Multiphase Flow
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Lecture No. # 10
Compressible Flow A Recapitulation

Well today, what I had decided was that till yesterday we had completed the basics of homogeneous flows how to find out homogeneous then different approaches of calculating the frictional factor the sorry the frictional pressure drop because, the other pressure drops more or less they were quite straight forward. Depending upon whether there is going to be an area change or whether there is going to be a quality change or whether the compressibility effects are quite remarkable we will have an acceleration pressure drop. If it is a non horizontal we will have a gravitational pressure drop and frictional pressure drop at all times it is going to be there. So, the thing which I had concluded in the last class was more or less we have completed the homogeneous flow model the only thing which was left was a discussion on the denominator part, so regarding that since I have found that many of you are not very conversant with compressible flows.

So, I thought I will just brush up a few things of compressible flows, and maybe I will take maximum one or two classes not more than that. And then from there once you understand compressible flows when we did when we write down the momentum balance equation for compressible flows we find that for that particular situation also denominator term comes. So, we are going to first understand the significance of the denominator term for compressible flows then accordingly we can understand the significance of the denominator term for two phase flow under homogeneous equilibrium conditions. You will notice as we go subsequently for this separated flow model there also a denominator term is going to appear, it will always appear whenever the compressibility of the phases have to be considered, because it comes from the fact that v is a function of p. So, therefore, after this also whenever for separated flow or for such other conditions we get the denominator term it will be easy for us to correlate it with equivalent useful parameters.

So, I will start from the basics some basics of compressible flows. So, that it maybe it is may maybe a repetition for some of you, but anyhow more or less I will just touch upon the basics and then we will proceed further.

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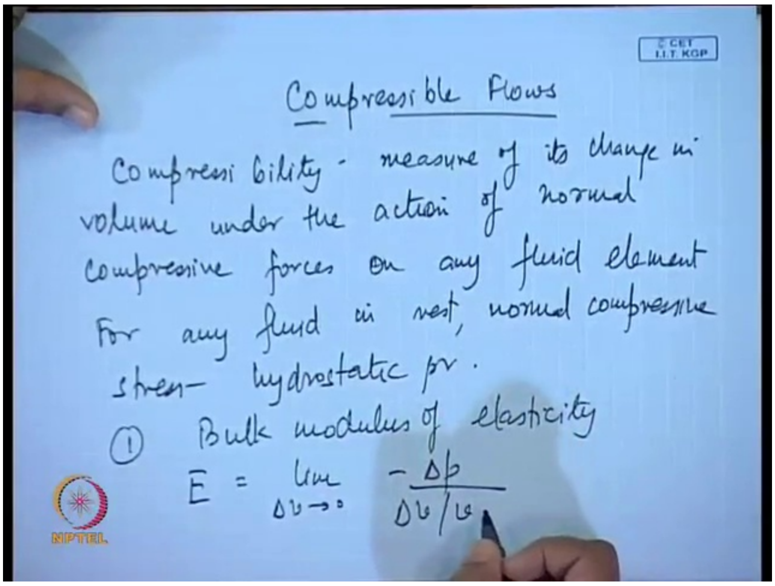

So, suppose we take up say now whenever we talk of compressible flows, the first thing is what is compressibility? So, how do you define compressibility, any idea how do we define compressibility of any particular substance. What is the way by which we define the compressibility of a substance.

Very true all of you are more or less to some extent it is correct inverse of bulk modulus, the change of density with pressure and so on and so forth.

So, the basic definition is it gives you a measure of its change in volume, gives the measure of change in volume under the action of this actually I should have prepared a slide, but anyhow under the action of normal compressive forces, this is what is important. So, it is the measure of the change in volume of the substance under the action of normal compressive stresses on any fluid element. So, this is the thing and what are these normal compressive forces now for any fluid element at rest the normal compressive stress is its hydrostatic pressure.

So, for any fluid in rest normal compressive stress is nothing but hydrostatic pressure. So, therefore, the degree of compressibility it can be characterized it is usually characterized by two parameters. The first parameter is the bulk modulus of elasticity the two parameters which characterize the degree of compressibility of a substance is first thing is bulk modulus of elasticity, which is usually defined as you know these definitions probably.

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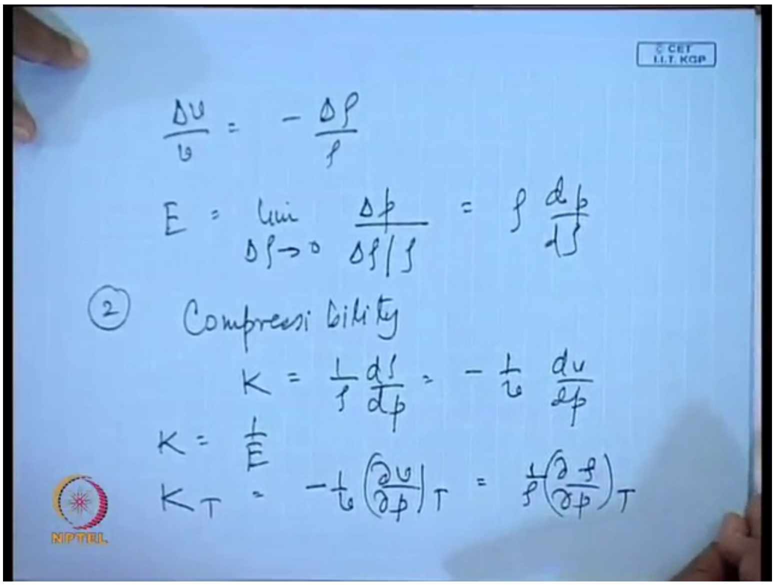

This is defined in this particular form. So, it is the negative sign naturally it shows that increase of pressure causes decrease in the volume, or in other words this delta v by v it is we know that it is nothing but delta v by v is nothing but minus delta rho by rho, but v is a specific volume, but rho is a density. So, from there we get e equals to limit we can also write it in terms of density in this particular form. In other words rho d p d rho this can be one particular thing this is one, but usually for fluid cases apart from bulk modulus of elasticity we prefer to use a second definition. Any idea what that definition is we prefer to define it in terms of compressibility, kappa that this compressibility it is nothing but one by rho d rho d p or rather minus one by v d v d p. So, from these two comparing these two you can very well understand that kappa is nothing but equal to one by e.

Now, remember one thing whenever we are dealing with any gaseous substance and for a gaseous substance, whenever we try to compress it we find that a change in pressure changes it is volume and it also changes its temperature. So, unless temperature is maintained constant or unless the temperature is specified this compressibility term has got no meaning. So, therefore, if we just define kappa it is not very useful we have to define the isothermal coefficient of compressibility. On others words this is defined in this particular form where this can be written down as del v del p at constant t, this is the correct definition or in other words, this can be written down as one by rho del rho del p constant t.

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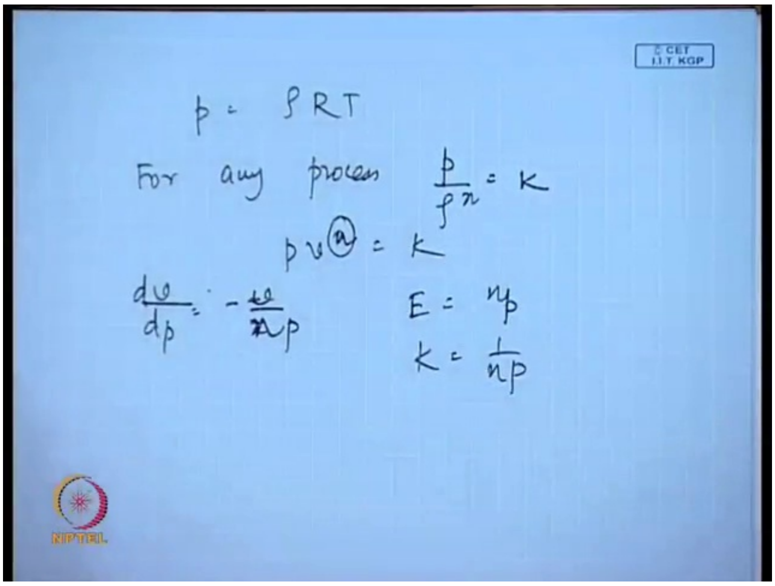

I hope you can differentiate between rho and p that I have written down. So, therefore, from here we find that what real gases which are far removed from their liquid states for such gases usually the ideal gas equation is quite valid for which we can write p equals to rho r t we know it, and we know that whenever the gas undergoes any particular process then for that particular circumstances we can describe the process in terms of p by rho n equals to say a constant k isn't it. Where depending upon the value or rather p v to the power n equals to k, where depending upon the value of k or rather the value of n it depends upon the process that we adopt, all of us know it n equals to one for isothermal process n equals to gamma for adiabatic processes and so on and so forth. So, for adiabatic processes more or less we know that d v d p it is equal to minus v by x p or n p sorry.

It is d v d p equals to minus v by n p and. So, from there we are we can very well define that your e the bulk modulus of elasticity this is nothing but n p and kappa equals to one by n p, this we know very well and we can define. Now whenever this is for fluids at rest whenever the fluids are at rest what happens it is density or its specific volume it changes with pressure, we can define it either with bulk modulus of elasticity or with the coefficient of compressibility. Whenever we define it in terms of compressibility since change in pressure builds about a change in volume as well as a temperature it has to be an isothermal coefficient of your compressibility. Now whenever a fluid is moving now under that circumstances, what happens under that circumstances we know very well you know it from Bernoulli's equation a very well known fact that the pressure head and the velocity head they have to be conserved isn't it.

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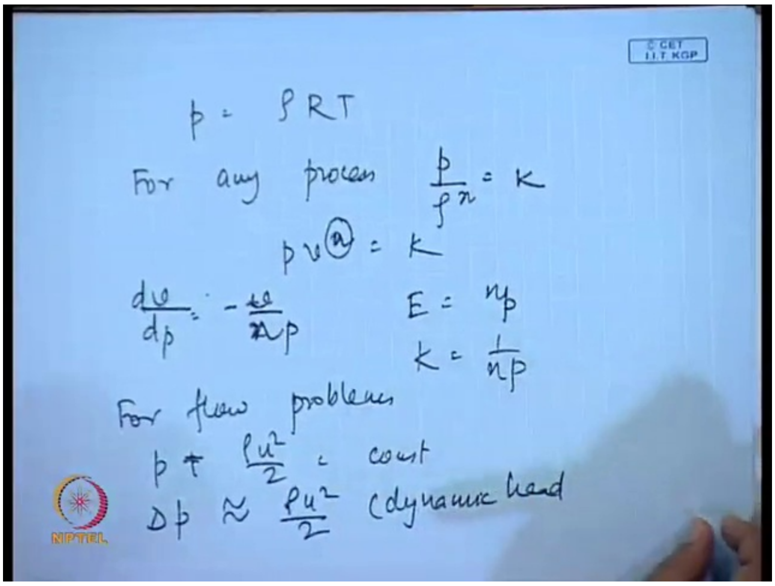

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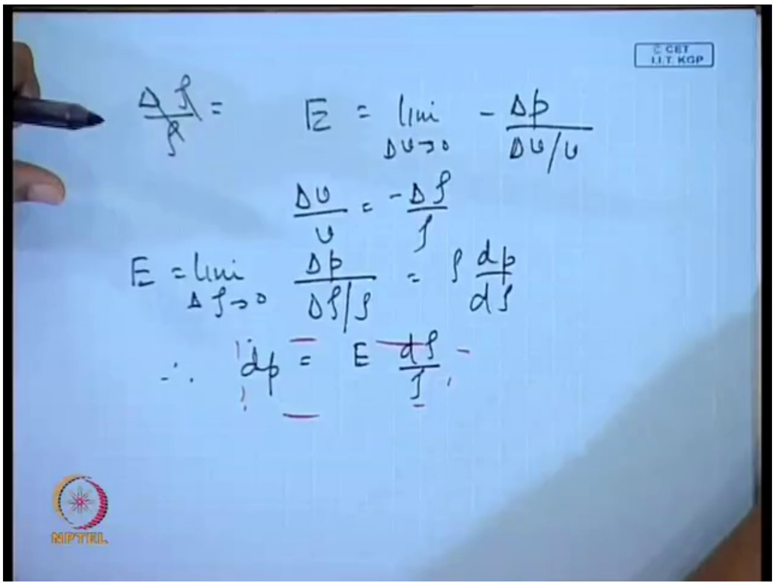

Or in other words for flow problems what we already know is, we know that p it is or rather p plus rho u square by two this is equal to constant. Or we can write down that delta p in any particular flow field that is more or less equal rho u square by two which is nothing but the dynamic head. Now we know that delta rho by rho this is nothing but equal to I will I will derive a little it will be easier for you. If how did we define e it was limit delta v tends to zero minus delta p delta v by v, is it not? And just like I have written it down this is nothing but equal to this particular term is it not? Now we know e in terms of density if we have to define then e will be equal to this I had already done this is delta p by delta rho by rho or it is rho by d p d rho. So, therefore, we can write d p is nothing but equal to e d rho by rho can we write it down from the basic definition of the bulk modulus of elasticity.

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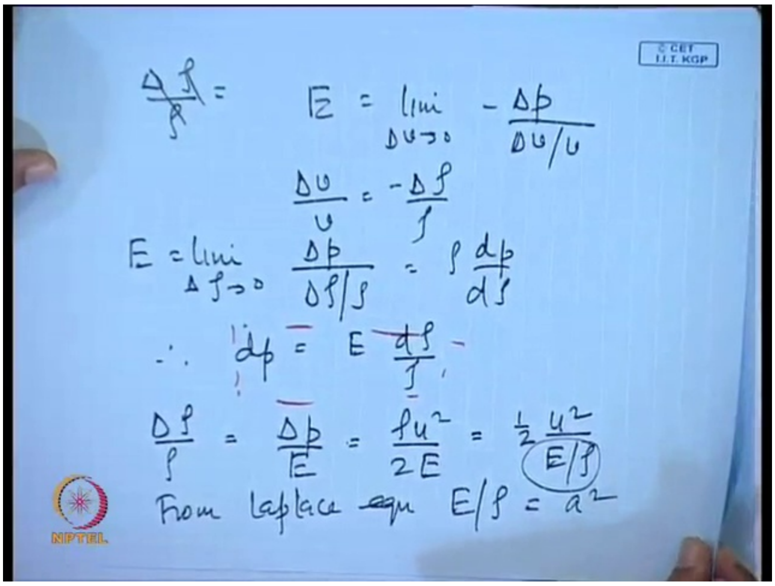

We can we can very well write down this particular equation agreed and we can substitute this particular this particular d p in the d p which we had obtained from Bernoulli’s equation. So, we can substitute this particular d p in the expression which is obtained from the Bernoulli’s equation is it not? So, therefore from here what do we get we get that delta rho by rho this is nothing but equal to delta p by e this is rho u square by 2 e .

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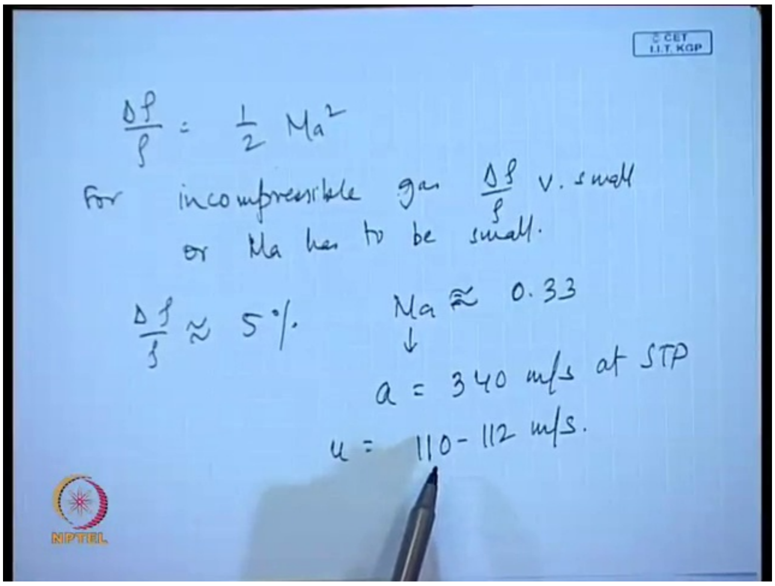

Or in other words this is half u square by e by rho can we write it down in this form yes or no. I am just doing basic substitution and what is this e by rho from Laplace's equation any idea what is this e e by rho, e by rho very good e by rho this is nothing but a square where a is the acoustic velocity or the velocity of sound in that particular medium. So, therefore, what do we get we get delta rho by rho is nothing but equal to half m a square correct. So, therefore, from this equation what do we get? If delta rho by rho is very small; that means, the density is not affected by pressure and the fluid is incompressible you agree with me. So, therefore, delta rho by rho is very small means, Mac number is very small. So, now do you understand you all of us you are already known that Mac number it is the most important characters and parameter for compressible flows, this equation explains why this is. So, agreed.

So, therefore, we find that in order to define the compressibility or the non compressibility you have to see how by how much amount the density changes for unit changes in pressure. And how much amount is it changes that can be related with the Mac number under those particular conditions agreed. So, therefore, we know that for incompressible flow or for incompressible gases, delta rho by rho has to be very small or Mac number has to be very small. Now, we know one thing that if we consider that we can allow a 5 percent change of delta rho by rho, see even for that phi if we allow that 5 percent change of delta rho by rho under that condition also, and if you substitute it here then your m a it is almost equal to 0.33 agreed. And for corresponding to this particular m a the a the velocity of sound we know it is three hundred forty meters per second under s t p or three hundred and forty meters per second at s t p. So, therefore, this gives u it is more or less about say one hundred and ten-one hundred and twelve per meters second.

Do you get? So, even for a five percent change in density we find that the velocity of flow has to be greater than about one hundred and ten meters per second, in order to manifest compressible flow characteristics. So, for most of the cases we can ignore the compressibility of the gaseous phase even, because usually under normal circumstances we as chemical engineers or mechanical engineers we probably we do not go to such high flow conditions for aerodynamics etcetera it is very important. So, from this we understand why under most circumstances we can deal or rather we can treat the gas phase as incompressible, but if you do. So, you can do. So, you can complete the calculations, but at the end you have to show that the density changes were negligible for any problem you do you can assume incompressible flow you can you can do the entire problem we will be doing a problem tomorrow which will enable you to understand this, but at the end you have to show that compressibility effects are very very small that has to be shown at the end.

So, from here we find out that number one how to define compressibility and number two why Mac number is so very important for compressible flows. The next thing which we should find out what is it we know that for in order to define or in order to quantify compressible flows, we need to know and rather we need to know the Mac number under that particular conditions, for knowing Mac number we need to know the velocity of flow and the velocity of sound under that particular condition. So, now let us see how to express velocity of sound in terms of measurable parameters agreed.

So, if we want to find out a measure of the velocity of sound, now suppose we take maybe any particular compressible flow. Now when we have an incompressible fluid and in that particular incompressible fluid we stand a pressure pulse, what happens it displaces the particles whenever it is propagating it is displacing the particles these particles displace more and more particles then overall all the particles they get displaced. For compressible flows what happens as the pressure pulse is travelling it displaces the particles moment it displaces the particles, the particles move ahead and then it increases the density of the adjoining area, as it increases the density of the adjoining area then it again increases the density of the next adjoining area, in these particular way when a pressure pulse propagates in compressible flow we find that more all less it propagates by increasing the density of the fluid through which it is travelling

Initially if the density was rho then now its rho plus d rho, so as the pressure pulse is travelling all the properties they change say pressure p to from p to p plus d p t to t plus d p rho to rho plus d rho in this particular way the pressure pulse propagates. And when it is propagating through a fluid at rest then it also, imparts some velocity to the fluid which is much less as compared to the acoustic velocity or much less to the velocity of the pressure pulse.

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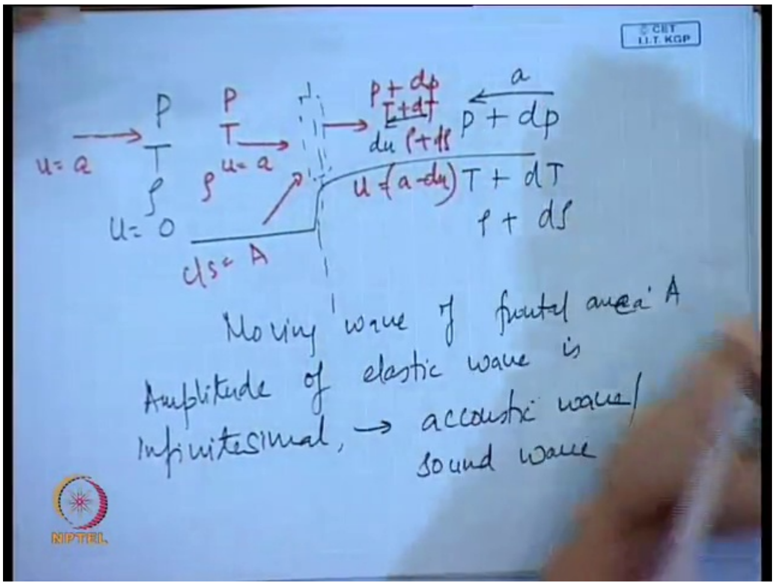

If we assume that the pressure pulse is of infinitesimal strength then we can approximate it as a acoustic wave right elastic wave or an acoustic wave. So, now let us see under such circumstances let us picturize the situation, say in this particular case we find that the pressure pulse it is travelling in this particular direction, maybe from the right to left it is travelling at a velocity u. Now while it is travelling it displaces a particle and when it displaces the displaced mass it compresses and increases the density of the neighboring mass. This increases the density of the adjoining mass and in this particular way it travels. So, now since the pressure pulse it is travelling in this particular from this particular direction it has not reached this particular portion. So, in this particular this portion on the left hand side the fluid is at rest it is undisturbed it is not affected by the pressure pulse till now.

So, here the properties will be p t rho and u will be equal to zero and since it has already travelled from here it is going to be p plus d p t plus d t rho plus d rho. And since it is travelling it imparts a very small velocity due to the fluid. So, what I have represented it is a moving wave of say frontal area, the cross sectional area through which it is moving frontal area a. So, we find that the disturbance it is travelling in the form of a elastic wave or a pressure wave through the medium. Now if the amplitude of the elastic wave, assuming that the amplitude of the elastic wave is infinitesimal. So, then amplitude elastic wave infinitesimal then it can be we can assume it be an acoustic wave or a sound wave as a result of which it is velocity can be defined as a the acoustic velocity in that particular medium under that particular condition ok. Now, when this infinitesimal pressure pulse is propagating at a speed a towards still fluid at the left. So, the properties I have defined as I have shown here, now for steady state analysis what we can do for steady state analysis we can superimpose another velocity a from this particular direction. So, that what happens your pressure pulse it becomes stationary at one particular location isn't, it we can do it we can superimpose a velocity a from this particular direction.

Now, moment we do this what happens we find out that in here if you superimpose then we find that more or less the pressure pulse it becomes constant at any particular location pressure pulse which has a cross sectional area of a. Now for this circumstance what happens we find that for this particular section, we find that fluid is entering from this direction fluid is moving out from this direction and in this direction fluid is entering at a this fluid is under conditions of p t rho. And the fluid is going out from this particular control volume the outlet conditions are p plus d p t plus d t rho plus d rho, and here the velocity was u equals to a and in this case it will be u equals to a minus d u. yes or no did you understand this particular portion for just for performing the mass balance and the momentum balance we want a particular control volume. So, what we do we enclose that infinitesimal area which is by which the pressure pulse is enclosed and that particular small area it is actually travelling.

What we do we superimpose another velocity from the opposite side. So, that that small area it becomes stationary and then we can assume fluid is entering into that area from the left hand side fluid is going out from the right hand side. So, now, for this particular fluid entry and out in that particular small cross sectional area under steady state conditions we can perform the mass balance, as well as the momentum balance you got my point.

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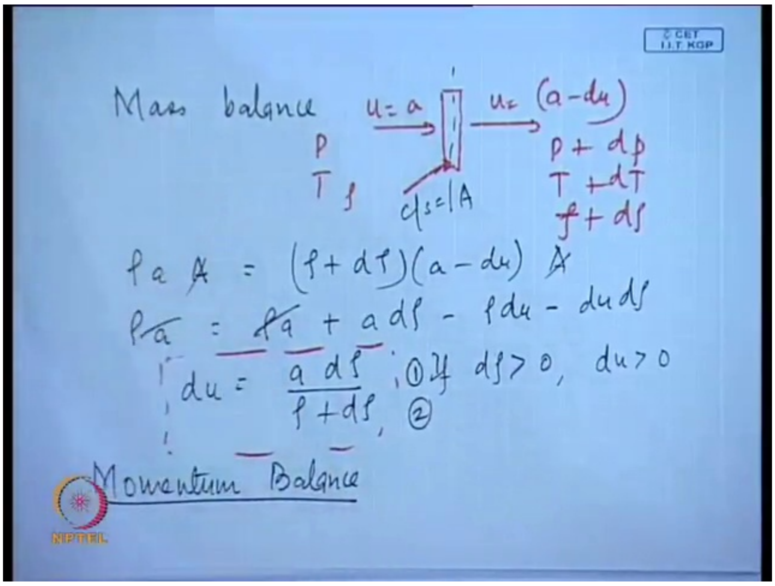

So, therefore, for this particular case let us perform the mass balance and the momentum balance and see what happens for these particular cases. Now for mass balance I will just draw it up once more. So, that it is easier for you fluid is entering at u equals to a fluid is going out at u equals to a minus d u here, it was p t rho here, it was p plus d p t plus d t rho plus d rho. And of course, I have already told you that the cross sectional area that is equal to a. Now, from mass balance what do we get mass entering rho a a mass going out rho plus d rho a minus d u a any portion you do not understand it is very simple you can tell me you can just tell me to repeat it. So, therefore, we know that a a cancels out this gives rho a plus a d rho minus rho d u minus d u d rho.

So, these also cancel out and we get d u equals to a d rho by rho plus d rho is not it? This is the thing which we get. Now, from this particular equation there are two things that we understand, first thing is if d rho is positive if d rho is greater than 0, d u is greater than zero right. If d rho is greater than 0 d u has to be greater than 0 agreed, or in other words whenever such a flow occurs whenever a compression wave it travels through a fluid it definitely leaves the fluid moving in the direction of the wave. Yes.

Multiplication of…

You can we will be neglecting it at the end you can just write it down as a a d rho by a by rho d rho definitely we will be doing it at the end, but I have just written it down if you want you can cancel it at this particular very beginning by assuming d rho tends to d rho definitely we will be doing it. So, for this particular case we find that from this equation we get two information, what is one that definitely it is when a pressure pulse or a compression wave it travels through a fluid it leaves the fluid which is travelling at a small velocity in the direction of the wave, because d u has to be greater than zero when d rho is greater than zero this is the first thing that we observe from here. And then we know that within the framework of infinitesimal strength of the wave we know a itself is also very small. So, this was about the mass balance, now let us take up the momentum balance and then we can do it in this particular page as well if we perform the momentum balance.

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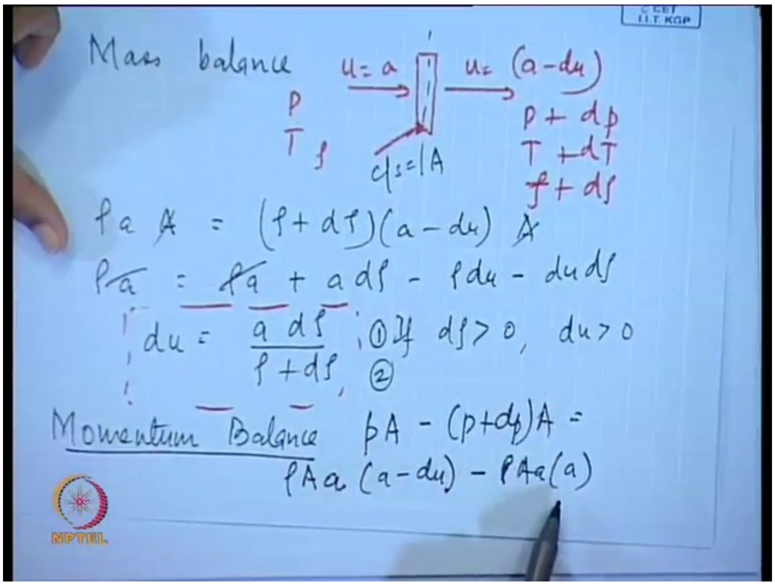

So, momentum balance what do we get momentum in my your force in minus force out equals to rate of accumulation of momentum. So, from here we get p A if you see that force which is entering it is p a I will just write it down here. So, you can compare and find out minus p plus d p A. This is equal to rate of momentum in minus rate of momentum sorry yeah rate of momentum out minus rate of momentum in.

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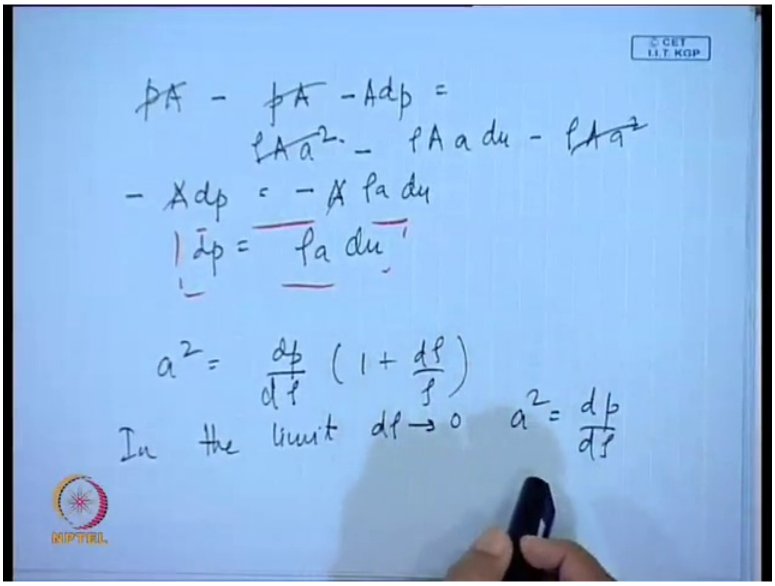

So, how much momentum mass into velocity mass I have already written it down rho A a. So, it is rho A a into a minus d u minus rho A a into a, if you compare this picture this is going to be the momentum balance agreed all of you. So, on simplifying this we find that we get p A minus p A minus A d p this is equal to again rho A a square minus rho A a d u minus rho A a square they cancel out these cancel out. So, finally, what we have landed up with we have landed up with minus a d p equals to minus a rho a d u a's a's also they cancel out or in other words d p is nothing but equal to rho a d u this is the thing sorry very sorry this is the thing that we get and d u for the expression of d u we can substitute d u from this particular expression is not it. So, if we do it then on substituting what do we get we get that a square you just substitute it and you find you get d p d rho into one plus d rho by rho ok.

You can cancel out this term in the very beginning or you can do it at this stage as well in the limit d rho tends to 0 what do we get a square is nothing but equal to d p d rho agreed. You are already probably familiar, with this particular expression although you do not know whether you had known the derivation earlier or not. Now remember one thing this process firstly was adiabatic and reversible. First thing the frictional effects they were confined to the inside of the pressure wave itself. Since frictional effects are confined to the inside. So, therefore, we can. So, therefore, there were no velocity gradients on either side of the waves and we can assume the process to be reversible. In addition there were no temperature gradients except inside the wave. So, therefore, we can assume that the process was adiabatic as well.

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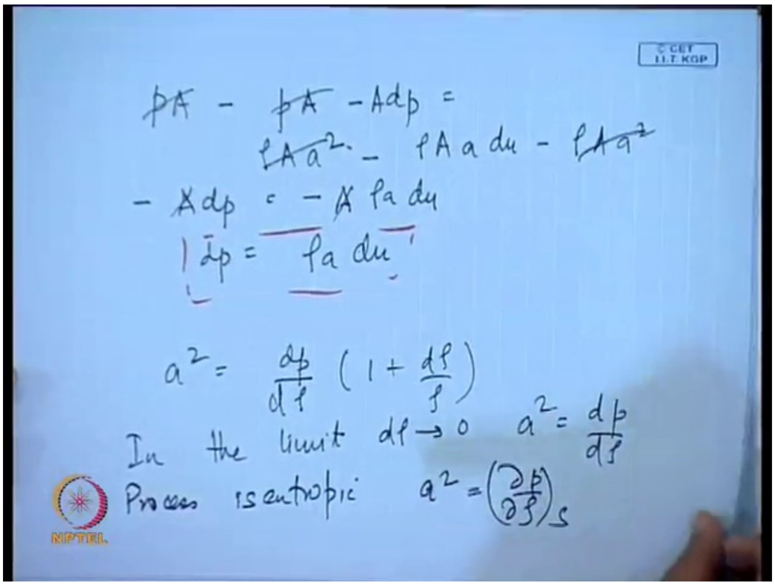

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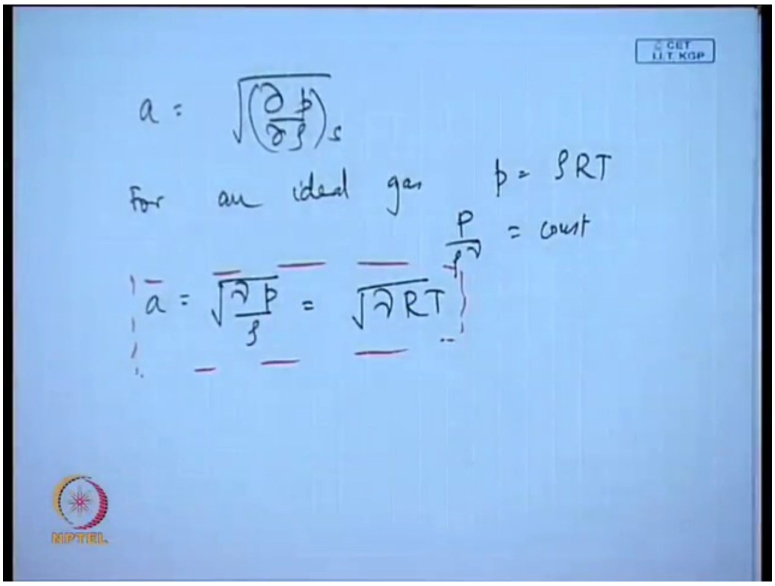

So, therefore, remember that we have done this derivation under the condition of adiabatic and reversible process or in other words, this derivation was done for an isentropic process. So, therefore putting this condition here, we can write down a square is nothing but equal to Del p Del rho under isentropic conditions. Or in other words a equals to root over of which you already know d p d rho at constant s. Now, for a perfect gas or for an ideal gas as we say, for most of the conditions as I have already mentioned if the gas is far removed from its liquid state we know p equals to rho r t. So, therefore, if we perform the differentiation we find that p by rho gamma this is equal to constant. So, therefore, a in terms of measurable properties it is nothing but gamma p by rho or in other words it is gamma r t. So, by this particular process we can first what we did we find out what to how to define compressible flows.

We found that Mac number is important. Now in order to calculate Mac number what we need we need velocity of sound. If we can relate velocity of sound under those particular conditions with measurable parameters it is going to be very convenient. So, the next things which we did is we related a or we expressed a in terms of the absolute temperature or in terms of pressure and so on. So, we obtained a is nothing but equal to root gamma r t. These expressions probably all of you knew from the beginning what you did not know was probably the how we arrive why was Mac number important for compressible flows and why is a equals to root del p del rho at constant s, probably these things you are not very clear earlier. Now whenever we define remember one thing the next thing which I would like to do, whenever we define compressible flows whenever we define an in compressive, suppose water is flowing through a pipe how do we characterize the water, we tell that its density is one gram per c c the it is at say room temperature thirty degrees or twenty five degree centigrade it said one atmospheric pressure.

So, moment we tell this we know under this maybe its volume floor it is. So, and other things. So, moment we it is define it is temperature pressure density may be enthalpy etcetera etcetera the state of the water becomes fixed. Any point from that water if if we measure the property at any point, that is constant or that is the same entire water can be characterized by that property. Now, what about compressible flows compressible a flow at every point pressure is different, at every point density is different at every point temperature is different. So, what to do about it whenever such a thing happens we have to refer to some standard conditions, without standard conditions we there has to be a datum with which we can compare, and we can identify or we can characterize the properties.

Any idea what are the reference conditions for compressible flows. What are the properties which we refer to as, because just property does not mean, anything for compressible flow, which properties any idea you whether you have heard about the name I would like to know that how do we what are the standard or the datum properties in compressible flows to which we refer to the property under the present situation. You may have heard about datum have you heard about stagnation properties and sonic properties. ok

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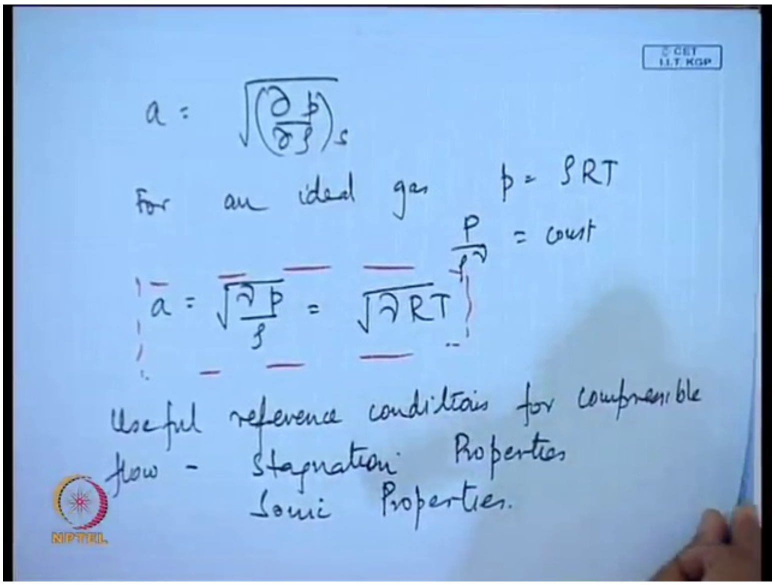

We will be discussing about you have heard about these things is not it. So, therefore, the useful reference conditions, remember one thing compressible fluid and compressible flows are not always the same, useful reference conditions for compressible flows are the stagnation properties, they are one set of properties and the sonic properties. Now what are these let us define them first stagnation itself it means, that somewhere the fluid should be at rest and the properties under those conditions, but remember one thing how the fluid has been brought at rest that part is also important. For compressible flows the path is also very important how we have brought the fluid at rest. So, therefore, when the fluid is brought isentropic ally to rest conditions then the property fits the fluid has under those rest conditions, they are referred to as the stagnation properties. Usually how do we denote them we denote them with a subscript 0 along with the property, like pressure the stagnation pressure is p 0, the stagnation temperature is t 0, the stagnation density is rho zero agreed. So, therefore, the stagnation properties are those properties they are defined as those properties.

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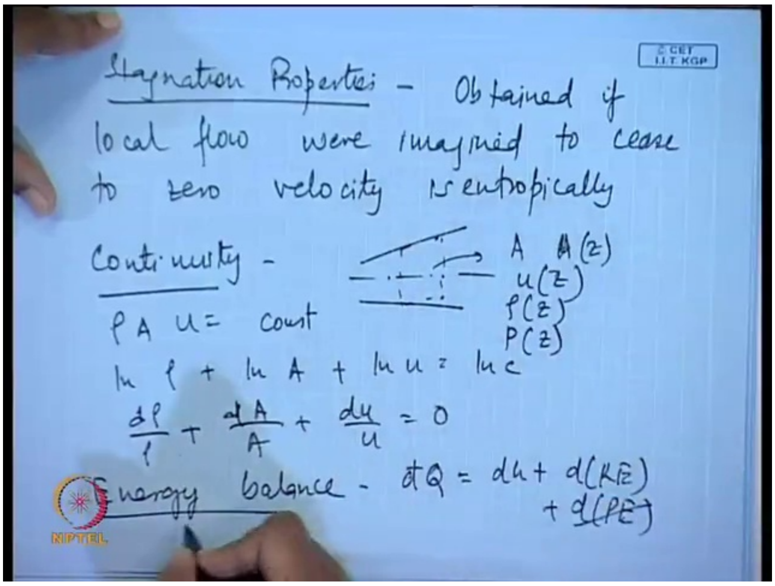

They are defined as those properties which are obtained if local flow were imagined to cease to zero velocity is entropically. This means that whatever equations you have learnt in your thermodynamic or reversible adiabatic flows all of them can be used in this particular case. So, therefore, we know certain things from energy balance we know that more or less our see for such a type of flow suppose, we would like to write down just I will go a little back and I would like to write down certain things we will be using these things. For such type of flows say may be compressible flows it is flowing a in a one particular cross sectional areas, say in this particular cross sectional area a where the curvature at the central line can be neglected.

So the properties will be varying at each and every point. So, therefore, they are a function of your axial distance. So, therefore, properties are written down in this particular fashion the velocity is u z is the area of course, it is a z. So, from continuity what do we know we know rho a u equals to constant, w equals to mass flow rate is constant that you already know? A useful form of this equation is to take the log and to perform the logarithmic differentiation from here we get d rho by rho plus d a by a plus d u by d equals to 0.

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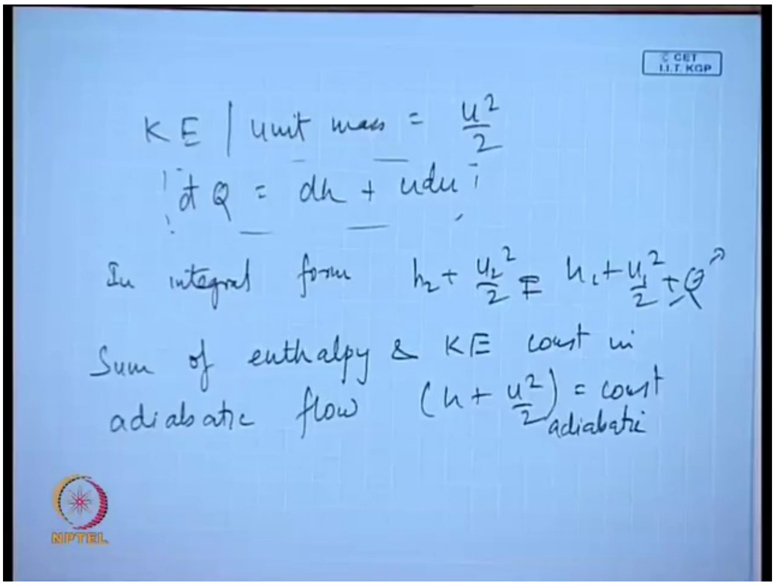

And from energy balance what do we get. From energy balance it is just the first law for of thermodynamics for open systems, from which we get d q d w we are going to neglect this is equal to d h, plus d kinetic energy plus d potential energy. Now usually we know that the potential energy that can be neglected, because in this particular case it can be neglected. So, therefore, and the kinetic energy I will go to the next page, the kinetic energy per unit mass this is nothing but equal to u square by 2. So, if this is substituted in the energy balance in this particular energy balance equation, then we get d q equals to d h plus u d u isn't it. So, the energy equation remember this particular equation this is valid even in presence of friction or non equilibrium condition in integral form. You can write it down as h 2 plus u 2 square by 2 sorry equal to h 1 plus u 1 square by 2 plus q.

We can write it down in this particular form which tells us that sum of enthalpy and kinetic energy it is constant in adiabatic flow isn't it. This term becomes equal to zero. So, therefore, we find h plus u square by 2 this is constant under adiabatic conditions. From energy balance equation we get it you can also get it from the steady form of from the second law of thermodynamics also, you can use it and you can get it by knowing that t d s equals to d h minus v d p isn't it, by applying that also you can get it ok. So, therefore, we find that for adiabatic conditions even if there is friction even if there are irreversible effects, we know that under adiabatic condition the sum of enthalpy and the kinetic energy of a fluid is constant agreed. So, therefore, at any particular point in a pipe h plus u square by two equals to constant agreed. Now if this particular fluid is isentropically brought to rest, then what happens isentropic ally means, it is adiabatic reversible. So, under that condition u becomes equal to 0 h becomes equal to h 0 agreed. H 0 is the stagnation and enthalpy do you get my point what do I what do I know from energy balance I have come to know that under normal conditions we know that the summation of h and the kinetic energy is equal to q.

For adiabatic conditions what I know the sum of the enthalpy and kinetic energy this particular sum it always remains constant at any particular portion of the fluid; that means, whenever the flow is occurring from in a under whatever condition at each and every cross section h by u square by 2 that has to be h 1 by u 1 square by 2 has to be equal to h 2 plus u 2 square by 2 has to be equal to h 3 plus u u three square by 2. Now, at any particular point if u equals to zero. Then in that case; that means, it has isentropically being brought to zero. So, under that condition what do we get we get h is reduces to h zero, the stagnation property how did we define the stagnation property it is those particular properties which are formed when the fluid ceases to move or if when a fluid is brought to rest isentropic ally you agree do you get the point.

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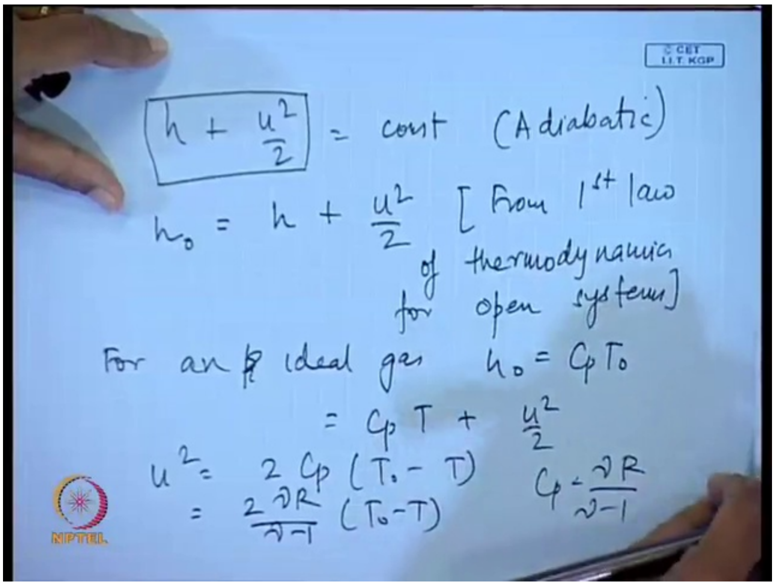

So, therefore, we can write it down in this particular form that for all these circumstances for adiabatic flow, this particular term this is constant this we can write isn't it. So, therefore, the stagnation enthalpy h zero this is nothing but equal to h plus u square by 2 isn't it. This we have got from first law of right we have already reduced it. So, therefore, we get h 0 equals to h square plus h plus u square by 2. Now, for a perfect gas or for an ideal gas I should say h 0 equals to c p t 0 is not it. T 0 is the stagnation temperature this will be equal to c p t plus u square by 2 you agree with me. Or in other words we can write it down we can write down u square it is equal to 2 c p t 0 minus t agreed, we know what c p gamma r is by gamma minus one. So, therefore, this u square can be written down as 2 gamma r by gamma minus 1 t 0 minus t yes or no.

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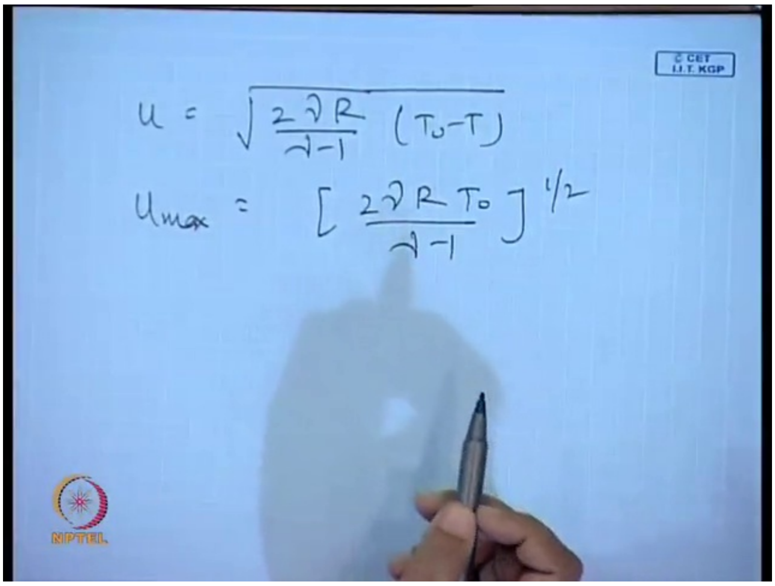

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And therefore, we can write down the velocity as root 2 gamma r by gamma minus 1 into yes correct, which gives you u max maximum velocity it is nothing but equal to 2 gamma r t 0 by gamma minus 1 whole to the power half can we write this yes or no, so therefore, from this particular equation from this equation and from this equation. What do we deduce from these two equations, we deduced that the total enthalpy and the t 0 are conserved if the process is adiabatic? Remember these two equations we did not put any condition of reversibility. So, for any adiabatic process we can write down the total enthalpy and the stagnation temperature t 0 they are conserved if the process is adiabatic. And what about this gives us a relation between the fluid velocity and the local temperature under adiabatic conditions, everything just adiabatic we are talking. So, this just gives you a local velocity a relation between local velocity and the local temperature under adiabatic conditions right.

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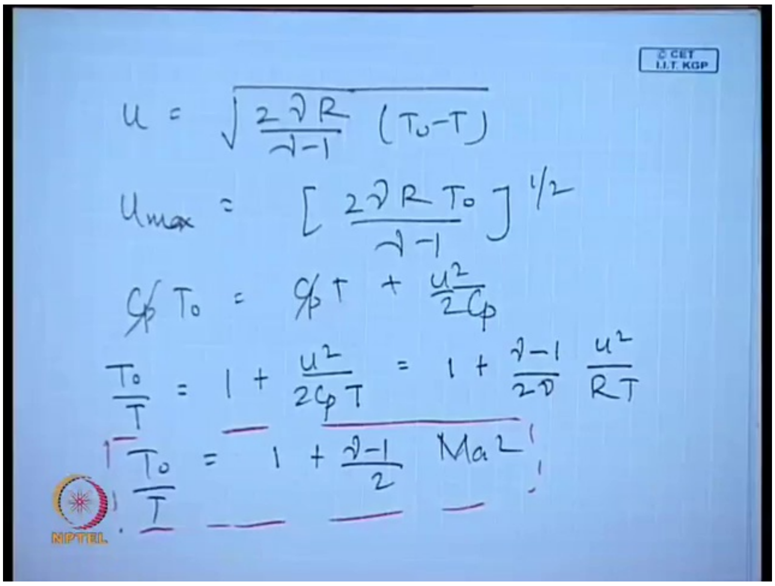

Now, once we have obtained a particular way of deducing rather finding out t zero. So, in the same way or rather if we just proceed further from this particular equation what do we get, we get c p t 0 equals to c p t plus u square by 2. Or in other words we can write it down in this particular way or we can write it down in this particular way or we can write down t 0 by t this is nothing but 1 plus u square by 2 c p t. Substituting c p by gamma r by gamma minus 1 we get this is nothing but 1 plus gamma minus 1 by 2 gamma u square by r t yes. We have just substituted c p we have not done anything else we can get it or in other words this can be written down as 1 plus gamma minus 1 by 2 m a square can we write down this equation, yes or no you tell me we can write it down. So, what have we done?

We have expressed our actual our local temperature in terms of this stagnation temperature. right And we find that the relationship between the local temperature and stagnation temperature is a function of Mac number. Now remember for this equation also it is just for adiabatic flow. Now once we could we could find out a relationship or we could find out an expression to express t 0 by t using the equations of adiabatic reversible flow, the relations connecting p p to t rho to t we can find out rho 0 by rho we can find out p zero by p and. So, on is not it.

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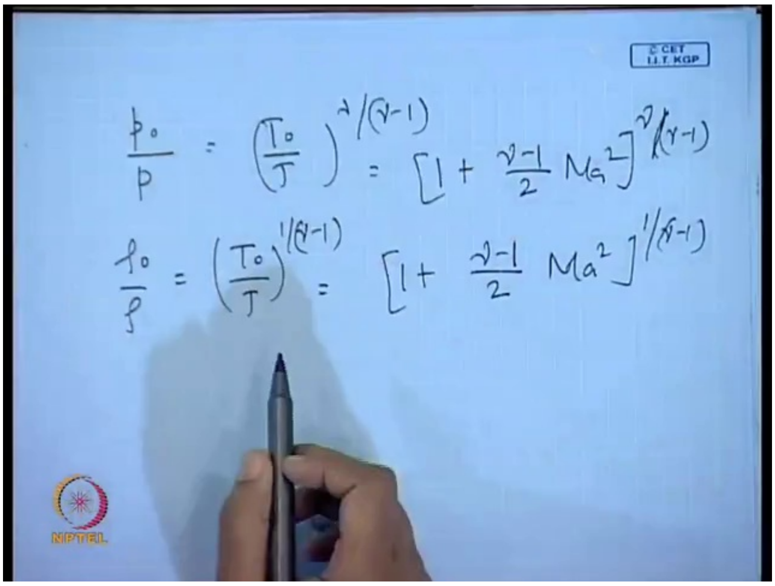

So, if we write them down what do we get the same terms we get p 0 by t this is equal to t 0 by t into gamma by gamma minus 1 which is equal to 1 plus gamma minus 1 by 2 m a square, gamma by gamma minus 1 is not it. Same way we can write rho 0 by rho equals to t 0 by t whole to the power 1 by gamma minus 1, and this is equal to 1 plus gamma minus 1 by 2 m a square, 1 by gamma minus 1 is not it. So, therefore, we find out that we are able to relate all the local properties in terms of the stagnation properties. Get started with the first law of open of thermodynamics are open system, we had first considered adiabatic systems, using that we could express or rather we could find expressions for h 0 and t 0. Then we use the expressions of your isentropic conditions and then from there we found out the relationships between p 0 and t 0. Now remember one thing that whenever a fluid is flowing the stagnation properties they should vary from point to point depending upon their actual properties is not it

So, from for each and every point they should have different stagnation properties. Now suppose there is no heat transfer and we can neglect the frictional loses, then under that conditions we can say that the fluid flow inside the pipe occurs under isentropic conditions can we do it we can do it is not it. So, the thing is that if we can assume that generally what we find we find that the stagnation properties they can vary throughout the flow field, but if we assume that the flow is adiabatic or rather then we find h plus u square by 2 is constant throughout the flow field. Now if h plus u square by 2 it is constant throughout the flow field, even in the presence of friction then we can say that all the stagnation properties have to be constant along an isentropic flow yes or no. So, therefore, we find that for each and every condition each and every position, if the condition is maintained isentropic then under for that situation even if the properties are varying along different cross sections, but isentropic ally if the properties are brought to rest under for each and every condition, then the stagnation properties will be the same.

And what will be the stagnation properties they will be the properties which the fluid will have if it started from rest in a stationary tank or a reservoir suppose the flow starts from a reservoir. So, from that reservoir the flow is flowing to the pipe and in that pipe you have maintained isentropic conditions. Then then in that case the property of the fluid in the reservoir are nothing but the stagnation properties is it clear to you. So, therefore, it is not so very impossible that stagnation properties are varying throughout the flow and therefore, how to find them it is of no use it is not that the if for most of the cases you can assume isentropic conditions and for that particular case the flow in the reservoir or rather the reservoir where the fluid is at rest, the properties of the fluid in that condition gives you the stagnation properties agreed.

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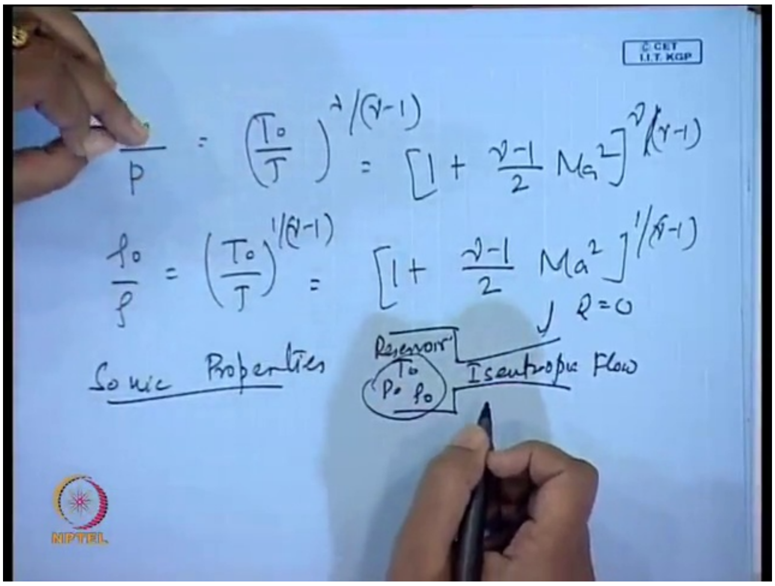

Now, next I would like to tell you about sonic properties. And remember one thing that is just one thing that I want to say, if we have a reservoir through which the flow is going then in as I have already mentioned in this reservoir it is t 0 t 0 and rho 0. In this particular case isentropic flow occurs therefore, q equals to 0. So, if we measure the properties here we know the isentropic properties. And remember one thing total enthalpy and t 0 is conserved for an adiabatic process. Irrespective of whether there are frictional loses or not. If there are frictional loses then we find that t 0 and h 0 does not change, but in contrast p 0 and u 0 they decrease if there is friction. So, for isentropic conditions h 0, p 0, t 0, rho 0 nothing changes, if it is adiabatic h 0 t 0 is conserved, but p zero and rho zero they decrease.

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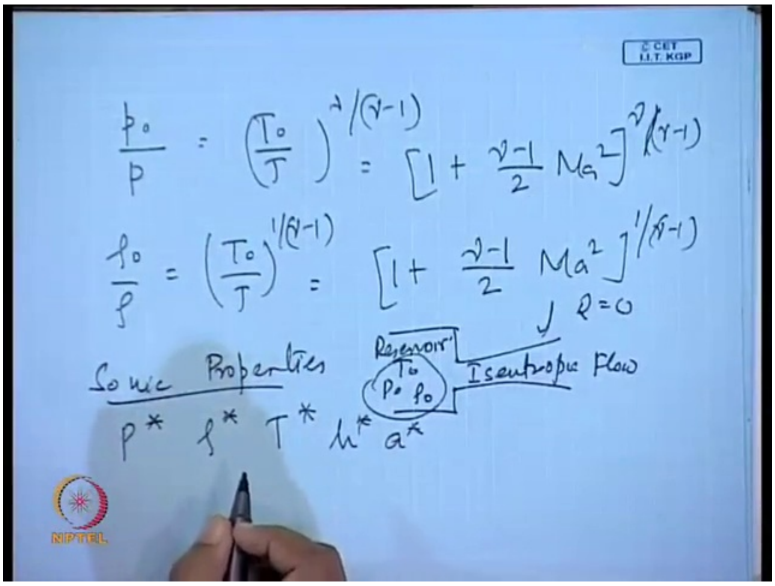

Next is sonic properties, sonic properties are the properties when m a becomes equal to one is not it. So, and these properties they are denoted by an asterisk. So, they are denoted by p star they are denoted by rho star, t star, h star, a star everything. So, how to express this in whatever equations we have obtained if m a is reduced to one for m a equals to one we can get the corresponding p star, rho star, t star everything value. So, accordingly these properties are attained if the local fluid is imagined to expand or compress adiabatically or isentropic ally till it reaches m a equals to one there till the fluid is brought to rest isentropic ally in this particular case the fluid is either imagined to expand or compress isentropic ally till m a becomes equal to one. So, therefore what do we get t 0 by t star simply what we do instead of t it is t star.

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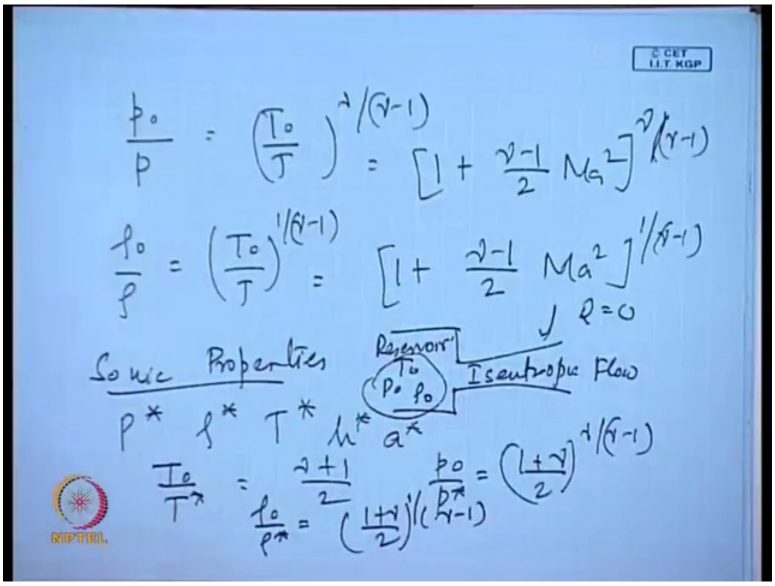

Where moment it is t star m a becomes equal to 1 when m a becomes equals to 1 it is just 1 plus gamma minus 1 by 2 or in other words this can be written down as gamma plus 1 by 2. Same way p 0 by p star this is nothing but 1 plus gamma by 2 into gamma by gamma minus 1. Same way rho 0 by rho star it can be written as 1 plus gamma by 2 1 by gamma minus 1 agreed. And we know for diatomic gases gamma is equals to 1.4. So, accordingly you can find out the values of p star p star rho star and so on and so forth. Now these were the important reference properties for compressible flows, stagnation properties, sonic properties we have found out how to express the velocity of sound in terms of measurable parameters, why Mac number is, so very important etcetera etcetera. Tomorrow we will be discussing about the chocked flow conditions we will be discussing how the effect of or rather how area change influences your flow properties. We already know that whenever there is an area change there is acceleration even for incompressible flows is not it the pressure changes the velocity changes the velocity changes that happens for incompressible flows as well.

In this particular case your with a area your velocity will change your pressure will change your density will change. So, how the situation becomes when we are dealing with area changes in compressible flows we will find out we will be discussing the chocked flow conditions, and we will proceed in this particular manner thank you very much.