Multiphase Flow
Prof. Gargi Das
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture No. # 12
Chocked Flow Conditions for Homogeneous Flow

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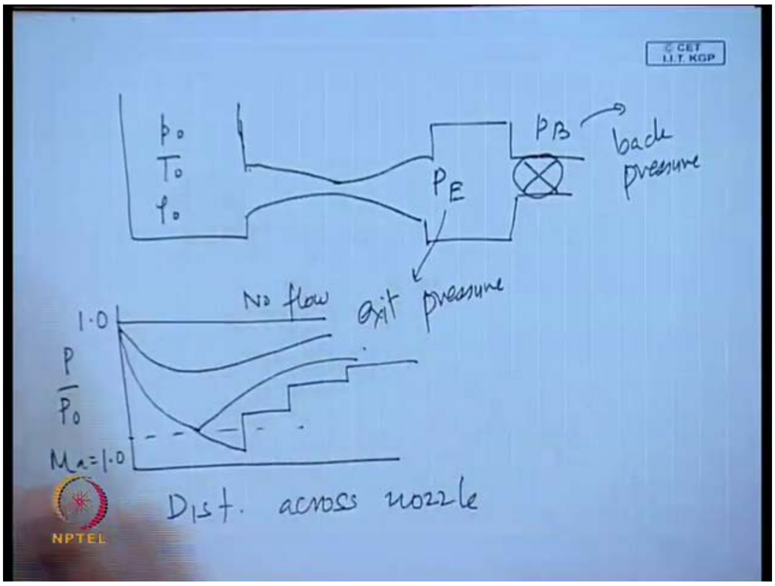

Well. So, before going to the chocked flow condition for homogeneous flow there is one particular question. So, I would like to clarify it that instead of converging nozzle if we have a converging diverging sort of a nozzle. So, under that condition what happens? Now, let us see again we have the tank and from that particular tank if we have something of this sort. So, here we know that the conditions are stagnation conditions; P 0, T 0, rho 0. Here it is again this is connected to this; actually I did not have plans to teach this, but we do not know whether it is going to get little more complex or not.

So, this is just the same particular situation which was there, but there we had a converging nozzle; here we have a converging diverging type of a nozzle. Now, here the upstream conditions are maintained; this is known as the exit pressure and this is known as the back pressure now. So, we find that the nozzle it discharges to this particular back pressure. Now, initially, again the valve is closed everywhere the pressure is P 0 and there is no flow just like what we had seen in the previous case.

Here also I would like to plot your say P by P 0 with distance across nozzle same thing. So, initially it is 1.0 here we do not have any particular flow. Now, what we do? We keep on reducing P B as we reduce P B, P E equals to P B and flow starts. So, therefore, we find that it is something of this sort.

We keep on reducing P B P equals to P B and we get a flow something of these goes on till we reach say M a equals to 1.0. So, after that what we find is that we keep on lowering P B in such a way that finally, we get sonic conditions at the throat.

So, we get something of this sort and then moment we get sonic conditions at the throat the flow rate is maximum here; just like it was in the previous case moment. We have sonic conditions at the throat then supersonic conditions here. So, under this particular condition the flow rate is maximum for this particular given nozzle; for this particular stagnation conditions, we find that we cannot reduce rather we cannot increase the flow rate any further.

Now, if we keep on reducing the pressures slightly more. So, we try to reduce back pressure more. Now, then what happens? Upstream the flow is not affected. Upstream the flow remains the same it does not respond, but in this diverging section what happens is flow initially, it becomes supersonic sort of a thing and then it tries to adjust itself by a series of shockwaves; the normal shock which are standing inside the nozzle.

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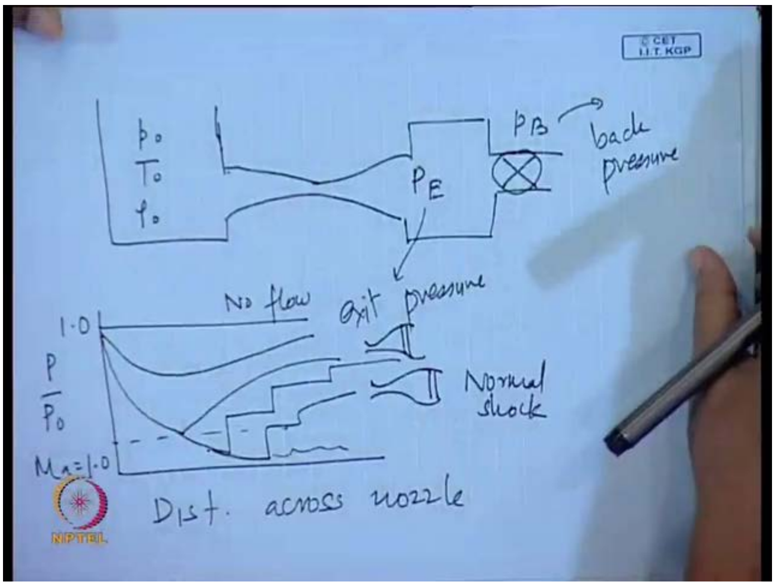

I would not like to go into details of shockwaves because compressible flow is not my topic. My topic is multi phase flow. I just wanted to teach a little amount of this. So, that some portion or rather you have some ideas of compressible flows and you can correlate it when you come back across that. So, the point is what happens initially, when initially there is no flow; when P B is closed everywhere the pressure is constant.

Gradually, we open it; as you open it P B reduces; when P B reduces P E also reduces, but P E equals to P E and so, P E is also controlled by this particular valve. And, as this happens obtains the graph which we had obtained in the previous case. Now, this keeps on continuing till we get the maximum particular flow rate at M a equals to 1. This is the maximum for the given stagnation conditions and the given nozzle design.

Now, if we reduce the back pressure further then we find that the flow upstream of the throat that does not respond at all, but in the downstream section or the diverging section the flow it initially become supersonic and then it adjusts itself to the back pressure P B because finally, it has to come to this pressure P B. So, therefore, then it adjusts itself to the back pressure P B by means of shockwaves.

And, in these cases we find that the position of the shock it moves downstream as P B is decreased. As we keep on reducing P B we find that the upstream pressure it does not change, but the downstream pressure lot of shockwaves etcetera they come this shockwaves; they are initially standing and then they gradually propagate downstream. So, in these particular cases the position of the shock, it moves downstream as P B is decreased and finally, normal shock which find they stand right at the exit plane. And the flow in this particular section, it is now supersonic and after that we cannot adjust it any further.

So, therefore, the same thing happens, but in this particular case we can go from this subsonic to the supersonic zone; but that to also till a particular point when chocked flow conditions has restraint moment chocked flow conditions has reached in the neck region we cannot do anything more. Although the flow in the upstream region is not affected, but in the downstream region we find normal shocks are there and these shocks they start propagating and the flow rate cannot be increased any further.

So, if you see these particular cases here the condition it is something like, in this particular way these shockwaves they are generated and these normal shockwaves they are propagated here, and in these particular then gradually oblique shocks they start and so on and so forth. So, that goes into the area of shockwaves. So, this was all that I had to tell you about compressible flows.

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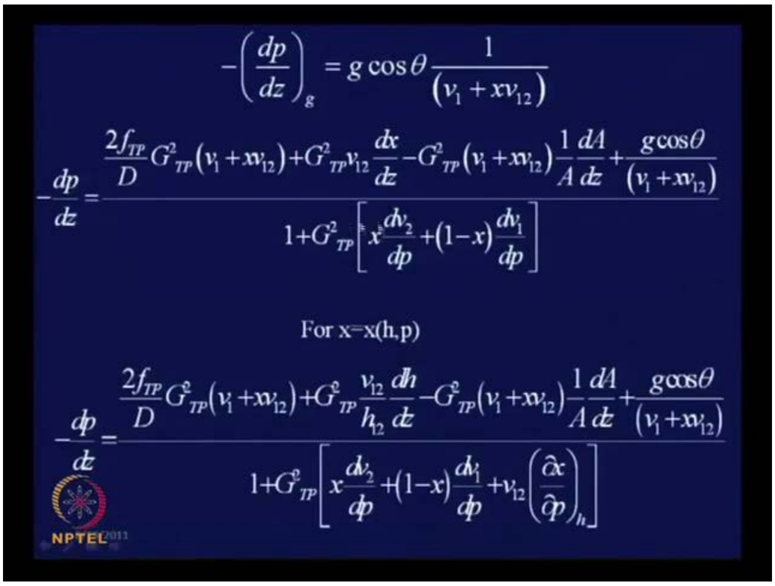

Now, our main idea about teaching you all these things they started just because we had a denominator if you remember in the homogeneous flow situation. We had this for the homogeneous flow situation if you see; we had this particular denominator and we tried to find out that for this particular denominator it signifies. Now, if you remember, in our introduction chapter when we were doing the introduction what I had tried to do?

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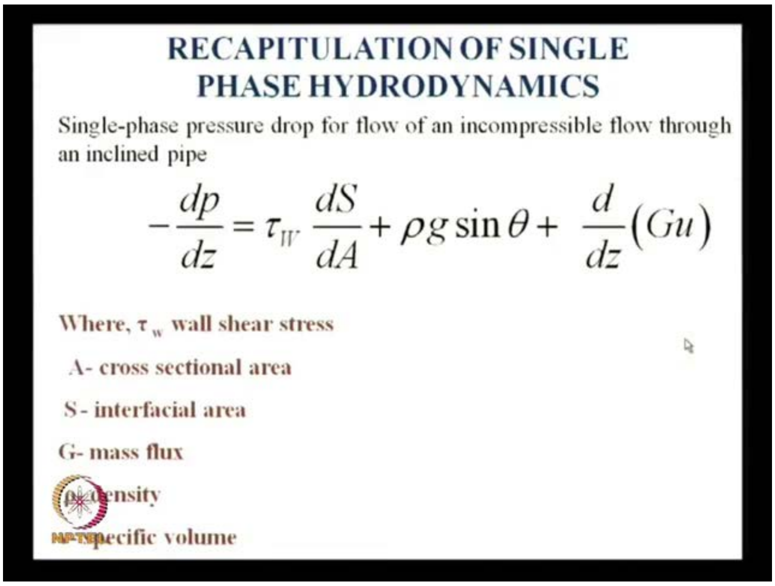

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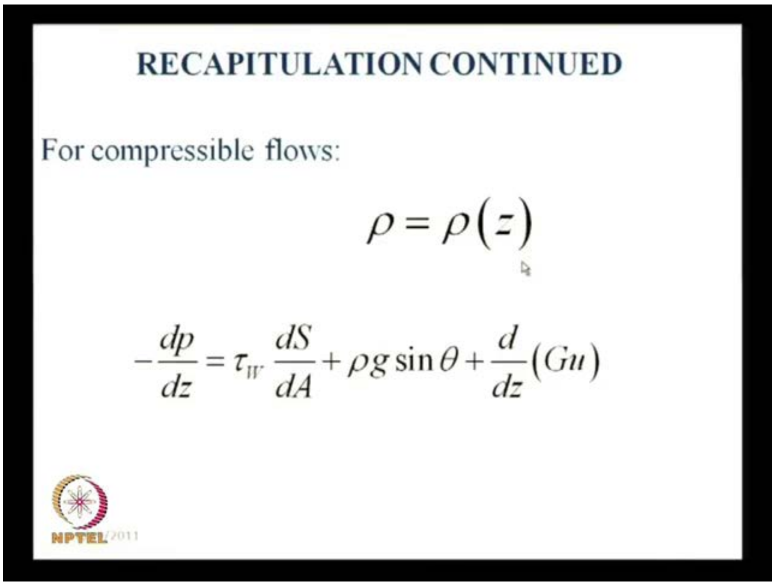

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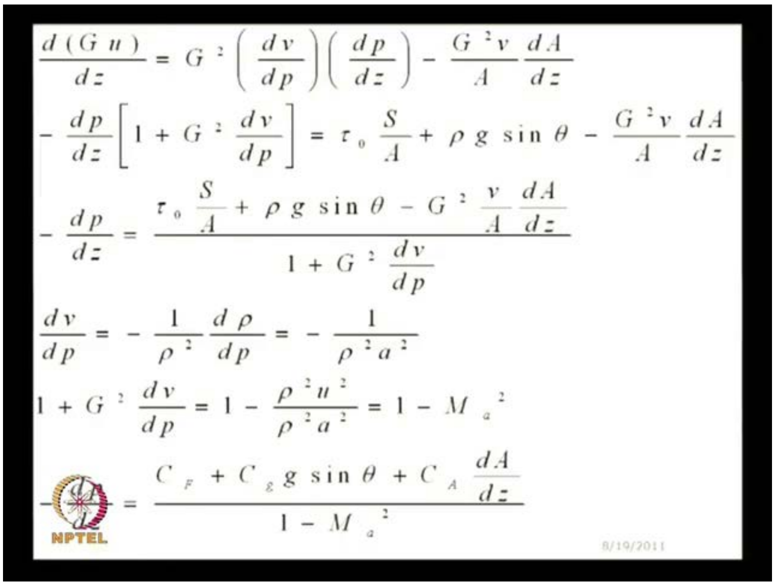

I had tried to deduce the continuity and the momentum equation for incompressible flows, I had tried to deduce it and this was the case. And, then I told you that for compressible flows what happens? In this particular case, your rho the G u, u is a variable pi because rho varies and since rho varies with z. So, therefore, accordingly the acceleration pressure drop, it takes some different forms and finally, we had come to this particular denominator.

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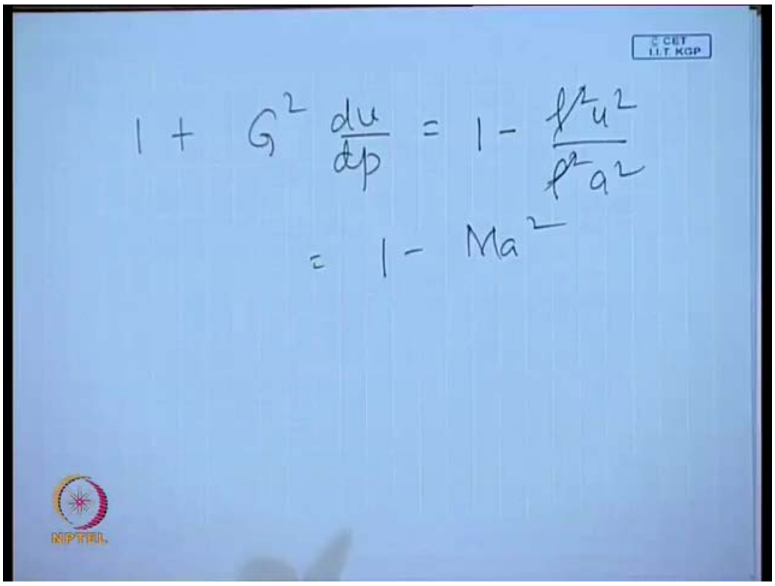

Now, if you notice this particular denominator let us see for this denominator, I have it is already done, but I would like to do and explain this further to you; this was the denominator for compressible flows. Now, what is G equals to? We know G equals to rho u that we already know or G square equals to rho square u square. And, what is this d v d p equals to? This is nothing, but equal to 1 by rho square d rho d p or this nothing, but equal to, can we write it in this particular form and what is this equal to d p zero a square right. So, therefore, these are nothing, but minus 1 by rho square a square.

So, therefore, your 1 plus G square d v d p this is nothing, but equal to 1 minus rho square u square by rho square a square; or in other words, we find the denominator which I had already told you that we will be discussing the significance of the denominator later this is 1 minus M a square. So, therefore, what do we find? We find that the denominator in this particular case, this gives you an idea about the condition where M a equals to 1 or this gives you an idea about the chocked flow condition.

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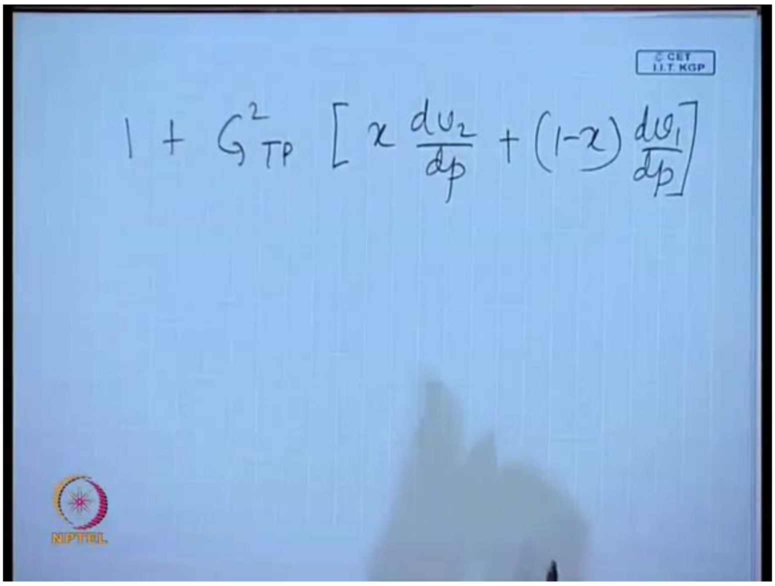

So, therefore, the denominator for single phase compressible flows this corresponds to the choked flow condition or it M a equals to 1. Now, we find that if we look at the denominator which we had obtained for the homogeneous case. What was the denominator that we had obtained for the homogeneous case? Let us see now, that was 1 plus G square 2 phase x d v 2 d p plus 1 minus x; this was the denominator which we had obtained.

Now, the point is, if the denominator under that condition had corresponded to chocked flow conditions then in this particular case also this denominator should also correspond to a Mac number of two phase flow under homogeneous flow conditions. Do you agree with me? It is just simply fluid flow; we have simply done what we had done for the compressible flow case. What extra have we done; we have taken into account certain averaged parameters or average properties and instead of G rho we have G T P rho T P and so on and so forth.

And, we have a mass fraction quality here, but the same approach was used. We have used the continuity equation; we have used the momentum equation and we have considered the acceleration pressure drop due to the density changes of the fluid. Usually, one fluid is compressible; the other is not, but to keep matters generalized we have considered compressibility of both the phases.

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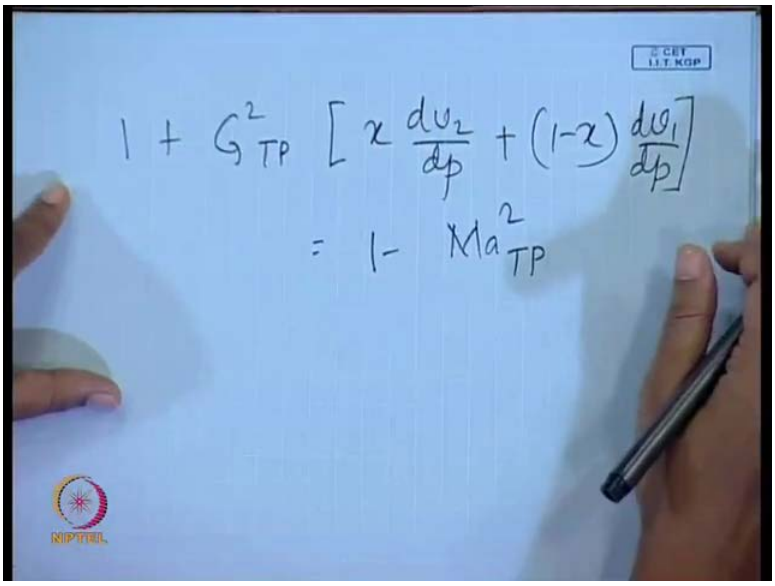

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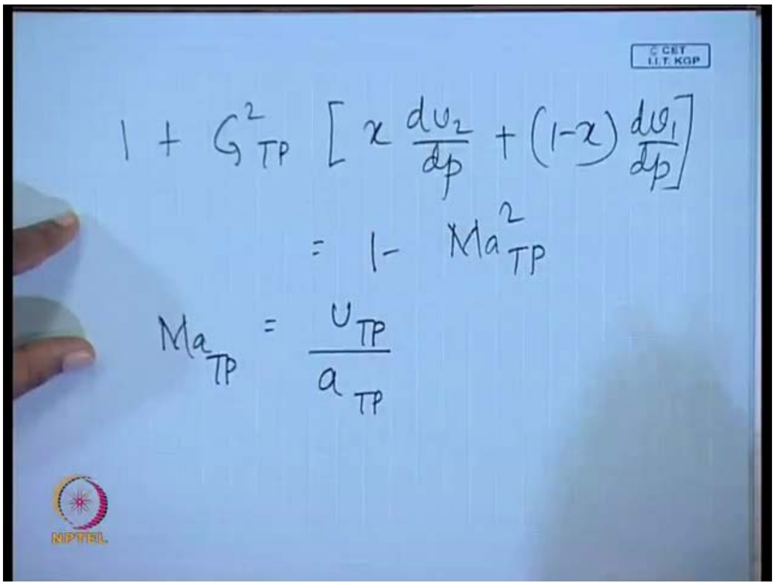

Finally, we have arrived at the denominator which is of the same form that we had obtained for the compressible fluid flow cases. So, therefore, if this corresponds to 1 minus M a square for single phase compressible flow; this must correspond to 1 minus M a square for two phase homogeneous flow. So, therefore, just like we know that the Mac number for compressible flow is equal to U by a. In this particular case, this particular Mac number for two phase this should have a form of U T P by a or a T P.

There one thing you remember, we found that velocity of sound a, it was a constant; it depends upon that particular material for air; it is one particular value for water; it is one particular value. So, a is a constant; we do not knowing a in this homogeneous particular case what is going to be a T P, but definitely a T P must correspond to the velocity of sound in this two phase flow under homogeneous flow conditions; under the present circumstances.

Under this particular pressure, temperature, composition etcetera the speed with which sound will be propagating in this two phase flow medium under homogenous flow of condition that should correspond to a T P. This part all of you agree with me. Now, let us see what is the expression of a T P? Is it a constant like single phase flow or does it depend on any other parameter? And, if it depends on any other parameter what are the parameters? How to evaluate? How to estimate or rather how to quantify a T P in terms of known measurable parameters?

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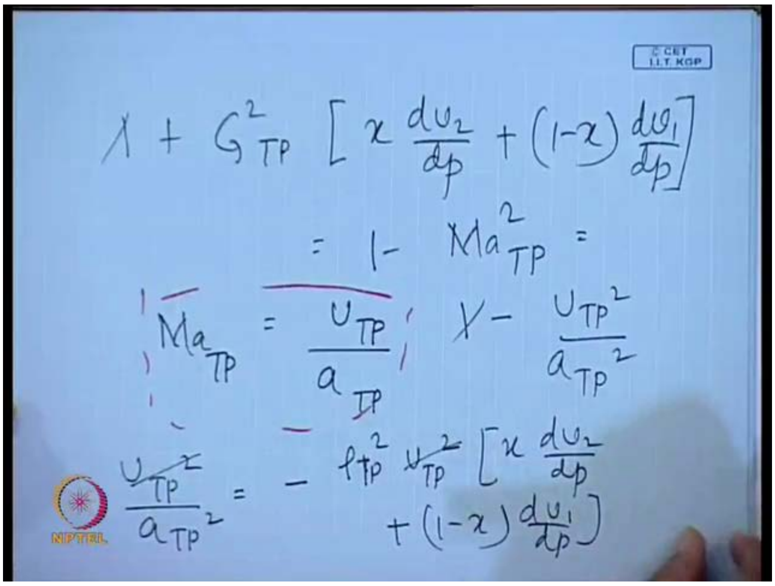

So, therefore, what do we find that in this particular case? If this expression has to be equal to M T P; this has to be 1 minus U T P square by a T P square or in other words, we find U T P square by a T P square, this should be equal to minus G square T P which is again rho T P square U T P square into x d v 2 d p plus 1 minus x d v 1 d p. This is acceptable again we can cancel the two. So, that can get a T P in terms of certain measureable things. So, what do we get?

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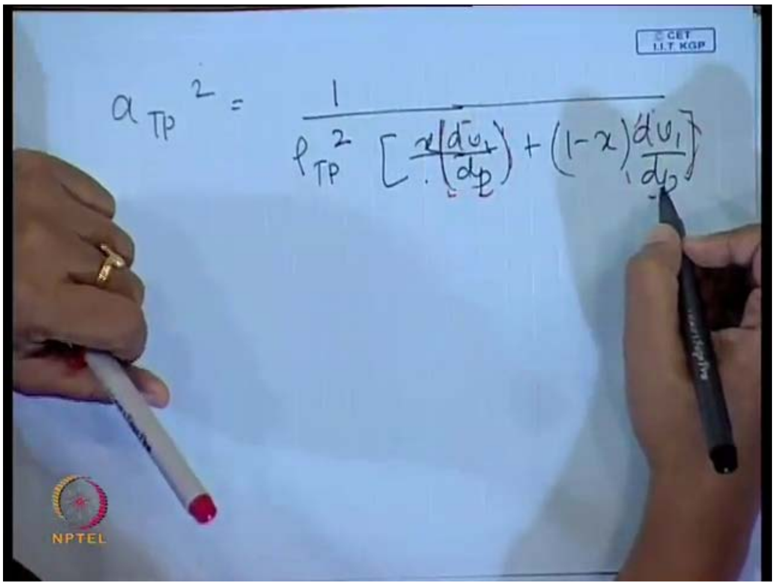

Then we get here 1 a T P square, this should be equal to 1 by rho T P square x d v 2 d p plus 1 minus x d v 1 d p we should get something of this sort. Now, what about these terms? This d v 2 d p and d v 1 d p we have already derived d v d p same form it should come. So, therefore, these should signify the velocity or the acoustic velocity in fluid one and fluid two under these pseudo homogeneous conditions.

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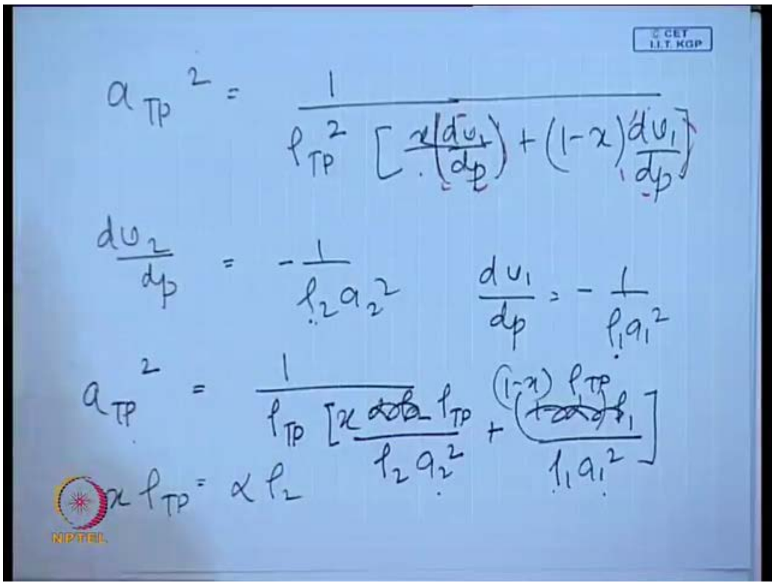

That means instead of this two face flow, if fluid one would have been flowing under the same conditions of two phase; same conditions of temperature, pressure etcetera. Then the velocity of sound which would have been there is represented by d p d rho 1 in this particular case and d p d rho 2 for fluid two. So, therefore, how can we define these particular things? This d v 2 d p this is nothing, but equal to minus 1 by rho 2 a 2 square.

And d v 1 d p is nothing, but minus rho 1 a 1 square, where a 2 and a 1 are the acoustic velocities in fluid two and fluid one under this pseudo homogeneous conditions or under the conditions of this homogeneous flow. So, therefore, from here we get that instead of what we have written it down here. So, in place d v 1 d p and d v 2 d p we can substitute these particular terms yes, now let us substitute them and let us find what we get?

If we substitute we get a t p square this is equal to 1 by rho t p, let us take 1 rho t p inside and we get alpha rho two by how do we get it I am going to tell you. What have I done? Let me do one thing this will not be easy for you let me simply take 1 rho t p. Here, I can write it in this particular form yes or no. You first tell me I have just substituted d v 2 d p and d v 1 d p in terms of these two these two parameters this I have already derived just now.

And on substituting them we find that instead of d v two d p I have written rho 2 a 2 square and instead of d v 1 d p I have written rho 1 a 1 square, and here I have taken just 1 rho T P inside. So, it is x rho T P and this is 1 minus x rho T P. Till this much I hope you do not have any problems. Now, if you remember I had derived one particular expression if you remember that expression x rho T P equals to alpha rho 2.

How did I get that? What did I tell you? That rho t p is the weight of say one unit volume of two phase mixture. So, therefore, k g of two phase mixture in say 1 liter two phase mixture. What is x rho T P? It is the amount of phase two in this particular 1 k g mixture. Try to think and tell me what is x into rho T P? Rho T P is the total weight of unit volume suppose I have 1 liter or 1 meter cube of mixture then what is the total weight of this 1 liter mixture it is rho T P.

Now, in this 1 liter mixture, the weight is rho T P. What is the weight of phase two here? It is naturally the mass fraction of this rho T P k g which comprises of phase 2. Therefore, if the total mass of the two phase mixture is rho T P k g's, then the mass of phase two is X into rho T P k g's where x is the mass fraction of phase 2 in this particular amount.

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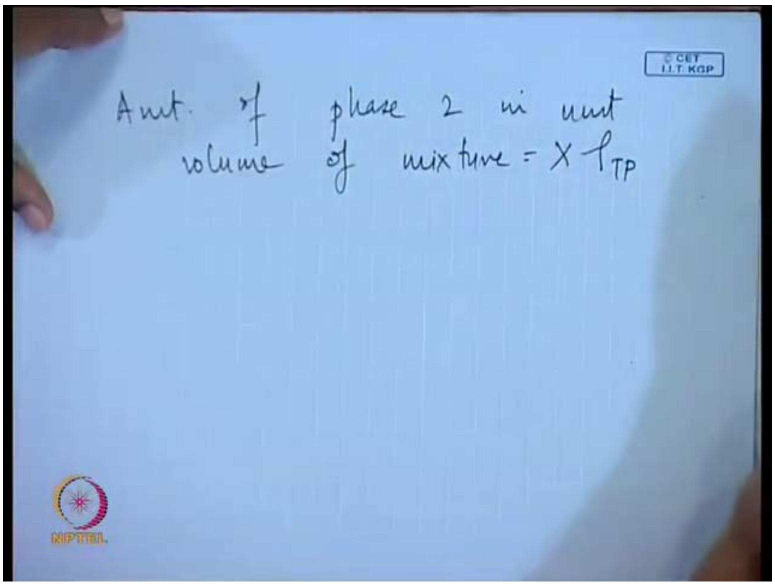

So, therefore, amount of phase 2 in unit volume of mixture is the total weight of the mixture and the mass fraction of it which is there. Now, in this particular unit volume what is the volume of phase 2? Alpha, think and say. What is the total volume of phase 2 in unit volume of the mixture? It is the volume fraction of phase 2 in the mixture; X is mass fraction; alpha is volume fraction.

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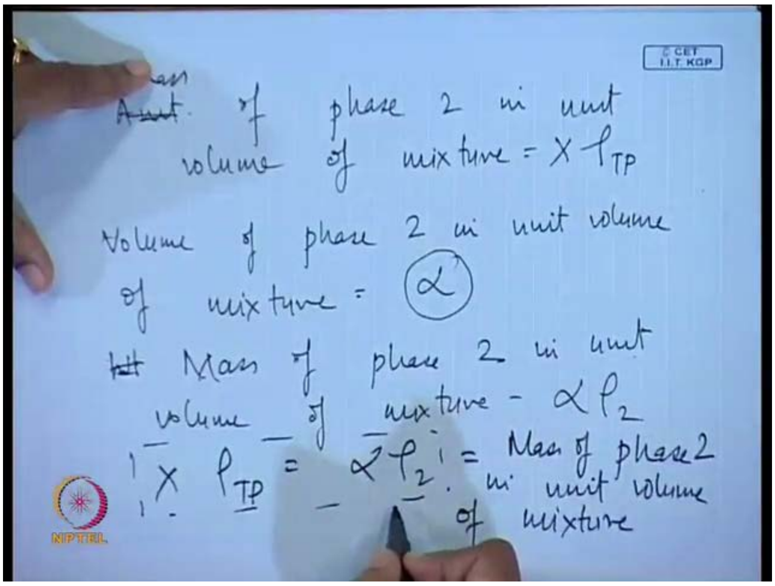

So, therefore, the volume of phase 2 in unit volume of mixture this is equals to alpha c liter whatever it is. What is the weight of this alpha meter cube or alpha liter of phase two? The weight or mass of phase 2 in unit volume of mixture; in unit volume of mixture, the volume of phase 2 is alpha. What will be the mass of this alpha? Alpha into rho 2. So, therefore, we find that mass of phase2 in unit volume of mixture is alpha rho 2 and this is the mass of phase 2 in unit volume of mixture is also X into rho T P. So, therefore, we find X rho T P has to be equal to alpha into rho 2.

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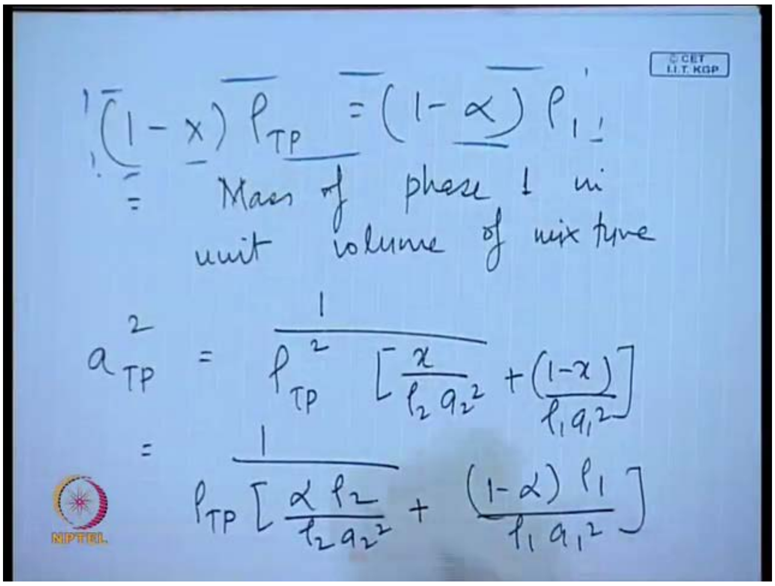

Do you get my point? Because both of these X rho T P and alpha rho 2, both of this signify mass of phase two in unit volume of mixture. This particular relation is very important; it will help you to solve several problems. So, these are equal and they are equal to mass of phase 2 in unit volume of mixture. So, I can deduce this particular relationship; same way I can also write down 1 minus X rho T P equals to 1 minus alpha rho 1 where this signifies mass of phase 1 in can I do this. So, therefore, these are the two relationships that I have deduced and these two relationships are very important.

Now, let us look at a T P square, the two phase your acoustic velocity this particular expression. What was the expression? It was rho T P square x by rho 2 a 2 square plus 1 minus x by rho 1 a 1 square. This was the expression which I have obtained; probably there should have been a minus sign, minus has cancelled out. So, this is the expression; now, if I take 1 rho T P inside then what do I get rho T P into x and that is nothing, but equal to alpha into rho 2.

So, therefore, I take 1 rho T P inside and then I get 1 by rho T P alpha rho 2 rho 2 a 2 square again instead of 1 minus x rho T P, I can write 1 minus alpha rho 1 by rho 1 a 1 square. Can I do this? Yes. So, these cancel out and these cancel out, and this rho T P also I can simply write it down as alpha rho 2 plus 1 minus alpha rho 1. Why am I doing all these things? So, that I can express two phase acoustic velocity in terms of certain known parameters and in terms of the minimum possible number of known parameters. So, just for that reason I am trying to do this entire endeavor.

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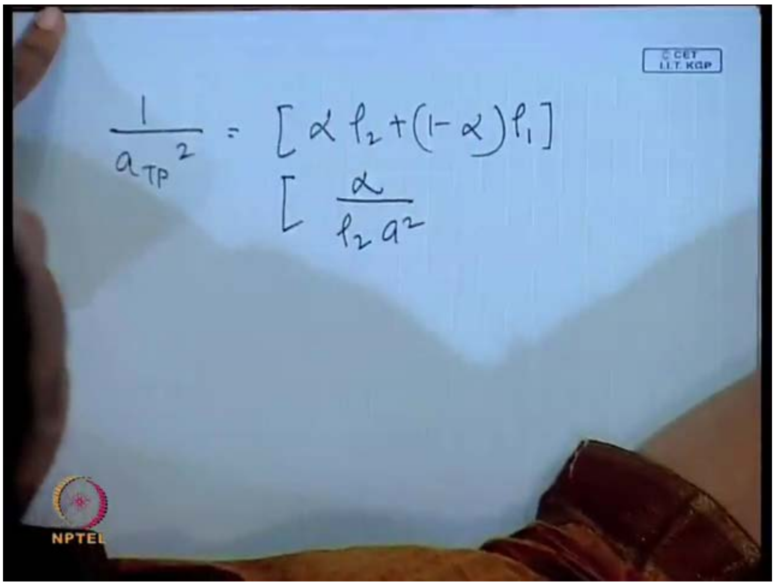

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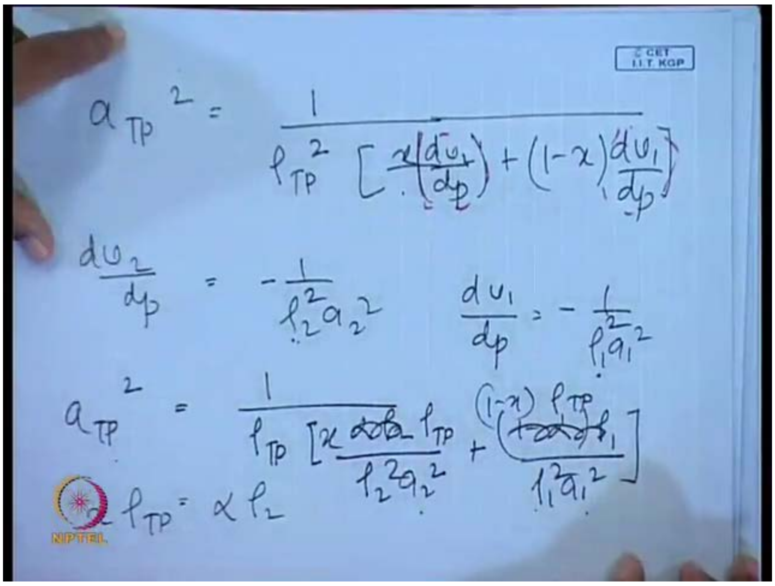

So, therefore, I can substitute this particular rho T P in terms of alpha rho 2 plus 2 minus alpha rho 1. Let me do it and let me see what I get. So, therefore, I get say 1 by a T P square that is alpha rho 2 plus 1 minus alpha rho 1 into alpha by rho 2 a 2 square; there were squares here. So, did I make any more mistakes anywhere let me see? Here also there should have been squares; please correct these things; these were all squares; just correct these things. There should have been squares I missed out them.

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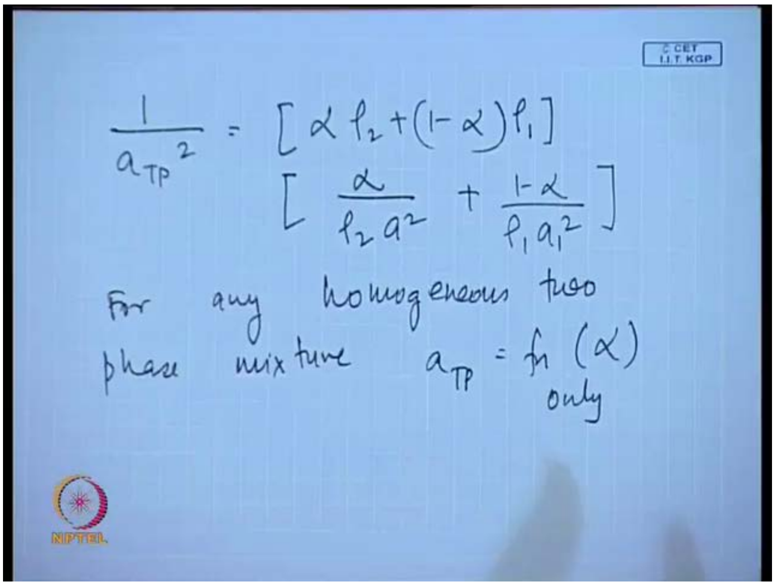

They are all squares I had missed this out anyhow. So, therefore, just correct all of these are squares I had already deduced them I believe when I had, see here itself I had deduced to rho square rho square. So, this was the thing d v d p equals to minus 1 by rho square a square. So, from there it had emanated. So, therefore, I find that a T P square equals to alpha rho 2 a 2 square plus 1 minus alpha rho 1 a 1 square.

So, this is the expression which I have obtained. So, what do I see from these expression we see rho 1, rho 2 they are constant. So, a T P it is definitely a function of alpha only. So, for any homogeneous two phase mixture, this is just for homogeneous flow conditions only or nothing else, a T P is a function of alpha only. So, therefore, if we change alpha under homogeneous flow condition my two phase sound velocities is going to change number one. Number two, this shows that unlike single phase flow your acoustic velocity for two phase flow condition is not a constant, it is a function of composition.

This is very important for single phase flow; it was a constant characteristic of that particular medium. In this particular case, it is not a constant characteristic of that particular medium; it depends upon the acoustic velocities of single phase flows, but along with that it also depends upon the composition of the two phase flows. Now, if it depends upon the composition of the two phase flow then definitely with compositions a T P should change or the two phase acoustic velocity should change. And, there definitely must be one particular value of alpha for which a T P will be a maximum or a minimum.

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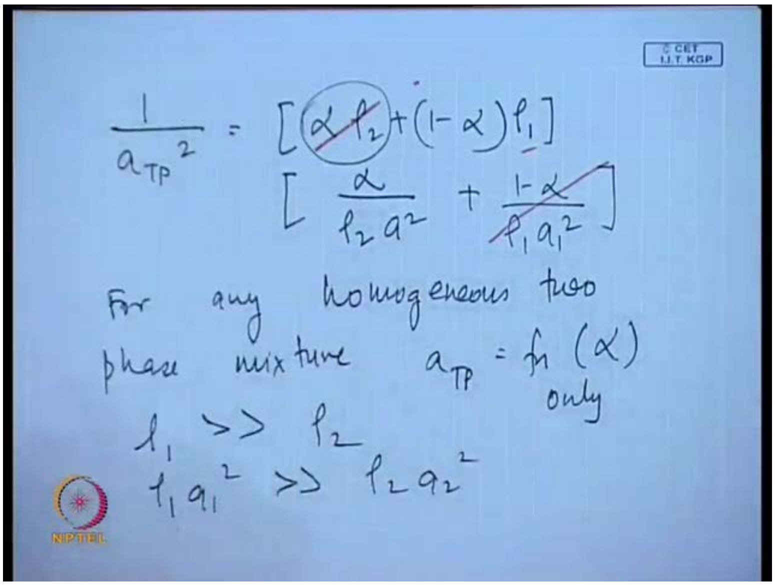

Do you agree? So, therefore, we can actually manipulate or we can actually control a T P, or we can actually modify, or we can have a control over a T P just by adjusting the compositions of the two phases constituting the two phase flow. And, how to get that particular optimum value of alpha? Definitely, for that particular optimum value of alpha d a T P d alpha has to be 0. Now, let us observe this particular equation; now, in this particular equation just know that usually it is a gas liquid or a vapor liquid mixture rho 1 has to be much greater than rho 2.

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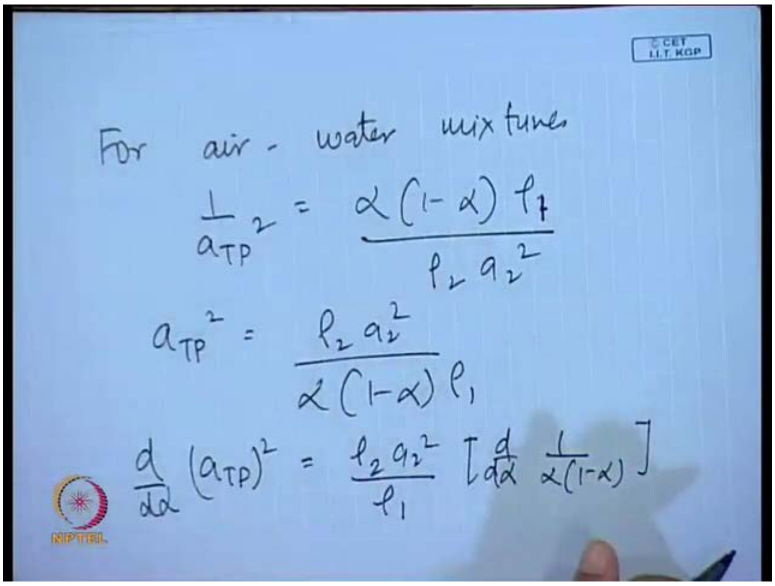

Do you agree with me? And, rho 1 a 1 square has to be much greater than rho 2 a 2 square. So, if that is the case then this particular term it disappears off, because rho 1 is much greater than rho 2. And, this particular term should also disappear off because since this is much greater than this. So, therefore, usually we find that 1 by a T P square for air water mixtures; this is not for anything else just for air water or vapor liquid mixtures only we get this.

What do we get? 1 by a T P square then this is nothing, but equal to 1 minus alpha rho 1 into alpha by rho 2 a 2 square. So, rearranging and writing we get alpha into 1 minus alpha rho 1 by rho 2 a 2 square. Now, this is a function of alpha since this is a function of alpha therefore, what we get that a T P square this is equal to rho 2 a 2 square by alpha into 1 minus alpha into rho 1.

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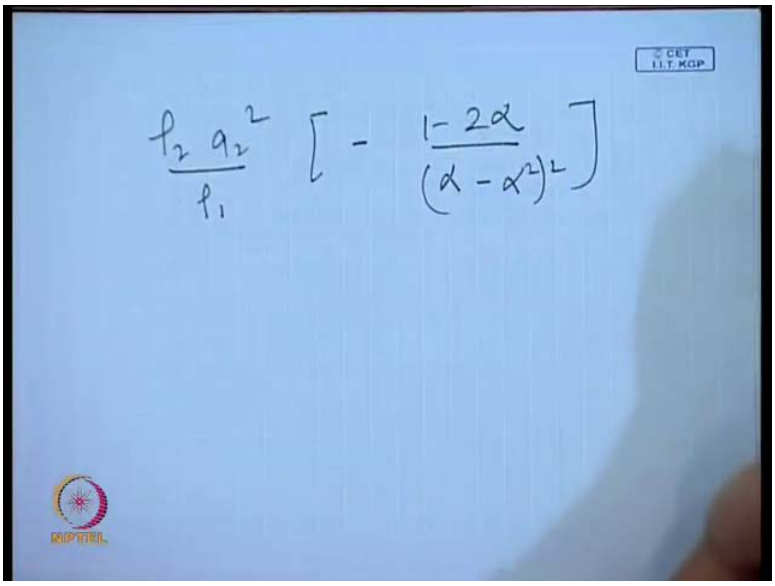

And, what is d a T P square d alpha? Now, this we know, this will be equal to just simply if we differentiate it we get rho 2 a 2 square by rho 1; you can perform the differentiation and you can find for yourself it is just d by d alpha of 1 by alpha into 1 minus alpha, because a 2 a rho 1 rho 2 all of those they are constants. So, therefore, this gives us rho 2 a 2 square by rho 1 minus 1 minus 2 alpha by alpha minus alpha square whole square.

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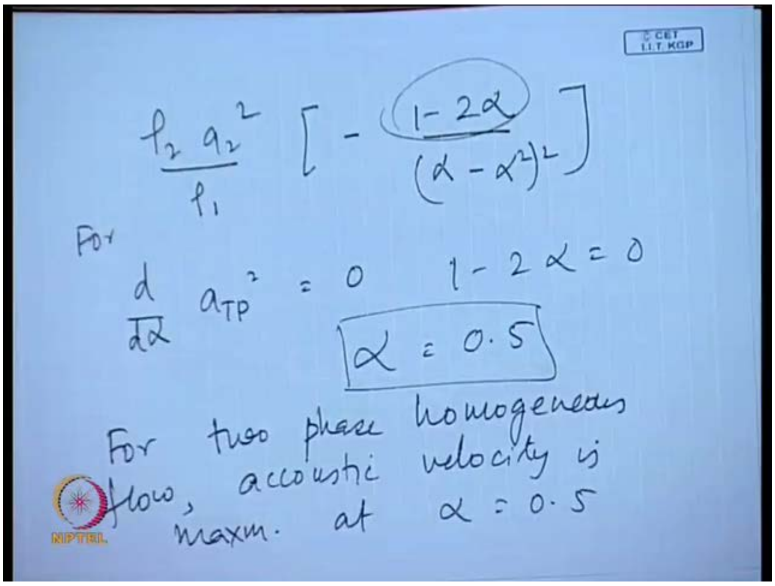

You see this difference whatever I am doing, I am just doing it; you are copying it down. Please remember you have to perform these differentiations on your own; these computations on your own. So, that you can perform them in the exams; this is a must. So, therefore, if I have d by d alpha of a T P square equals to 0; for this particular case, what do I need? This can be equal to 0 only when this term equal 0 or in other words 1 minus 2 alpha equals to 0 and what does it give you? Alpha equals to 0.5.

So, from this what do we get? We find out that for two phase homogeneous flow, acoustic velocity is maximum at alpha equals to 0.5. So, therefore, acoustic velocity is not constant under these conditions, it is a variable and it is a function of alpha; interestingly, we find that for 50 percent void fraction for air water system; remember we have made several approximations. Number one is air water system otherwise vapor liquid system.

Otherwise we cannot write rho 1 is much greater than rho 2, rho 1 a 1 square is much greater than rho 2 a 2 square. This was the first approximation we had made if you remember; we had from this particular expression; we had reduced to this and then finally, from this expression we obtained this particular expression.

So, first thing is there is rho 1 a 1 square is much greater than rho 2 a 2 square; rho 1 is much greater than rho 2. So, this is number one therefore, air water or vapor liquid then next is homogeneous flow condition otherwise, we could not have replaced rho T P with your alpha rho 2 plus 1 minus alpha rho 1. This we could do for homogeneous flow condition.

But in under that condition, alpha equals to beta which becomes an input parameter. So, therefore, we find that for homogeneous two phase flow of vapor liquid or gas liquid mixtures, we find that the maximum acoustic velocity is obtained for a void fraction of 0.5. So, this completes our discussions of the homogeneous flow theory and in the next class we are going to start the drift flux model.

But before that I had just wanted to discuss, I will be giving you tutorial sheets and you will be solving them out on your introduction part on the homogeneous flow model and so on and so forth, but before that I just wanted to discuss one or two problems. So, that we can see when we make some sort of simplifying assumptions then under that condition maybe analytical solutions are possible; much simplified solutions are possible.

So, just I would like to discuss one or two problems. So, that you can do them as your home assignment and find out the results. So, those problems if I discuss let me see whether I have those problems or not. So, I would like to discuss these problems. So, suppose we have water which is flowing through a pipe say at a particular velocity under saturated flow conditions.

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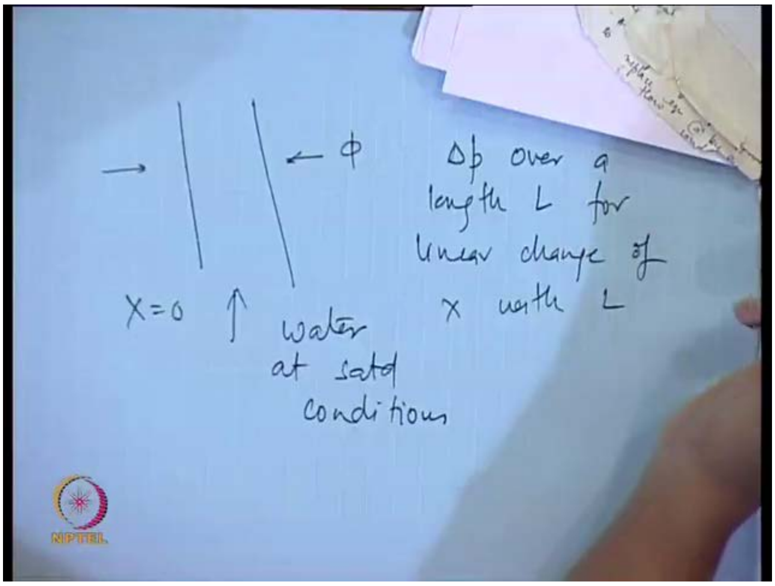

So, suppose we have a condition something of this sort; we have a vertical pipe through which water is flowing and initially it is water at saturated conditions. So, here we are having X equal to 0 and then gradually as it flows we have a constant heat flux say phi, and so it is gradually starts vaporizing and we have a vapor liquid mixture inside this.

So, I would like you to find out the pressure gradient delta p over a length L for linear change of X with L. Is my question clear to you? My question is that we have a vertical pipe implying that more or less all your gravitational force frictional forces acceleration forces all of them is there.

And, in that particular vertical pipe I have introduce water under saturated conditions; that means, X equal to 0, but moment it enters since there is a constant heat flux. So, under this condition some amount of vaporization starts and as we go up the quality of the mixture it increases, and gradually we get a higher and higher quality of vapor liquid mixture.

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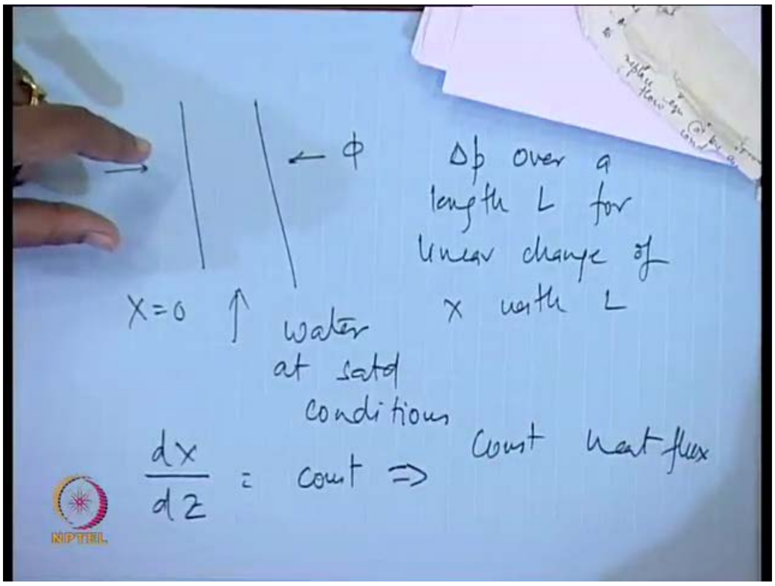

Now, over a particular length L I would like you to find the pressure drop when we assume that the change of quality with length is linear. Can you tell me under what conditions we will have a linear change of quality with length? And, how do we signify or how do we quantify a linear change of quality with length? It is denoted as d X d z equals to constant and this happens for constant heat flux conditions.

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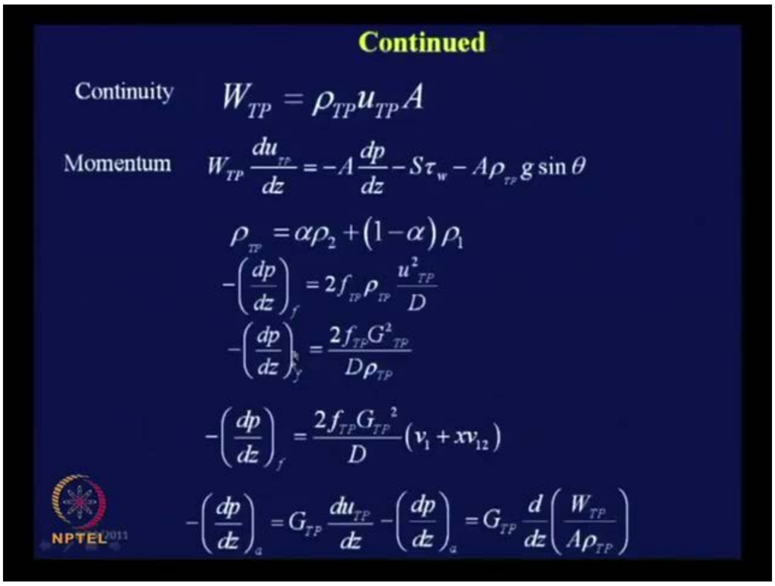

When the heat flux is constant, the area is constant etcetera. Under those conditions, we have d X d z equals to linear. Now, how to proceed? Now, naturally my starting equation should be from here I should obtain the frictional pressure gradient and from here the acceleration pressure gradient can be obtained, and the gravitational pressure gradient can be obtained. If I add all the three, then I should get the total pressure gradient. Once I integrate it over length I should get the total pressure drop right.

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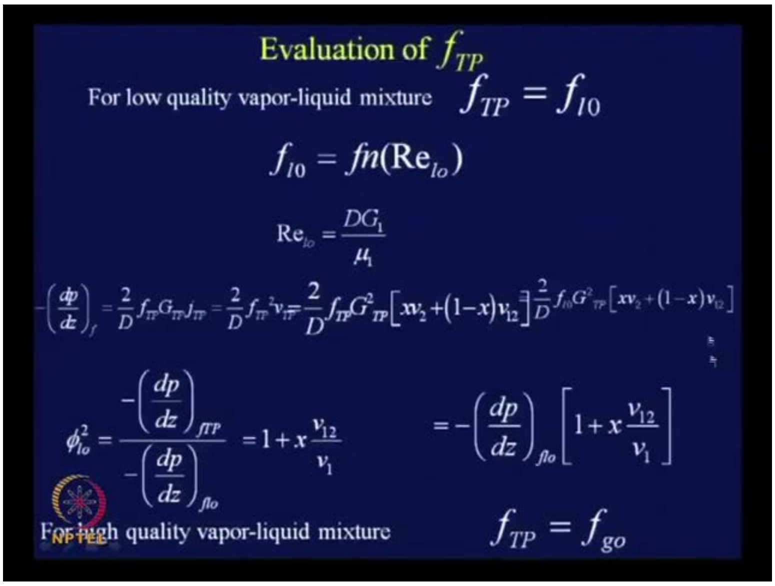

Now, notice certain things. We find that for the acceleration pressure gradient in this particular case, it can change due to a change in density; it can change due to a change in area. In our case, we do not consider the change in area at all; it is a vertical circular pipe. So, this particular term is no longer there we have this particular term; we have the gravitational term; and we have this particular term where it is already specified in the problem that d X d z equals to constant.

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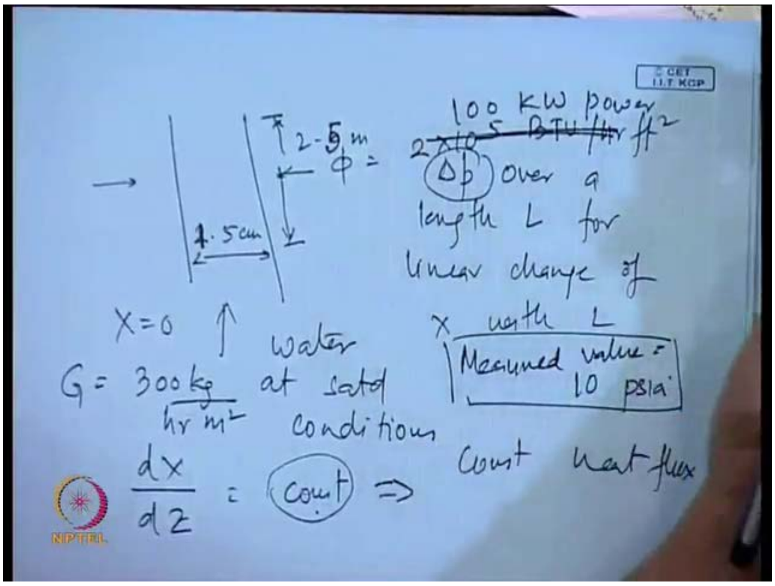

So, therefore, for this particular case if we write down the different terms then what do we get? In fact, I will give you two certain values. So, that you can work out the problem say for example, in this particular case the water at saturated conditions the G let me put as three hundred k g per hour meter square. So, that you can actually put values and you can work it out. It is entering into a pipe of say 2.5 centimeters diameter and the length is 2.0 meters.

Or we can put it 1.5 centimeters diameters, it is 2 meters length and the water is flowing at 300 k g per hour meters square. And, in fact, I have already told you that d X d z equals to constant and therefore, it implies a constant heat flux. See the constant heat flux if I give you the value then in that case, what you can do? You can find out the d X d z equal to constant this particular constant value you can find it out.

So, therefore, the heat flux you can take it down as 2 into 10 to the power 5 this is in B T U I forgot to convert it anyhow, I think I do not have the converted value anyhow. So, therefore, this is at 100 kilowatt power; you do not take this; you take it as 100 kilowatt power.

So, therefore, this is a 1.5 centimeters pipe; this is make it a 2.5 meters length and it is being uniformly heated with a 100 kilowatt power and this flow rate it is I have already given you as 300 k g per hour meter square. So, you are required to calculate the total pressure drop over a length of 2.5 meters and then the measured value. Let me tell you people have measured and they have hound out the measured value as say 10 p s i a.

So, you are supposed to compare with the measured value. Definitely, you will not get very good comparisons. So, you have to comment on why you have not got a very good comparison? Why I have given you these values? I have given you these values that you can perform the complete derivation and then you can substitute these values, and we can actually work out the problem.

Now, since I have given you the total pressure under which have I give you the total pressure under which it is operating? Just take up the pressure under which it is operating is saturated water, it enters at 400 p s i a. So, inlet conditions are 300 k g per hour meter square of water entering at saturated conditions at 400 p s i a in a tube which is 1.5 centimeters in diameter and 2.5 meters long, it is uniformly heated with 100 kilowatt power; and the saturated water enters the base at four hundred p s i a.

And, we are suppose to calculate the total pressure drop and compare with the measured value of 10 p s i a. Now, in this particular case, how do we proceed? Firstly, since its water you can refer to steam tables and you can find out all properties under the inlet conditions. So, you can find out the specific volume v 1; you can find out v 2; you can find out h 1 2; you can find out v 1 2 and so on and so forth.

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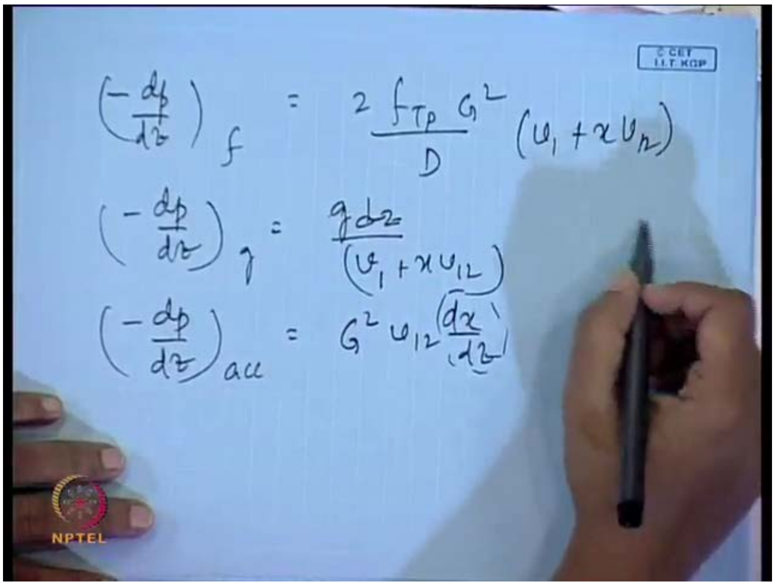

Now, what about the different expressions of pressure gradient that we have the frictional pressure gradient, the acceleration pressure gradient as well as the gravitational pressure gradient? So, in this particular case suppose we mention d p d z frictional. So, this can be obtained as just like we had obtained it in this particular case; this can be obtained from the expression which is written down here. So, it is 2 f T P G square by D v 1 plus x v 1 2.

So, from this we can obtain your frictional pressure gradient. What about your gravitational pressure gradient? This is simply g d z by v 1 plus x v 1 2; rho g d z. And, what about your acceleration pressure gradient? Your acceleration pressure gradient, you can obtain it as G square v 1 2 d x d z where you know that this particular d x d z is linear which I have already mentioned.

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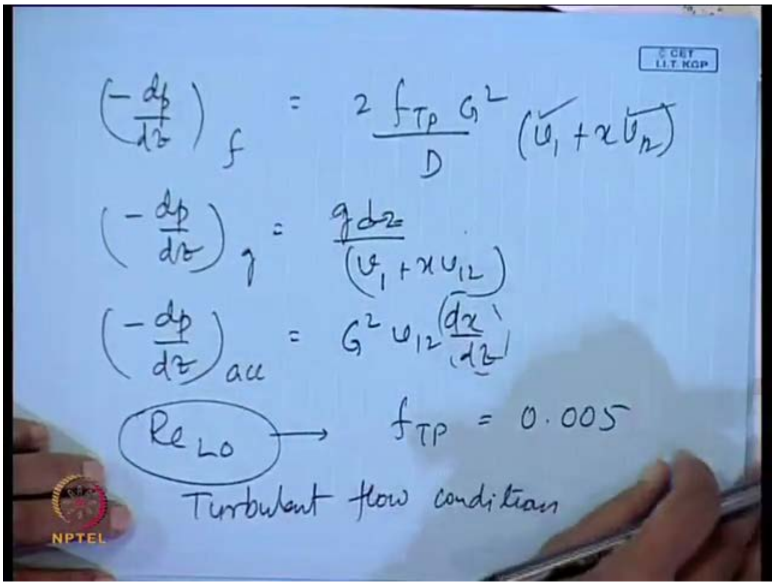

So, you know that more or less everything except f T P; several students were having problems with f T P. So, how to define f T P more or less in this particular case, what you can do is you can find out the Reynolds number; and you will find that the Reynolds number is very high. So, for that particular case and one more thing since here water is entering at saturated condition and flowing up. So, for this particular case we can find out R e liquid only and then we can more or less assume because at the entry condition it is just liquid.

And, based on this R e L o we can find out the f T P; means usually for the condition that I have given where R e L o will lie under the turbulent flow conditions when nothing is given simply assume f T P to have a constant value of 0.005; just to make matters simpler for you. So, nothing if something is specified, then it is fine when nothing is specified then you can simply assume f T P equals to 0.005.

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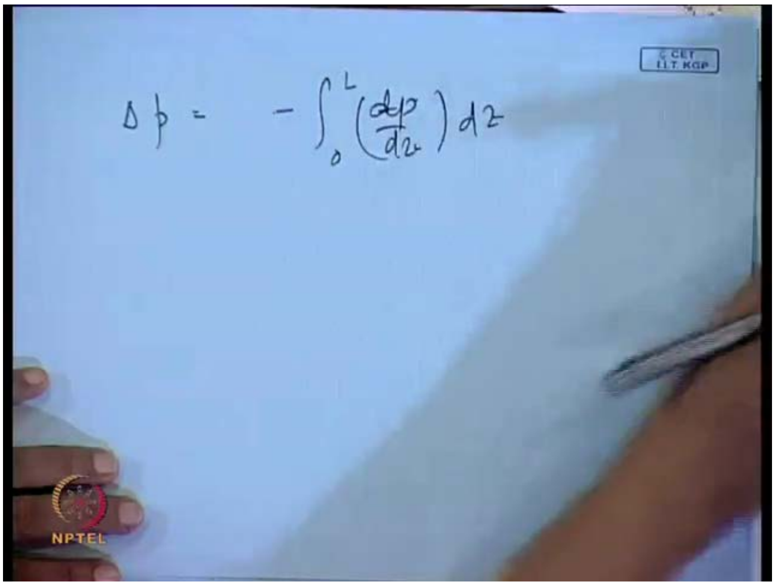

So, moment you know this; you know v 1; you know v 1 2. So, therefore, more or less you can find out the frictional pressure gradient. Similarly, the gravitational pressure gradient also you can find it out; and the acceleration pressure gradient also you can find it out. Now, how to find out the pressure drop from the pressure gradient? That also you know very well. Your delta p is nothing, but equal to minus integral 0 2 z d p d z into d z. So, therefore, d p d z by adding all these three you can find out d p d z and then if you integrate them with respect to z you can find out delta p.

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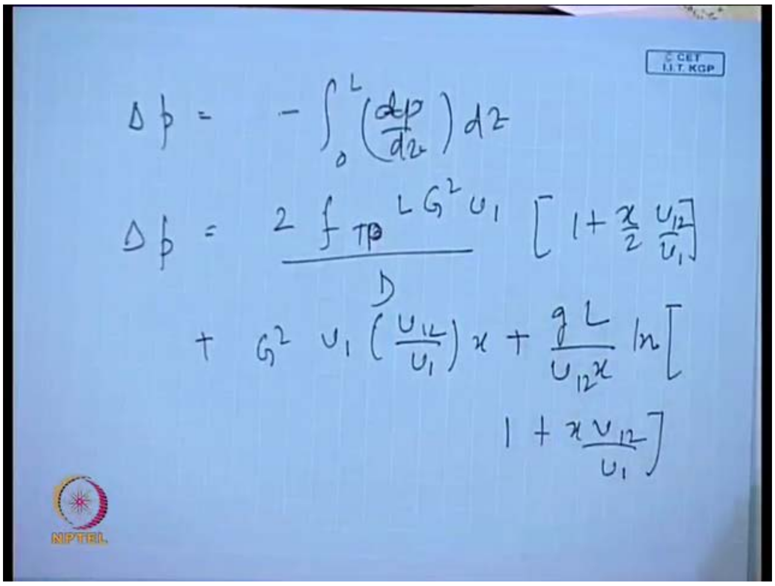

And, you find that the only variable in this particular case, the things which varies with z is just x nothing else varies with z. If you see the expression, you find that G square v 1 v 1 2 nothing else varies with z; only your x varies with z and d x d z equals to its linear therefore, d x d z equals to a constant c; you can find out the c from heat balance equations. So, therefore, if you substitute these and if you perform this particular integration more or less the expression which you are suppose to expect is 2 f F P you please do it and you see whether you have got it or not.

The final expression which you are expected is something of this sort v 1 2 by v 1 into x plus g L by v 1 2 x l m; write it down here 1 plus x v 1 2 by v 1; just see whether you get this particular expression or not. So, this is your home assignment do it and let us sees whether you get it or not. So, therefore, this completes our homogeneous flow theory and if we have certain simplifying assumption then the situation becomes simpler. Accordingly, we can proceed; we will be doing a few more problems. So, that the situation becomes much more clear to you. Thank you very much.