Multiphase Flow
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Module No. # 01
Lecture No. # 15
Drift Flux Model (Contd.)

Well, to continue with our discussions on the drift flux model. So, what we had done in the till the last class was, we found out how and why the drift flux is so very important and then how to express drift flux model. And finally, I had written down two particular equations.

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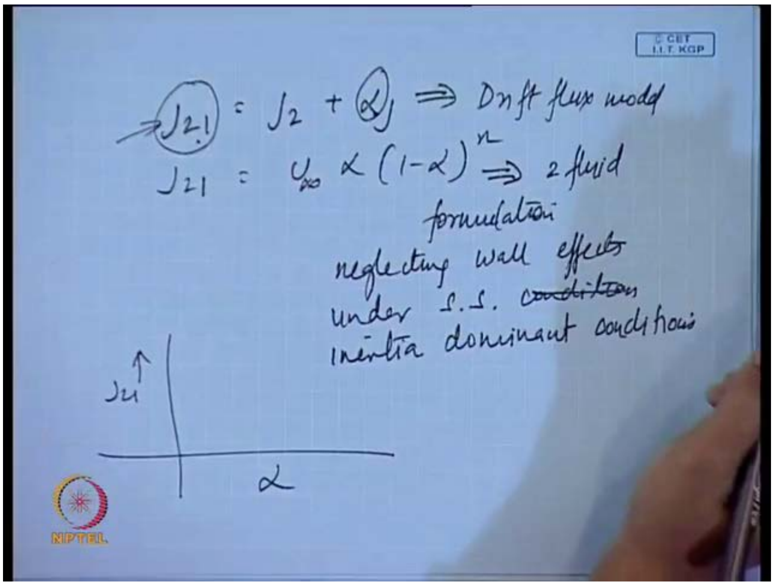

If you observe these two equations, then you will find that here I have written down there is one particular equation which has come from the drift flux model and the other equation which has come from the two fluid formulation. Where we have neglected or rather this particular equation we have obtained it from the two fluid formulation, neglecting wall effects under steady state conditions rather under steady state inertia dominant conditions.

So, for that particular condition we had obtained this and we find that this equation it expresses j 2 1 in terms of alpha and this is independent of the phase flow rates. It does not depend on how the two phases are behaving; it just depends upon how the two phases are distributed, what is the geometry etcetera, etcetera. Based on which n and u infinity can be calculated and this is one which we have obtained from the drift flux model. Naturally, two equations two unknowns so therefore, we can solve them simultaneously and we can do it.

Now, a better technique or a simpler thing is, instead of solving them simultaneously if we can draw the curves resembling this particular equation. Draw the curves resembling this particular equation and naturally the point of intersection is going to give us alpha and j 2 1. So, what sort of curves do you expect? The curves should be something of j 2 1 verses alpha is it not?. Now, tell me from the first equation, what type of curve do you expect or what type of relationship do you expect between j 2 1 and alpha?

Straight line.

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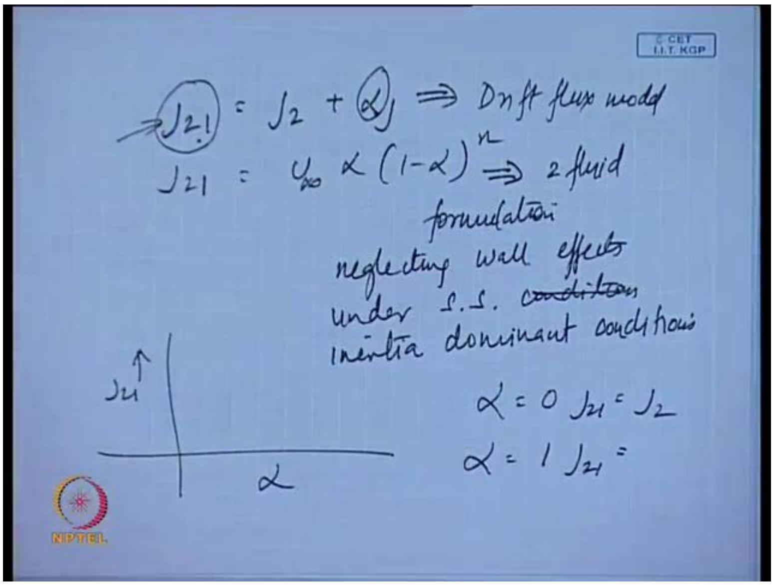

Straight line, very correct. What will be the intercepts of the straight line? If you know the end points or if you know the slope, you can find out the, you can draw the straight line very easily. What will be the intercepts of the straight line at the alpha equal to zero? So, you know at alpha equals to 0, j 2 1 equals to j 2, alpha equal to 1, do it, do it and tell me. If alpha is equal to 1, then in that case this becomes j 2 plus, I think this is minus, I have made some mistake well let me see, May be I have made a mistake in this particular case, let me just check up with the power point thing was it plus or was it minus.

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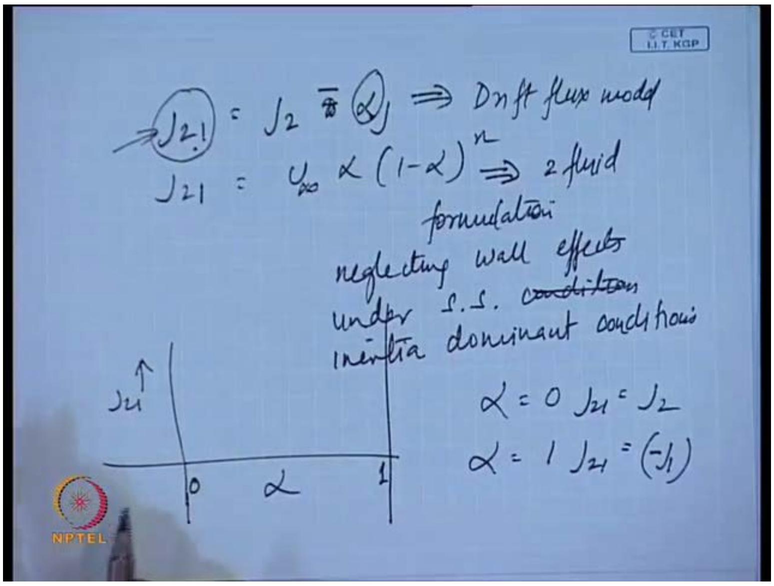

This is the j 2 minus sorry, sorry, sorry. So, this is j 2 minus very sorry, very sorry minus alpha j. So, therefore, when alpha equals to 1, this becomes j 2 minus j and j is j 1 plus j 2. So, therefore, this becomes minus j 1. So, therefore, if you plot j 2 1 verses alpha say from 0 to 1 if we take then therefore, you are supposed to get straight lines depending upon the magnitude of j 2 and j 1 depending upon the signs of j 2 minus j 1. Do you understand?

So, therefore, you will have different type of curves depending upon whether both j 2 and j 1 are positive; that means co-current up flow. Both j 2 and j 1 are negative; that means co-current down flow or else one is positive, one is negative. Depending on that you will have different curves in different particular quadrants. Now, let us see what type of curves you are going to expect and what you are going to get.

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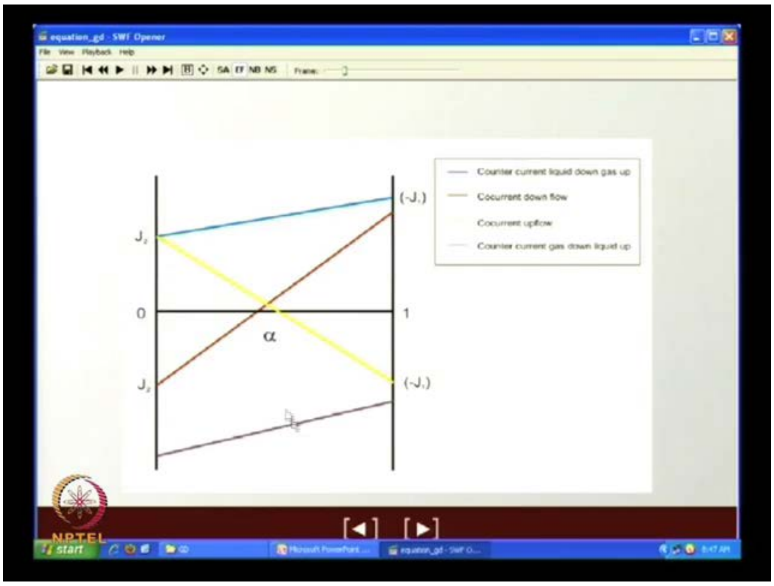

Now, see in this particular case suppose you are having co-current up flow; that means, if you just co-current up flow j 2 is positive and j 1 is positive or minus j 1 is negative, yes. So, therefore, this is the curve for co-current up flow. Now, you try to understand certain things, see suppose your j 2 is constant and j 1 is increased. So, what happens? The straight lines, they are, they intersect the alpha equal to zero axis at lower and lower values, the implications we will see later.

Now, let us see if it is co-current down flow, down flow means, we have already assumed the upward direction as positive. So, down flow means j 2 is negative, j 1 is negative or in other words j 2 is negative and minus j 1 is positive. So, therefore, this red colored line this shows your co-current down flow. Do you agree with me? Now, the other thing counter-current flow, counter-current flow can be two types, one is gas up liquid down, liquid up gas down. When it is gas up liquid down then in that case what happens j 2 is positive, j 1 is negative, minus j 1 is positive. So, therefore, the straight line is the blue line which lies entirely in the first quadrant. Do you agree with me?

Another counter-current gas down liquid up, gas down means j 2 is negative liquid up j 1 is positive minus j 1 is negative. So, therefore, this violet color line which lies in the fourth quadrant. Do you get my point? So, therefore, co-current down flow, co-current up flow they intersect the alpha equals to rather the x axis. That means what co-current up flow and co-current down flow? Naturally, both of them are have the same sign or in other words j 2 is positive, j 1 is positive or j 2 is negative, j 1 is negative. As a result of which we get j 2 positive minus j 1 negative co-current up flow, j 2 negative j minus j 1 positive co-current down flow, agreed.

So, therefore, what do we find? We find that for different particular flow situations we can have different straight lines. This was when we plotted the first equation which I have written down in this particulars case. The first equation which I have written down for this particular equation we have obtained the graphs which or rather the straight lines which I have shown.

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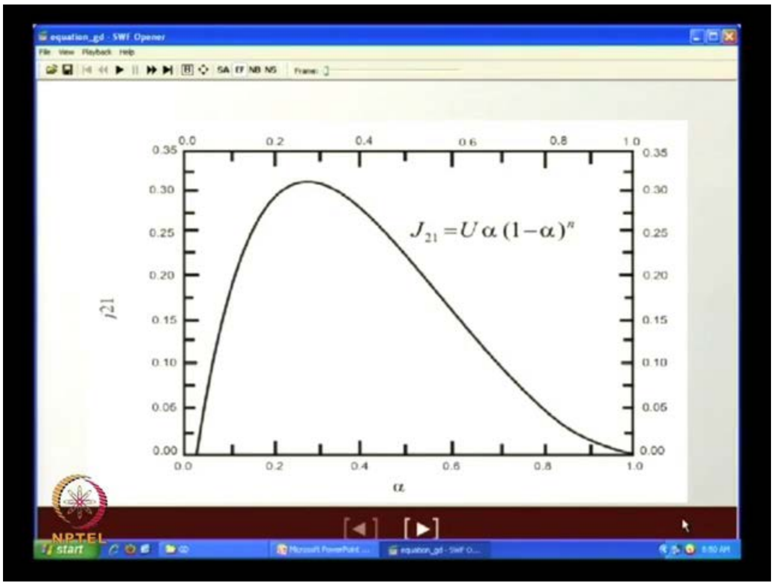

Now, if you plot the second equation, we will be doing problems and then you will find for the second equation, if you plot we get a curve something of this sort. We find that the curve is something of this sort or different values of n and alpha we get this particular curve. Now, simultaneous solution means what? In the same graph, we are suppose to this has slightly mistake this should have originated from alpha equals to 0, this by mistake has been displaced slightly towards the left. So, therefore, if we on the same graph paper if we plot this curve and if we plot the straight lines which I have shown, then the intersection of the curve and the straight line will give us a value of j 2 1 and alpha. Yes or no?

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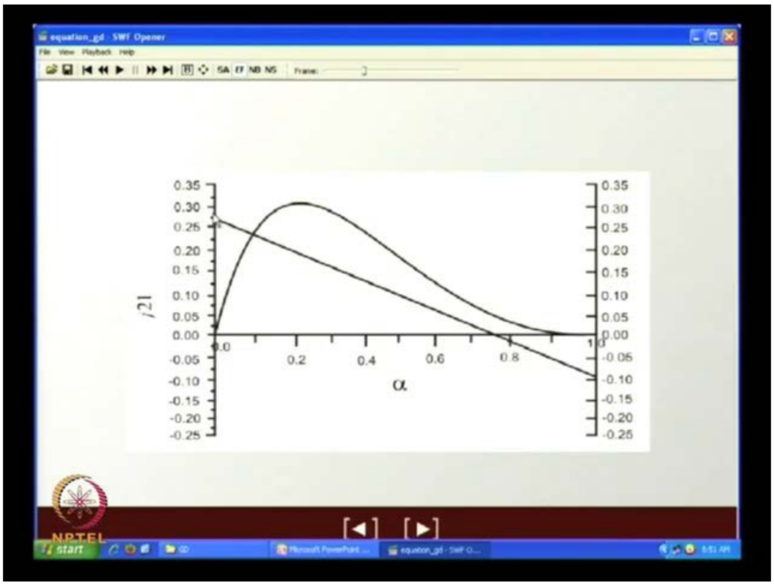

Now, let us see what we have done this is the curve which we have got, these are the straight lines that we have got. Now, let us see, this is for co-current up flow. Co-current up flow, you remember this was the straight line which we had, this was the curve which we have. So, therefore, the point of intersection that gives you the value of alpha 0.1 in this case and that gives you the value of j 2 1.

Notice one thing very interesting that in this particular case you find that no matter how much j 1 is increased or how much j 2 is increased, you will find that always the point of intersection lies along this particular portion or in other words it lies for a small value of alpha, where bubbly flow exists. Because what this particular j 2 1 this curve we have drawn that is for a particular u infinity and n, we have drawn it for bubbly flow pattern, accordingly we have selected j u infinity and n. Therefore, the point of intersection should be such that it lies in the bubbly flow pattern. When does bubbly flow occur? Low gas high liquid. So, therefore, we will always have the point of intersection on between alpha equals to 0 to alpha equals to 0.3 or something of this sort.

And what next do we observe? We observe that suppose we increase j 1; that means, you increase the liquid flow rate. Just observe for and keep j 2 constant, what happens? The curve will be intersecting at lower and lower points when this has to happen from a fixed j 2. Naturally, we find that the point of intersection will slightly shift towards lower and lower alpha. Quite expected higher liquid velocities, naturally, in that case we will have a lower alpha. Similarly, if we keep j 1 constant; if we keep on increasing j 2; what will happen? You see in this particular case, if you keep j 1 constant and we increase j 2 we find that alpha shifts higher and higher values. Quite expected alpha which is the gas void it will increase with gas velocity it will decrease with liquid velocity this was the case for counter sorry co-current up flow.

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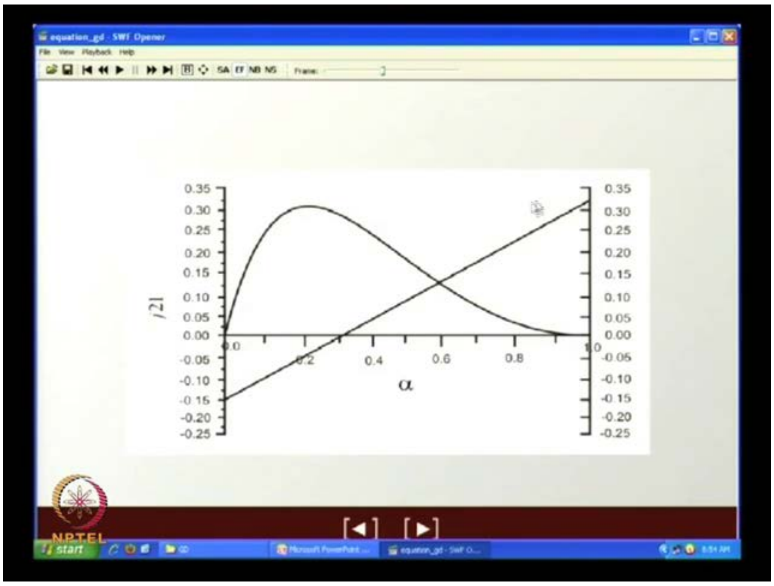

Now, if we take the next case, sorry, very sorry, very sorry. Now, if we take the next case, this was co-current down flow, now, see that situation for co-current down flow. In this particular case we find that more or less the point of intersection occurs somewhere here. Now, if we increase or rather we will keep j 2 constant increase j 1 then we find gradually alpha shifts to lower and lower values. If we keep j 1 constant and increase j 2 we find they shift to higher and higher values as expected. Here we find that alpha occurs at a higher gas velocity because for co-current down flow if gas velocity it is not quite high we cannot obtain a stable flow pattern. So, this was the point of intersection for co-current down flow.

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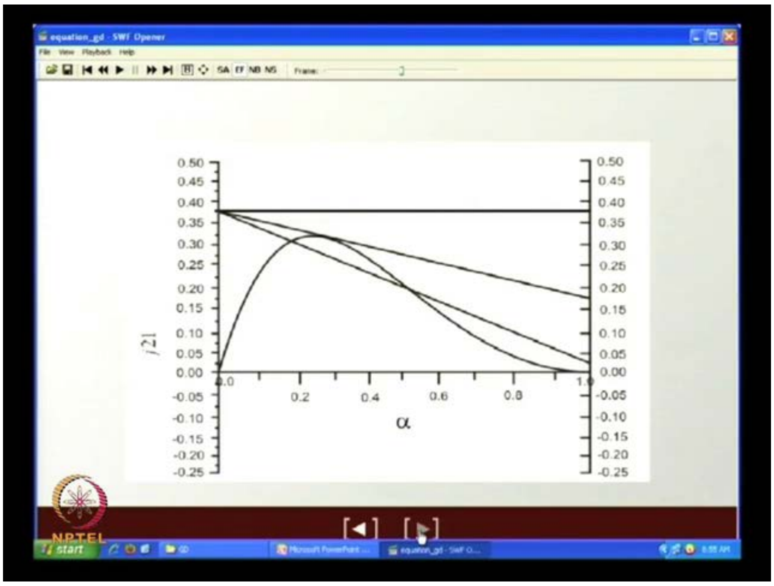

Now, let us take up counter-current flow, counter-current flow with gas up liquid down. So, therefore, gas up means j 2 positive, liquid down means j 1 negative minus j 1 positive, is this portion it is very simple. So, it is nothing very difficult to understand. So, for this particular case, what do we find?

We find that the straight line lies entirely in the first quadrant and if you observe you will find that the straight line cuts the curve at two particular points. If you go for lower and lower j 1 we find that for every case it cuts for two particular points. If we keep, if keeping j 2 constant, if we keep on increasing j 1 or we keep on increasing minus j 1 we find finally, a point comes where it just touches the curve. Under this particular situation, there is only one point of touching or intersection whatever it is. Now beyond that particular situation we find that the straight line lies above the curve there is no point of intersection. Do you get my point?

So, what do you get under this particular situations? From this situations we find that below a critical value of alpha, what do you have? You have two points of intersection; that means, what down flow, no sorry; for counter-current flow with gas up liquid down there can be two possible flow patterns where the flow can be stable. One is that low alpha, what does it denote bubbly flow liquid higher proportion gas lower proportion and high alpha sort of a droplet flow or may be annular type where droplets are dispersed in the gas core. So, therefore, one solution you get at a low alpha, other solution you get at a higher alpha, clear to all of you.

Now, you keep on increasing the air flow, sorry, the liquid flow rate; what happens? At one; or in other words you also keep on increasing the gas flow rate as well, does not matter. You can keep this constant and you can keep on increasing the gas flow rate also. By any means, what you find a point comes when the line becomes tangent to the curve, what does it denote? It denotes the limit of counter-current operation, when does the limit come under flooding conditions. Is it clear? So, therefore, this point of tangency this gives you the flooding condition beyond which the counter-current operation is not possible.

So, beyond that if we increase either of the flow rates we find that there is no point of intersection between the straight line and the curve. So, and it is quit expected that beyond the flooding point there is no stable operation either there will be a change in the flow pattern due to which this curve is going to change. Or there will be some amount of rejection of material from the flow passage whatever it is the flow is not going to be stable are or it is not going to retain the same interfacial characteristics as it had retained earlier.

So, for counter-current flow with gas up liquid down we all know that it operates within a particular stable operating range and that range it is limited by the flooding condition on one case and the weeping condition and the other. So, therefore, below that particular; below the flooding point, we find that we can get counter-current operations under two conditions; one is at low alpha, the other is at high alpha. The point at lower void fraction that denotes the bubbly flow situation, the point at higher void fraction it denotes the droplet flow or may be the wispy annular sort of flow condition. Now, when we are operating below the flooding point we find stable operation at two values of void fraction.

At the flooding point that is denoted by the point of tangency or the point the just the curve rather the straight line touches the curve at one particular point. This is the point of tangency, this denotes the flooding point, this denotes the limit of counter-current flow operation. Beyond this, we will not get a counter-current flow operation. How do we understand it by seeing this particular graph? If we observe this graph we find that beyond this the line passes above the curve, there is no point of intersection between the two.

From here you get another very important thing, what is that? See in order to generate this curve, what we had done? We had used the equation which I had already written down I have written it down here we have used this particular equation. Now, in order to use this particular equation, we need to know the flow pattern we need to know u infinity and n.

Now, by observing this counter-current operation, what did we come to know? We have to come to know that there can be an alternate way of constructing this curve. What is the alternate way? Alternate way is to construct the curve from the flooding data. If we have flooding data then for different alpha values there will be different combination of gas and liquid velocities under which flooding will occur. Do you get the point?

For each particular alpha there will be one set of j 2 and j 1 under which flooding will occur. So, for each alpha if we get j 2 and j 1 then we can construct these particular or rather we can locate each and every point, is it not. We will find out what is j 2, we will find out minus j 1 and then at that particular alpha the point of intersection of this alpha with the straight line will give us one particular point in the curve.

Thus, for the entire range of alpha we can get the from the flooding data for different values of alpha, we can locate different points on this particular curves. And if we join the points then in that case we can generate this particular curve from flooding data. Even if we do not have an idea about the flow pattern and even if we do not have an idea about u infinity n etcetera, etcetera. This can give you an alternate way of constructing this curve as well, clear to you. Well now, let us go to the final operation it was counter-current operation with gas down liquid up. Gas down means j 2 is negative, liquid up means j 1 is positive minus j 1 is negative so that means, both j 2 is negative minus j 1 is negative.

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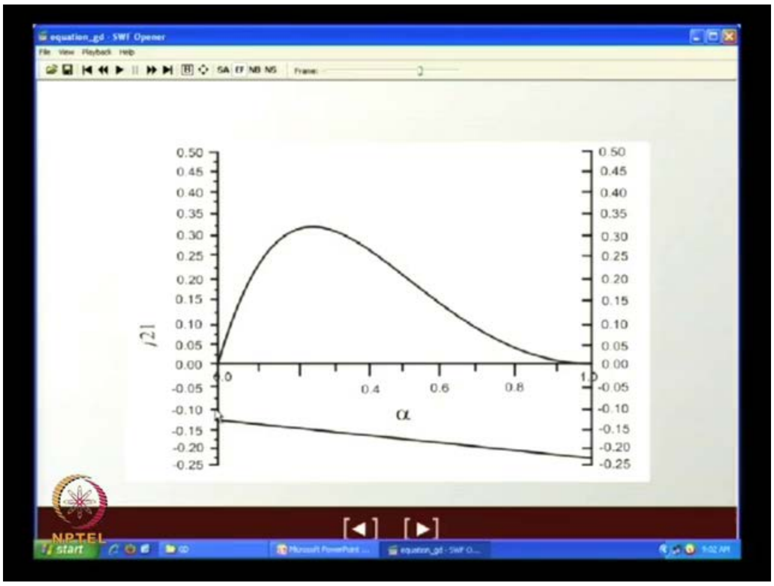

So, therefore, the straight line will lie in the fourth quadrant, is not the straight line will be something of this sort. And if you observe this particular p p t you find that the straight line will lie here. And therefore, what do we find? For such a flow situation there cannot be any point of intersection between this curve and the straight line. Quite expected, have you ever found any counter-current operation with gas down liquid up? This cannot be a stable situation, we cannot have anything where the gas is going down and the liquid is going up, counter-current means gas up liquid down. So, from this particular curve we find that such a situation is not possible which you already know.

So, from these particular curves what we have found out? From these particular curves, we have found out that from the point of intersection of the straight line as well as the curve we can superimpose them on different graphs. And we can find out the corresponding value of alpha and j 2 1 depending upon different values of phase flow rates or depending upon different values of q 1 and q 2.

And what have we found out? We have found out that for co-current up flow, co-current down flow we find that there is one particular point of intersection. And their all circumstances this point it shifts to higher alpha when we increase j 2 and we decrease j 1 as expected. And the opposite behavior occurs for the; if we decrease j 1 and increase j 2 naturally alpha increases, if we decrease j 1 and increase j 2 then opposite thing happens.

Then we next we found out that for counter-current operation, gas up liquid down we find either two solutions are possible. Under one particular situation, only one solution is possible, beyond that no solution is possible or in other words counter-current operation with gas up liquid down can occur only under a set of conditions only. Beyond the flooding point it is not possible and we found out that for counter-current operation with gas down liquid up it is never going to be possible, agreed.

So, and also from this particular curve, we had got an idea regarding an alternative way of constructing this curve. Because constructing the straight line is not a problem, we just have to know the end points. What are the end points? Here this is j 2, here this is j 1. What is j 2? q 2 by a, we get it from the volumetric flow rate. What is j 1? q 1 by a, So, q 1 q 2 an input parameters. So, from them, we can find out j 2 we can find out minus j 1 constructing this straight line is not at all a problem.

Constructing this curve is a problem we need to know data on u infinity and n. Apart from this particular curve; we find that if we exploit the points of tangency or different values of alpha then we can construct this curve from flooding data. So, if we have flooding data for different values of alpha then also this curve can be constructed from the flooding data.

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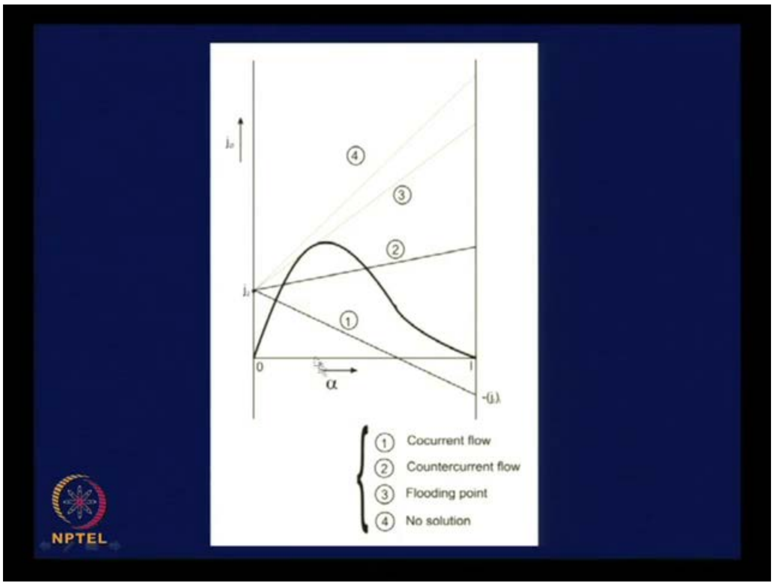

Now, there is one I have a consolidated form of; just a minute. So, the entire thing I can show it, in one particular curve, one particular data I have. Here I have shown the entire thing, in one particular curve, I have shown the entire thing, this is for co-current down flow, this is for your counter-current flow. Where counter-current flow with liquid down gas up and I have shown all the things in this particular case. After this, the next thing which I would like to show is one more thing I would like to tell you for this, under this particular flow situation. Now, remember one thing when we are doing it for gas liquid systems more or less we find that this particular curve it can extend from alpha equal to 0 to alpha equals to one, agreed.

If we consider fluid particle systems, then in that case what happens is? We find out that for fluid particle system we find that there is a point of discontinuity at one particular alpha. This particular curve which I have shown here this cannot extend from alpha equal to 0 to alpha equal to 1. Can you tell me why?

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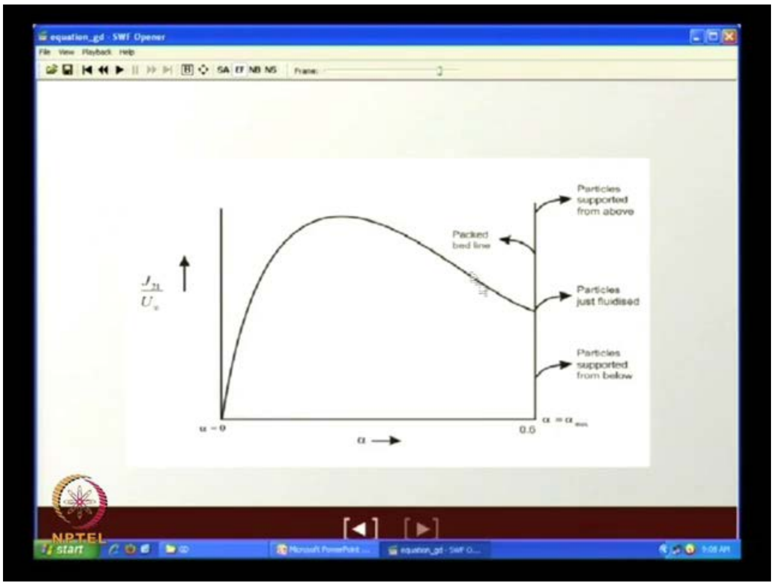

For, gas particle system this is not a problem, but for fluid particle system we find that, sorry, gas fluid-fluid systems this is not a problem, but for fluid particle system we find that there is a point of discontinuity beyond which this curve cannot exist, the curve look something of this sort.

It exists till a maximum alpha, this maximum alpha it depends upon the nature of the fluid and the solid. If it is a flocculating solid then the alpha maximum can be as low as 0.1. If it is totally non flocculating, it can go to higher values. Usually for rigid incompressible particles we find, that usually alpha max it varies between 0.58 to 0.62. In this particular case, we have assumed alpha to be 0.6 in this particular case.

Can you tell me, why For gas solid cases or for fluid solid cases we cannot have the curve extending till alpha equals to 1 as we can have for gas liquid cases, Any idea why did it does not happen? Now, remember one thing, the problem in this class is I cannot give you time to think, I can just throw the question and I have tell the answer myself that becomes a problem in this particular class.

See remember one thing, when the particles beyond a particular packing they come close to one another. When they come close to one another, particle-particle interaction becomes very very important. Now, if you remember how we had derived this particular equation? You remember in that case what we did, if I just go through the derivations you will understand, how did we drive this particular equation.

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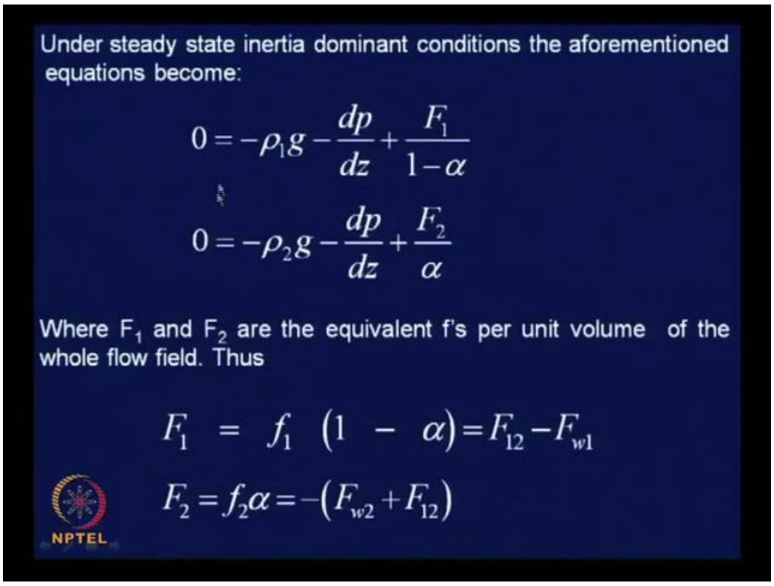

If you notice this particular equation, what we did? First we had written it down for steady state inertia dominant conditions. Then in f 1 f 2 we had f w 1 and f w 2 and f 1 2. Note, there was no term considering f 2 2 here, if we have particle-particle interaction there should have been 1plus f 2 2 in this particular case. Is it correct?

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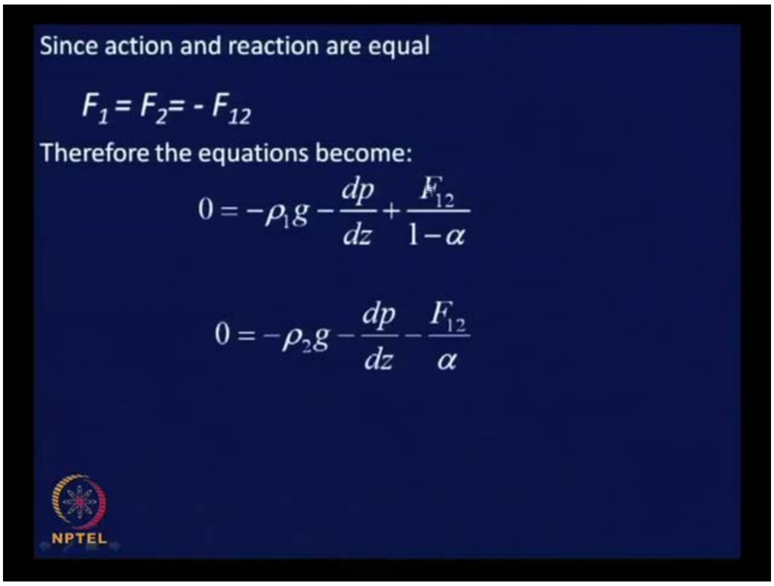

Here we have considered only, what are the cases we have considered? We have considered your interaction between phase 1 and phase 2 wall and phase 1 wall and phase 2. Then after that we have also neglected the wall term here. So, this two equations which we have derived they are just for steady state inertia dominant with the hydrodynamic drag being important even the wall interactions are not important. And definitely in under no condition did we did I include f 2 2 in this particular case.

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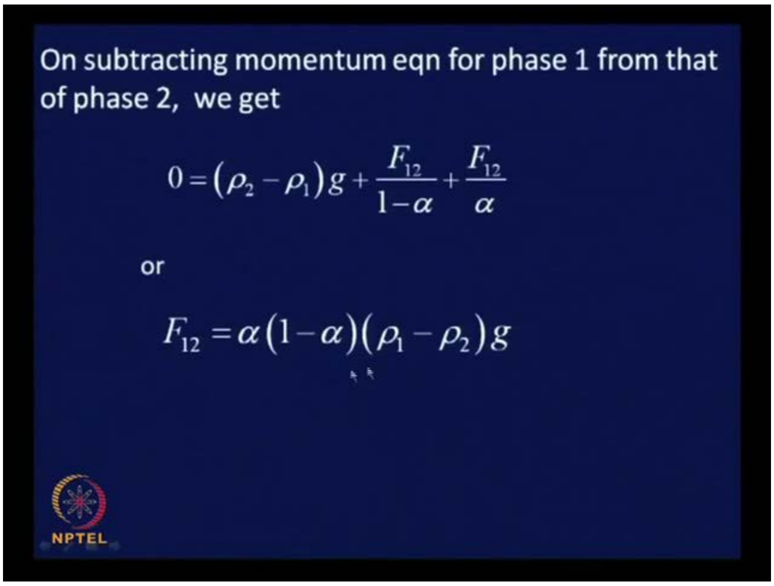

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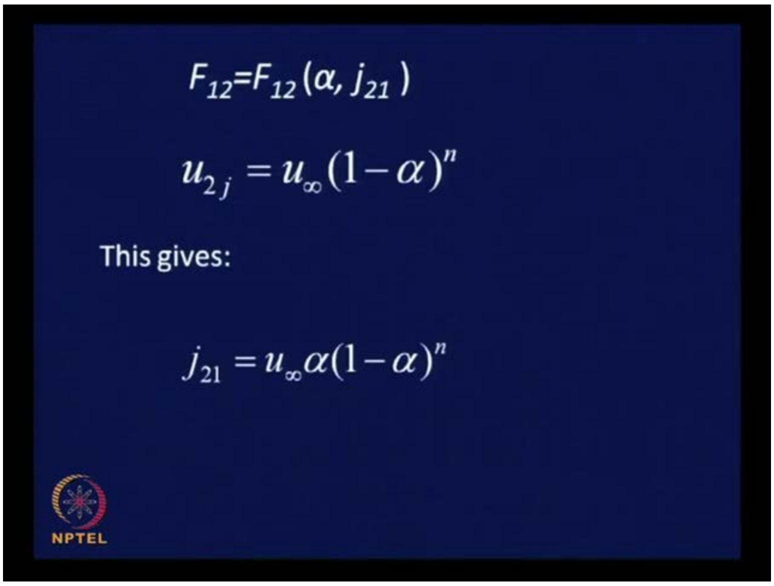

So, therefore, finally, from here we got this particular expression from which I have obtained this equation. So, this particular equation was obtained under the condition of steady state inertia dominant where absence of wall interaction effects, absence of particle-particle interaction.

So, now, when the particles come very close to one another then your particle-particle interaction becomes much more important. And when it becomes important the basic assumption on the basis of which, I had derived the equation for the curve that itself feels. So, therefore, when particles come closer to each other such that particle-particle interaction becomes comparable or more important as compare to the hydrodynamic track. Under that condition what I have to do? I have to go back to the original momentum equations include f 2 2 there and then I have to do the entire derivation. In that case the form of the equation changes, the curve which I have got that is no longer applicable. Is it clear to you?

So, therefore, for the fluid particle system, what we find? For fluid particle system as I have already shown you that we can go till a particular point beyond that it is not possible. So, therefore, this gives you the limit of the packed bed line. Where at this particular point particles are just fluidized, here particles are supported from below, here particles are supported have need to be supported from above.

So, this is one particular point which I wanted you to remember that for gas solid cases since they are very flexible for such flexible cases the curve can extend from alpha equal to 0 to alpha equals to 1. But for fluid particle cases, they cannot extend beyond the critical value of alpha which gives you the maximum limit of solid packing. Beyond that solid-solid interactions becomes important and the basic assumptions based on which I had made the derivation is no longer valid. So, therefore, beyond that we cannot proceed. Now, this value of alpha critical it depends upon the nature of the particles, if they are flocculating type then it can be as low as 0.1 but, generally it varies between 0.58 to 0.62 so that, we take alpha critical equals to 0.6 for most of the cases, agreed.

Now, certain other things regarding the graphical technique also I would like to mention. Now, the first thing which I have already mentioned is that how to construct the flooding line so that we had already mentioned. Now, there is now if you want to denote the equation of the flooding line analytically then how can we do it?

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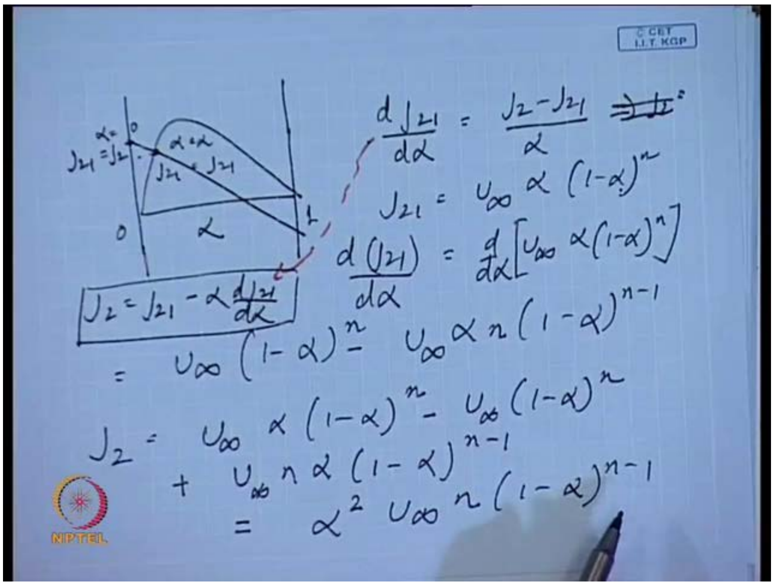

Now, depending upon this particular line which I had got, this was j 2 1 this was alpha. So, and this was the curve if I say and may be if we consider up flow then this is the line. So, therefore, at any particular point you say alpha equals to alpha, here alpha equals to 0, is it not. Here j 2 1 equals to j 2 and here we have j 2 1 equals to j 2 1, correct.

Now, what is the slope of this line? The slope of this line d j 2, at this particular point d j 2 1 d alpha this is nothing but j 2 minus j 2 1 by alpha minus 0, correct. This portion it is correct. Now, at the point of intersection, what do we know? We know at the point of intersection j 2 1 equals to u infinity alpha into 1 minus alpha whole to the power n.

So, at this particular point, if i differentiate j 2 1 j alpha in terms of this so, what do I get? This is d d alpha of u infinity alpha into 1 minus alpha whole to the power n. This equation if I differentiate it with respect to alpha, then I get a slope of this curve. And this slope I can substitute it here and then I can get an analytical expression of relating j 2 and j 1 under conditions of flooding. So, if we perform this differentiation what do we get? From this particular differentiation we get this is u infinity into 1 minus alpha whole to the power n minus u infinity alpha n into 1 minus alpha whole to the power n minus 1, just simply I have differentiated this part.

Now, moment I have got this. So, from there I can get if you substitute it here then I get what is j 2 equals to it is u infinity alpha into 1 minus alpha whole to the power n minus u infinity 1 minus alpha whole to the power n plus u infinity; any point if you do not understand you tell me. This should come as j 2 because from this particular equation we know j 2 equals to, probably you cannot see it here. From this particular equation, we know that j 2 equals to j 2 1 minus alpha d j 2 1 d alpha, is it not. From this equation we can we know this.

Now, d j 2 1 d alpha I have already found it out j 2 1 I know it already if I substitute this and if I substitute this in this particular equation I get an expression of j 2. So, at this can be written down as alpha square if you just you will find out n into. So, j 2 you can be obtained by an equation as alpha square u infinity n into 1 minus alpha whole to the power n minus 1, correct. Similarly, what is j 1 equals to? j 1 equals to, we know that this from this particular graph this is minus j 1, is it not. So, from here if we do then here we know that minus j 1 minus say j 2 1 this will be alpha equals to 1, this is alpha equals to alpha. So, we can find out the slope from these two points, we can also find out the slope from these two points, is it not.

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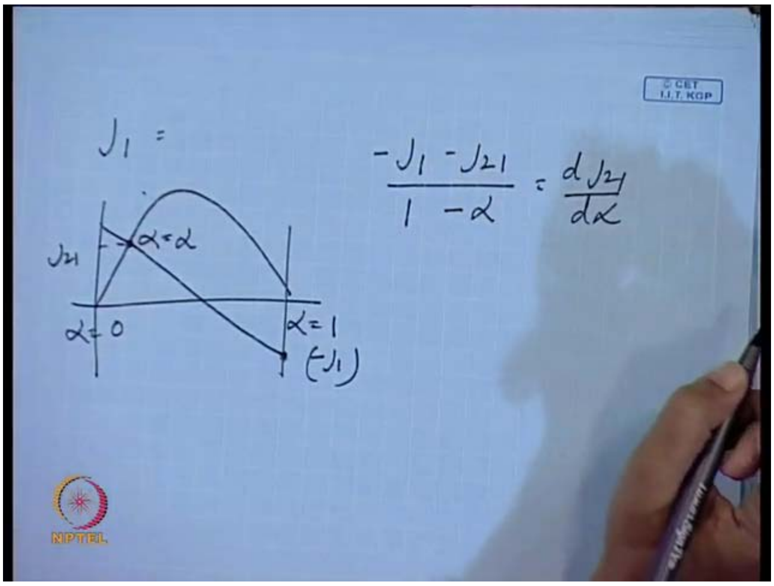

So, therefore, if we do that let me draw it once more, this is alpha equals to 0 alpha equals to 1 this is the curve this is the line. In this line this is alpha equals to 0, sorry, alpha equals to 1, this is alpha equals to 1 alpha this is minus j 1 and this is j 2 one. So, what is the slope in this particular case? The slope will be minus j 1 minus j 2 1 by 1 minus alpha this will be the slope d j 2 1 d alpha, correct. Again this d j 2 1 d alpha this can be substituted from this particular equation. We can substitute this in here in d j 2 1 d alpha and then accordingly we can find out the j 1 under flooding conditions or we can find out j 1 j 2 for the corresponding points in the curve.

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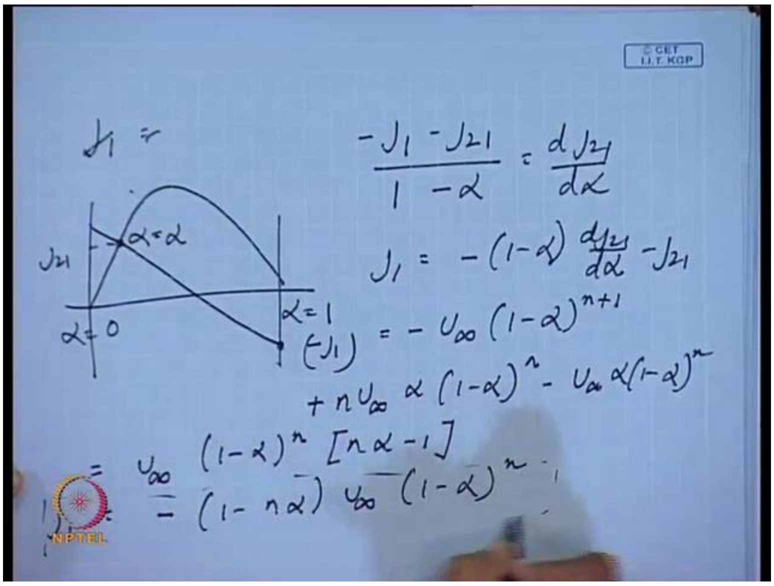

So, if we do it then in this case, what do we get? We get, just substitute it and you see what is the thing that you are going to get. This j 1 it is nothing but minus of 1 minus alpha d j 2 1 d alpha minus j 2 1, is it not. Or this gives you if you write it down you get minus u infinity; please perform all these your whatever derivations I am doing, please perform all these derivations in your home and you find out that whether you are getting the expressions, there can happen that I am making some mistakes. So, therefore, unless you do the derivations on your own it is going to very difficult. n u infinity alpha 1 minus alpha whole to the power n minus u infinity alpha into 1 minus. Please do them and check up that I have done the correct thing or not. It can always happen that I have made mistakes or even I have made mistakes while I am doing it at the present moment.

So, this gives you u infinity into 1 minus alpha whole to the power n into n alpha minus 1 or in other words this can be minus of 1 minus n alpha u infinity 1 minus alpha whole to the power n. So, therefore, what do we find? This is the j 1 at flooding point and this is the j 2 at flooding point.

So, therefore, what are we require to do? We are require to locate this j 2 and this j 1 for difference, see they are functions of just alpha n n u infinity. For both the cases if you find you will find that they are functions of alpha n n u infinity. So, therefore, n n u infinity they depend upon the flow patterns interfacial distribution they are going to be different for bubbly slug churn etcetera etcetera.

So, for the corresponding flow pattern you find out n and u infinity. Then for each alpha you can find out j 2, you can find out j 1. Accordingly, for each alpha you can locate the n points of this particular curve and then from there you we can find out each and every point and we can construct this particular curve, agreed. So, this was an analytical way of doing it. By this particular method you can locate this particular curve. Either this method can be used or the flooding data can be used, got my point.

Now, there is a second way of representing the data or the relationships which we have got. Same thing, we start form the same particular relationships that from this particular equation itself we can start. And we can find out a relationship between j 1 and j 2. And we can represent it in a curve of j 1 verses j 2. Let us see, how to do it? This is simply a second way of representing the curve.

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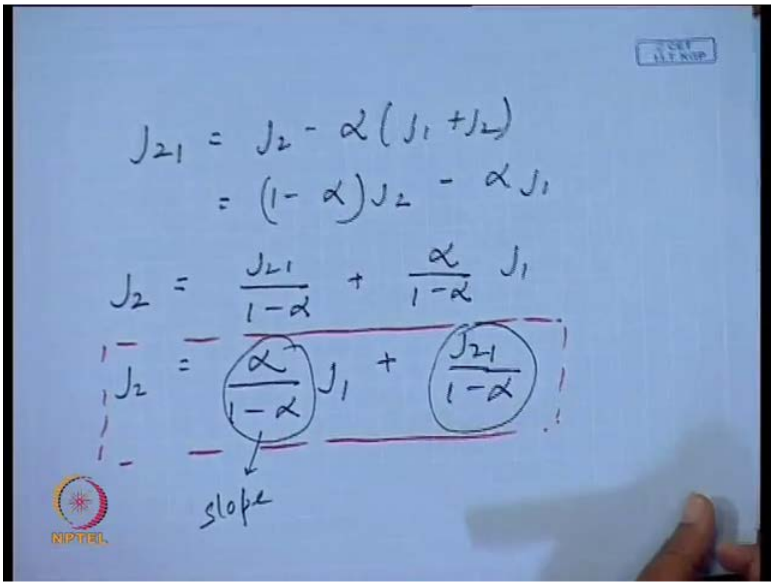

Now, in this particular situation, what do we have let me minimize it let me see if I have it or not? (No audio from 41:02 to 41:16) So, this you do not see now, let me see now what is the second way of representing it? Second way of representing it is I have this equation j 2 1 equals to j 2 minus alpha j or minus alpha j 1 plus j 2, is it not. Or this can be written down as 1 minus alpha into j 2 minus alpha into j 1, can I do this? Or in other words I can write j 2 equals to j 2 1 by 1 minus alpha plus alpha by 1 minus alpha j 1. Just I will I will just rearrange it for my convenience. Now, if you see this particular equation. Now, this particular equation if you notice what do you find? That from the drift flux model we can get a relationship between j 2 and j 1.

Now, observe that for constant alpha, what sort of a curve do you expect between j 2 and j 1? It will be a straight line. Straight line with the slope of alpha by 1 minus alpha and the intercept is going to be this.

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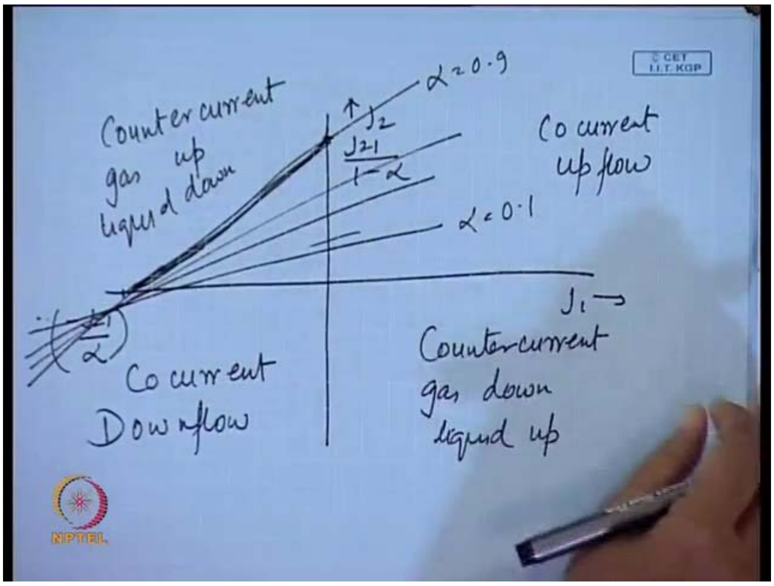

Now, you tell me suppose we draw this, we draw the straight lines for different conditions. Let us take the straight line, let us see how it resembles? If we draw this is j 2, this is j 1, j 2 and j 1 they can have different positive as well as negative values, is it not. Now, for a particular alpha, you tell me what will be the intercept here and what will be the intercept in the x axis? x axis means j 2 equal to 0. So, when j 2 equals to 0, what will be the intercept? Intercept for j 2 equals to 0 or the intercept minus j 2 1 by alpha. So, therefore, it is always going to be some sort of a this intercept is going to be minus j 2 1 by alpha.

What will be the intercept on the y axis? j 2 1 by 1 minus alpha. So, therefore, what are we going to get? We are going to get a straight line something of this sort. This is for one particular alpha. If we go for lower and lower alpha we find that we get something of this sort. Again we go for a higher alpha we get something of this sort. In this particular way our straight lines, we get different, different straight lines for different, different alpha values. May be this is for alpha equals to 0.1 this is for alpha equals to 0.9 and so on and so forth.

So, I could not draw it very well. The thing is if you draw it properly, maybe I will give you a problem in which you can draw it and you can find out, that in this particular region where both j 2 and j 1 are positive; that means, this is co-current up flow always a solution is possible. In this particular region where both j 2 and j 1 are negative; that means, this is co-current down flow. Here also you find that always a solution is possible. Here this one is j 2 is positive, j 1 is negative. So, therefore, j 2 positive j 1 negative means it is counter-current with gas up j 2 positive liquid down.

You will find that since it I have not drawn it very well, but if you draw it, you will find that more or less. You will find a some sort of a limiting region where here solution is possible, beyond that solution is not possible. And this will be sort of a curve at this curve will be limited by the flooding line.

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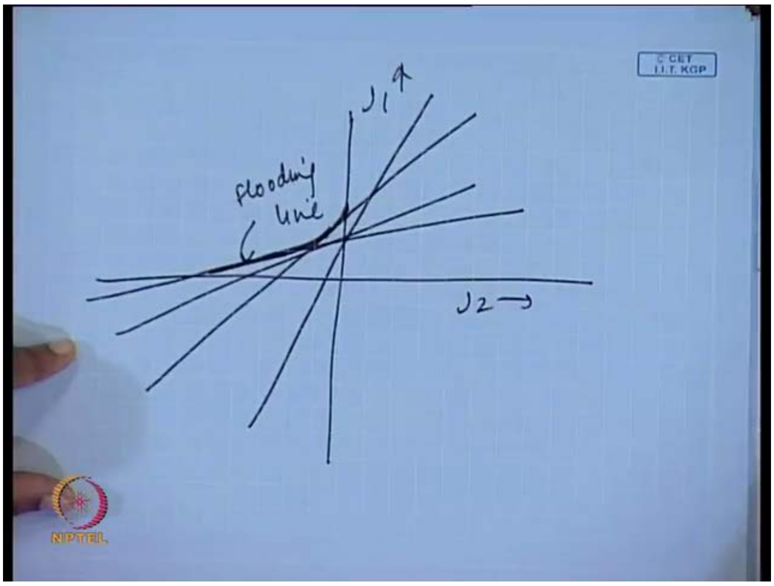

And here it will be counter-current with gas down j 2 is negative j 1 is positive and liquid is up, here no solution is possible. This one I could not draw it very well, but if you draw it then you will find that more or less all these for different alpha they are go into coincide under different conditions. It will be sort of this more or less if you draw it; it will be something of this sort. Your for very low alpha, it is going to be something out this sort or very high alpha it is going to be something of; then for lower cases you get something of this sort. So, more or less from these intersections you get a curve something of this sort which is the flooding line and you know that your limited operations will occur within this particular flooding line. Here you are always going to get a solution this is j 1 this is j 2. Here you are always going to get a solution. Here you get a solution under limited conditions. Here you do not get any solution at all.

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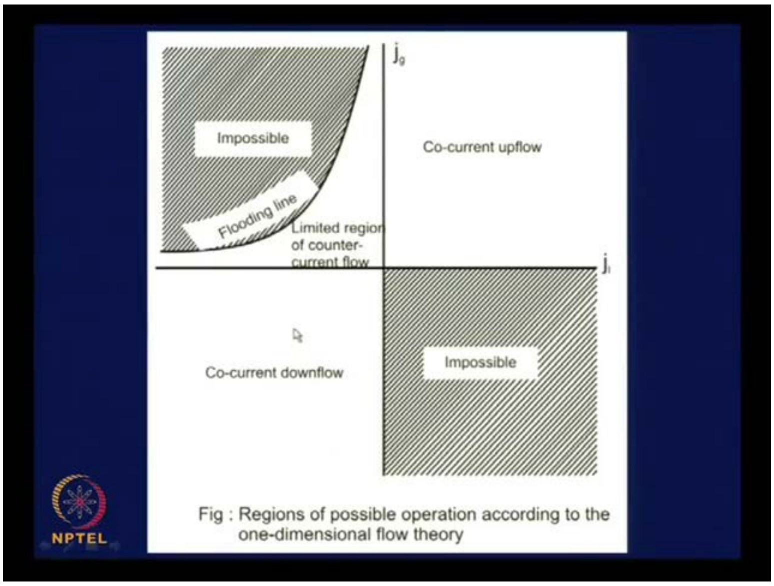

Fig : Regions of possible operation according to the one-dimensional flow theory

So, this I have represented it in the form of a p p t. Which I am going to show you this particular p p t which tells you that in co-current up flow there is always a solution, in co-current down flow there is always a solution. If you plot the curves we will be doing it when I give you more problems you find that there is a limited region in counter-current flow with gas up liquid down. This limited region there is a solution and beyond the flooding line there is no solution. And in this particular case, you find that there is no solution for gas going down and liquid going up.

So, therefore, in order to conclude this particular portion, what are the things that we have discussed? First thing I have discussed is the drift flux model how to obtain the different rather, how to obtain the different mixture parameters with respect to the drift flux model. Definitely we had expressed alpha rho t p in local velocities in terms of the drift flux model. We found in all the cases our or the expressions that we obtained they were a function of j 2 1 non dimensionalised or normalized with the individual volumetric fluxes. We for all the terms, we had j 2 1 by j 2 or j 2 1 by j something of that sort.

Now, if we have an independent method of finding out j 2 1 it is going to be very advantageous. Normally, for the flow situations which we have there is an independent way of evaluating j 2 1. That particular method we took from the two fluid formulation. From the two fluid formulation we found out that more or less j 2 1 can be expressed in terms of certain governing parameters. And we found out that for steady state inertia dominant conditions where mutual hydrodynamic drag is important. More or less j 2 1 can be a function of alpha only for a particular system, for a particular flow pattern.

The functional form of j 2 1 and alpha is going to be different for different flow patterns. So, accordingly by studying the different flow patterns a generalized expression was proposed, where it had two parameters u infinity and n which vary with the different flow parameters; different flow patterns. So, depending upon the flow pattern a suitable u infinity and n had to be incorporated and accordingly we could get a proper relationship between j 2 1 and alpha for different flow patterns. That particular equation, it yielded a curve. So, we found the best way of finding out alpha and j 2 1 walls a graphical solution.

So, we discussed the graphical solution and we found out that there is always a solution possible for co-current up and down flow. Two solutions possible till a critical particular flow conditions for counter-current flow with gas up liquid down beyond which there is beyond the flooding point no solution is possible. And there is never a solution possible for counter-current flow with gas down liquid up.

Then we discussed different ways of representing the data using the drift flux model, one was j 2 1 verses alpha, the other was j 2 verses j 1. And finally, what we did? We found out that generating the curve is not always very simple. And therefore, this curve can also be generated from flooding data, when the flooding data can be obtained from for different values of alpha. We can find out analytically the different values of j 1 and j 2. If we plot them then we can generate the curve over the entire range of alpha from alpha equal to 0 to alpha equals to 1.

Other point which has to be noted was that for fluid-fluid combinations the curve can go from alpha equal to 0 to alpha equal to 1. But for fluid particle combination usually there is a discontinuity beyond a critical value of alpha. And when there is beyond the discontinuity, we find that the drift flux model under this condition is no longer applicable because under that condition particle-particle interactions become important. So, the basic equation has to be modified. Now, in the next class we are going to discuss, how 2 and 3 dimensional effects can be incorporated in the model? How it can be made more generalized? And that will complete our drift flux model. Thank you very much.