

Multiphase Flow
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Lecture No. # 16
Drift Flux Model (Contd.)

Well, good morning to all of you. So, today we have come almost to the final portion of the drift flux model, see in the last classes what we had derived is we had derived what is a drift flux model, and then the concept of drift flux and then how the drift flux can be incorporated into different expressions of the mixture properties like the mixture density the in situ velocities and so on and so forth.

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$$J_2 = J_{21} + \alpha j_{TP}$$

$$\langle J_2 \rangle = \langle J_{21} \rangle + \langle \alpha j_{TP} \rangle$$

For any property p

$$\langle p \rangle = \frac{\int_A p dA}{\int dA} = \frac{1}{A} \int_A p dA$$

\Rightarrow Area averaged value for property p averaged over the cross sectional area

Void fraction averaged mean value = $p_2 = \frac{\langle \alpha p_2 \rangle}{\langle \alpha \rangle}$

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And finally, we had derived the final expression of the drift flux model which gives us as J_2 , this is equal to J_{21} plus αj . This was for the final expression we had derived from which we had calculated α , and as accordingly that α was substitute in the different expressions. Now, the first thing which I would like to tell you in this class is when we had written down this particular expression, this particular expression this assumes that J_2 , J_{21} , they are constant over the entire cross sectional area. Is it not? This was the first thing that we had assumed that well we take an averaged value of these particular parameters averaged over the entire cross sectional area.

Now, usually we find that this can be a very drastic assumption for most of the flow situations for most of the flow situations we find that usually they keep on varying across the cross section. So, corrected expression for the in order to incorporate the average quantities averaged over the entire cross section the best thing will be to average it out. Now, whenever we write down averaged over any over the cross section we usually denote it by this particular symbol what we mean is that actually whenever we are writing this expression what we mean is that all these quantities are averaged over the entire cross section.

Where this particular quantity say J_2 or for any particular property p for any property p your this particulars, this squarish brackets they basically mean $\frac{\int p dA}{\int dA}$ by integral over dA integrated over the entire cross sectional area or in other word this is $\frac{1}{A} \int p dA$ integrated over a integral $p dA$.

So, therefore, you can very well understand that when I put this brackets across the individuals terms of these expressions what I mean is, this is the average volumetric flux of component to averaged over the entire cross section this is nothing but basically $\frac{\int J_2 dA}{A}$ this is the average drift flux which is averaged over the entire cross section and this is the product of volumetric concentration and volumetric flux which is averaged over the entire cross section.

Well just like to make a correction they are all referred to the two phase values now therefore, we find that all of them are the average quantities where as I have told you this particular bracket for any property this simply denotes the area averaged value averaged over the cross sectional area and the void fraction there are two terms one is since for single phase flow this is a sufficient definition the area averaged.

Now, value of each property or rather for any property this is nothing but the area averaged value for property p averaged over the entire cross section averaged over the cross sectional value, **sorry** averaged over the cross sectional area and when we are having two phases then we know that both the phases do not occupy the entire cross sectional area, see phase one occupies A_1 part of the area phase two occupies A_2 part of the area under that condition there is one additional definition which is void fraction averaged weighted mean value.

Or void fraction averaged mean value the void fraction weighted mean value means it is weighted against the void fraction this is for any particular property for phase I or k whatever it is void fraction weighted mean value of property p this is $\overline{p_k}$ this is nothing but αp say if it is p_2 then αp_2 by α .

If it is p_1 then it is $1 - \alpha p_2$ into by α . So, this is one particular term which is very important. So, if you observe this term you find that here αj_{Tp} will nothing will be equal to nothing but α into $\overline{j_{Tp}}$ isn't it something of this sort. So, what it is that we will be noticing or α into p_2 sorry αj_{Tp} yeah this is going to be in the other particular fashion.

So, therefore, we find that there are 2 particular definitions which we have to we have to keep in mind this we have already got in your single phase flows for any property p. The area averaged value when the area is averaged over the entire cross sectional area, this can be given by this particular form and the weighted mean value of the property that is weighted over the entire or the void fraction weighted mean value it is can be expressed in this particular form agreed.

So, accordingly we find that if this is true then accordingly the different mixture properties which we get they should be averaged over the entire cross sectional area. So, therefore, just like I have written it down for J_2 similarly for ρ_{Tp} also I can get the expression.

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$\langle f_{TP} \rangle = \langle \alpha \rangle p_2 + (1 - \langle \alpha \rangle) p_1$
 $p_1, p_2 = \text{constant because transverse pr. gradient within a channel is relatively small.}$

$\alpha = \langle \alpha \rangle$
 Axial component of weighted mean velocity of phase 2 $\overline{u}_2 = \frac{\langle \alpha u_2 \rangle}{\langle \alpha \rangle} = \frac{\langle J_2 \rangle}{\langle \alpha \rangle}$
 $\overline{u}_2 = \frac{\langle J_2 \rangle}{\langle \alpha \rangle} \neq \langle u_2 \rangle = \frac{\langle J_2 \rangle}{\alpha}$

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Say suppose, I get it for ρ_T what was it this was $\alpha \rho_2 + (1 - \alpha) \rho_1$ isn't it. Now, for this particular case it is the mixture density averaged over the entire cross section this will be $\alpha \rho_2 + (1 - \alpha) \rho_1$. Where we assume that ρ_1 and ρ_2 they are constant over they do not vary over the cross section and this particular assumption remember this is a valid assumption, because the transverse pressure gradient constant, because we find that the transverse pressure gradient within a channel is relatively small.

So, therefore, this is a reasonably this ρ_1 ρ_2 being constant is a reasonably valid assumption and under that condition the mixture density. The correct expression is in this particular form remember after we finish the drift flux model we will not be putting curly brackets everywhere just because it is very cumbersome, but we will imply that when we just say α it is the area averaged values it is implied that when after this topic is over when we write α we mean this.

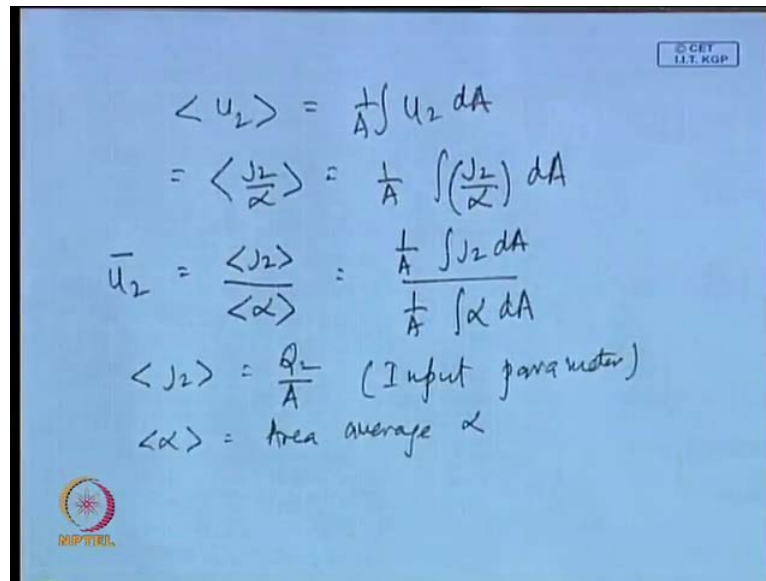
Unless otherwise stated because every time writing this down is difficult. So, when you are just expressing it α in this particular way it is the mean averaged values. In the same way we find say the Axial component of the weighted mean velocity suppose the u_2 Axial component of weighted mean velocity of phase two. So, this is going to be \bar{u}_2 if we write this is $\alpha \bar{u}_2$ divided by α agreed.

Now, we know what is $\alpha \bar{u}_2$ equal to $\alpha \bar{u}_2$ equals to this is the void fraction multiplied by the in situ velocity of component two. So, therefore, gives you J_2 so therefore, please remember that Axial component of the weighted mean velocity of phase two \bar{u}_2 is the average J_2 divided by the average α , please remember this is not equal to this \bar{u}_2 which is equal to this.

This is not equal to averaged u_2 this is the weighted mean this is the cross sectional average the void fraction weighted mean velocity this is the cross sectional averaged velocity, these two are not equal why this is equal to the cross sectional average of J_2 divided by the cross sectional average of α whereas, this means J_2 by α over which the cross section is taken. Are these concepts clear to you?

Please try to understand these concepts they are extremely important the thing is in this particular case, we get this is the weighted mean average velocity and this is the cross sectional average velocity.

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$$\begin{aligned}\langle u_2 \rangle &= \frac{1}{A} \int u_2 dA \\ &= \left\langle \frac{J_2}{\alpha} \right\rangle = \frac{1}{A} \int \left(\frac{J_2}{\alpha} \right) dA \\ \bar{u}_2 &= \frac{\langle J_2 \rangle}{\langle \alpha \rangle} = \frac{\frac{1}{A} \int J_2 dA}{\frac{1}{A} \int \alpha dA} \\ \langle J_2 \rangle &= \frac{Q_2}{A} \quad (\text{Input parameter}) \\ \langle \alpha \rangle &= \text{Area average } \alpha\end{aligned}$$

What does this mean this \bar{u}_2 it means this means integral $u_2 dA$ 1 by A or in other words, what it means if this is equal to J_2 by α this means $\frac{1}{A} \int J_2$ by α dA that means, for each particular point you measure J_2 you measure α you divide the two get the product then get the quotient. And then that quotient you integrate over the entire cross sectional area divided by the area that gives you the weighted mean average velocity, and when you talk of \bar{u}_2 that gives a cross sectional average velocity and when you talk about the weighted mean average velocity this means J_2 by α which is nothing but equal to $\frac{1}{A} \int J_2 dA$ by $\frac{1}{A} \int \alpha dA$ now, by comparing these two expressions. Is it clear why this is not equal to this?

This is the cross sectional average velocity of component two this is the weighted mean velocity of component two. So, they are grossly different for this what you have to do, you have to measure the local or the in situ J_2 at the every point, the local or the in situ α at every point, divide the two find the quotient then, that quotient you integrate in this particular case it is much more easier what you do you find the cross sectional average J_2 .

Finding out cross sectional average J_2 is very easy this is nothing but Q_2 by A . So therefore, this is an input parameter or this is a measurable quantity similarly this α it is just the average α . So, it is the area average α which you can very easily measure either by the conductivity probe technique or the radiation accumulation

technique and so on and so forth. So, therefore, this is a much more friendly quantity or a much more determinable quantity as compared to this. So, therefore, these concepts have to be very clear.

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$J_2 = J_{21} + \alpha J_{TP}$
 $\langle J_2 \rangle = \langle J_{21} \rangle + \langle \alpha J_{TP} \rangle$
 $\langle p \rangle$ For any property p
 $= \frac{\int_A p dA}{\int_A dA} = \frac{1}{A} \int_A p dA$
 Void fraction \Rightarrow Area averaged value
 property p averaged
 over the cross sectional area
 Weighted mean value
 of property p

Now, remember one thing certain concepts you have already learnt in single phase flows for example, this area averaged value this you have learnt in single phase flows is not it. In single phase flows very frequently you are you are suppose to find out the area average velocity for which you have given a velocity profile, you integrate the velocity profile over the entire area divided by the area, you get the your cross sectional averaged velocity or in other words you integrate the entire profile, you get the volumetric flow rate of that particular fluid over that particular area.

So, this area averaged value of this of this property the properties averaged over the entire cross sectional area you are already familiar with is not it, but the weighted main value the this is also a pretty I should say a pretty straight forward it is just the weighted main value which gives you the void fraction means, the total distribution divided by the void fraction or divided by the area over which it is distributed for p_2 it is αp_2 by α if it would have been p_1 it is $1 - \alpha p_1$ by α .

So, therefore, this particular property considering the effective area over which it is divided, the previous one was over the total area this is the effective area over which it is distributed or divided. So, that particular effective area is considered and from there we

get the void fraction or the weighted mean average property of any particular parameter. So, accordingly considering these particular things or these particular concepts what has been done the different mixture properties they can be defined.

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$$\langle \rho_{TP} \rangle = \langle \alpha \rangle \rho_2 + (1 - \langle \alpha \rangle) \rho_1$$

$$\rho_1, \rho_2 = \text{constant because transverse pr. gradient within a channel is relatively small.}$$

$$\alpha = \langle \alpha \rangle$$
 Axial component of weighted mean velocity of phase 2

$$\bar{u}_2 = \frac{\langle \alpha u_2 \rangle}{\langle \alpha \rangle} = \frac{\langle J_2 \rangle}{\langle \alpha \rangle}$$

$$\bar{u}_2 = \frac{\langle J_2 \rangle}{\langle \alpha \rangle} \neq \langle u_2 \rangle = \langle \frac{J_2}{\alpha} \rangle$$

For example, the rho Tp it can be defined the local in situ velocities can be defined.

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$$\bar{u}_1 = \frac{(1 - \langle \alpha \rangle) u_1}{\langle (1 - \alpha) \rangle}$$

$$\bar{u}_{TP} = \frac{\langle \rho_{TP} u_{TP} \rangle}{\langle \rho_{TP} \rangle} = \frac{\langle \alpha \rangle \rho_2 \bar{u}_2 + (1 - \langle \alpha \rangle) \rho_1 \bar{u}_1}{\langle \rho_{TP} \rangle}$$

$$\langle J_{TP} \rangle = \langle J_1 \rangle + \langle J_2 \rangle = \langle \alpha \rangle u_2 + (1 - \langle \alpha \rangle) u_1$$

Similarly, you can define a u_1 well in this particular case u_1 would have been given as it is nothing but $1 - \alpha u_1$ or rather it is better written as $\bar{u}_1 = \frac{1 - \alpha u_1}{1 - \alpha}$. So, at each point you find it out and then you do it

similarly suppose, we would like to define u_{TP} the two phase mixture velocity it is nothing but $\rho_1 u_1 + \rho_2 u_2$ divided by ρ_{TP} . So, therefore, this is given as $\alpha u_2 + (1 - \alpha) u_1$. This is the thing and in the same way if you find out the volumetric flux the volumetric flux say j_{TP} this is $J_1 + J_2$ this is nothing but $\alpha u_2 + (1 - \alpha) u_1$. So, in this particular way we can continue we can define all the different properties. Now, let us see how easily we can introduce some particular correction to the three dimensional drift flux model in order to arrive at the one dimensional flow situation.

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$$J_2 = J_{21} + \alpha j_{TP}$$

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For any property p

$$\langle p \rangle = \frac{\int_A p dA}{\int_A dA} = \frac{1}{A} \int_A p dA$$

\Rightarrow Area averaged value
for property p averaged
over the cross sectional area

Void fraction for property p

$$\overline{p_2} = \frac{\langle \alpha p_2 \rangle}{\langle \alpha \rangle}$$

Weighted mean value of property p

So, the basic definition which I had put it in this particular case is that this was the actual drift flux model which we can define is not it. From this particular model or from this particular concept now, the very rational approach to obtain the one dimensional drift flux model will be to integrate the three dimensional drift flux model over a cross sectional area, and then we can introduce proper mean values. We can integrate this by using this particular formulae over the entire flow field and over the entire cross sectional area and then we can introduce proper mean values.

Now, remember one thing whenever we have to deal with this particular equation finding out J_2 is very easy it is nothing but Q_2 by A as I have already written down here this particular J_2 this is nothing but Q_2 by A J_{21} it can be found out by it is just the average drift flux averaged over the entire area. So, this also can be found out easily now for

finding out alpha into jTp what you have to do for each particular point you have to measure alpha at each particular point you have to measure jTp you have to multiply the 2 and then you have to integrate the entire functional form. Now, this is not very easy. So, a simplified approach is that if we can replace this by some other easily measurable or input parameters. Now, how to do this now for this particular thing now, certain things that I have told you this needs to be replaced.

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$$\langle J_z \rangle = \langle J_{z1} \rangle + \langle \alpha J_{TP} \rangle$$

$$\langle \alpha J_{TP} \rangle \neq \langle \alpha \rangle \langle J_{TP} \rangle$$

" $\frac{Q_1 + Q_2}{A}$

Momentum flux in a pipe with velocity profile and uniform density

$$\rho \int u^2 dA = A \rho \langle u^2 \rangle$$

$$A \rho \langle u^2 \rangle \neq A \rho \langle u \rangle^2$$

$\frac{A \rho \langle u^2 \rangle}{A \rho \langle u \rangle^2} =$ correction factor

$\langle u \rangle \rightarrow$ = 1 for truly 1d case
NOT far from unity for general case

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Now, if I write down the equation once more J21 plus alpha jTp now, this term we know that definitely alpha jTp this is not equal to this term isn't it and this is a measurable parameter where this alpha jTp is nothing but Q1 by plus Q2 by A and alpha can also be measured. So, instead of this if we can replace it by this term along with some other corrections then probably my equation becomes a little more friendly, is it not? So, if that can be done then it becomes slightly little more easier. Now, how to do this now if you remember in single phase flow see single phase flow whatever wherever we get stuck we refer resort to single phase flows what has been done for single phase flows accordingly we would like to do it here.

Now, in single phase flows if you remember that when we had considered your the momentum flux and say suppose when we considered the momentum. If you remember the momentum correction factor and the kinetic energy correction factors alpha beta in single phase flows there what we did just to keep the matters simple, what we did was

momentum flux say in a pipe with velocity profile and uniform density. So, this is given as $\int u^2 dA$ where this is nothing but $A \rho u^2$ is not it now, we knew that this $A \rho u^2$ this is not equal to $A \rho \bar{u}^2$ it and, but this is a much more easily measurable parameter.

So, why because you just measure the velocity profile and then you integrate it find out the profile and then you square it, but for this particular case you have to measure the velocity at each and every point you have to square it then, you have to find out the profile then you have to integrate it. So, in single phase flows what we did we just took the ratio of these two and expressed it as a correction factor if you remember that this was the exact thing which we were done in Bernoulli's equation.

If you remember Bernoulli's equation the basic equation it is just $\frac{u^2}{2} + \rho g z + \Delta p$ by ρ plus $\frac{\Delta u^2}{2}$ plus and then we introduced a correction factor α there is it not? The α was the kinetic energy correction factor why did we introduce it, we introduced it just for this particular reason just for because if we introduce a correction factor then we can take up the average velocity and we can square it and we can use it.

But otherwise it is actually the velocity profile for each particular point the velocity has been to be measured it has to be squared. Accordingly, the profile changes that profile has to be determine that has to be integrated and then we can introduce it remember this. So, therefore, in single phase flows what we had done for kinetic energy even for momentum flux when the density is constant, and there is a velocity profile what we did how to express those things we had introduced a suitable correction factor.

What was the correction factor there for simplicity a correction which was nothing but the ratio of this divided by this particular ratio this was introduced as a correction term keeping in mind that this particular correction factor will be equal to unity for truly one dimensional case and not far from unity for the general case. So, it has to be $\frac{\int u^2 dA}{A \bar{u}^2}$ alright.

So, therefore, usually what is the concept of a single phase flows? In single phase flows we find that more or less the ratio of these they are not very far from unity and for truly one dimensional case they become unity, because for truly one dimensional case. What is the case the velocity is the same for all points in the cross section, so therefore, this

correction factor which is equal to 1 for truly one dimensional case and not far from unity for the general case. So, therefore, in these particular way the correction terms were introduced. Similarly, we the similar approach was taken for this particular term here this αj_{TP} instead of αj_{TP} if we can replace it as α average j_{TP} averaged into a correction factor then, situation becomes simpler provided the correction factor equals unity for truly one dimensional case it is not far removed from unity for the general case.

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$$\frac{\langle \alpha j_{TP} \rangle}{\langle \alpha \rangle \langle j_{TP} \rangle} = \text{Correction factor}$$

$$\langle j_2 \rangle = \langle j_{21} \rangle + \langle \alpha \rangle \langle j_{TP} \rangle \text{ Correction factor}$$

$\frac{Q_2}{A}$ $\frac{Q_1 + Q_2}{A}$

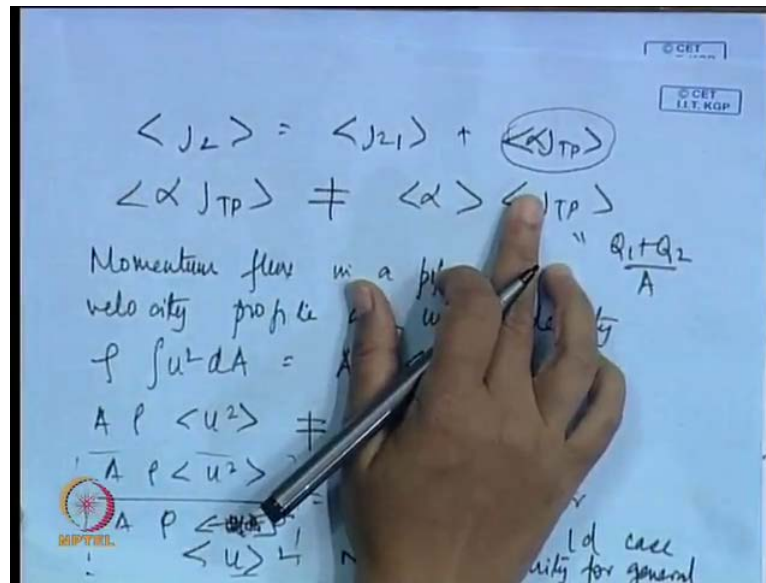
Distribution parameter = C_0

$$= \frac{\langle \alpha j_{TP} \rangle}{\langle \alpha \rangle \langle j_{TP} \rangle} = \frac{\frac{1}{A} \int (\alpha y) dA}{\left[\frac{1}{A} \int \alpha dA \right] \left[\frac{1}{A} \int j dA \right]}$$

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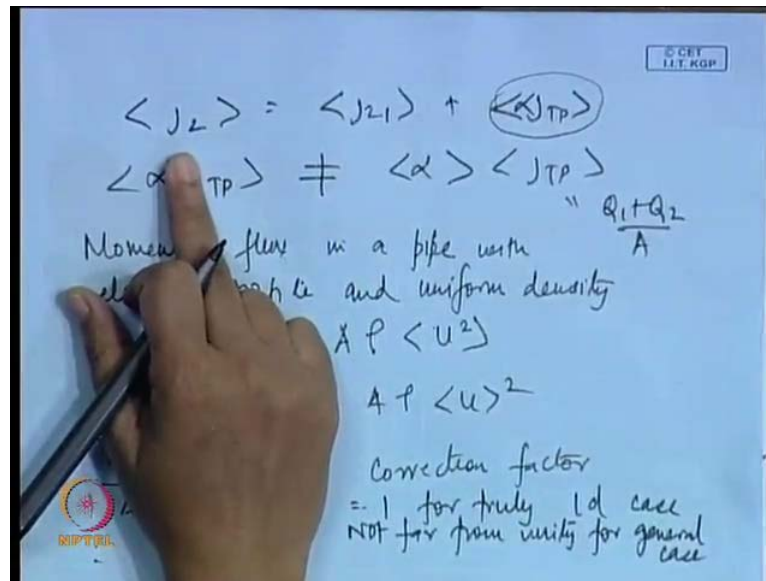
So, therefore, keep it just by using this particular concept it was thought that in the same way suppose we can refer this as the ratio this particular. This can also be introduce as a correction factor and then using the correction factor we can write down the equation as J_2 equals to J_{21} plus say αj_{TP} into the correction factor, this factor will be equal to unity for the general case for the truly one dimensional case not much removed from unity for the general case, and then we find that in this particular equation this can be obtained from Q_2 by A . A measurable parameter this is Q_1 plus Q_2 by A again a measurable parameter α can be obtained from your simply void fraction measurements and j_{21} from the drift flux correlations, we had already discussed the different drift flux correlations they are all for truly one dimensional cases.

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So, therefore, this becomes much more easier now, this correction factor was first it was proposed by your Zuber and Findlay this is usually known as a distribution parameter. This is defined as C_0 which can be defined as the C_0 can be defined as αJ_{TP} by this is the definition or in other words if you break it down this is 1 by A integral $\alpha j dA$ divided by 1 by A integral αdA into 1 by A integral $j dA$ see, if this particular till this portion it is clear to you or not, is it clear to you? Just to make corrections to the simple one dimensional theory what we did we first wrote down the actual equation in the form of average quantities. Now all these quantities there area average quantities, where this area average quantities can be defined in this particular form.

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And next after this what we did once this could be defined then we found that more or less this can be obtained from a easily measurable parameter this is also the area averaged value this creates a problem. Now, this is a product of two particular quantities which vary which might vary across the cross section. So, therefore, what has to be done the volumetric concentration has to be measured on each and every point your the overall volumetric flux has to be measured at each and every point then the product has to be made and then it has to be integrated.

For engineering applications this is something very difficult we would always like to keep matters simple while not sacrificing much on the accuracy of it. We would not like to go into something very complex do a lot of mathematics waste a lot of time and then get a very accurate results where a little less little loss on accuracy would not have been much loss to us, if the matters could have been made simple and we are doing it in several engineering applications.

In single phase flow also we have done it for the momentum correction factor kinetic energy correction factor etcetera etcetera. We had introduced a correction factor keeping in mind see whenever you introduce a correction factor something has to be kept in mind that for it reduces to some particular boundary conditions for example, the correction factor equals to 1 for truly one dimensional case and for other cases it is not very far suppose the correction factor becomes 5, 7, 10, 11 then it is not worth.

If it is not very far removed from unity then these particular concepts it works very well; that means, the correction is not very much is not it. It is a very small correction you can get this small correction by a larger number of mathematics or you can you can introduce some particular constant where that particular constant has to be evaluated accurately. So, these are the general concepts which we use even for single phase fluid flow. So, based on these concepts what we decided was since finding out these particular term becomes difficult.

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$$\frac{\langle \alpha J_{TP} \rangle}{\langle \alpha \rangle \langle J_{TP} \rangle} = \text{Correction factor}$$

$$\langle J_2 \rangle = \langle J_{21} \rangle + \langle \alpha \rangle \langle J_{TP} \rangle \times \text{Correction factor}$$

$$\left[\frac{Q_2}{A} \right] = \left[\frac{Q_{21}}{A} \right] + C_0 \langle \alpha \rangle \left[\frac{Q_{TP}}{A} \right]$$

Distribution parameter = C_0

$$= \frac{\langle \alpha J_{TP} \rangle}{\langle \alpha \rangle \langle J_{TP} \rangle} = \frac{\frac{1}{A} \int (\alpha_j) dA}{\left[\frac{1}{A} \int \alpha dA \right] \left[\frac{1}{A} \int J dA \right]}$$

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So, if we can break it down into two terms and then introduce a correction factor which is nothing but the ratio of the two then we can get this expression in a much more simpler much more user friendly form. And this particular correction term this is defined as a distribution parameter this or a Zuber and Findlay parameter. Since, they were the first researchers to propose this particular concept for 2 phase flow and accordingly this distribution parameter has been defined and this makes the above equation the form of the equation now becomes something of this sort, so this becomes the form of the equation now it becomes something of these sort.

So, therefore, in these particular term we find that if we can find out C_0 and if can find out j_{21} then more or less this equation can be used for finding out alpha or for all practical purposes. This equation can be used once you can find out alpha from this equation this alpha can be substituted in $\rho_{TP} u_2, u_1$ Kinetic energy terms the

Momentum flux terms etcetera etcetera and all the terms can be deduced. Accordingly and these terms when they are when they are substituting the pressure drop expression that we have obtained from the homogeneous flow model we can get a much more accurate estimation of pressure drop as well.

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Handwritten mathematical derivations on a blue background:

$$\langle J_z \rangle = \langle J_{z1} \rangle + C_0 \langle \alpha \rangle \langle J_{TP} \rangle$$

$$\frac{\langle J_z \rangle}{\langle \alpha \rangle} = \frac{\langle J_{z1} \rangle}{\langle \alpha \rangle} + C_0 \langle J_{TP} \rangle$$

$$\frac{\langle \alpha u_z \rangle}{\langle \alpha \rangle} = \bar{u}_z = \frac{\langle J_{z1} \rangle}{\langle \alpha \rangle} + C_0 \langle J_{TP} \rangle$$

$$\bar{u}_z \neq \langle u_z \rangle \quad \frac{Q_z}{A \langle \alpha \rangle} = C_0 \frac{Q_1 + Q_2}{A} + \frac{\langle J_{z1} \rangle}{\langle \alpha \rangle}$$

$$\bar{u}_z = \frac{\langle J_z \rangle}{\langle \alpha \rangle} = \bar{u}_{z1}$$

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So, therefore, the next discussion goes on how to estimate this C_0 once we have certain ideas about estimating C_0 it becomes much more easy. Now, before that I would like to write these particular expression is usually written in a slightly different form. What is the form the original expression is something of this sort usually dividing throughout by alpha what do we get something of this sort.

Where we know the this particular term what is this? This is nothing but alpha u_2 by alpha isn't it which is nothing but u_2 bar the weighted mean velocity as I have already defined and so, therefore, u_2 bar equals to j_{z1} by alpha plus C_0 . So, therefore, this can be defined and we know that this is a much more convenient term as compared to u_2 bar this is a much more convenient term because this can directly be related to input parameters like the overall volumetric flow rate and the mean volumetric concentration and so, therefore, it is a much more means input parameter as compared to this and so, therefore, we define this particular equation by considering the weighted mean average velocity.

This is given as u_2 is nothing but Q_2 by A or in other words in terms of measurable parameters if you write this is nothing but Q_2 by A alpha this is equal to $C_0 Q_1$ plus Q_2 by A plus J_{21} by alpha. In this particular way we can write it down or in other words we can write it as \bar{u}_2 equals to j_2 by alpha equals to \bar{u}_{2j} this can again be written down as \bar{u}_{2j} is not it, because this is alpha into \bar{u}_{2j} if you remember the basic definition of J_{21} what was the basic definition of J_{21} alpha into \bar{u}_2 minus J which is nothing but alpha into \bar{u}_{2j} fine.

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$$\frac{\langle J_{21} \rangle}{\langle \alpha \rangle} = \frac{\langle \alpha u_{2j} \rangle}{\langle \alpha \rangle} = \bar{u}_{2j}$$

$$\bar{u}_2 = \frac{\langle J_{21} \rangle}{\langle \alpha \rangle} = \bar{u}_{2j} + C_0 \langle J \rangle$$

$$\langle \alpha \rangle = \frac{Q_2 - A \langle J_{21} \rangle}{C_0 (Q_1 + Q_2)}$$

- ① Dependence of J_{21} on α
- ② Variation of α across cross section

$$\langle J_{21} \rangle = \frac{1}{A} \int J_{21} dA$$

So, therefore, if you write down J_{21} by alpha this is nothing but alpha \bar{u}_{2j} by alpha it is nothing but \bar{u}_{2j} bar isn't it. So, therefore, the previous equations which we were writing this particular equation or this particular equation we can also write it down as \bar{u}_2 bar is nothing, but it is \bar{u}_{2j} bar plus $C_0 j$. So, in this particular way also we can write down the equation in terms of two weighted mean average values. The weighted mean average value of the velocity of phase two and the weighted mean average value of the drift velocity. So, from this particular expression we can get the expression of alpha this is equal to Q_2 minus $A J_{21}$ divided by C_0 into Q_1 plus Q_2 . So, therefore, if Q_2 and Q_1 they are measured A is known if J_{21} is known and C_0 is known alpha can be very well determined till this part. Is it clear to all of you?

So, for finding out J_{21} what do you need for finding out J_{21} you need two things one is dependence of J_{21} on alpha and the two thing is variation of alpha across cross section if

you know these two things then in that case because this J_{21} this is nothing but 1 by A j_{21} dA. So, therefore, you have to find out two things one is how j_2 depends upon α . And how α varies across the cross section if these two data are known therefore, your J_{21} can be easily found out and one more thing that we also know is suppose say J_{21} is much smaller as compared to Q_2 . If j_{21} is much smaller then what happens to this particular expression.

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$$\frac{\langle J_{21} \rangle}{\langle \alpha \rangle} = \frac{\langle \alpha u_{21} \rangle}{\langle \alpha \rangle} = \bar{u}_{21}$$

$$\bar{u}_{21} = \frac{\langle J_{21} \rangle}{\langle \alpha \rangle} = \bar{u}_{21} + C_0 \langle J \rangle$$

$$\langle \alpha \rangle = \frac{Q_2 - A \langle J_{21} \rangle}{(Q + Q_2)}$$

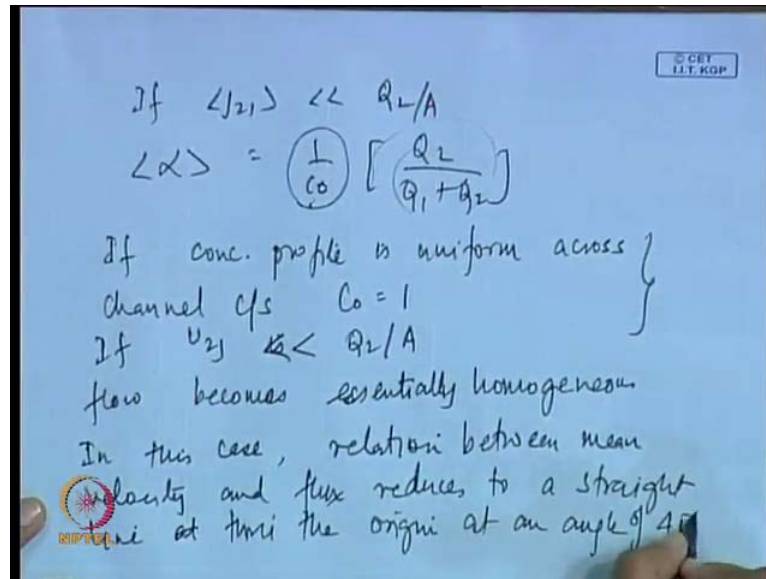
(1) Dependence of J_{21} on α
variation of α across the section

If your j_{21} much less than Q_2 or Q_2 by A then in that case your α becomes 1 by C_0 Q_2 by Q_1 plus Q_2 or in other words if the drift velocity or drift flux is very small that means, the two phases are moving at almost equal velocity under that circumstances we just need a distribution parameter in order to account for the non uniform velocity and void age profiles. And therefore, under that circumstances simply the homogeneous value has to be modified by the term one by C_0 in order to account for the non uniform velocity and the void age profiles and accordingly the this particular correction it is found out, but under normal circumstances we find that generally we have this term J_{21} this is usually not very small we have to account for it.

And when we find that your j_{21} is also very small compared to Q_2 by A and the more or less the velocity and void age profiles are flat across the cross section under that circumstance what happens a circumstance where j_{21} is much smaller as compared to Q_2 by A that means, both the phases are moving at a more or less uniform velocity. And the

your C_0 is near to unity that means, the velocity and the void age profiles are more or less uniform that means, more or less it is a flat velocity profile and the dispersed phase is more or less uniformly dispersed in the channel or in the conduit geometry under that conditions C_0 equals to 1 if u_{2j} is **much** less than Q_2 by A for that for these two conditions what happens.

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The expression of alpha reduces to you can observe the expression of alpha where this is very small this term goes away and this is one term. So, therefore, it becomes the homogeneous flow value is not it. So, therefore, you please understand what does the drift flux model do it accounts for the relative motion of the two phases it accounts for the non uniform velocity and the void age profile. So, this is the thing which I would like you to know that for if concentration profile is uniform across channel cross section C_0 equals to 1 if you u_{2j} is much less than Q_2 by A then for these two cases we get flow becomes essentially homogeneous.

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$$\frac{\langle J_{21} \rangle}{\langle \alpha \rangle} = \frac{\langle \alpha u_{21} \rangle}{\langle \alpha \rangle} = \bar{u}_{21}$$

$$\bar{u}_{21} = \frac{\langle J_{21} \rangle}{\langle \alpha \rangle} = \bar{u}_{21} + C_0 \langle J \rangle$$

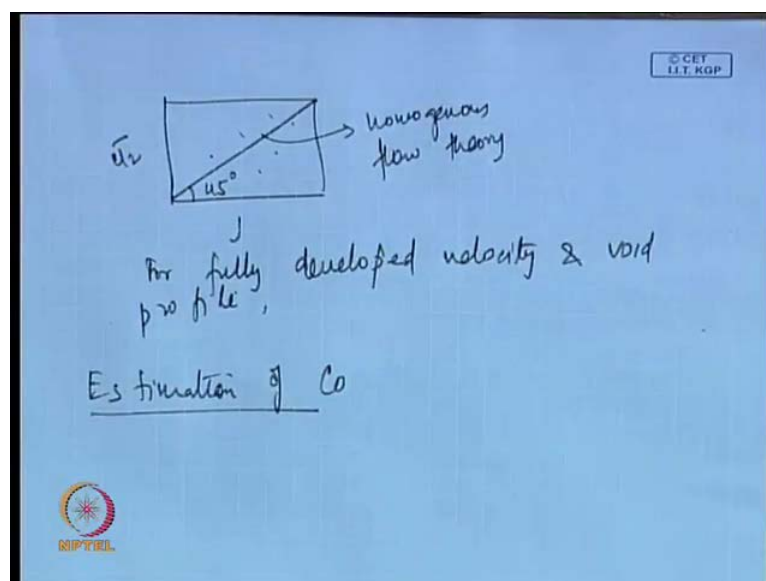
$$\langle \alpha \rangle = \frac{Q_2 - A \langle J_{21} \rangle}{C_0 (Q + Q_2)}$$

{ ① Dependence of J_{21} on α
 { ② Variation of α across C/c

$$\langle J_{21} \rangle = \frac{1}{A} \int (J_{21}) dA$$

And then what happens in this particular case the relation between your when this happens under that particular circumstance what happens if we go a little further, we find that this has become equal to unity this term has gone off then the relation between this and this it gives you a straight line passing through the origin at an angle of 45 degrees to the x axis or the y axis. So therefore, under that circumstance what happens relation in this case relation between mean velocity and flux reduces to a straight line through the origin at an angle of 45 degrees or in other words for such a circumstance.

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If we plot your u_2 bar with j we get a straight line through the origin isn't it we get such a straight line. And the thing is what we usually do we and this gives you the homogeneous flow theory now, for this particular case what we get is that usually what we do whenever we get in a experimental data we try to plot it on this particular curve with the 45 degrees line on it and then we see this spread of this data from this spread of the data, we try to understand how much it has deviated from the homogeneous flow theory accordingly we introduce corrections.


Do you understand usually whenever we have any particular experimental data what we do we usually with when it is for fully developed velocity and void profile for fully developed velocity and void profile, what we try to do is we make such a plot and then in that particular plot we substitute the data around this 45 degrees line and then we find out that usually data point will be clustering around this particular line. The amount of spread it has from this line that particular spread tells us how much it has deviated from the drift flux theory and whether we should go or how much it has deviated from the homogeneous flow theory and whether we should go for some correction or not and in what way should we introduce the correction whether C_0 is important or whether j_2 is important usually we find that j_2 is important and some particular j_2 has to be incorporated there. Now, the next thing which I would like to discuss is the estimation of C_0 how we can estimate C_0 or how the C_0 can be obtained for different particular flow situations.

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It may be noted that

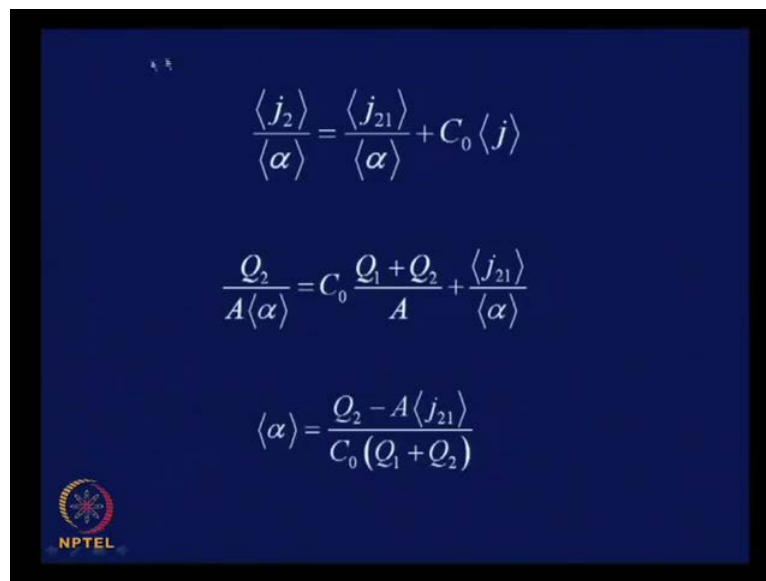
$$\langle \alpha j \rangle \neq \langle \alpha \rangle \langle j \rangle$$

Since

$$\langle \alpha \rangle \langle j \rangle = \left[\frac{\int \alpha dA}{\int dA} \right] \left[\frac{\int j dA}{\int dA} \right]$$


Now, if you see here I have already written down in this ppt. if you observe then you find that in the ppt I have written down the basic equations if you observe this ppt then you find that in the particular ppt I have written down the basic equations you can always go through them and you can find out that more or less the basic things are written here. So, C_0 is the ratio of the average of product of flux and concentration to the product of the averages.

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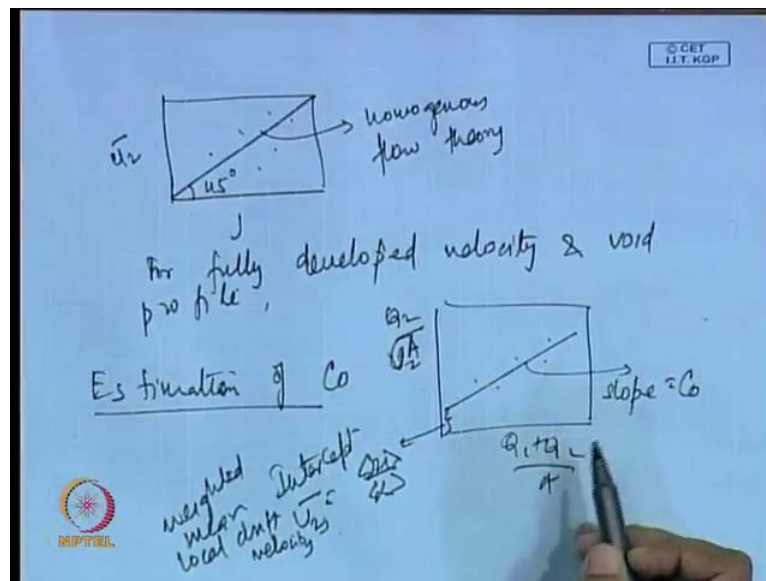
$$\frac{\langle j_2 \rangle}{\langle \alpha \rangle} = \frac{\langle j_{21} \rangle}{\langle \alpha \rangle} + C_0 \langle j \rangle$$

$$\frac{Q_2}{A \langle \alpha \rangle} = C_0 \frac{Q_1 + Q_2}{A} + \frac{\langle j_{21} \rangle}{\langle \alpha \rangle}$$

$$\langle \alpha \rangle = \frac{Q_2 - A \langle j_{21} \rangle}{C_0 (Q_1 + Q_2)}$$

This is the basic definition of C_0 and then using this particular definition we come across this particular friendly equation based on input parameters from where alpha can be obtained by using this expression. Here we find if j_{21} and C_0 is close to unity for fully may be when the velocity and the concentration profiles are flat and when j_{21} is much less compared to Q_2 by A then, alpha reduces to the homogeneous flow value. Now, the usual trend to find how much your actual situation deviates from the homogeneous flow value. We usually plot this particular term with this particular term and then the deviation of the data from the 45 degrees line passing through the origin gives us an idea about the magnitude of C_0 and j_{21} by alpha.

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So, next is estimation of C_0 now, what are the different ways by which you can estimate C_0 now, there are two ways the first thing is you construct such a type of plot now, if you construct such a type of plot then in that particular plot usually what we find is that it usually. Since, I have told you C_0 is much not much removed from unity is not it. So, we find that if we make such a plot or in other words if we make a plot of say Q_2 by Q_1 plus Q_2 by A then usually, we get a straight line the best fit curve is usually a straight line and can you tell me the slope of this straight line what it gives you and the intercept of this straight line what it gives you. You can see the basic equation and you can tell me what we get from the slope and the intercept of this particular straight line.

Slope gives you C_0 and this gives you j_{21} by alpha isn't it. So, therefore, very simply usually we find that for fully developed velocity and void age profiles, please remember this for fully developed velocity and void age profiles usually we find that when we do such a plot it gives a best fit curve is a straight line. When the local drift velocity is constant or it is negligibly small in that particular case your slope it gives you the slope of this best fit line this gives you the value of the distribution parameters C_0 and the intercept gives you the value of the weighted mean local drift velocity, this gives you the weight or in other word this gives you u_{2j} which is nothing but the weighted mean local drift velocity.

So, this is one particular way by which we can find it out and we have done it several times what we do whenever we have to analyze experimental data first, we refer to such

a plot we find how much is the deviation. When we find the deviation is quiet large then we resort to this particular plot from where we can find out C_0 we can find out u_{2j} bar and once we find these out we can find out all the other qualities all the other parameters and, but this is usually done for fully developed velocity and void age profiles when it is varying with time definitely for temporal variations we cannot resort to this, but apart from this we find that there are certain other ways also by which we can find out C_0 what are the other ways, if we go to the basic definition of C_0 what was the basic definition.

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① $\frac{Q_{12}}{A} = \frac{\langle u_1 \rangle}{\langle \alpha \rangle} + C_0 \frac{Q_{12}}{A}$

② $C_0 = \frac{\langle \alpha \rangle}{\langle \alpha \rangle \langle J \rangle} = \frac{\frac{1}{A} \int (\alpha y) dA}{\left[\frac{1}{A} \int \alpha dA \right] \left[\frac{1}{A} \int J dA \right]}$

Power law profiles

$\frac{J}{J_0} = 1 - \left(\frac{r}{R}\right)^m$

$\alpha - \alpha_0 = 1 - \left(\frac{r}{R}\right)^n$ at the center

$\alpha - \alpha_w = 1 - \left(\frac{r}{R}\right)^n$ at the wall

$C_0 = 1 + \frac{2}{m+n+2} \left[\frac{m}{1-\frac{m}{n}} \right]$

So, the ways of finding out C_{12} is from this particular equation which is Q2 it gives you so, this particular equation and we plot this versus this then we can get C_0 the other thing is what is the basic definition. Basic definition is αJ by αJ or in other words if we go to the actual definition this is this is the other definition. So, what we can do from the velocity profile and the concentration profile also we can find out C_0 this can be the other way is not it. Usually, what we assume we assume parabolic power law profiles usually the type of profiles, which we assume are power law profiles which give you J by J_0 equals to $1 - \frac{r}{R}$ whole to the power m $\alpha - \alpha_0$ by $\alpha_0 - \alpha_w$ equal to $1 - \frac{r}{R}$ whole to the power n .

Usually we have such power law profiles even in single phase flow also you have found out that usually we deal with the power law type of profiles. So, when we have such type

of profiles where you remember that alpha is the average volumetric concentration at any particular radial location r alpha 0 is at the center and alpha w. It is at the wall it is usually equal to 0 same thing j0 is at the center and this is the radius and this is the radial distance. So, for such a situation we find that C0 it can be given as 1 plus 2 by m plus n plus 2 into 1 minus your alpha by alpha. So, this I have already written down in the ppt.

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Estimation of C_0

Assuming power law profiles for α and j

$$C_0 = 1 + \frac{2}{m + n + 2} \left[1 - \frac{\alpha_w}{\langle \alpha \rangle} \right]$$

For fully developed bubbly flow (Ishii)

$$C_0 = C_0 \left(\frac{\rho_2}{\rho_1}, \frac{GD}{\mu_1} \right)$$

NPTEL

If you observe the ppt then you find that this particular C0. It has been derived for power law profiles of alpha and J several researchers have walked on different aspects of your drift flux model, and how to find out C0 and it has been found several researchers have proposed different particular values of C0. Some people have said that C0 it is a function of density ratio as well as the Reynold's number.

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For flow in a round tube


$$C_o = 1.2 - 0.2 \sqrt{\rho_g / \rho_l}$$

For flow in a rectangular channel

$$C_o = 1.35 - 0.35 \sqrt{\rho_g / \rho_l}$$

For developing void profile ($0 < \alpha < 0.25$)

$$C_o = (1.2 - 0.2 \sqrt{\rho_g - \rho_l}) (1 - e^{-18\alpha}) \quad \text{round tube}$$
$$C_o = (1.35 - 0.35 \sqrt{\rho_g - \rho_l}) (1 - e^{-18\alpha}) \quad \text{rectangular channel.}$$

 NPTEL

And accordingly, several different definitions of C_0 have been proposed which you can use for as the case may be they are imperial co-relations which have been derived for a large number of data for example, for fully developed flow in a round tube please remember C_0 has been expressed as a function of ρ_g by ρ_l again for fully developed flow in a rectangular channel. It has been developed in this particular way when it is not fully developed then in that case the expressions are given here and accordingly I have written down a large number of expressions from where your C_0 can be obtained under different conditions, but please remember these were all for more or less parabolic type of void age profiles where we assume that the void age is much more higher at the center as compared to the walls. But if the situations change say by some example may be say there is a heat flux and there is intense vaporization at the walls. So, under that condition the profile instead of becoming this it becomes this particular shape from concave it becomes convex.

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For boiling bubbly flow in an internally heated annulus


$$C_o = \left[1.2 - 0.2 \sqrt{\frac{\rho_g}{\rho_l}} \right] \left[1 - e^{3.12 < \alpha > 0.212} \right]$$

In downward two-phase flow for all flow regimes

$$C_o = (-0.0214 < j^* > + 0.772) + (0.0214 < j^* > + 0.228) \sqrt{\frac{\rho_g}{\rho_l}} \text{ for } (-20) \leq < j^* > < 0$$

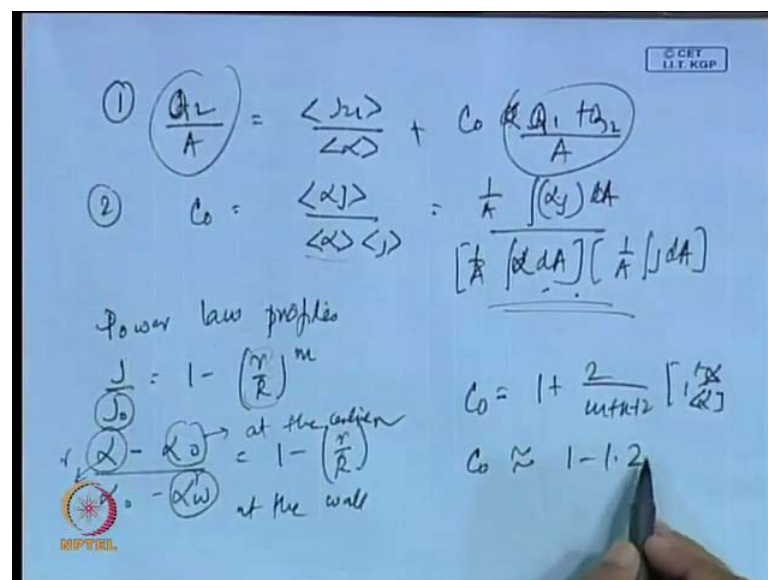
$$C_o = (0.2 e^{0.00848 [< j^* > + 20]} + 1) - \left(0.2 e^{0.00848 [< j^* > + 20]} \sqrt{\frac{\rho_g}{\rho_l}} \right) \text{ for } < j^* > < (-20)$$

Where $< j^* > = \frac{j}{u_{2j}}$



So, in such particular cases we find that the C₀ calculation has to be done from the basic equation that I have written down here.

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① $\frac{Q_w}{A} = \frac{\langle \alpha u \rangle}{\langle \alpha \rangle} + C_o \frac{Q_w}{A}$

② $C_o = \frac{\langle \alpha \rangle}{\langle \alpha \rangle \langle j \rangle} = \frac{\frac{1}{A} \int (\alpha_j) dA}{\left[\frac{1}{A} \int (\alpha) dA \right] \left[\frac{1}{A} \int j dA \right]}$

Power law profiles


$\frac{j}{j_0} = 1 - \left(\frac{r}{R} \right)^m$

at the center $\alpha_0 - \alpha_c$

at the wall $\alpha_0 - \alpha_w$

$C_o = 1 + \frac{2}{m+2} \left[\frac{\alpha_c}{\alpha_0} \right]$

$C_o \approx 1 - 1.2$



From this particular basic equation it has to be done for two particular cases one is when there is heat flux and there is intense vaporization at the walls. So, therefore, there is a larger vapor concentration at the walls as compared to the center the other is when we are introducing air through say a porous wall then in that case what happens in the

developing region there is greater concentration of air bubbles at the wall as compared to the center.

For such particular cases we cannot use the C_0 values which I have written down in my ppt they are mostly for fully developed flow cases or for those particular cases which have a convex sort of a profile. So, for anything else C_0 has to be derived from the basic equation which I have written down and one more thing also which I would like you to remember in this particular case is usually by whatever way you find out C_0 . Usually, C_0 it varies from 1 to 1.2 as I have told you it does not deviate much from unity it varies it lies close to unity and usually from normal circumstances it has a maximum value of say 1 to 1.2 or 1 to 1.1 it does not vary more than that.

And finally, the last word of caution which I would like to give you regarding the drift flux model is see in this particular model we considered the volumetric flux velocity everything has vectors. If that is the case then the direction becomes very important for example, for all the cases co-current up flow co-current down flow I have been considering the upward direction as positive that is why j_2 was positive for counter current flow j_1 was negative when liquid was flowing down etcetera. So, remember 1 thing sign convention is very important as far as the drift flux model is concerned.

There are several sign conventions which we take the first one is usually the direction of flow is taken as positive or the upward direction is taken as positive. Now, we find that most of the two phase flow analysis what we do we give more importance to the dispersed phase that is why we have defined α as the volumetric concentration of the dispersed phase is not it.

We do not define $1 - \alpha$ we talk in terms of α we usually talk in terms of j_2 rather than j_1 isn't it. So, usually we find that the direction of motion of the dispersed phase or the discontinuous phase is taken as positive there can be situation where the direction of gravitational force is taken as positive you can do whatever the case may be. But remember one thing whatever direction you are taking as positive you have to you have to define that in the beginning of the problem considering this direction as positive the following momentum analysis follows. So, this has to be referred to before you do any problem sign convention is very important for the drift flux model.

So, this completes our drift flux model and from the next class we are going to start the two fluid model or the separated flow model thank you very much.