Multiphase Flow
Prof. Gargi Das
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture No. # 18
Separated Flow Model (Contd.)

Well, so we will be continuing our discussions regarding the separated flow model. Is it not? This was one extreme of the situation, one extreme was the homogeneous flow model where we considered that both the phases are intimately mixed with one another, and the other is when they are totally separated from one another. So, as we as I had already discussed in the last class more or less that in this particular case the best thing is that we consider the two phases separately.

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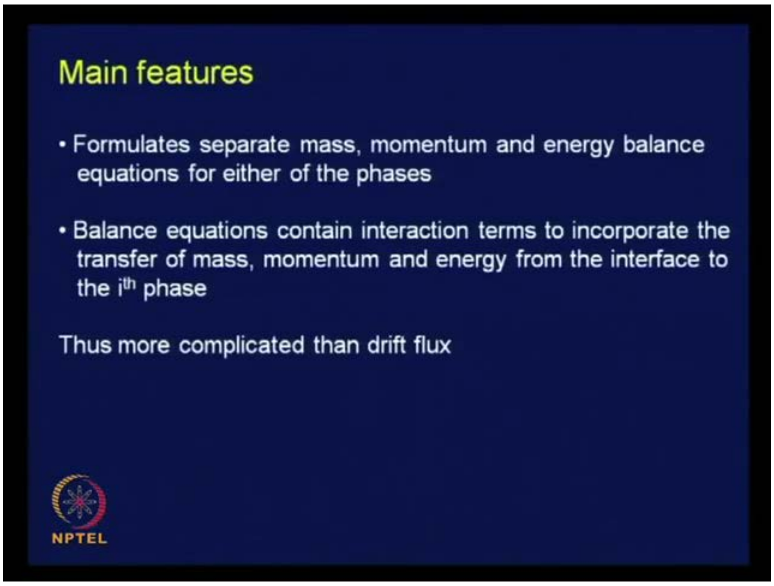

And we formulate separate mass momentum, and energy balance equations, and these balance equations they have to contain interaction terms to incorporate the transfer of mass momentum and energy from the interface to the i th phase.

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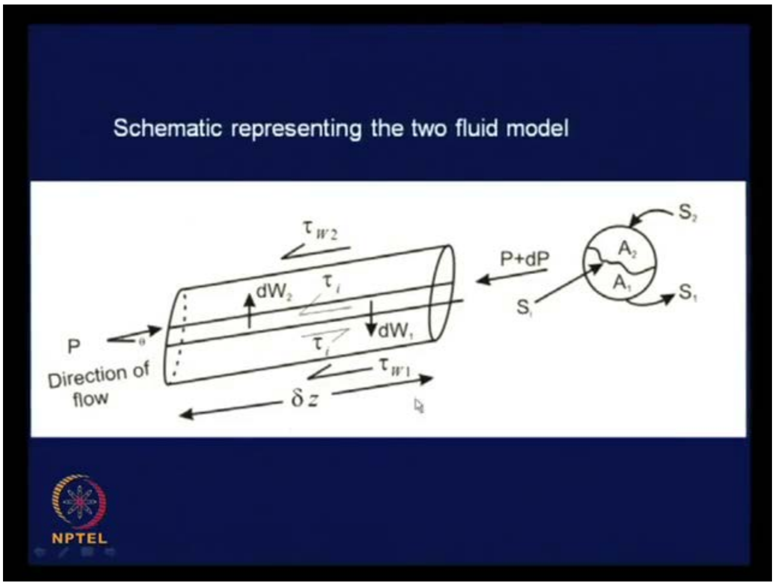

Accordingly we have written down the equations and by considering that both the phases they flow separately, and therefore there is 1 tau w 1 the interaction between phase 1 with the wall, tau w 2 interaction of phase two with the wall.

And tau i the interaction between their two phases which we assume to be in the direction of motion for the lighter sorry for the heavier phase, and opposite to the direction of motion for the lighter phase, because quite natural the lighter phase flows faster than the heavier phase. Is it not? So, accordingly what we had done we had these are advantages etcetera, what we had done? We had discussed the equation of continuity under different conditions, and then we had discussed the momentum balance equation.

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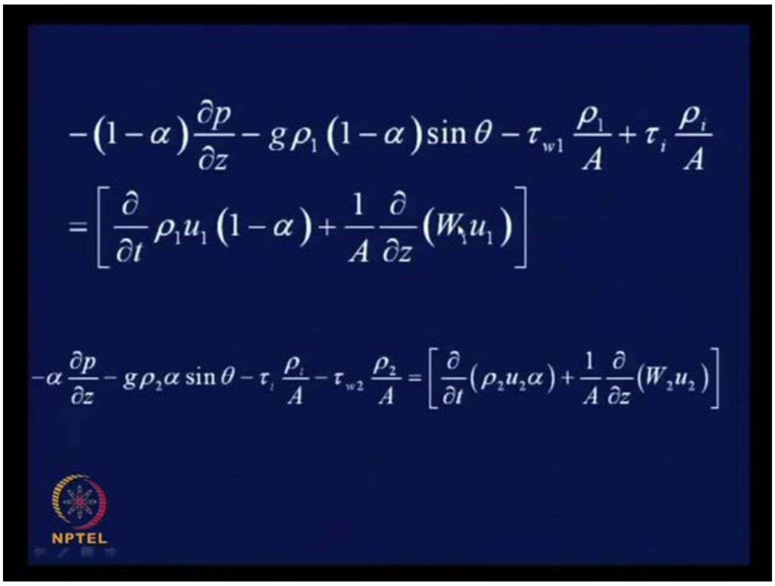

And we had finally derived the momentum balance equation separately for phase 1, and then separately for phase two. Is it not? And after that we had combined them in several ways, 1 way what we did was dividing this particular equation with 1 minus alpha, dividing this particular equation with alpha, and then we had subtracted second equation from the first one, so from there what we got we found out that the relative sorry the pressure drop term was no more there.

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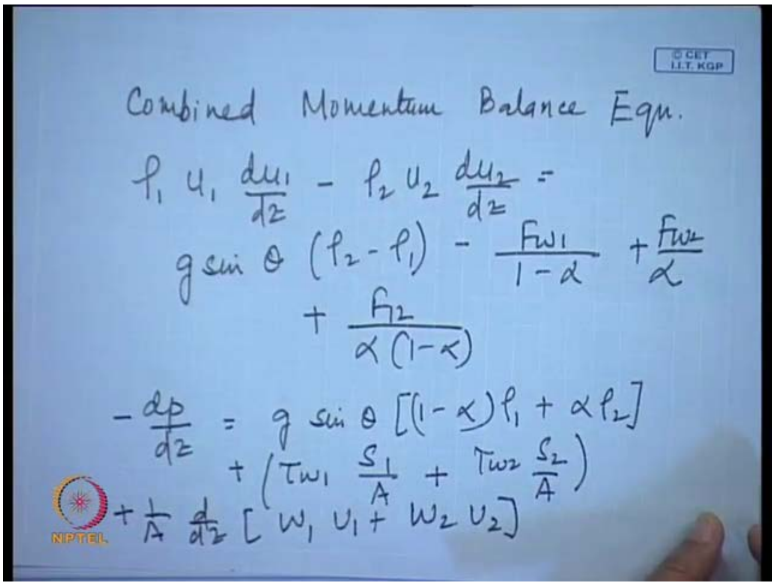

And it was re obtained or relative velocity sort of a thing, where we found that it does not include the pressure gradient, and it could be considered as a relative motion equation, in case we have forgotten I will just write down the equation once more. The first equation which I had got by means by dividing the first equation here by 1 minus alpha and dividing this by alpha and then subtracting the second equation from the first we found we had got the combined momentum equation.

We had already done these things, combined momentum balance equation, so by this particular process we had obtained a equation d u 1 d z this you already have it that I will just for the recapitulation part I will be doing it, this is equal to g sin theta rho 2 minus rho 1 minus 1 by 1 minus alpha, plus F w 2 by alpha, plus F 1 2 by alpha into 1 minus alpha, this was 1 thing that we did.

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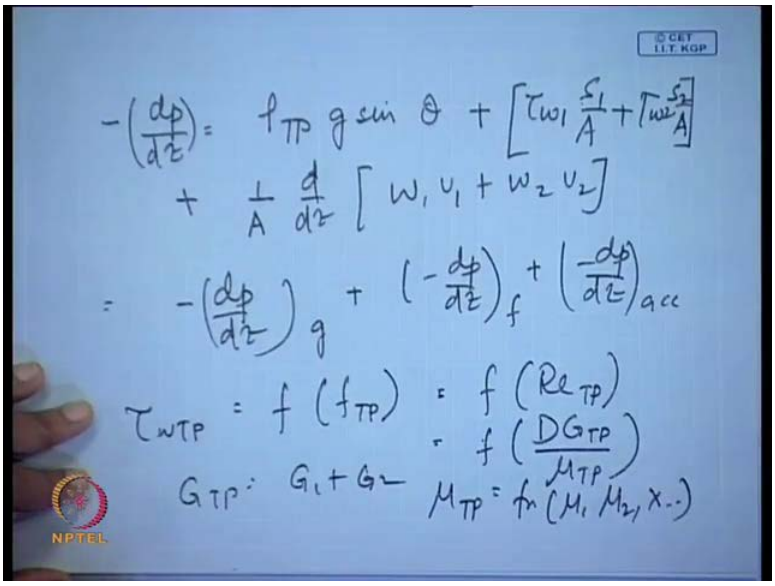

The other was simply adding up the two equations which the momentum equations which you have written down for phase one and for phase two, so if we simply add them up then what we had got? We had got minus dp dz equal to g sin theta into say let I will write it down fully 1 minus alpha rho 1 plus alpha rho 2, plus there was a term tau W 1 S 1 by A, plus tau W 2 S 2 by A which comprises of the frictional pressure gradient, and then there was plus 1 by A it is d dz of W 1 U 1 plus W 2 U 2.

So, this was all that I had got from simply adding the two equations which I have written down in this particular transparency. So, from that we had got this, or in other words we can just write it down as minus dp dz this is equal to rho two phase, this particular expression that I have written down here 1 minus alpha rho 1 plus alpha rho 2 this is nothing but the two forth density rho TP.

So therefore, rho TP g sin theta plus this particular frictional pressure gradient tau W 1 S 1 by A, plus tau W 2 S 2 by A, plus 1 by A d d-z of whatever we had got, where we find that the first term this particular expression it can be written down as the summation of a gravitational component, plus a frictional component, plus an acceleration component is not it? Just like what we had done in the for the homogeneous flow model.

What did we do for the homogeneous flow model? We found out that for the homogeneous flow model what we did we expanded the acceleration pressure gradient. Why? Suppose, we would like to find out an expression or the pressure gradient or the pressure drop term from the equation which we have derived we find rho TP if you know alpha we can find it out is not it? For the frictional pressure gradient, we would like to know how the two phases interact with the wall.

What is the proportion of the wall perimeter or the sorry for the proportion of the wall wetted area which is occupied by phase 1 and the proportion which is occupied by phase 2, or in other words S 1 S 2 we need to know, tau W 1 tau W 2 we need to know, this part just like in, see in homogeneous flow theory what we did? We found out there it was just tau W two phase is not it? S by A, so for tau W two phase we had devised several methods or several techniques we found out, to find out tau W TP based on our knowledge of single phase flow, either we had defined a FTP based on a Re TP which is again based on a mu TP and a rho TP.

Or in other words, we assume that if a very small amount of vapor is there; very large amount of liquid is there, then more or less we can consider that the whole phase is flowing as a liquid and vice versa. If there is large amount of vapor small amount of liquid, then we assume the other thing accordingly we had defined two phase multipliers, here also we have to do something but there the situation was easier, why? Because this whole thing we could take up as tau W TP S by A.

So, in that particular case we did not have to know S 1 S 2, and there was no separate S 1 S 2 there at all, just we had to find out tau w t p is not it? So, there were certain things that we did, first thing which we did was there may be tau W TP this was a function of f two phase which is in turn a function of Reynolds number two phase, and which again depends on or this Reynolds number was DG TP by mu TP, so DG TP was nothing but it was G 1 plus G 2, so what we had to do we had just to find out mu TP as a function of mu 1, mu 2, X and so on and we had to do it.

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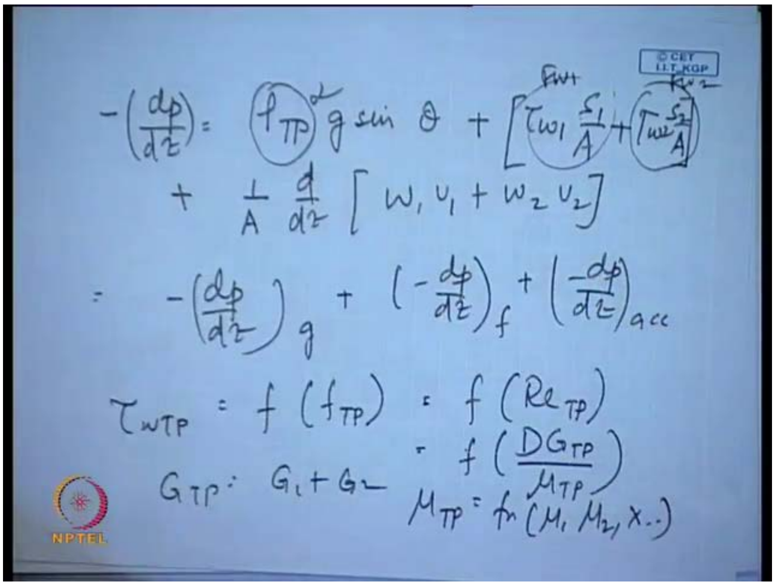

The other thing which we did is we assumed that if one phase is in much larger proportion as compared to the other phase, then in that case we assumed the frictional pressure gradient dp-dz frictional as the frictional pressure gradient for either liquid flow or may be the gas for only for only gas flow into may be a some sort of a correction factor which we defined as phi l square, in this particular case it was phi g phi g square.

They were all two phase multipliers, they were based on single phase pressure gradient when either the liquid component flows alone in the pipe, or the gas component flows alone in the pipe, or the entire mixture flows as liquid entire mixture flows as liquid, in which case this becomes phi square LO, or may be which we do not used much this is entire mixture flows as gas, so these were the things that we had defined there.

So, in these particular two ways we had defined the frictional pressure gradient either in terms of tau W TP S by A, or two phase multipliers. So, in this particular case also we have to device some particular way of finding out the frictional pressure gradient, but in this particular case we cannot consider the mixture to be a mixed flow or something, so therefore, some particular device has to found out we have to know either tau W 1 tau W 2 S 1 S 2.

Or maybe we can combine it in some particular way and we can do something, we can assume this to be F W1 interaction of phase one with the wall, this as F W2 and then we have to do something to find out F W1, F W2 agreed, but apart from there we find there is one more thing, to find out rho TP what do you need? What is the expression of rho TP? We need to know alpha. So therefore, there was no need of knowing alpha separately, alpha was equal to beta and beta we could find out from input parameters. This particular case alpha is a variable, it varies with x; it varies with other phase physical properties and we have to device ways of finding out alpha as well.

And the second thing is this is the same thing as in homogeneous flow theory what we found that W 1 W 2 they are input parameters, but U 1, U 2 they are not input parameters they are in situ velocities, they have to be expressed in terms of certain input parameters only after that we can write down this particular equation in terms of known measurable properties.

So, first what we will do? We will first try to derive this equation in a form which will enable us to find out the pressure drop from known input parameters. For that we will be dealing with the acceleration pressure gradient just as we had done for the homogeneous flow theory, now moment you know whenever we try to expand this U 1, U 2 normally what happens we get some terms containing dp-dz is not it? Why? Because V 1, V 2 etcetera they vary with pressure.

So, when they are brought on the left hand side, we find that the final pressure drop equation it has a denominator from there we go to derive the condition of choking. Is it clear to all of you? So, first what we will do? We will be trying to find out the condition of choking from this particular equation, and after that we will be discussing methods of finding out alpha and finding out the frictional pressure gradient right.

But before I do anything at all let me just remind you one particular fact, that fact is if you notice the two equations which I have written down on the PPT, these two equations for the component balance we find there is one tau i here, there are certain mistakes which I have told in the last class these are going to be S 1, S 2 the interfacial and the so these are some typographical errors we will be correcting them.

So, we have a tau i here and we have a tau i here also, now remember one thing we are considering the separated flow model which interacts at the interface, but there is no mass transfer between them, and there is no phase change between them agreed. Now in presence of mass transfer what do you expect? We first discuss this and then we will go for expressing the pressure gradient in terms of known input parameters. So, for the time being can you tell me that when we are having or in presence of mass transfer what sort of change do you expect in these two equation?

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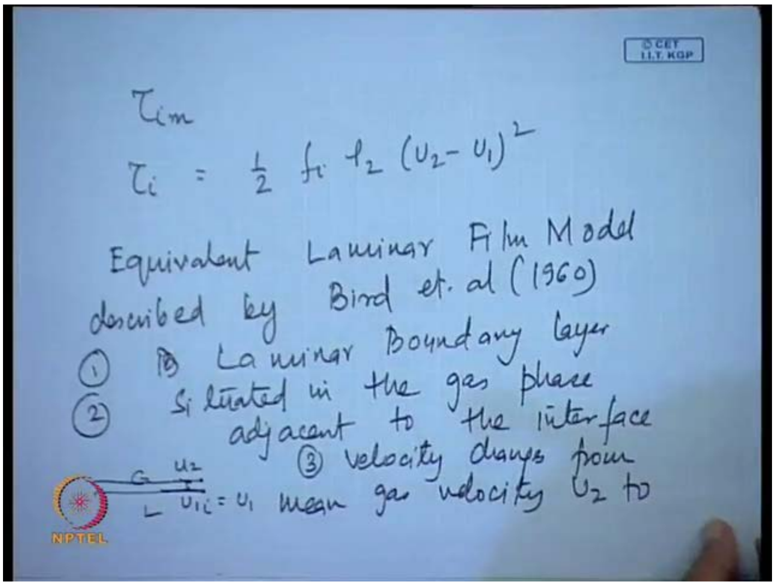

If we can find out the changes in these two equations and then accordingly we can incorporate the changes. Now none of the terms are going to change is not it? The only things which is going to change is tau i here; and tau i here, only tau i is going to change. Now in this particular case we find out that the… Normally, we do not have much data on this. So, what we do? We assume that in presence of mass transfer say the tau i can be expressed in terms of tau im, and in absence of mass transfer, we can express it in terms of tau i which we have already defined.

Now, this tau i what is this? We can define it as half f i rho 2 into U 2 minus U 1 square, let me tell you first there is thing which I would like to tell you that is in order to express tau im in terms of tau i what we have done is we have assumed the laminar equivalent laminar film model which was described by… So, this particular equivalent laminar film model has been assumed and on the basis of this model we have tried to express tau im in terms of tau i.

Now, what does this model assume? First thing the model assumes is the boundary layer is laminar. So, the things which the model assumes the first thing is laminar boundary layer, and this particular boundary layer that is situated in the gas phase close to the interface. Is it clear? So therefore, if we have a liquid, if we have a gas this is situated in the gas phase close to the interface, and here the velocity changes from the mean gas velocity U 2 to the interfacial velocity of the liquid, say U 1 i as a first approximation we take this as U 1. This is the assumption of this equivalent laminar film model.

What does it assume? It first assumes that the boundary layer is laminar and secondly, this boundary layer it is situated in the gas phase adjacent to the interface, and in order to find out the and then the other thing is that within this particular thin boundary layer the velocity changes from mean gas velocity that is U 2 to mean gas velocity U 2 to the liquid interfacial velocity which is assumed as U 1 as a first approximation.

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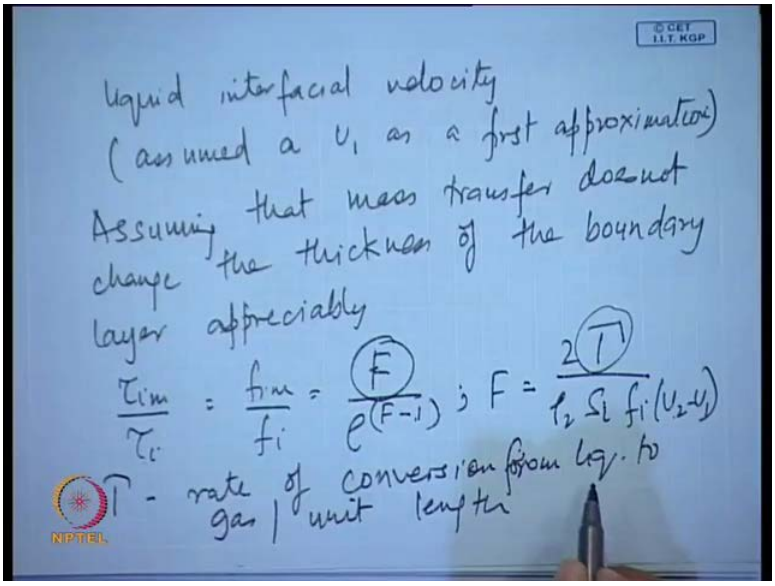

So therefore, what we find that these are the basic equivalent laminar film or these are the basic postulates of the equivalent laminar film model which has been described by your bird et al, the first thing is that it assumes the boundary layer to be laminar and it is situated in the gas phase adjacent to the interface, and within this particular thin laminar boundary layer the velocity changes from the mean gas velocity to the interfacial liquid velocity, which can be taken as U 1 as the first approximation. So, and on this particular basis and if we assume assuming that mass transfer does not change the thickness of this particular boundary layer; that means, this particular thickness delta this is not much affected due to mass transfer. So, if we make this particular supposition that the thickness of the mass transfer does not change the thickness of the boundary change the thickness of the boundary layer appreciably, so under these conditions what we can do? That we can write down that tau im.

So, once we make these a suppositions, so we can write down tau i equals to this in a similar way your tau im equals to f im rho 2 into U 2 minus U 1 square, yes or no? Because this is the change in velocity which is happening and since this is located in the gas side so therefore, it is rho 2 and f im is the equivalent friction factor for tau im; f im is the equivalent friction factor for tau i. So therefore, on this particular basis if we assume that the your the thickness of the boundary layer is not affected appreciably by the change of mass, then in that case your tau im by tau m what it becomes?

All these things cancel out and this becomes f im by f i right and we know that from this particular supposition tau im by tau i equals to f im by f i, and this is nothing but equal to f by capital F by e to the power F minus 1, where this particular F this particular F this can be written down as 2 gamma by rho 2 S i f i U 2 minus U 1, where we know that this particular gamma, this is nothing but the rate of phase change or may be rate of conversion rate of mass transfer rate of conversion from liquid to gas per unit length and S i, it is nothing but the interfacial perimeter rho 2 density U 2 minus U 1 is the relative velocity.

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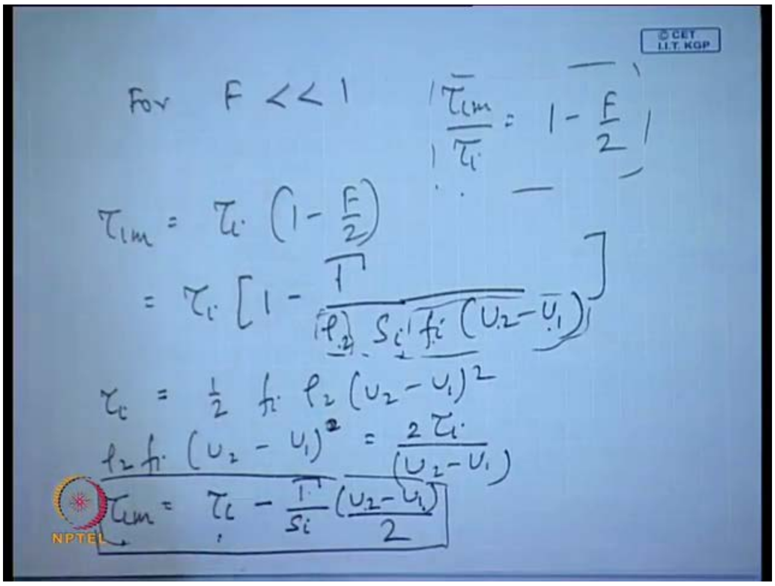

So therefore, we find that tau im by tau i can be expressed in this particular form where F can be written down as 2 gamma rho 2 S i f i U 2 minus U 1, where S i this is sorry the S i is the interfacial perimeter and your gamma is the rate of conversion from any particular change of mass from liquid to the gas phase per unit length. Now, this gamma it can be expressed as, if it is condensation or vaporization then naturally it can be expressed in terms of your heat fluxes etcetera, if it is mass transfer then accordingly we have to express it.

Now, we know that for F much less than 1 we know tau im by tau i it can be expressed as 1 minus F by 2. Now, if we substitute this particular equation here, this particular equation if we substitute in here, then in that case what do we get? In that case we get tau im equals to tau i into 1 minus F by 2 agreed. Now, what is F? F, I have already written down your F is 2 gamma rho 2 S i f i into U 2 minus U 1 agreed.

So therefore, instead of F we can write it down as tau into 1 minus 2 2 cancels out, so it becomes gamma rho 2 S i f i U 2 minus U 1; I can write it in this particular way, and I know what is this rho 2 f i into U 2 minus U 1, if you see here your rho f i rho 2 into U 2 minus U 1 what will it be? It will be 2 tau i by U 2 minus U 1, is not it? Is this part clear to all of you?

So therefore, what can I do if I consider this rho 2 f i into U 2 minus U 1? So therefore, we had already defined tau i as half f i rho 2 U 2 minus U 1 square. So therefore, rho 2 f i

U 2 minus U 1 square, so this part it becomes 2 tau i by U 2 minus sorry U 2 minus U 1 yes or no? So instead of this particular term, I can substitute it with 2 tau i into U 2 minus U 1. Can I do this? Agreed?

So therefore, what do I get? I get tau im that is equal to tau i minus gamma by S i U 2 minus U 1 by 2 this is the thing which I get. This is this particular portion clear to all of you? So therefore, what do I get? I find out that tau im how it is not equal to tau i, it depends upon the relative velocity and the rate of your mass transfer or rate of phase change whatever it is per unit interfacial area, because this is per unit length and S i is interfacial perimeter, so this is per unit interfacial area.

So, we find that if it is a change of phase say may be it is an evaporation; that means, evaporation means the change of phase occurs from the liquid to the vapor phase, for that particular condition the interfacial shear is reduced by an amount which corresponds to the product of the evaporation rate per unit interfacial area and half the velocity difference, and if it is condensation then in that case gamma is negative and therefore, the interfacial shear is enhanced.

So therefore, when there is a change of phase from liquid to vapor or liquid to gas then your interfacial shear is reduced, when there is an opposite in mass transfer in the opposite direction tau im is increased. So therefore, whatever we had derived it was assuming that there is no mass transfer there is no phase change, if there is mass transfer then tau im or rather the interfacial shear gets modified accordingly.

Similarly, we can discuss what happens when there is change of phase. Can you suggest can you just look at the two equations that I have given in the PPT, and can you suggest what will be happening when we have a change of phase? What happens when we have a change of phase? Your frictional pressure gradient that remains constant, yes or no? gravitational pressure gradient only the alpha it keeps on changing otherwise the expression of gravitational pressure gradient that also remains constant is not it? What happens then?

Then, we will find out if we go a little further in this particular case, we find that there will be some amount of rate of creation of momentum or rate of loss of momentum why? Because whenever some amount of it is changing its phase; what is happening? It is going from one phase to the other. Say suppose some particular fraction of it say eta fraction of the liquid phase is being getting converted to the gas phase.

So therefore, W 1 into eta this is the amount which is getting converted, and this particular amount this is changing its velocity from that of the liquid phase to the gas phase, or in other words it is changing its velocity from U 2 to U 1. So therefore, this mass flow rate into the change in velocity gives you the rate of change of momentum. So we have to take that into account accordingly these particular expressions are going to change under that particular condition.

Now, if we deal with that situation flow with phase change usually what we do is I will just do the basics because unless we learn something else, we cannot deal with this completely. So in this particular case if you remember the very basic equation which I had written down the very basic equation rate of creation of momentum I think I do not have the very basic equation here anyhow leave it. So the very basic equations which I had written down there, that is rho 1 if you remember the equation del U 1 del t plus U 1 del U 1 del z.

This was equal to b 1 plus f 1 minus del p del z, do you remember from there itself we had started the derivation, there is one particular request please come prepared in the class otherwise this is going to be very difficult for you, because now the derivations are getting tough and it is for me to interact with you in the class and to means to just to brush up what we had done in the previous class, so unless you are prepared it is going to be very difficult.

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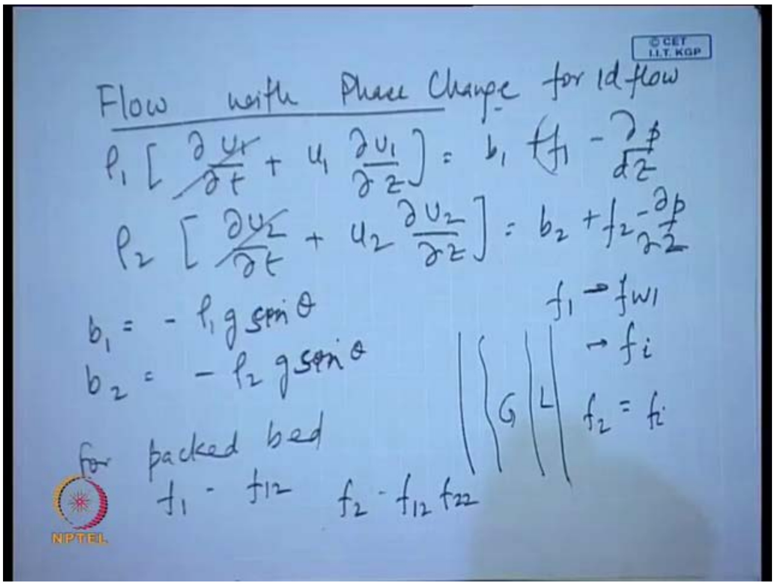

So, this was the basic equation that we had written down for flow with phase change for one dimensional flows. And similarly, we had rho 2 del U 2 del t plus U 2 this was equal to b 2 plus f 2 minus del p-del z, and there at that time if you remember what I had told you b 1, b 2 are the body forces; this was written per unit volume of phase 1; this was written per unit volume of phase 2. Is not it?

And from there what do we get? b 1 I had told you it was minus rho 1 g cos theta assuming the upper direction to be positive, and then we had b 2 minus rho 2 minus g cos theta cos or sin I think I will put up sin so those things were there, and for under steady sate condition we find these two terms they cancel out; these are the pressure gradient terms, and I had told you f 1, f 2 are just the left over forces which are included in order to keep the account straight.

Do you remember those things? I had told you that f 1 and f 2; f 1 is the left over forces per unit volume of component 1; f 2 is the left over forces per unit volume of component 2. So they were just included in order to keep the account straight, and I had told you that they take into account the interaction between phase 1 with the wall interaction of phase 2 with the wall.

Interactions between the two phases the hydrodynamic drag and if there is change of phase the change of phase will also have to be accounted by f 1, f 2. Whatever if it is particle particle interaction it has to be accounted by f 1, f 2. So remember, one thing this particular these two equations they are common for all two phase flow situations, we can also write down two such equations even when there is a drift when the two phases have a strong coupling among one another.

Because, they we are just considering the two phases and we are writing down the momentum equation, is not it? But remember, one thing the important part in this particular case is that here in order to keep the account straight whatever extra force which is not included in b 1 which is not included in pressure gradient everything is included in f 1 and f 2. So therefore, how do we account for different flow situations whether it is drift flux or a separated flow f 1 and f 2 are going to be different.

Even in separated flow for annular flow you will have one type of f 1 and f 2, if there is a fluidized bed you have different type of f 1, f 2. Is it clear to all of you? So therefore, we find that suppose it is annular flow say suppose it is suppose say it is a vertical annular flow, so there is a gas coat; there is a liquid; there is a liquid film. So, what will f 1 comprise of? It will comprise of fw 1 interaction of liquid with the wall agreed, f i interface between gas and liquid fine.

And what will f 2 comprise of? f 2 will comprise of just f i, because there is no f it will just comprise of f i. Now suppose, we have packed bed liquid is flowing through the packed bed then in that case what will happen? In that case your f 1 will comprise of f 12, say if liquid is phase 1 and solid is phase 2, is not it? So for packed bed what do we have? For packed bed your f 1 it will comprise of f 12, if we neglect the wall interactions as compared to the interactions between the solid and liquid, and what will f 2 comprise of? f 12 or f 21 whatever it is. Anything else?

If it is a packed bed, then will f 2 comprise of anything other than f 12, f 22 particle particle interaction. In this particular case it becomes important, for gas liquid cases it is not important, but for particle particle cases it is very important. So therefore, it will comprise of f 12 and f 22, is it clear? We will be doing some problems on it as well, but please remember one thing that whenever we have any particular whenever we have any particular two phase flow situation, we can write down the momentum equation for phase 1 in this particular way, momentum equation for phase 2 in this particular way, b 1 b 2 are the body forces they can be very easily evaluated, for steady state these two go away and whatever for different flow situations what is going to differ your f 1 and f 2 are going to differ. We will be taking up the annular flow and the packed bed situations in our tutorial classes and then it will be much more clear to you, that whatever happens if f and f 2 are going to be different, is not it?

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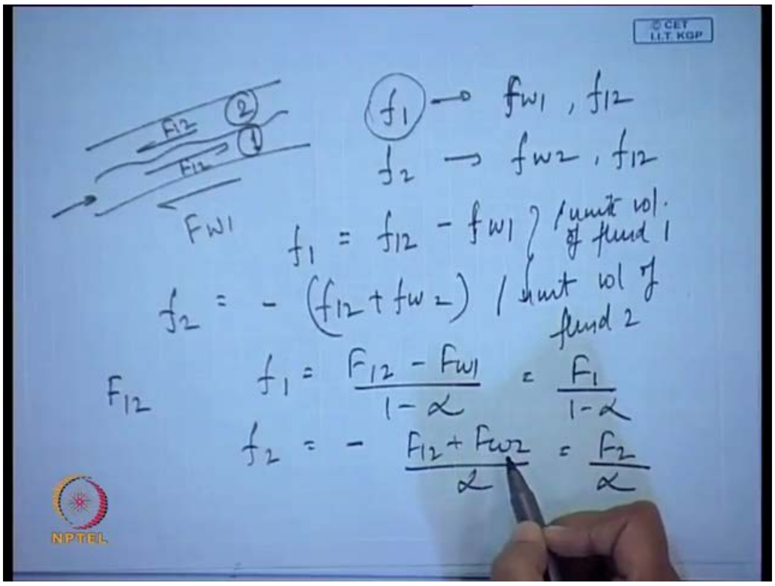

And accordingly, the different phases are going to come in. So therefore, the particular case which I was dealing with where we have the two separated flow situations which they interact at the interface, in this particular case this is phase 1; and this is phase 2. So f 1 should comprise of what f w1 has to be there, right and f 12 has to be there, yes or no? f 2 what should it comprise of? f w2 has to be there; and f 12 has to be there, is not it?

So accordingly, we had written down the equations and what did we get? So therefore, b 1 and b 2 I have already written down, b 1 is this; and b 2 is this. Now, if we can write down f 1 f 2 then we get a proper momentum balance equation from which we can proceed. Is this part clear to all of you or do you find it slightly difficult? This was the thing see.

There what initially what I did? I just took up a separated flow sort of a situation and I actually derived a momentum balance equation for the simplest case or the simplest case I had derived it and accordingly we had started to work. Now, as we go more and more complex the derivation from the first principle becomes slightly more elaborate. It is not that it is impossible to do it, but it becomes slightly more elaborate. So therefore, what we do? We just take up the basic equation the momentum balance equation for phase 1; basic momentum balance equation for phase 2. If you can substitute all the terms accurately you can simply add them up and you can get the momentum balance equation for the mixture clear to all of you. Now therefore, whenever we write it down the basic equations as I have written. Now depending upon the flow situation; depending on how to two phases interact with one another how the two phases interact with the pipe wall; depending on these particular things your f 1 and f 2 are going to change. Is it clear to all of you?

So, when f 1 and f 2 changes, so therefore, for the simplest case which I had taken under that particular situation we found out that this particular f 1 which is the left over forces for phase 1 it should comprise of the interaction of the phase 1 with the wall, is not it? and since the flow direction here this will be in this particular direction, and it will also comprise of f 12. Now since 2 moves faster than 1 therefore, f 12 will be in the opposite direction for phase 2 and in the direction of motion for phase 1.

That we had already done when we were doing the simplest possible case, is not it? And one more thing please remember, whenever I had written down these two equations I had repeatedly told you this is per unit volume of fluid 1; per unit volume of fluid 2. Now remember, when we take unit volume of the total mixture there we do not have unit volume of fluid 1 or unit volume of fluid 2, what we have, alpha amount of fluid 2 and 1 minus alpha amount of fluid 1.

So therefore, if it is expressed in terms of unit volume, if these things are expressed in terms of unit volume then there has to be accordingly modified using alpha and 1 minus alpha so that we can account for the total forces per unit volume of the flow or per unit volume of the mixture. Is this clear to all of you? So accordingly therefore, what did we get? f 1 it comprises of for the simplest case it comprised of f 12 minus f w1, is not it? Because we take the direction of flow as the positive direction.

Any doubts anywhere you get it clarified. What was f 2 equal to? This is minus of f 12 plus f w2, yes or no? So, this part if you see… So therefore, we find that it becomes f 1 f 1 becomes f 12 minus f w1, and f 2 becomes minus of minus of f 12 plus f w2 fine. So and if we have to consider it per unit volume this these two are this is per unit volume of fluid 1 this is per unit volume of fluid 2.

These things we had already discussed I believe in the last class. So therefore, in order to describe it as per unit volume of the total fluid mixture what we had defined? We had defined that capital F 12 all that capital F it was per unit volume of the mixture. So accordingly, if we do then in that case we found out then f 1 would be equal to your capital F 12 minus f w1 by 1 minus alpha, and f 2 is minus of F 12 plus f w2 by alpha, where these capital F they are per unit volume of mixture flow.

So therefore, your f 1 into 1 minus alpha becomes capital F 1, this is capital F 1 by 1 minus alpha; this is capital F 2 by alpha. So, simply we had just in order to account for the total forces per unit volume of the total mixture we had defined some additional quantities capital F, which I had already discussed when we were dealing with the drift flux model. This part these things I had already discussed when we were dealing with the drift flux model if you remember.

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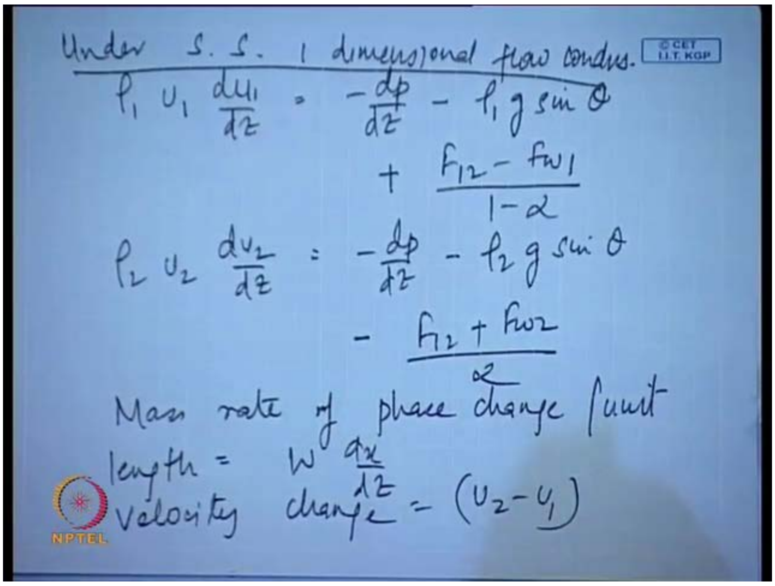

So therefore, there also I had written the two fluid model and there also I had discussed it. So therefore, accordingly F 1 and F 2 can written down in this particular form, and then if we substitute it then finally, what do we get? We get rho 1 U 1 du 1 dz; this is minus dp-dz one dimensional flow, so therefore, everything becomes in this particular form minus rho 1 g sin theta, plus F 12 minus F w1 by 1 minus alpha.

If you remember, this we had already derived when we were dealing with a drift flux model, is not it? And then I had said that wall shear stresses are negligible, so we have just F 12 and from there under inertia dominant conditions we found out that F 1 2 was a function of alpha j 2 1 etcetera, in the drift flux model we had done it, is not it? Please go through all these things otherwise it is going to be very, very difficult.

So, this is minus dp-dz minus rho 2 g sin theta, minus of F 12 plus F w2 by alpha, so these were the two things that we had got under steady state conditions, steady state one dimensional flow conditions. This was the thing that we had derived or this was the thing that we had obtained. Now you tell me when there is a change of phase what do you expect? What is going to change in these particular equations in order to account for the change of this? Same thing one dimensional flow you take up steady state conditions also, so that these cancel out and you tell me, what is the thing what is going to change when there is change of phase? Somehow we have to incorporate it within f 1 and f 2. Now, what now tell me when change of phase occurs then under that condition what happens? There is some portion we do not know what portion, there is some portion which changes from say the velocity of one phase to the velocity of other phase.

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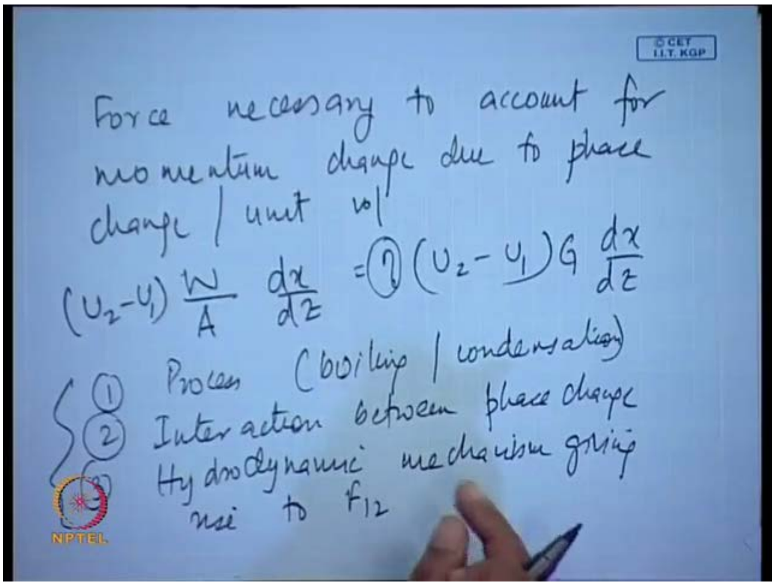

Now, suppose we assume that the mass rate of phase change per unit length, this is equal to say W dx-dz, yes or no you tell me? Is not it? Per unit length what is the mass rate of phase change? It is dx-dz per unit weight of the two phase mixture. So therefore, for the total two phase mixture it is going to be W dx-dz, is not it? And what is the velocity change for this? This is U 2 minus U 1, you agree with me? Now therefore, what is the total force which is necessary, the total force necessary to account for momentum change or yeah momentum change due to phase change per unit volume.

For per unit volume of say any of the phases what is the total force that is there? Total force must be the mass rate of phase change into the velocity change that will give you the force which is associated with phase change, do you get my point? So therefore, what is that? That is going to be nothing but per you see remember one thing, this mass rate of phase change per unit length was W dx-dz. So therefore, if it is per unit volume it becomes W by A dx-dz, clear to you?

So therefore, force necessary to account for momentum change due to phase change per unit volume this becomes W by A dx-dz into U 2 minus U 1, or in other words this is nothing but U 2 minus U 1 G dx-dz. So this is the total force which is necessary to account for momentum change due to phase change per unit volume. There is a total force which is associated. Now you tell me one thing how much of this force should I assign to phase one how much of this force should I assign to phase two?

This is the total force the total force was G dx-dz into U 2 minus U 1, the mass rate of phase change per unit volume of flow was G into dx-dz, and the velocity change was U 1 minus U 1. Now, this is the total force which is associated with phase change per unit volume. Now how much of this force should I give to phase 1, how much of the force should be ascribed to phase 2?

Why alpha how do you know that this force will be associated with the proportion of the area that they are occupying?

G is fine, definitely G is there agreed, dx-dz is also there, so this much amount of mass is going, but at this juncture you have no idea of how this force is shared by the two by the two phases. And importantly, this force how it will be shared it depends upon a large number of factors, whether it is boiling; whether it is condensation and so it depends upon a large number of factors. The factors on which the fraction of this force which will be shared by which will be taken by phase 1 which will be taken by phase 2, they depend upon number of things, one is what is the process? Is it boiling? Or is it condensation?

Next thing is how the two phases are interacting during phase change, interaction between phase change this also you have to know, and other thing is what is the hydrodynamic mechanism giving rise to F 12. So all these things unless you know you cannot find out what fraction of this force should be given to phase 1, phase 2. For example, say suppose there is an evaporating droplet for that particular case drag force will depend upon the rate of evaporation, is not it? Suppose the droplet is evaporation then on what will the drag force depend?

It will depend upon the rate of evaporation, is not it? So therefore, at this juncture we are not in a position to find out what fraction of this particular force should be given to phase 1, what fraction should be given to phase 2. So just to keep matters general what do we assume? We assume that eta amount of this force is given to phase 2, and 1 minus eta amount of this force is given to phase 1. This eta this is a variable this depends upon all these factors and therefore, how we are going to choose eta is going to differ for different systems. Is it clear to you?

So therefore, what do we get now for the time being? We get that whatever this force is there from that eta amount goes to phase 2, and 1 minus eta amount goes to phase 1 correct? This portion clear? So therefore, for phase change we this was steady state one dimensional flow, for phase change what do we have? We will have two one term here; and one term here, is not it? Those two terms we can find out it will be eta of this particular term.

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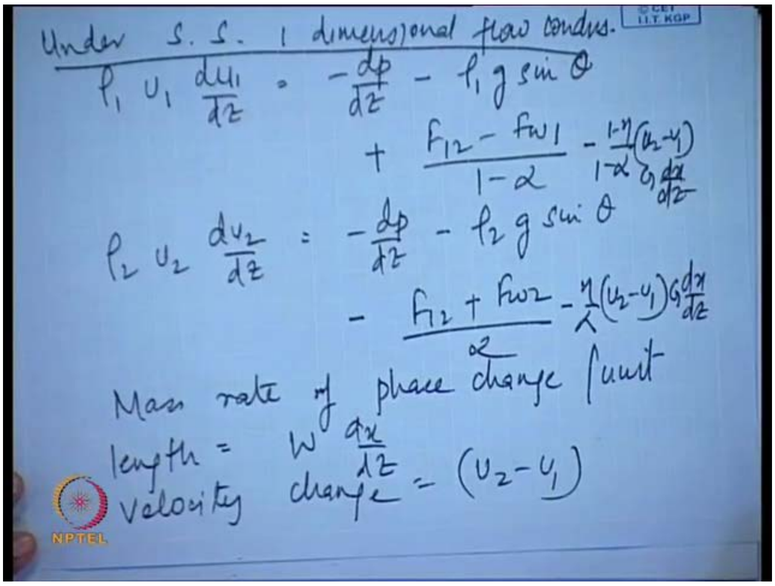

For phase 2 1 minus eta into this particular term for phase 1, do you agree with me? Therefore, for phase change what will we have? Apart from whatever we have this is 1, so here we will have minus 1 minus eta by 1 minus alpha U 2 minus U 1 G dx-dz yes, and here what we will have? Here we will have minus eta by alpha U 2 minus U 1 G dx-dz, say this portion is clear to you or not? So therefore, what I did for phase change? We have one additional term which has come from the f 1 and f 2 which we had defined in the original momentum equation.

So, whatever I had defined in the last class, in the last class when I have defined your when I had combined the two equations or in other words well I have it here I believe. Just a minute sorry. So, whatever equation I had here, apart from here we shall be having two additional terms in this particular case; one of the terms is going to be with this the things which I have written down, and in the other case it is going to be this yeah.

Why do we put a minus sign here? See this is again we do not really know what is going to happen whether it is boiling or whether it is condensation, so depending upon this situation we are going to put a minus sign, when do we put a minus sign? When it is opposite to the direction of motion. Here we really do not know what is happening, what sign will actually we put here that depends upon the rate of change of mass transfer, and exactly what is happening whether it is boiling whether it is condensation etcetera, it is going to depend on that.

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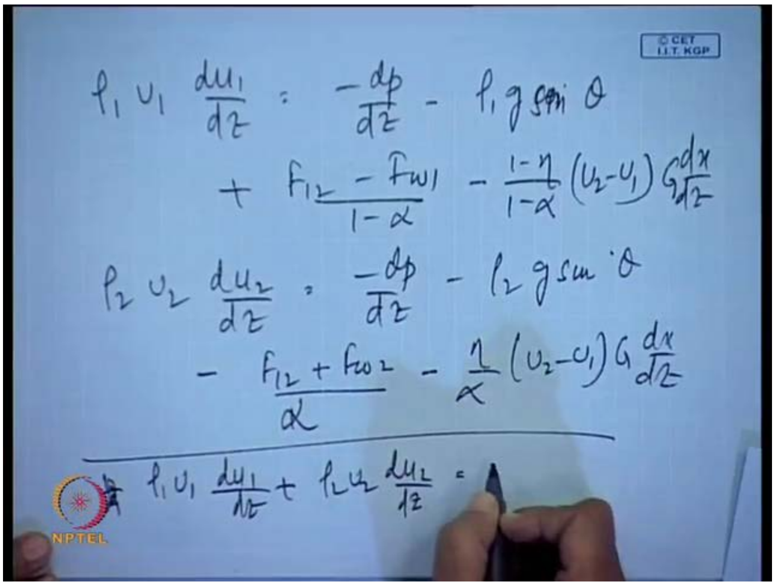

But the point is it usually it tends to offer a resistance. So therefore, just like resistive forces we have to do the minus sign this is the only reason there is nothing else. So therefore, we find that if we write it down properly then we find that the final equations which we have derived for this particular cases this is going to be rho 1 U 1 du 1 dz finally, if I write down the equation this is minus dp-dz minus rho 1 g sorry sin theta, plus F 12 minus F w1 by 1 minus alpha minus 1 minus eta by 1 minus alpha U 2 minus U 1 G dx-dz this is number one, and the number two is just the same equation just written down for phase 2.

So minus of course, the only difference is the sign change of F 12 and F 21, minus eta by alpha U 2 minus U 1 G dx-dz. Now just I had as I had discussed in the last class they can be combined in different ways, you multiply first one by alpha; the second one by 1 minus alpha, and then you add up the two you get the mixture momentum equation, and if you subtract one from the other your dp-dz goes off and you finally, get the equations which we had got.

So therefore, if you simply add them up what do we get? We get 1 minus a you just add them up and finally, the equation which we will be getting in this particular case, it is just the equation which we had got in the last class, it will be rho 1 U 1 du 1 dz, plus rho 2 U 2 du 2 dz this will give you… You have to multiply both sides by this by 1 minus alpha; this by alpha and then finally, which you if you complete it then you are going to get a equation something of this sort.

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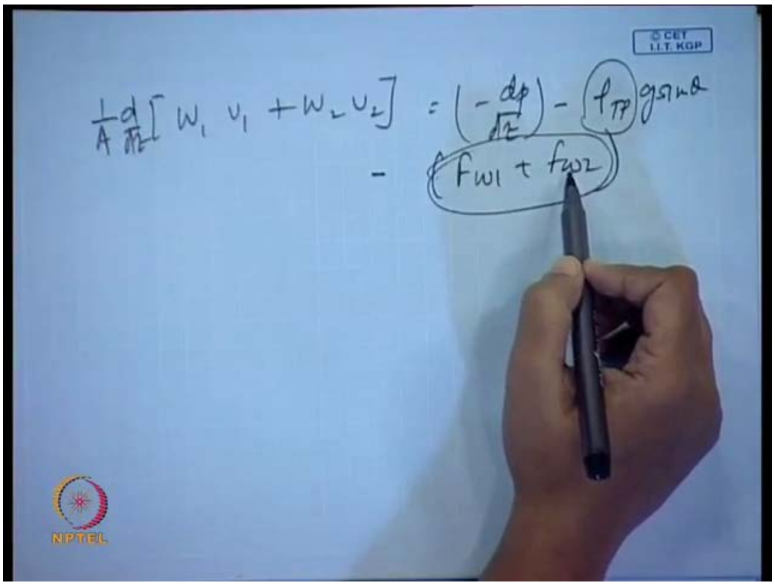

Let me write it down, you just try it out in your hostels and you will find it you get 1 by A d just the equation which I had got, W 1 U 1 plus W 2 U 2 equals to minus dp-dz minus rho TP g sin theta minus f w1 plus f w2, because what happens all the other terms they simply get cancelled out. And finally, we arrive at the equation which I had already derived here in the which I had already derived for the two phase flow under separated flow conditions clear. So therefore, we find that for change of phase how the momentum equations have changed those things we have observed, what are the different changes that we have observed, so and next and what happens when there is mass transfer, what happens when there is phase change those things we have already discussed.

In the next class what we will do? We will simply try to express the pressure gradient in terms of known input parameters, then we will go off to find out the condition of chocking, and then from this particular basic equation we find that in order to find this particular term or the frictional pressure gradient term we need to know how to express tau W 1, tau W 2 etcetera, how to find out alpha without alpha we cannot derive this particular term, so those things there are certain empirical approaches those things we will be proceeding after this. Thank you very much.