Multiphase Flow
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Lecture No. # 20
Separated Flow Model – Condition of Choking

Well, so today we will be continuing with our discussions on the separated flow model. Till yesterday, what we did? We deduce the two fluid model and then we combine them, you obtain the mixture momentum equation, we found that in that particular equation there was a numerator part, there was a denominator part if you remember. And then naturally, with analogy from your single phase, compressible flows, which I would like to show you here, not this one, this one yeah, with the single phase compressible flows, if you remember in this particular case.

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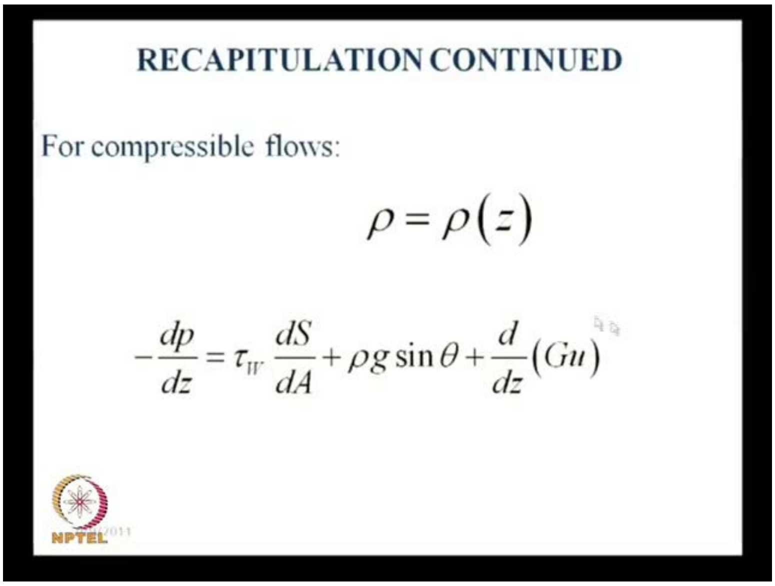

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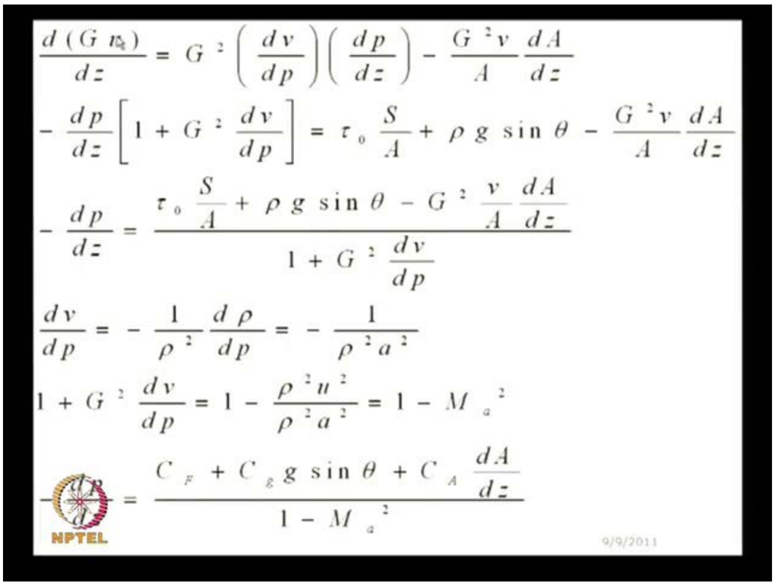

So, we have also derived a equation something of this sort, where we have a frictional pressure gradient, we have a gravitational pressure gradient and we have a acceleration pressure gradient as well. And after that, we found that since u is a variable in that particular case, so we had substituted u accordingly and these all derivations we had we had got it, we found out that finally, for compressible flows, the pressure gradient also has a numerator term and a denominator term. Just like, it has it had for homogeneous flow, and now we have deduced the same thing for the two fluid model.

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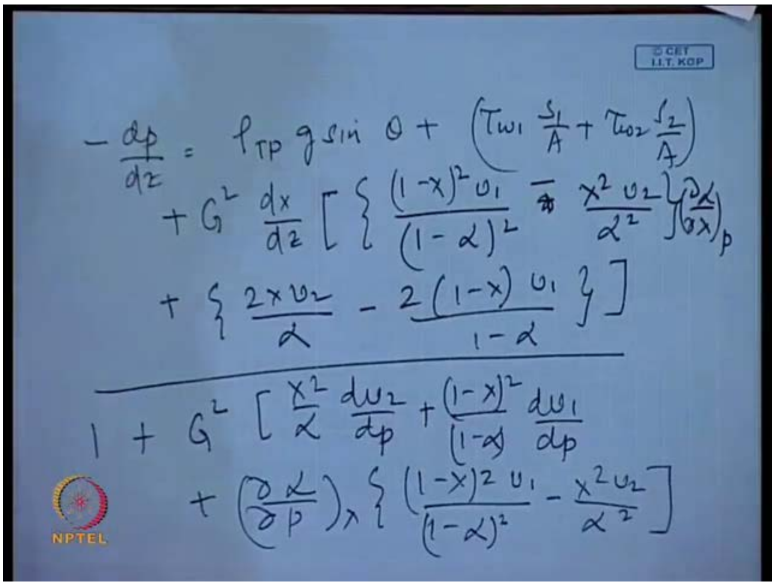

Now, if you remember in the last class, we had deduced the final expression. It was a huge expression, I would just like to write it down once more, so that you can and I think it is there with you. So, you can just check up the denominator term which we have, this is the numerator term, this I have written down several times actually plus g square d x d z 1 minus x whole square v 1 by 1 minus alpha whole square sorry minus x square. Just check up with the expression, I have derived in the last class, and let me know if I am just making some mistakes in writing it down plus 2 x this was already deduced.

So, you need not note it down anymore. So, this was the numerator part divided by 1 plus G square x square by alpha d v 2 d p 1 minus x whole square by 1 minus alpha.

So, this was all the and and we had already deduced it. So now, we find out that if we compare what we have in the transparency we find that the denominator part is 1 plus G square d v d p, here also we have 1 plus G square. Since, there are two phases we have 1 d v 1 d p 1 d v 2 d p till this much we already had in the homogeneous flow model. Now, in this particular case since alpha is also available we have one term concerning alpha.

And then if you look at the ppt, you will find that in this particular case, we deduced that this 1 plus G square d v d p this was nothing but 1 minus the Mach number corresponding to that particular fluid flow, is it not? So, therefore, this the denominator term this was equal to 1 minus M a square. Now, if we start from the same analogy then we we are in a position to comment that this particular denominator, if you see with there a denominator, which we have deduced in the last class for the pressure drop equation or the pressure gradient expression that we had derived. This should also correspond to 1 minus the two phase mach number for this particular fluid, is it not.

This should also correspond or in other word this should be equal to 1 minus M a T P square and from suitable modifications or simplifications of this particular expression which I have written down, we should be in a position to arrive at the condition of choking for from the two fluid model which is applicable for the separated flow situations, is it not. So, therefore, now what I am going to do? I am going to deduce the condition of choking for from the two fluid model considering the mixture momentum equation.

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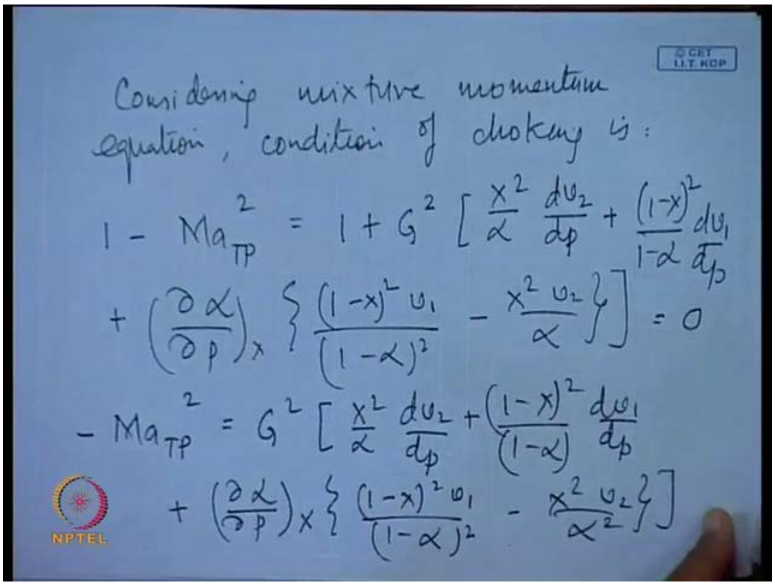

So, considering mixture momentum equation condition of choking. What is the condition of choking by considering the mixture momentum equation? We find that the condition of choking, in this particular case naturally, then this becomes 1 minus M a T P square, this is the two phase square, two phase MaTp number this should be equal to the denominator which I had obtained in this particular case. So, therefore, this is equal to 1 plus g square x square by alpha d v 2 d p plus 1 minus x whole square by 1 minus alpha d v 1 d p plus del alpha del p at constant x 1 minus x whole square by 1 minus alpha d v 1 d p plus del alpha del p at constant x 1 minus x whole square v 1 by 1 minus alpha whole square minus x square v 2 by alpha.

So, therefore, now, for the condition of choking 1 minus M a T P square this should be equal to 0 or in other words we get the condition of choking for a unit value of MaTp number. That is what we had deduced for the single phase compressible flows, is it not. So, in this particular case also it is expected that the left hand side 1 minus M a T P square that should be equal to 0 or in other words this whole expression should be equal to zero in order to obtain the condition of choking. So, equating the denominator to zero we should be in a position to obtain the MaTp number for or rather from the two fluid model under the separated flow conditions.

Now, let us see what are the other sort of simplifications or modifications which we can do in this particular case. And from here consequently, we can obtain an expression of the mach number as obtained from the two fluid model this is nothing but G square into X square by alpha d v 2 d p plus 1 minus X whole square by 1 minus alpha d v 1 d p plus del alpha del p at constant X 1 minus X whole square v 1 by 1 minus alpha whole square minus X square v 2 by alpha. So, therefore, the two phase mach number should be the negative of this particular expression.

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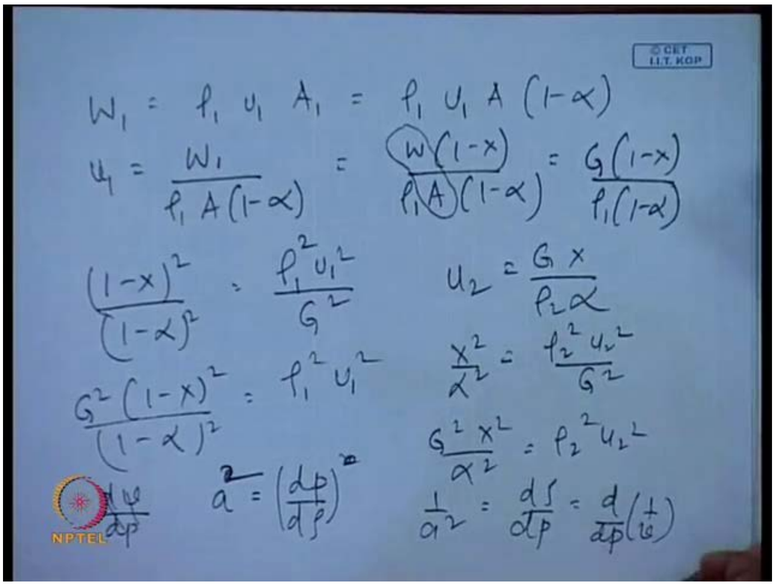

Now, let us see whether we can express this in certain other simpler forms so that it will be easy for us to assume. Again we will start from the basic equations which we have already derived in chapter 4; which this one, sorry, this is alpha square. So, therefore, again we will start from the basic equations which we had already derived in our chapter 4, the equation of continuity and the basic definitions of W 2 etcetera, etcetera. So, what were the basic equations that we derived? W 1 equals to rho 1 u 1 A 1. Everyday I write down these equations so that they get into your head and you do not forget how to start your derivations or in other words this is nothing but this. So, from here we can get what is u 1 equals to it is nothing but w 1 by rho 1 A into 1 minus alpha or in other words this is nothing but W into 1 by minus x by rho 1 A into 1 minus alpha.

Now, we know this W by A this is nothing but G. So, therefore, this is G into 1 minus x by; see these things have to be there in your mind very very thoroughly, otherwise it is going to be difficult. So, therefore, u 1 equals to this. So, here we had something like 1 minus x whole square by 1 minus alpha square into v 1 rho 1 means this becomes v 1.

So, therefore, from here you you know what is this particular term, is it not. We can substitute it here and we can find it out that 1 minus x whole square by 1 minus alpha square this is nothing but equal to rho 1 u 1 square by G square. So, we instead of the this particular term we can substitute this particular term here.

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rho rho 1 square sorry sorry sorry yeah yeah. So, we can substitute this particular term similarly, just like we have expression for u 1, u 2 also we can write it down it is just g x by rho 2 alpha. If we derive it for one the other one becomes very easy. So, therefore, from here, we have got 1 minus X whole square by 1 minus alpha square in this particular form or in other words g square into 1 minus x whole square by 1 minus alpha square equals to rho 1 square u 1 square. In the same way we would like to get x square by alpha square it is nothing but rho 2 square u 2 square by G square and similarly, G square X square by alpha square equals to rho 2 square u 2 square.

So, these things so, therefore, here G square into 1 minus X whole square this particular term we can substitute X square alpha by alpha square into G square, this particular term we can substitute, correct. The other thing is d v 2 d p and d v 1 d p, these things we had already derived. If you can tell me, what is this d v 1 d p and d v 2 d p, any idea? We had already derived these things, d v d p equals to what is the velocity of sound? Say for medium one and medium two. What is the basic definition of sound, you tell me? A for any particular fluid equals, d p d rho is it not or del p del rho under isentropic conditions.

Since, here, we have very less amount of friction and very less amount of irreversibility we can consider this to be an isentropic flow. So, therefore, what is the thing that we knew? We knew that a equals to d p d rho or in other word a square equals to this wholes; sorry, very sorry, very sorry, very sorry, a square equals to d p d rho or in other words 1 by a square equals to d rho d p del for constant isentropic conditions. Since, I have already assumed that this is del del of s and our flow is occurring under isentropic conditions so, therefore, I am removing that delta part here. Right all of you, any doubts?

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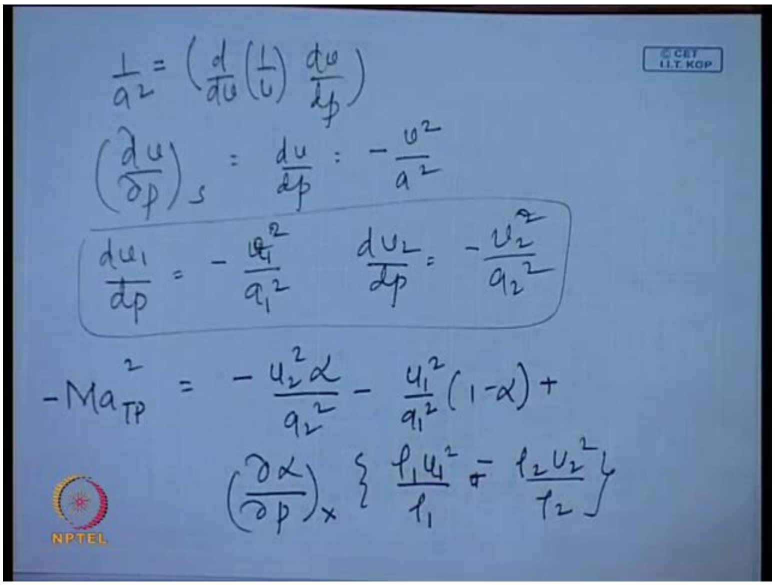

So, or in other words this becomes equal to d d p of 1 by v, agreed. Or in other words these derivations I had already done in the class previously. Or in other words, we can write it down as 1 by a square this is equal to; sorry, this this is equal to d d v of 1 by v into d v d p. So, therefore, we find d v or rather del v del p at constant s which is in this particular case d v d p, this is minus v square by a square, yes or no. So, therefore, what we can do? Whatever we have substituted here this particular term then the other one is this one this particular thing and then d v 1 d p is minus v 1, sorry, u 1 square i think, is it not; v 1 square; sorry, sorry, v 1 square by a 1 square and d v 2 d p equals to minus v 2 square by a 2 square.

So, this these things so, if you make all these substitutions into the denominator of the momentum balance equation which we had derived, then what do we arrive at; just do it and tell me what do we get under that circumstances? Do it, just do it and then tell me what are the things that you are getting under those circumstances? Simply, if you make the substitutions we get the mach number of the two phase flow. Just make the substitutions and you tell me what are the things that we have going to get? If you make the substitutions or in other words if I put minus M a T P square, then in that case what are the things that I am going to get in this particular case.

This becomes equal to you just see whether you are getting these terms or not minus u 1 square by a 1 square into 1 minus alpha plus del alpha del p at constant X rho 1 u 1 square by rho 1, this is u 1 plus this rho 1 means from v 1 I have made it 1 by rho 1 minus rho 2 u 2 square by rho 2. If you make all the substitutions, if you substitute your X square by alpha substitute d v 2 d p, substitute 1 minus X whole square by 1 minus alpha, substitute d v 1 d p, then substitute these terms, these terms substitution I have already told you. So, if all of these are substituted and then finally, you arrive at the derivation which I have obtained in this particular case.

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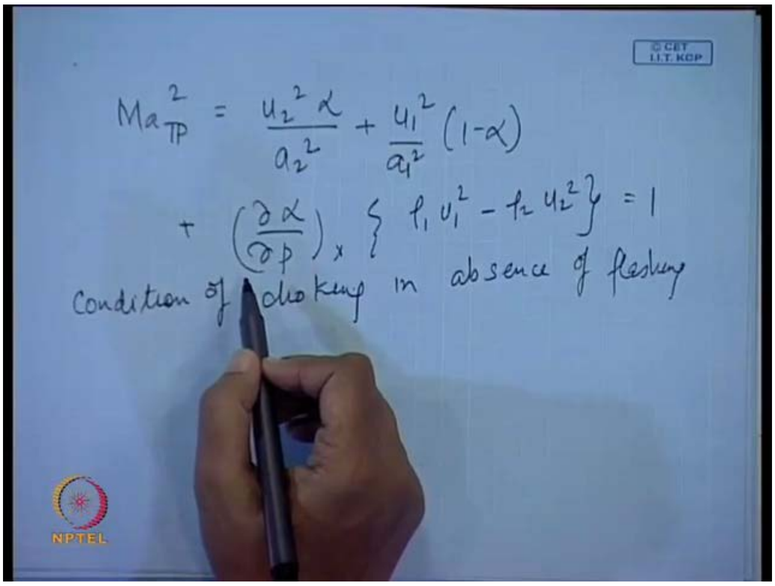

So, therefore, this gives you the expression of MaTp number for the two fluid as obtained from the two fluid model this is equal to u 2 square alpha by a 2 square plus u 1 square 1 minus alpha by a 1 square plus del alpha del p at constant x rho 1 u 1 square minus rho 2 u 2 square. And what is the condition of choking? For condition of choking, this has to be equal to 1. Is this portion, clear to all of you? Please perform these derivations once again so that the entire thing is clear to you. Now, can you tell me the basic thing under what assumption we have derived this particular condition of chocking?

Louder, isentropic; see let me tell you one thing when you have derived bernoulli's equation, what was the thing that we had assumed, friction less in which it flows which is nothing but isentropic flow. So, therefore, for fluid flow usually, we assume it is isentropic because usually, we consider friction when there is no friction and what were irreversibility that is confined in a very narrow portion. So, usually we can, we assume isentropic flow. But can you tell me that under what condition this particular equation has been derived? If you look at the basic at the, in the equation, the expression of d p d z which we had derived in the last class and I had written it down once more here it might give you a hint on what was the basic assumption which we took to derive this particular expression. See, if you look at this expression I had assumed that v 1 v 2 these two things they vary with pressure is it not. And what is alpha it varies with pressure?

So, alpha it varies with quality and pressure and the specific volumes they vary with pressure. Let me tell you for most of the cases, this particular term will be disappearing off. So, you just keep in mind when you are given a derivation in your exam, this is a just a generalized form I have written then in that case depending upon the problem in question, we will be eliminating the basic terms from the very beginning so that your assumption or rather your expression or your derivation become simpler. So, therefore, we had just assume these two things. We had assumed that alpha it varies with quality as well as pressure very justified and we had assumed that both the specific volumes or the densities they are function of pressure.

What else can vary with pressure which we have not considered here? Something else can also vary with pressure if you see this particular expression. See the first thing, I cannot give you much time to think that is a tragedy here see I have taken d x d g to be constant or in other words your x does not vary with pressure. When does it happen? It happens under ordinary circumstances in absence of flashing. So, in this particular case, we have assumed that flashing does not occur. If you remember for the homogeneous flow model also what we did initially, we derived it for this particular condition only and then we assumed that if flashing occurs then X will also be a function of enthalpy and pressure. For the present case that I have assumed x is a function of h only or in other words x can be obtained from the enthalpy balance or the heat balance equation, is it not.

But for flashing X is a function both of enthalpy as well as pressure. So, therefore, the equation which I have derived in this particular case, this particular equation it is applicable the condition of choking in absence of flashing. Now, when we have flashing under that circumstance, what happens? Under that circumstances, this d x d z this has to be written down as function of h and p, as we had done in the previous case. And therefore, in the denominator certain terms like del x del h at constant p or something;

sorry, del x del p at constant h along with an associated terms will also come and therefore, accordingly the condition of choking will be different in presence of flashing.

So, that I am not deducing that has been left as a home assignment for your case, to deduce the condition of choking in absence of flashing. The other thing which I would like to tell you is that see in this particular expression we will have something like del alpha del p at constant x. That means, when the quality is constant, how alpha varies with pressure. Now, this particular; your this particular differential this is usually derived from correlations which are obtained at moderate values of pressure. Now, when we talk of moderate values of pressure gradient this means when the frictional forces dominate the inertial terms. And when the friction forces dominate the inertial terms then naturally, alpha is not a very strong function of p. So, therefore, in that particular term, this equation is slightly misleading.

Only under conditions where rho 1 u 1 square equals rho 2 u 2 square and this term is no longer exist it is fine. But for other cases we find that this equation is slightly misleading because the term del alpha del p at constant x. This is usually derived from a correlation which is obtained at moderate values of pressure gradient when the frictional forces dominate over the inertial terms. So, if we have to arrive at a much more accurate expression, what will we have to do, for a much accurate expression what we have to do? We have to go to the basic momentum equations.

The basic momentum equations what did we do? We first derived the basic momentum equation for phase 1, we derived the basic momentum equation for phase 2. If we just consider these two basic momentum equations then there we would find that we would get one particular term like 1 minus u 1 square a 1 square for phase 2 we would get 1 minus u 2 square by a 2 square. So, they would give us the condition of choking separately for phase 1 and separately for phase 2. Then if we combine these two momentum equations there we find that some alpha term will come.

And if we observe that alpha term we find that the just the choking of phase 1 or the choking of phase 2 or the choking of both phase 1 and phase 2, does not guarantee compound choking of two phase flow. Why? because alpha can adjust itself and prevent the condition of choking. So, therefore, now, what we are going to do is? We are going to consider the two phases that separately, we are going to write the momentum equation which we have already derived in the last class. And we will just take up the final expressions that we had got and again we will make some simplifications we will be deriving the conditions of choking of the individual phases.

Then we will combine the two equations and we will try to arrive at the condition of compound choking. Now, this is usually slightly it involves quite a good number of mathematical computations. And the most importantly, it again requires all the derivations or all the expressions, which we had all the nomenclatures, which we had defined in chapter 4. So, let us start with that and let us actually arrive at the condition of choking for two fluid model from the two fluid analysis, which we had started.

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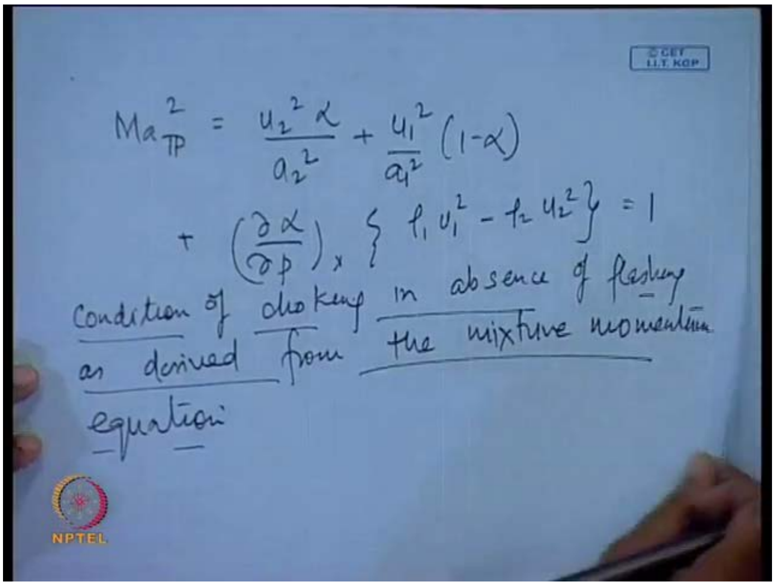

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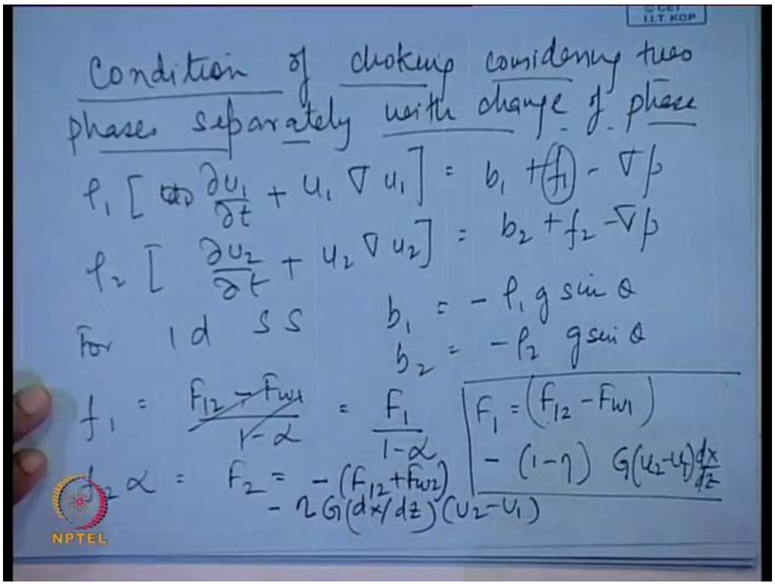

Now, just to keep matters simple, we would I would like to prefer or to I would like to take that particular two fluid expression where we had considered the change of phase as well. This is just to keep matters simple if there is no change of phase in the problem which which you have to deal with then simply you ignore the change of phase terms and you can proceed accordingly. So therefore, let us derive the conditions so this was the condition of choking in absence of flooding as derived from the mixture momentum equation, just remember this. So, next what we would like to do is? We would like to derive the condition of choking, considering two phases separately with change of phase. So, let us find out that what will be the condition of choking under this particular situation.

This is going to be the means almost the last of the complicated derivations which we have because this will involve a good amount of things. So, just try to follow it very well, the way I am trying to do, how I am trying to substitute. Now, final results you will get in several text books, but the intermediate derivations will not be there. So, those derivations you can just; that is why I am going to go into the details of the derivations so that you can follow them and you can understand how the final derivations have been arrived at.

So, if you remember in the last; not last, last to last class we had derived the basic equations. For the two fluid model, if you remember the basic was rho 1 u 1 del u 1 del t, sorry, u 1 delta u 1 this was equal to b 1 plus f 1 minus delta P, is it not. From there we I will just recapitulate a few things so that you remember this is rho 2 del u 2 del t plus delta u 2 equals to b 2 plus f 2 minus delta p. So, for one dimensional case, one dimensional steady state case, where we knew that b 1 is nothing but minus rho 1 g sin theta, b 2 was equals to minus rho 2 g sin theta and we knew that f 1 it comprised of f 1 2 f w 1 or in other words if I write it down by 1 minus alpha. Why did this come about? somebody had asked me this question this was because this entire expression this is written for unit volume of fluid one, this is written for unit volume of fluid two.

So, therefore, this f 1 it refers to the left over forces per unit volume of fluid one, agreed. But usually, it is difficult when the two phases are flowing, we can very well identify unit volume of the flow element which will comprise of fluid one and fluid two. It is very difficult for us to separate unit volume of fluid one or unit volume of fluid two and perform; and do the necessary things. So, what we had done is? This entire expression was unit volume of fluid one, this was unit volume of fluid two.

Now, for combining what we did? That suppose something is for unit volume of fluid one. Now, we have the entire or rather we have unit volume of the two fluid mixture. Out of this, how much of fluid one is there? Alpha amount is fluid one, 1 minus alpha amount is fluid two. So, therefore, if the effective force per unit volume of fluid one is f 1 then then per unit volume of the total two phase mixture, what will it be? It will be say f 1 by 1 minus alpha is it not. So, per unit volume of the two fluid mixture was taken as capital f 1 where this one was f 1 by 1 minus alpha, did you get my point. So, therefore, I will just cut it down.

So, f one into 1 minus alpha is the total left over forces in fluid one per unit volume of the flow filed. Clear to you, same way, f 2 into alpha equals to f 2. And what is f 1 equals to? f 1 it comprised of f 1 2 minus f w 1, if you remember the interaction between the wall and fluid one, interaction between fluid one and fluid two and when there was a change of phase what extra we had? For change of phase, we had the total force u 2 minus u 1 d x d z. G d x d z was the mass rate of evaporation into u 2 minus u 1, it was the velocity change which occurred due to mass mass transfer, sorry, due to phase change. You remember those things we had done already.

This was the mass rate of phase change G d x d z that into u 2 minus u 1 gives us the total force which is associated with phase change. And out of this total force we had assumed that 1 minus eta of the total force goes to phase 1 and eta amount of the total force goes to phase 2, remember those things. So, therefore, we find f one equals to f 1 2 minus f w 1 minus of 1 minus eta into G u 2 minus u 1, these are all u 2 minus u 1 d x d z. The same way we can write f 1 this is equal to minus of f 1 2 plus f w 2 minus this was f 1 minus eta G d x d z into u 2 minus u 1. So, these things we had already derived.

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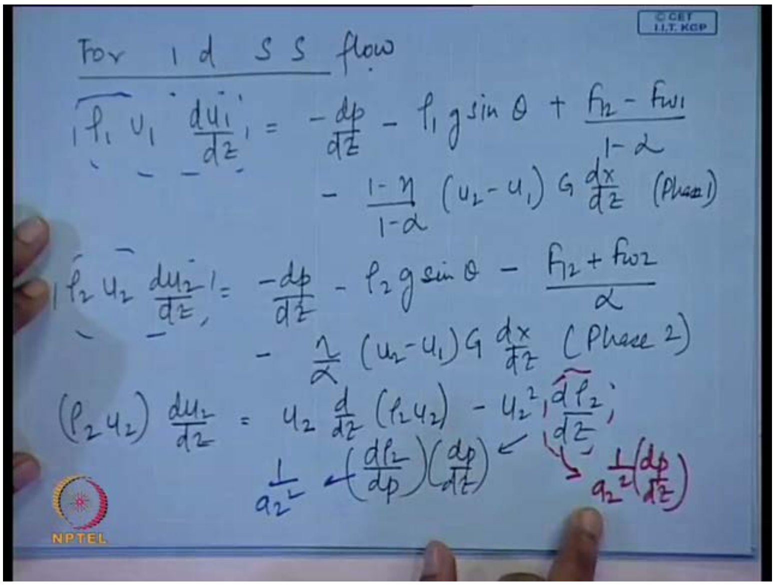

So, now if you substitute this f 1 and f 2 in this particular expression and assuming steady state one dimensional flow analysis. So, for that particular case what do we get? The final expressions we get are, for one-dimensional steady state flow, we get rho 1 u 1 d u 1 d z there is no delta here, minus d p d z minus rho 1 g sin theta plus f 1 2 minus f w 1 by 1 minus alpha minus 1 minus eta by 1 minus alpha u 2 minus u 1 G d x d z. The same way, for this was for phase 1 similarly, for phase 2 again we have rho 2 u 2 d u 2 d z this is again same thing we can write it down, just the subscript one will be substituted by subscript two and we have to take into account the directions in which the forces are acting in each of the phases, u 2 minus u 1 G d x d z, this is for phase 2.

So, these were the two equations, which we had already derived. If you remember these were the two equations which we had already derived in the last class. By considering the momentum balance equation for phase 1 and by considering the momentum balance equation for phase 2. Now, from here in order to arrive at the conditions of choking, again we have to substitute u one's with something etcetera etcetera. Now, let us see how we go about for these particular substitutions. Now, let us take this term for example. So, let us take this particular term and let us take this particular term.

Now, we can write it down as just see what I am doing rho 2 u 2 d u 2 d z can this be written as u 2 d d z of rho 2 u 2 minus u 2 square d rho 2 d z, why I am writing that you will understand shortly. This d rho 2 d z can again be substituted etcetera. So, this we can we can write rho 2 u 2 d u 2 d z in this particular form, yes or no. well, now, what is this d rho 2 d z? Now, if you observe this particular term this is nothing but this term is nothing but d rho 2 d p into d p d z, yes or no.

So, again we get a d p d z d z dependents on the on one particular side on the left or the right hand whatever the case may be, agreed. And now, what is d rho 2 d p again? Just now, I have derived what is d rho 2 d p. This part is 1 by a 2 square. So, therefore, this particular term which can be written down as; so, this d rho 2 d z can we write this particular term as 1 by a 2 square d p d z, can we do this? Yes, we can write it down in this particular form, I will be going a bit slow because this derivation is slightly little more complex.

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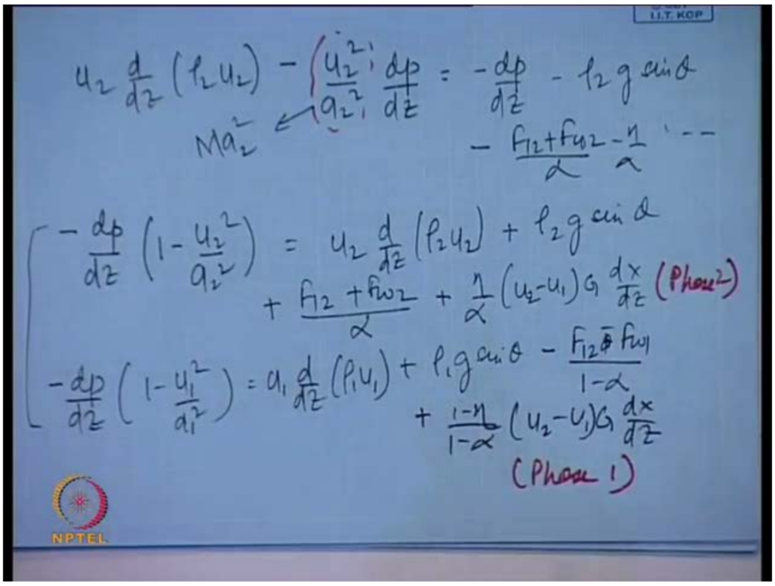

So, if I substitute these things then in that case, what do I get? Then if I substitute rho 2 d u 2 d z with u 2 d d z of rho 2 u 2 minus u 2 square by a 2 square d p d z, I can very easily do that, is it not. Now, doing that what do I get? doing that I get, u 2 d d z of rho 2 u 2 minus; this u 2 square by a 2 square is what? This particular term, this is m a 2 square right MaTp number of phase 2. So, this is nothing but m a 2 square. So, this into d p d z this is nothing but equal to minus d p d z into whatever the rest of the things that we have written down, is it not.

So, this is nothing but equal to minus d p d z minus rho 2 g sin theta minus f 1 2 plus f w 2 by alpha minus eta by alpha and all those terms which are there, fine. Now, I can bring both the d p d z terms on one side, the same thing which I have been doing for all the derivations. So, I can bring it on one particular side and on doing so, what do I get? I get just take up the signs which I am may be I might make some mistakes. So, just see whether I am writing it correctly or not. What I have done? I have taken this d p d z this side. So, I get minus d d p z 1 minus u 2 square by a 2 square, correct.

And this is equal to what I will bring all the other terms on the right hand side. So, I get this is equal to u 2 d d z of rho 2 u 2 plus rho 2 g sin theta; all the minuses here are going to get plus, plus f 1 2 plus f w 2 by alpha plus eta by alpha u 2 minus u 1 G d x d z. Can I do this, yes or no? This I did for phase 2, agreed. Similarly, I can just write down the same type of equation for phase 1 there is no need of deriving it separately. Just the same type of equation I can write it down for phase 1, yes or no? plus rho 1 g sin just take into account one thing the signs are very important.

Because here it becomes minus f one two sorry minus f w 1 by 1 minus alpha this you have to keep into account again plus 1 minus eta by 1 minus alpha u 2 minus u 1 g d x d z. Same thing I have written it down this is for phase 2 this is for phase 1, agreed. I can write down these two equations. So, from these two equations what do I get? I get that when is phase 2 choked when u 2 square equals to a 2 square we all of us know it and we have derived it from this particular equation, agreed. When is phase 1 choked when u 1 square equals to a 1 square this also all of us know.

When is this two phase system getting choked, we cannot say u 1 equals to a 1 and u 2 equals to a 2 will guaranty the condition of compound choking, because there is good amount of alpha part also here. Moment there is some alpha part this alpha is also a function of pressure. So, unless we can account for alpha being a function of pressure and bringing those things also on the right hand side, we cannot arrive at the condition of compound choking, is it clear.

So, therefore, remember one thing choking of phase 1, choking of phase 2 does not give you any idea about choking of the two phase mixture. Why because the composition the in-situ composition of the two phase mixture is very important and this particular this the in-situ composition how it varies with the pressure gradient and other factors is also very important, correct. Now, we have to go for that or we have to find out that how alpha varies, we have to substitute that and then only probably we can get a two equations, which we can combine, and we can get the condition of compound choking. For that what we have to do? Let us consider the equation of continuity.

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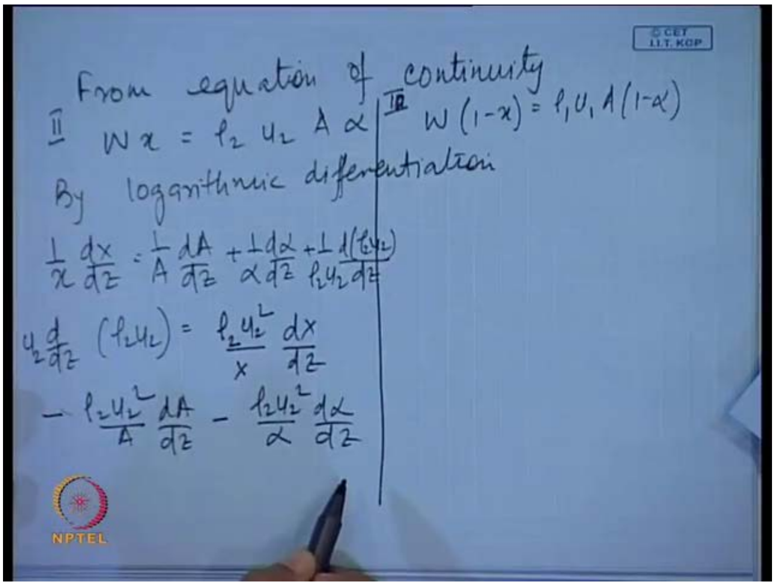

So, from equation of continuity what do we get? I am deriving this whole thing I have not prepared a transparency so that it becomes easier for you to follow the derivations. And if you do it along with me, when I am doing it will get some time, because I am also writing down the things. So, probably you will get a little more thorough this is the only reason. Otherwise, I could have prepared some transparencies and I could have shown it to you. My course coverage could have been faster under that particular circumstance any how.

So, from equation of continuity what do we know? We know w x equals to rho 2 u 2 a alpha, agreed. Same thing I will just write it down for phase 1 as well. So, this is for phase 2, similar things I will be writing, sorry, I will be writing for phase 1 as well. So, w into 1 minus x equals to rho 1 u 1 a into 1 minus alpha, agreed. Now, by logarithmic differentiation, this we have been doing quite a number of terms and we have also done it I believe in your other subjects as well.

So, by logarithmic differentiation, what do we get? For this particular case we get 1 by x d x d z 1 by a d a d z why I am doing this after write down the expression it will be clear to you plus 1 by rho 2 u 2 d rho 2 u 2 d z. So, can you understand what I want to do I want to substitute this rho 2 u 2 with things like say x a etcetera, etcetera alpha so that I can find out all sorts of alpha dependences in this particular equation and I can substitute those alpha dependences and then I can proceed. So, I have to find out what are the terms which depend on alpha, is it not. Then only I can bring the alpha dependences to one side and find out the condition of choking.

So, this is the thing which I can write it down or in the other words I can write d d z of rho 2 u 2, this is nothing but try to understand into u 2, then I get this term u 2 d d z of rho 2 u 2. This whole term if I want to substitute it in terms of say certain known parameters then this whole term what does it become, just check up what I am writing rho 2 u 2 square by x d x d z minus rho 2 u 2 square by a d a d z minus rho 2 u 2 square by alpha d alpha d z, yes or no. Please, flow the derivation carefully. So, I have simply substituted this and accordingly I have brought the things to the left hand side I have got this particular equation, agreed.

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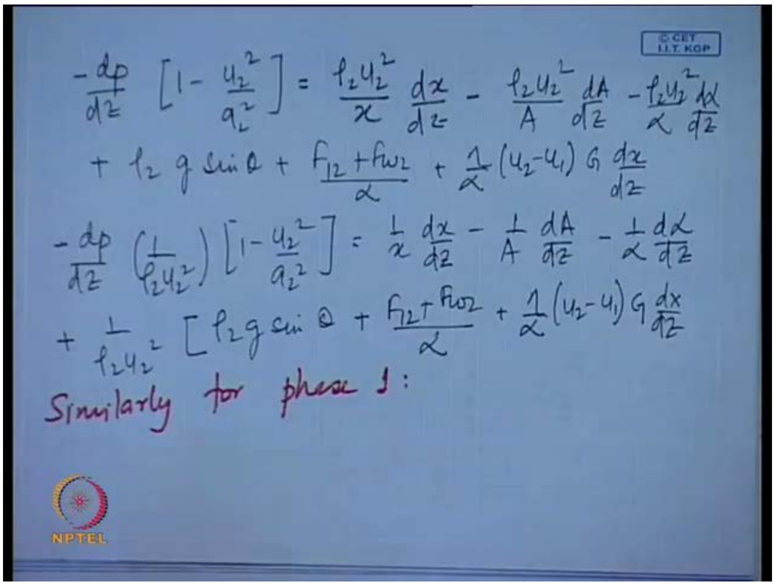

Now, if I substitute this particular equation here for phase 2 then what do I get? If I substitute just see the if you remember that equation it was minus d p d z 1 minus u 2 square by a 2 square. This will be equal to rho 2 u 2 square by x d x d z, agreed. I am just substituting that rho 2 u 2 with this the this particular term, this u 2 d d z of rho 2 u 2, I am just substituting this particular term minus rho 2 u 2 square by a d a d z minus rho 2 u 2 square by alpha d alpha d z, fine. I have substituted, then the original terms which were there, rho 2 g sin theta plus f 1 2 plus f w 2 by alpha, whatever terms I had there itself, fine.

So, this again can be written down as we have rho 2 u 2 square everywhere. So, therefore, we can write it down as minus d p d z 1 by rho to u 2 square 1 minus u 2 square by a 2 square, this is nothing but equal to 1 by x d x d z 1 by a d a d z 1 by alpha d alpha d z plus 1 by rho 2 u 2 square. See whether, I am doing it correctly or not plus f 1 2 plus f w 2 by alpha plus eta by alpha u 2 minus u 1 G d x d z see whether your are comfortable with the final expression that I have derived. Just see whether you are comfortable with the final expression which I have derived.

Then once I have derived it for phase 2 similarly, for phase 1 we can write down an identical equation. So, we have written down one equation for phase 2, in the same way we can write down an identical equation of phase 1 by substituting u 1 d d z of rho 1 u 1 in the same way from the equation of continuity and performing the your logarithmic differentiation, and we can arrive at one particular expression for phase 1. So, we will be doing it in the next class and then we will finally, arrive at the condition of compound choking for the two phases under using the two fluid model. So, thank you very much.