

**Multiphase Flow**  
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**Lecture No. # 13**  
**Drift Flux Model**

Well, good morning to all of you. So, today we are going to start a new topic the drift flux model. So, till now whatever we have discussed the different analytical models and then I had told you that well usually there are two extreme situations; one is the homogeneous flow model where we considered that the two fluids are intimately mixed with one another and we assume it as to be a pseudo fluid with suitable average properties. The next extreme is we assume that the two fluids are completely separated they may interact, they may not interact at the interface.

If you assume that they do not interact at the interface it is an gross assumption; at times we do do it to simplify calculations. The other extreme is the separated flow model. Now, the homogeneous flow model that is also a gross assumption what we assume that the two fluids they are flowing; they are intimately mixed and they are flowing as a pseudo fluid as if the none of the phases have a tendency to slip past the other fluid. This is a gross idealization this can happen only when the mixture velocity is very high.

Such that the slip velocity is negligible as compare to the mixture velocity. This normally it does not happen; it happens only under extremely idealized conditions for very high phase velocities. Otherwise, even if it appears that the two fluids they are totally mixed with 1 another, it will always happen that there will be a relative velocity between the two phases because there is always a density difference between them.

So, therefore, always the lighter fluid will have a tendency to slip past the heavier one. So, therefore, what happens? There has to be a slip velocity. Now, if we do not consider this; if we do not incorporate this, then naturally results are bound to be erroneous. So, therefore, an improvement of the homogeneous flow model can be suggested in terms of incorporating the relative velocity and modify the expressions of we had obtained for the homogeneous flow model.

Now, in that particular case what we do? We use the concept of drift flux, drift velocity which we had already defined in our nomenclature chapter; I think in chapter three we had already defined those things. So, we will be using the drift flux concept in order to incorporate the concept of relative velocity into different parameters which we defined for the homogeneous flow model; accordingly we will be modifying the homogeneous flow equations in order to account for the relative motion between the phases.

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### General Theory

Relative velocity between the phases taken care of by the concept of drift flux

Volumetric Flux  $J_{TP} = \frac{Q}{A}$      $j_1 = (1 - \alpha)u_1$      $j_2 = \alpha u_2$

Drift flux  $J_{21} = \alpha(u_2 - j_{TP}) = u_{21}\alpha(1 - \alpha) = j_2 - \alpha j$


$j_1 = (1 - \alpha)(u_1 - j_{TP})$      $j_1 = (1 - \alpha)J_{TP} - J_{21}$

$J_2 = \alpha J_{TP} + J_{21}$

$\alpha = \frac{J_2}{J_{TP}} \left( 1 - \frac{J_{21}}{J_2} \right)$

$\rho_{TP} = \frac{j_1 \rho_1 + j_2 \rho_2}{J_{TP}} + (\rho_2 - \rho_1) \frac{J_{21}}{J_{TP}}$

**Application-** Bubbly flow, slug flow, drop regimes of gas-liquid flow as well as to fluidized bed



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$$J = \frac{Q}{A}$$

$$\langle j_1 \rangle = \frac{Q_1}{A} \quad \langle j_2 \rangle = \frac{Q_2}{A}$$

$$J_{21} = \alpha(u_2 - j_{TP})$$

$$J_{21} = -J_{12}$$

$$J_{21} = \alpha(1 - \alpha)u_{21}$$


Now, if you recollect the concept of drift flux which we had already discussed you remember that we had started with the definition of volumetric flux. Volumetric flux is nothing, it is just the volume flow rate per unit cross sectional area and accordingly just if we take up a a single phase fluid in that case it is just  $J$  equals to  $Q$  by  $A$ , and for the two phases naturally it becomes  $J_1$  averaged over the cross sectional area; it is equal to  $Q_1$  by  $A$  and  $J_2$  this is equal to  $Q_2$  by  $A$ .

And then we had tried to relate the volumetric flux where the local in-situ velocity and these relations they have been given in this particular terms. And after volumetric flux next thing which we defined was the drift velocity and the drift flux. So, the drift flux how did we define it? We had defined that drift flux in something of this particular term the drift flux of component 2 with respect to component 1; this was defined as  $\alpha U_2$  minus  $J$  or the  $J_2$  phase if we consider the two phase mixture in this particular case; where we defined it as the velocity of 1 fluid with respect to a observer moving at the average velocity.

In that particular way we had defined and then what we had done? We had done several important derivations and finally, we had arrived at two important equations. One was a symmetric equation  $J_{21}$  equals to minus  $J_{12}$  which you must be remembering; the other was  $J_{21}$  it was  $\alpha_1 U_1$  minus  $\alpha_2 U_2$ . So, these were the two final equations that we had obtained when we had discussing the concepts of volumetric flux drift velocity and drift flux towards the end of our nomenclature chapter.

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$$\begin{aligned} J_{21} &= \alpha (U_2 - J) \\ &= \alpha \left( U_2 - \frac{J_1 + J_2}{\alpha} \right) \\ &= \alpha \left( \frac{J_2}{\alpha} - J_1 - J_2 \right) \\ J_{21} &= J_2 - J_1 \alpha - J_2 \alpha \\ &= J_2 - \alpha (J_1 + J_2) = J_2 - \alpha J \\ \boxed{J_{21} = J_2 - \alpha J} \end{aligned}$$

Now, from whatever we have defined during that time we come across certain interesting equations in this particular case. For example, as I had written down in the previous class the derivations are more or less done in this p p t, but we will be deriving them and then you can refer to the p p t as in when its require. So, starting from the basic definitions please remember always we will be starting from the basic definitions and then we will be proceeding. So, what does this give us? This gives us alpha into U 2 minus J 1 plus J 2. U 2 also we can substitute it in terms of J 2 and we get a equation something of this sort.

We have just substituted J and U 2 in terms of component fluxes J 1 and J 2. So, from here what do we get? J 2 minus J 1 alpha minus J 2 alpha equals J 2 1; or in other words we can get it as in fact, without doing so much also I could have got it to be very honest; from here itself I could have got it; it is J 2 1 equals to J 2 minus alpha J. Same thing I can get from this particular thing also it is J 2 minus alpha J 1 plus J 2; it is not required to do; it in this particular way, directly I wanted to derive 1 other equation. So, I started this way any how.

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$$\alpha = \frac{J_2 - J_{21}}{J}$$

$$= \frac{J_2}{J} \left(1 - \frac{J_{21}}{J_2}\right)$$

$$\alpha = \beta = \frac{Q_2}{Q_1 + Q_2} = \frac{J_2}{J_1 + J_2} \Rightarrow \text{Homogeneous Flow Theory}$$

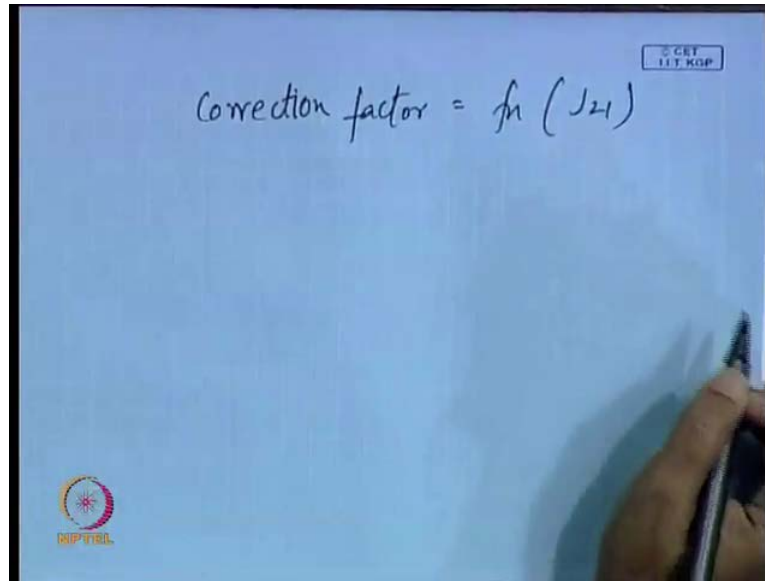
$$= \alpha_{\text{homogeneous}} \left(1 - \frac{J_{21}}{J_2}\right)$$

$$\alpha_{DF} = \alpha_{\text{hom}} \text{ (Correction factor)}$$

So, we can very well get this particular equation. Now, from this particular equation what is the value of alpha that you get? Alpha equal to just derive the value of alpha from this particular expression and see what you get; alpha will be equal to  $J_2$  minus  $J_{21}$  by  $J$ . Now, what is the homogeneous value alpha can you tell me? For homogeneous value alpha equals to beta. So, therefore, the homogeneous value alpha is how can we express the homogeneous value alpha or alpha obtained from the homogeneous flow theory? It is just the input flow rate of phase 2 divided by the total input flow rate; alpha equals to beta for the homogeneous flow theory we get this equal to  $Q_2$  and this is nothing, but equal to  $J_2$  by  $J_1$  plus  $J_2$ ; we get this from the homogeneous flow theory.

So, therefore, from the homogeneous flow theory alpha equals to  $J_2$  by  $J$ . So, this we can write it down as  $J_2$  by  $J$  into  $1$  minus  $J_{21}$  by  $J_2$ ; or this can be written as alpha homogeneous into  $1$  minus  $J_{21}$  by  $J_2$ . So, therefore, what do we find? Just by incorporating the concept of drift flux, what did we do? We have just modified the homogenous flow value alpha by multiplying it with a correction factor which is a function of  $J_{21}$ .

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So, basically we find that the alpha which has been obtained from the drift flux model that is equal to alpha homogeneous into some sort of a correction factor where the correction factor is a function of  $J_{21}$ . So, therefore, basically what has the drift flux model done then? It has simply modified the homogenous alpha value by incorporating the relative motion between the phases. So, that now we can expect that since the relative motion has been considered we can expect to get a more accurate value of the void fraction under practical situations.

Now, we will find that similarly what is state flux model does is it modifies all the mixture properties obtained from the homogeneous flow theory by incorporating a correction factor which is a function of  $J_{21}$  by  $J_2$  or  $J_{21}$  by  $J_1$  or  $J_{21}$  by  $J$  as the case may be, but it simply considers this particular relative motion in the form of drift flux; and by incorporating this drift flux in the expressions of the different parameters, say the different parameters can include alpha which you have already seen we will be seeing shortly how it modifies the expression of the mixture density  $\rho_{TP}$ ? How it modifies the expression of  $U_1 U_2$ ? Even it modifies the expressions of momentum flux, kinetic energy flux everything.

It modifies all the expressions which we had obtained from the homogeneous flow theory by either adding or multiplying a particular correction term where this particular correction term it is a function of  $J_{21}$  the drift flux model which we had derived.

So, in this particular way we find that the drift flux model it attempts to modify or improve the predictions of the homogenous flow model just by incorporating a correction factor either as a multiplicative or an additive term; and this particular term is invariably a function of drift flux  $J_{21}$  or  $J_{12}$  as the case may be. So, this is basically what it does? How it modifies the concept or rather the expression obtained for alpha? Alpha homogeneous is by this particular expression we can simply multiply alpha homogeneous with  $1 - J_{21} / J_2$  and we get the corrected alpha for the actual situation.

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Correction factor =  $f_1(J_{21})$

$$\rho_{TP} = \alpha \rho_2 + (1 - \alpha) \rho_1$$

$$\rho_{hom} = \frac{J_2}{J} \rho_2 + \frac{J_1}{J} \rho_1$$

$$\rho_{DF} = \frac{J_2 - J_{21}}{J} \rho_2 + \frac{J_1 + J_{21}}{J} \rho_1$$

$$\alpha_{DF} = \frac{J_2 - J_{21}}{J} \quad 1 - \alpha_{DF} =$$

Similarly, if we take up say the mixture density  $\rho_{TP}$ ; we know what is the mixture density? It was  $\alpha \rho_2 + 1 - \alpha \rho_1$ . From the homogeneous flow theory we know alpha equals to beta. So, for that particular case it is simply  $J_2 / J \rho_2 + J_1 / J \rho_1$ . Now, let us see for the drift flux model, what is the expression of  $\rho_{mixture}$  that we get? From the drift flux model, we know alpha can be obtained from this particular expression.

So, if we substitute this particular expression of alpha in the expression of  $\rho_{TP}$  then we should get the corrected expression of the drift flux or rather the corrected expression of the mixture density as obtained from the drift flux model. So, just substitute and then tell me what is the expression that you are going to get for this particular case? Instead of alpha we can write it down as  $J_2 - J_{21} / J$  into  $\rho_2$  plus  $J_1 + J_{21} / J$  into

rho 1. So, we had already defined alpha drift flux as  $J_2$  minus  $J_{21}$  by  $J$ . Similarly, if we define  $1 - \alpha$  drift flux what is the expression that we will be getting? For  $1 - \alpha$  drift flux again we can refer to the expressions of  $J_1$  there. In this particular case, we had obtained the expression in terms of  $J_2$ . Is it not?

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$$\begin{aligned}
 J_2 &= \alpha J + J_{21} \\
 J_1 &= (1 - \alpha)J - J_{21} \\
 J_{12} &= (1 - \alpha)(U_1 - J) \\
 -J_{21} &= (1 - \alpha) \left[ \frac{J_1}{1 - \alpha} - J \right] \\
 &= J_1 - (1 - \alpha)J \\
 \frac{1}{(1 - \alpha)} &= \frac{J_1 + J_{21}}{J} = HL
 \end{aligned}$$

From here we had obtained this particular expression. Now, if we express it in terms of  $J_1$  then in that case what do we get? Just like  $J_2$  equals to  $\alpha J$  plus  $J_{21}$ , in the similar way we can write down  $J_1$  equals to  $1 - \alpha$   $J$  minus  $J_{21}$ . You simply substitute these things and then you find it out; you are going to get this particular expression from where you get it we know  $J_1$  equals to  $1 - \alpha$  into  $U_1$  minus  $J$ .



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$$\alpha = \frac{J_2 - J_{21}}{J}$$

$$\alpha = \frac{J_2}{J} \left(1 - \frac{J_{21}}{J_2}\right)$$

$$\alpha = \beta = \frac{Q_2}{Q_1 + Q_2} = \frac{J_2}{J_1 + J_2} \Rightarrow \text{Homogeneous Flow Theory}$$

$$\alpha_{DF} = \alpha_{\text{homogeneous}} \left(1 - \frac{J_{21}}{J_2}\right)$$

$$\alpha_{DF} = \alpha_{\text{hom}} \text{ (Correction factor)}$$

And we know  $J_1 - J_2$  is nothing, but minus  $J_2 - J_1$ . So, this is equal to  $1 - \alpha$  by  $1 - \alpha$  minus  $J$ ; or in other words  $J_1 - 1 - \alpha$  into  $J$ . So, therefore,  $1 - \alpha$  this is nothing, but equal to  $J_1 + J_2 - 1$  by  $J$ ;  $1 - \alpha$  is nothing, but the liquid hold up  $H_1$ . So, therefore, we have derived the expression of  $1 - \alpha$  here and we have derived the expression of  $\alpha$  here;  $\alpha$  equals to this particular expression. Now, if we substitute the expressions of  $\alpha$  and  $1 - \alpha$ ; this expression of  $\alpha$  which we have got and this expression of  $1 - \alpha$  which we have got in the expressions of rho drift flux then we are going to get something of this sort. Substitute yourself and then you are going to find out that you are going to get something of this sort.

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$$\begin{aligned} \rho_{DF} &= \frac{J_2 - J_{21}}{J} \rho_2 + \frac{J_1 + J_{21}}{J} \rho_1 \\ &= \left( \frac{J_2}{J} \rho_2 + \frac{J_1}{J} \rho_1 \right) + \frac{J_{21}}{J} (\rho_1 - \rho_2) \\ &= \rho_{\text{homogeneous}} + \frac{J_{21}}{J} (\rho_1 - \rho_2) \end{aligned}$$

$U_1 =$   
 $U_2 =$

So, therefore, simplifying this particular expression what do we get? We get this rho drift flux; let me write it down again it was please do it in your note books and then see whether you are getting what I am getting or not. It is very difficult for you to copy these things and then understand them. This will be obtained as say  $J_2$  by  $J$  rho 2 plus  $J_1$  by  $J$  rho 1 plus  $J_{21}$  by  $J$  rho 1 minus rho 2. Now, what is this particular term can you tell me? This is rho for homogeneous case. I had already written down rho for the homogeneous case.

So, therefore, we find that this is nothing, but rho homogeneous plus  $J_{21}$  by  $J$  into rho 1 minus rho 2. So, what do we find we find that for the expression of alpha we had found that the correction factor was in the terms of a multiplicative term which was a function of  $J_{21}$ . And in this particular case what we find? We find that rho drift flux, it is again a correction factor; it is additive in this particular case which is again a function of  $J_{21}$  and that additive term when added to the homogeneous value it gives us the rho drift flux.

In the similar way we can expect to find out  $U_1$ ; we can expect to find out  $U_2$ . And once we find out  $U_1$   $U_2$ , we can expect to find out momentum flux and things like that; kinetic energy as obtained from drift flux; the momentum flux as obtained from the drift flux model we can find out everything just try and do  $U_1$  and  $U_2$ . And see how this  $U_1$   $U_2$  varies from the homogeneous  $U_1$  and  $U_2$  or rather the  $U_1$   $U_2$  values we had

obtained from the homogeneous flow theory. Just do it and let us see what you are going to get.

How to express  $U_1$ ?  $U_1$  is equal to  $Q_1$  by  $A_1$  or in other words in terms of  $J$  very true  $W$  by  $\rho_1 A_1$  in terms of  $J$ . Because see what we want? We want to express it in such a way that we can compare the  $U_1$  we have obtained from drift flux with the  $U_1$  that we have obtained from the homogeneous flow theory. From the homogeneous flow theory, no slip means  $U_1$  equals to  $J_1$ . So, in this particular case what is the expression of  $U_1$ ? Very true or in other words is it  $J_1$  by  $1 - \alpha$ ?  $Q_1$  by  $A_1$  is equal to  $J_1$ .

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$$f_{DF} = \frac{J_2 - J_{21}}{J} f_2 + \frac{J_1 + J_{21}}{J} f_1$$

$$= \left( \frac{J_2}{J} f_2 + \frac{J_1}{J} f_1 \right) + \frac{J_{21}}{J} (f_1 - f_2)$$

$$= f_{\text{homogeneous}} + \frac{J_{21}}{J} (f_1 - f_2)$$

$$U_1 = \frac{J_1}{1 - \alpha} = \frac{J_1}{J_1 + J_{21}} \cdot \frac{J}{\left(1 + \frac{J_{21}}{J}\right)}$$

$$U_1 = J_1 = J = U_2 = J_2$$

So, therefore, this is going to be  $J_1$  by  $1 - \alpha$ .  $1 - \alpha$  the expression I have already derived for  $1 - \alpha$  this particular expression. So, you can substitute this particular expression and then you can find out what is the value of  $U_1$  in this particular case. Just do it and tell me what is the expression that you are getting. This should be  $J_1$  by  $J_1 + J_{21}$  or in other words this is  $J_1$  by  $J_1 + J_{21}$ .

So, therefore, we know that for the homogeneous flow theory  $U_1$  would have been equal to  $J$ . Because under the homogeneous flow condition we get  $U_1$  equals to  $J_1$  equals to  $J$  equals to  $U_2$  equals to  $J_2$ . All the the 2 phases they flow at the same velocity due to which we say that there is no slip between the phases or in other words  $K$  the slip ratio equals to 1 relative velocity  $U_2 - U_1$  equal to 0.

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$$f_{DF} = \frac{J_2 - J_{21}}{J} f_2 + \frac{J_1 + J_{21}}{J} f_1$$

$$= \left( \frac{J_2}{J} f_2 + \frac{J_1}{J} f_1 \right) + \frac{J_{21}}{J} (f_1 - f_2)$$

$$= f_{\text{homogeneous}} + \frac{J_{21}}{J} (f_1 - f_2)$$

$$u_1 = \frac{J_1}{1-\alpha} = \frac{J_1}{J_1 + J_{21}} \cdot \frac{J}{\left(1 + \frac{J_{21}}{J_1}\right)}$$

$$u_2 = \dots$$

$u_1 = u_2 = J = u_2 = J_2$

So, therefore, U 1 equals to U 2 and since alpha equals to beta therefore, U 1 equals to J one. So, therefore, U 1 equals to U 2 equals to J 1 equals to J 2 which gives alpha equal to beta. From the drift flux model, what do we get? We get U 1 is the homogeneous velocity divided by a correction factor again which is a function of of J 2 1; in this case it is in a slightly different form.

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$$u_2 = \frac{J_2}{\alpha} = \frac{J_2}{\frac{J_2}{J} \left(1 - \frac{J_{21}}{J_2}\right)}$$

$$= \frac{J}{\frac{J_2 - J_{21}}{J_2}} = \frac{J}{\left(1 - \frac{J_{21}}{J_2}\right)}$$

*Homogeneous in-situ velocity of phase 2*

$$u_2 = u_1 = J_1 = J_2 = J \text{ (NO slip condition)}$$

$$\alpha = \beta = \frac{Q_2}{Q_1 + Q_2} = \frac{J_2}{J_1 + J_2}$$

Same way U 2 if you try to find out; U 2 equals to here it is going to be slightly congested I will do it in a different paper. U 2 this is equal to again J 2 by alpha. What is

alpha equals to? We have already found out the derivation of  $\alpha = \frac{J_2}{J_1 - J_2}$  by  $J_2$ . Substitute this value of alpha here, then what do you get? This is  $\frac{J_2}{J_1 - J_2} \cdot \frac{J_1 - J_2}{J_2}$ ; or in other words again taking up  $J_2$  on the top we get this as  $\frac{J_1 - J_2}{J_2}$ ; or in other words this is  $\frac{J_1}{J_2} - 1$ .

So, we find this is the homogeneous in-situ velocity of phase 2 phase 1 everything is the same. So, with this homogeneous in-situ velocity if this particular correction term is multiply then we can get the actual velocity and definitely it will be a much better approximation of the actual situation. So, what we have done? First we have tried to see how the drift flux concept it modifies the homogeneous flow theory? What it does? Homogeneous flow theory it assumes  $U_2 = U_1 = J_1 = J_2 = J$  or in other words this arises from the no slip condition. And from here what we get? We get  $\alpha = \beta = \frac{Q_2}{Q_1}$  by everything expressed in terms of input parameters, but we find that is grossly erroneous.

So, instead of that if we can introduce something some particular term which will be a function of the relative motion between the 2 phases then we will get a much more accurate value. How to do it? Now, for doing that what we have done? We have used the drift flux concept. And using the drift flux concept what we could is we could actually modify the expression of alpha; as compare to the expression of alpha which we get from the homogeneous flow theory. And then we found out that once the expression of alpha and  $1 - \alpha$  were modified accordingly, your mixture density has some particular or this mixture density can also be expressed as the homogeneous mixture density added or multiplied by some particular correction terms.

The local in-situ velocities again they are the homogenous flow velocity added, multiplied, divided by some particular correction term. So, from this what do we find? We could actually modify all the parameters that we have decided or rather that we can define for 2 phase flow. And these modifications all of them they are in terms of the normalized drift flux either with the volumetric flux of phase 1; or with the volumetric flux of phase two; or with the volumetric flux of the total mixture.

So, for all the cases that we have done till now you and you can try it for the momentum flux; you can try it for the kinetic energy, we will find that for each case the correction term it is a function of the normalized drift flux where it is normalized either with respect

to the volumetric flux of phase 1 or the volumetric flux of phase two or the total volumetric flux. So, therefore, we find that once we have some particular estimate of  $J_{21}$  where  $J_{21}$  can be estimated from some other equation in a straight forward manner.

Then that  $J_{21}$  can be substituted in all the equations that we have written down; for  $U_2$  for  $U_1$  whatever equations we have written down, wherever if we can substitute  $J_{21}$  from some other equation then in that case once we have an estimate of  $J_{21}$ ; we can find out all the parameters or we can modify all the parameters of the homogenous flow model to account for the relative slip between the phases and to give us a better estimation of the hydrodynamic parameters.

If this particular  $\alpha$  they are substituted in the pressure drop equation, then naturally that pressure drop expression is going to give us a better estimate of the actual situation. So, now, the next step will be how to find out  $J_{21}$ . Now, it should normally come from some particular type of a force balance sort of a thing. In that particular way we should be in a position of getting or estimating  $J_{21}$ . Now, usually drift flux model it is very suitable under certain conditions. What are the conditions? When the wall shear is negligible as compare to the interfacial shear.

Naturally, because when the wall shear is negligible only under that condition interfacial shear will be much more important. If the interfacial shear is much more important  $\tau_{12}$  if it is much more important, then naturally that arises to the relative motion therefore, drift flux or the  $J_{21}$  comes into picture. Usually, it is applicable for those conditions; it is much more applicable for gravity dominated situations; that means, usually for horizontal flow as compare to horizontal flow it is much more appropriate for vertical or near to vertical inclined conduits.

Now, before we go for discussing how to estimate the drift flux parameter? I would just like to discuss the advantages of the drift flux model. Now, see this is just an intermediate. For homogenous flow, what we had assumed? That everything is totally mixed up. So, accordingly we had one continuity equation, one momentum equation, one energy equation for the entire mixture; this is one extreme. Actually for the two fluid model, if it is a two phase system we should have a two fluid model, there we should have two continuity equation, two momentum equations, two energy equations.

And, in these momentum equations, energy equations the interaction parameter should be included. Now, drift flux model is just an intermediate between the two. It has a mixture continuity; it has a mixture momentum; it has a mixture energy and it has a may be a gas phase continuity equation. So, instead of six equations we have four equations here. So, how do we account for this loss of or the simplicity that we have incorporated by reducing two equations? We have to supplemented with additional constitutive relationships which will predict J 2 1.

Now, this is a simplification we are doing. Definitely, it will introduce some amount of inaccuracies. When these inaccuracies are not quite drastic as compare to the advantages it offers, we will definitely go for this model. If it introduces gross inaccuracies then we will not adopt it. Fortunately, we find that for most of the situations it is much more advantageous as compare to it is disadvantage is 1, it is simplified version of the two fluid model, because there are just four equations in place of six equations in the two fluid model and therefore, some constitutive equations have to be provided.

These constitutive equations they take into account the actual flow situation and accordingly these situations are provided. If more or less we have a proper estimate of the constitutive equations, then naturally it gives us much more accurate results and at the cost of much more simplified flow situations. So, let us see under what situations it is much more advantageous. First thing is definitely simplicity it goes without saying, it is much more simple as compare to the two fluid model. Secondly, it is applicable to a wide range of two phase flow problems of practical interest.

Now, remember one thing; see for example, the slack flow pattern, can you call it a completely separated flow pattern? Or can you call it a totally mixed flow pattern? It is actually if you think it logically it is neither, because if you think it logically you will find that in the slack flow there are bubbles which are going; these bubbles they move faster than the liquid through which it is flowing. So, definitely there is a relative motion in this particular case, but it is more or less you can say it is dispersed in the liquid phase. The two phases are not completely separated like they are in stratified or the annular flow patterns.

So, therefore, and interestingly this slack flow pattern it occurs over a very wide range of flow velocities. And more interestingly, see now actually we are going to the era of

miniaturization; we are going for smaller and smaller dimension tubes. When we reduce the dimension interestingly you will find slack flow occurs a much or rather it occurs over a much wider range of phase velocities. Now, what is slack flow particularly neither the homogeneous flow model nor the drift the two fluid model gives a very accurate results. The drift flux model is much better, because it takes into account the drift or the relative velocity of the Taylor bubble with respect to the liquid through which it is rising.

Same thing happens for the bubbly flow pattern, because whenever we talk of bubbly it is not always that very fine uniform bubbles are dispersed in the water or the liquid medium. If they are very fine bubbles, they uniformly dispersed that can occur only under very high phase velocities. Most of the situations what we find? You can even visit my laboratory and you can see there for the most of the situations it is the water phase and more or less large deformed bubbles are flowing through it. These bubbles they can be oblate spheroidal; they can be cap shaped; they can be elliptical.

But they are more or less defined and discrete bubbles. Now, moment the bubbles they have taken such a particular shape, they will have a relative velocity relative to the liquid through which it is rising. So, naturally under that condition homogenous flow model is grossly erroneous. Do you get my point?

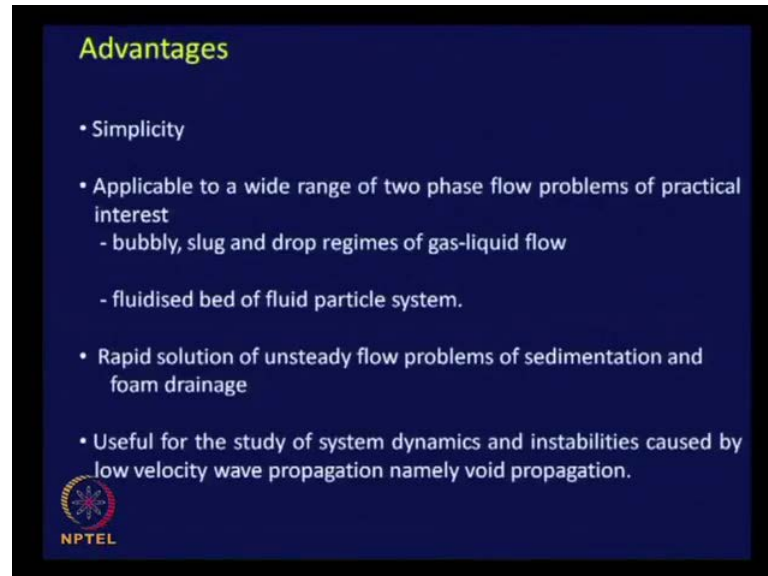
So, therefore, for the bubbly flow pattern, for the slack flow pattern, for the droplet flow pattern also droplet flow pattern usually we get it for heated tubes vapor liquid cases; and this droplet flow pattern they are always said when for the wispy annular cases. There also we find within the gas coat there are deformed droplets of liquid. For such flow situations we find that the drift flux model it is extremely advantageous and it is very simple; simplicity also we have to consider if it is very complex, if it takes a large amount of computation time then such models will not serve engineering purposes.

Event at the cost of or even at the loss of a small amount of accuracy also, if simplicity and faster computation time can be incorporated it is going to be much more advantageous. So, this is number 1; next thing is fluidize bed also for fluid particle system in fluidize bed also we find that the all the solid particles they are in a fluidized state either in the gas phase or in the liquid phase. There also there has to be a relative velocity between the solid and the fluid phase. Whenever, there is a relative velocity if



you ignore it and consider homogenous flow which we do at times just for the simplicity case if it gives more or less accurate results, but it does not serve our purpose.

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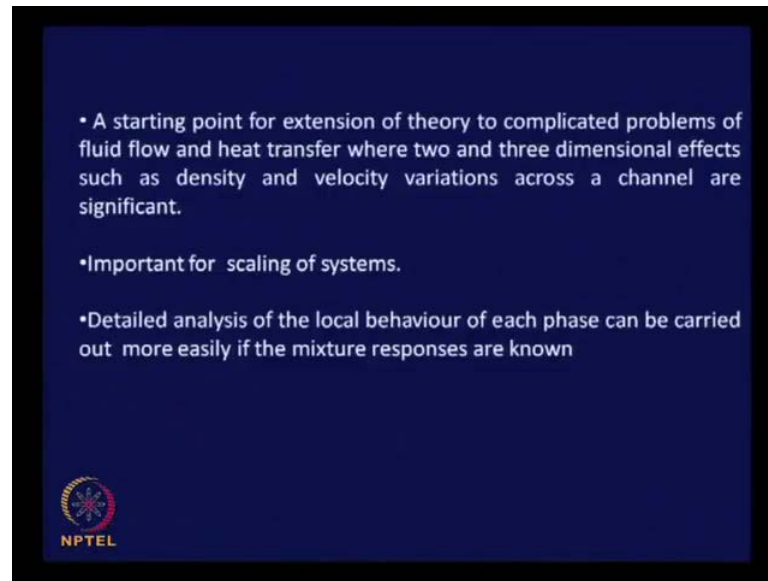


So, you will find that for most of the cases particularly for the intermittent flow pattern which we had defined in our flow pattern chapter; we find that for most of these intermittent patterns as well as a few of those dispersed pattern which we had discussed the drift flux model is going to be extremely advantageous. So, Firstly, simplicity; Secondly, it is applicable to a wide range of two phase flow problems which are practically encountered in industries. These bubbly slug drop fluidized bed they are actual actually these these things are encountered in industries.

Next we find that for rapid solution of unsteady flow problems.

Unsteady flow problems pertaining to foam drainage and sedimentation, we find that this gives us a very rapid solution for this. And again suppose there are instabilities and for the study of system dynamics, and instabilities which are cost by low velocity wave propagation namely may be a void fraction wave is being propagated. For the study of these things also it is very advantageous. How you will try? You will gradually understand as we go into the details of the drift flux model.

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Next, see 1 more thing which is very important which we do is in a drift flux model initially, what we have done? We have assumed 1 dimensional flow only. Accordingly, what we have assumed? We have assumed that everything varies in the flow direction itself; and there is no variation across the cross section. So, therefore,  $J_1$ ,  $J_2$ ,  $U_1$ ,  $U_2$  everything we assumed average over the cross sectional area.

But again this is not a very correct situation. So, therefore, we have to consider the variation across the cross section. Now, how do we do that? In the drift flux model, after I have completed the entire derivation part, after I have discussed how to find out  $J_2$ ,  $\alpha$  etcetera then we will see what are the corrections? Corrections also the idea we have taken from single phase flow itself. What are the corrections that has been incorporated to in order to account for the voidage profile velocity profile etcetera etcetera.

If they are not very drastic; if the corrections are not very much far removed from unity then they often give very accurate results. So, this is one more thing which drift flux model it enables us to account for the two dimensional and the three dimensional effects. For example, when there are velocity variations; you know for laminar flow, the velocity profile is is parabolic; when we go for turbulent flow, we assume a flat velocity profile. Now, when the velocity profile is parabolic or when the concentration profile in this particular case there are two phases.

Say if it is air water phase, most of the cases you will always expect that the air phase does not exist or its very less near the wall, because it is there is always a tendency of the liquid phase to wet the wall. So, naturally there will be a tendency for the gap phase to be concentrated towards the center. So, very frequently we find that voidage profile that also has a parabolic shape like the velocity profile. Now, if you assume that alpha is uniformly distributed over the cross section and to take that particular alpha. Will it give you a very accurate results?

Or considering the actual voidage profile and integrating it accordingly this particular profile that will give you a much more accurate results. So, the thing is we do not go into that much complicity, but we introduce some simple correction factors. Again whose idea we have got from single phase flows. So, once we can include what we get? We get an averaged alpha and then we multiply it by that correction factor. Correction just accounts for the fact that it is the properties not uniform across the entire cross section.

If the corrections are not very far removed from the unity, we find that the drift flux model the correction factors which we introduce they give us more or less accurate results. So, therefore, the next advantage which I have noted it down in the p p t is that it gives us or it provides us a starting point of the theory to complicated problems of fluid flow at heat transfer, where two and three dimensional effects across the channel are significant; there also it provides us accurate correction effects.

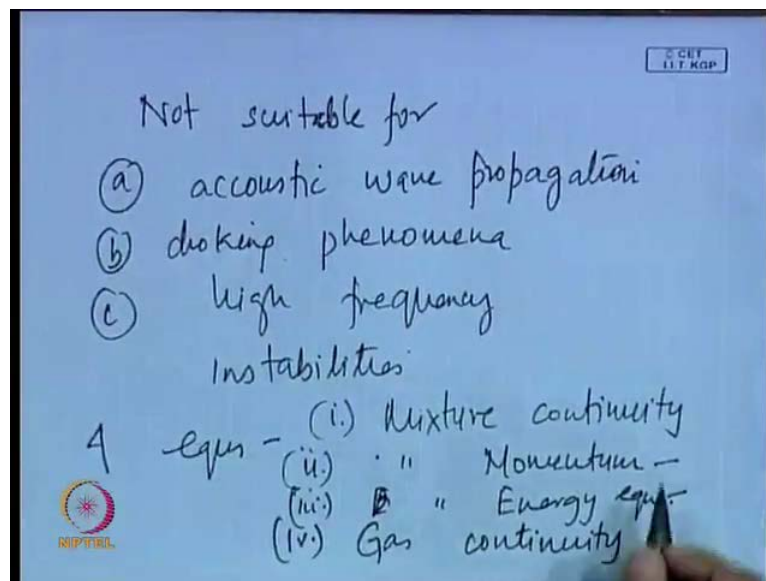
So, naturally when it has got so much versatility quite natural it is very important or very useful for scaling of systems; suppose we have performed experiments in a small scale in a pilot plant scale in a laboratory scale; we would like to scale it up for the industrial scales. So, therefore, if we have some way of accounting for the two dimensional, three dimensional effects. If we know how alpha can be related with  $J^{2/3}$ ; and we know that  $J^{2/3}$  it depends upon the fluid particle or the fluid wall interactions. So, therefore, in those particular cases the scaling up becomes very important; this is 1 more thing.

And the other thing naturally it has been found out that sometimes if we have to do a detailed analysis of the local behavior of each phase, it is probably sometimes much more easy if the mixture response is unknown. So, in this particular case we have the mixture responses since we know how the total mixture response. So, therefore, the detail analysis of the local behavior of each phase they can be carried out more easily.

So, due to its large amount of application coupled up with its simplicity, the drift flux model is going to be very very useful and we find that for most of the analysis of multiphase systems starting point is the drift flux model. It is not the homogenous flow model.

Definitely, if we go for the two fluid model where we consider the two fluids separately we will be doing this after completing this drift flux model. We will find that the equations are much more complex, you take much more time to solve them; they are much more elaborate and so naturally we will definitely do it, but that time you will see that well the drift flux model was a much simpler approximation; and the error introduced as a result of the simplification is not very large. But definitely, if you want to use drift flux model for stratified flow annular flow you will not get a correct result.

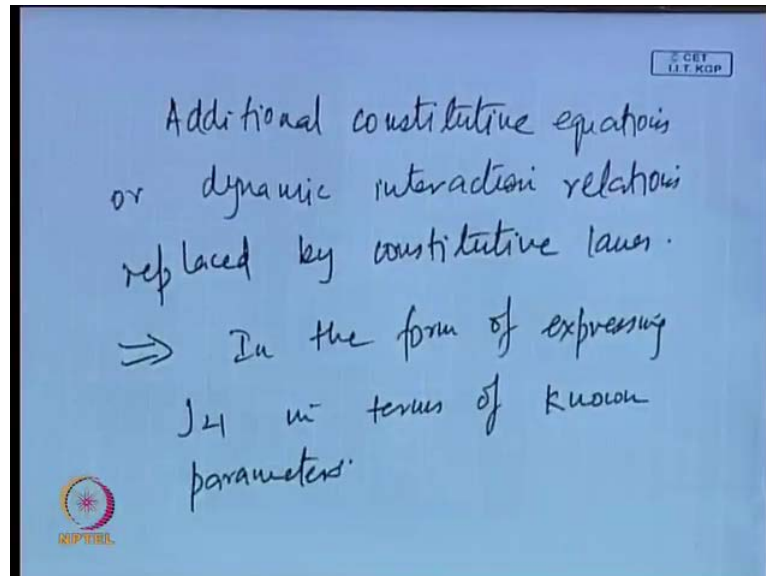
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So, therefore, these were the advantages of the drift flux model which I have written them down; these are the different advantages, but remember it is not applicable for all particular situations. For example, it is not suitable say acoustic these are slightly difficult things, but we do come across such particular situations in our recent researches and for these situations it is not at all suitable; one is for acoustic wave propagation then for the choking phenomena and for high frequency instabilities. For such situations, it is not at all suitable. So, therefore, remember one thing in the drift flux model, we have

used the drift flux concept and in this particular case we are having four equations. What are the four equations?

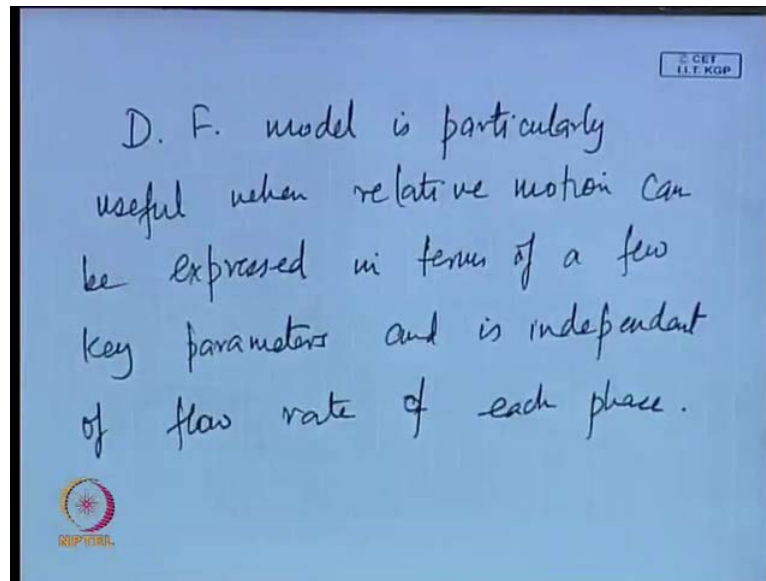
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It is the mixture continuity; next is the mixture momentum equation; next is the mixture energy equation and fourth one is the gas continuity equation; assuming that it is a gas phase or the dispersed phase continuity equation. And what are the equations that we have eliminated? We have eliminated 1 momentum equation and 1 energy equation in this particular case. Since we have eliminated two equations what we need to do is, we need to substitute these with additional constitutive equations or dynamic interaction relationships means since we have eliminated these two we have definitely considered their relative motion.

So, therefore, either we have to give additional constitutive equations or we have to give some dynamic interaction relations replaced by constitutive laws. Now, these particular constitutive equations they will naturally be in the form of expressing  $J_{21}$  in terms of known parameters. So, this will naturally be the particular approach; that the constitutive equations or the constitute laws which will be used they will naturally be in terms of expressing  $J_{21}$  either in some particular term it has to be expressed, but it has to be expressed in terms of known input parameters.

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And remember one thing that drift flux model is particularly useful when this relative motion can be determined by a few key parameters, it is independent of flow rate of each phase. So, just remember this thing, that the drift flux model I will just write down the sentences quite important. These are the keys by which we go to find out  $J_{21}$  since finding out  $J_{21}$  is the most important thing here. So, these are the clues by which we find  $J_{21}$ . Drift flux model, it is particularly useful when relative motion can be expressed in terms of a few key parameters and is independent of flow rate of each phase.

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So, this is one particular way by which you can do it. Now, usually we are finding out  $J_{21}$  there are two distinct approaches for finding out this relative motion. Now, the first approach is that we use the mixture field equations and then various constitutive axioms are directly applied to the mixture independent of the fluid model. This is one thing; we just choose the mixture equation and then from some constitutive axioms we just substitute them and we find out  $J_{21}$ . The other thing is we use the two fluid model.

And then from the two fluid model we try to find out the interaction terms and then substitute them and find out  $J_{21}$ . So, in the next class what we are going to do is we are going to discuss what are the advantages and disadvantages of the two approaches;

which approach is generally adopted and then using that approach we will try to find out some particular constitutive equation in order to determine  $J_2$ .

So, thank you very much.