you already know air water this is thousand k g per meter cube and rho 2 you can take it down as one point four k g per meter cube J one j two you can find out. So, you can find out an expression of alpha using this particular equation, once you can find out the expression of alpha then once you know j one j twosome j one you can find out u 1 from j two you can find out u 2 moment you know j one alpha and j two alpha. You can find out u 1 u 2? Once you can find out u 1 u 2 these are u 1 f u 2 f, because u 1 and u 2 initial were 0. So, from there you can find out u 1 f u 2 f once you know these two from using this equation you can find out delta p agree.

So, in the next class I would like you to come up with values of u 1 f u 2 f as well as delta p both for large forces such that it suppress the relative motion and both for almost no forces such that there is lot of relative motion. Under both the conditions I would like you to solve it and then come up with the answers. Once you come up with the answers I will be discussing why I have given you this problem and then we will proceed for discussing the imperial techniques for wall shear stress and so on. And so next class please come prepared with the problems rather please do the problem and come with the answer so, that we can make further discussions. So, thank you very much.