Microscale Transport Processes

Prof. S. DasGupta

Department of Chemical Engineering

Indian Institute of Technology, Kharagpur

Lecture No. # 37

Microfluidic Networks

So, we are going to cover a few topics, which are slightly different in this class. The first is towards the end of the last class, we were talking about imperfect transfer of momentum from a solid surface to a gas molecule, which is impinging on it. Now, if it is a dense gas, then what we would assume is that the transport of momentum between the solid and the gas molecule would be complete. Similarly, let us say, we have a heated wall in contact with a gas.

So, at the interface between the solid and the gas, we would assume that the complete thermodynamic equilibrium has achieved and as a result of which the temperature on the solid side of the interface would be equal to the temperature on the gas side of the interface. Or in other words, there would not be any temperature jump across the interface. So, any molecule which comes from the bulk and strikes the solid, will reach thermal equilibrium, thereby attaining the temperature of the solid before it bounces back to the bulk of the gas. But this concept would be would not be valid when the pressure of the gas is reduced, and when we reach the condition such that the continuum limit may not be adhered to.

So, whenever we reach the situation of a dilute gas or in the case, where the knudsen number is rather large, in that case, imperfect attainment of equilibrium can give rise to a temperature jump at the interface. So, the concept of accommodation coefficient, which was introduced in the last class, but I could not cover it completely.

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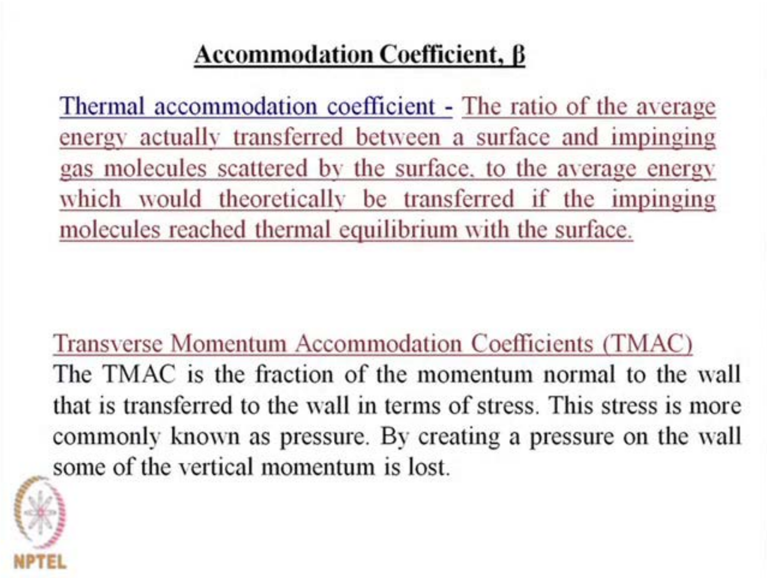

So, I am going to talk first about the accommodation coefficient and the equation that can be used for the temperature profile of a fluid, when it flows through the duct in Microsystems. And we are also going to talk about an interesting application of flow at the microscale, in the form of creating or fabricating a sensor for fluid velocity. So, how can we design sensor to measure the flow velocity, for fluid velocity? And what are the two different the, there are two different approaches. One is to measure the temperature of the sensor itself and the second is to measure the temperature of the surrounding fluid and from that, how one can calibrate the velocity of the fluid flowing past the sensor?

So, we will be discuss about that but the main emphasis of today’s lecture would be microfluidic networks. That means, if you would like to irrigate, let us say, an area with a number of microchannels. So obviously, there would be some microchannels, which would be the principle source of fluid. Let us say liquid towards that region and then you have to, have number of branches from the main artery so, to say from main channel to a number of branches and So on.

Now, we how do we branch? How do we make these branches? Should we divide them equally, first from one to two and then from two to four and so on, and if we do that, what would be? Is there a specific ratio, is there a specific geometric ratio that, we have to maintain, in order to achieve the best possible irrigation at the smallest pressure drop in the system. Because remember like the moment, we have a two channel being created from one single channel, there would be some pressure drop, there would be some entry pressure drop at the point, where the two channels, where the channel is bifurcated.

So, how do we reduce? How do we keep this pressure drop at a minimum? And that would be that, that is that, we are going to talk about in microfluidic networks. But first let us, talk about accommodation coefficient, which we define as beta and we are going to talk about two accommodation coefficients, one for the momentum and the other for energy. So, if we think about the thermal accommodation coefficient, the definition is it is a ratio of the average energy which gets actually transferred between a surface and the impinging gas molecules scattered by the surface, to the average energy which would theoretically be transferred, if the impinging molecules reached thermal equilibrium with the surface.

So obviously, the value of beta will be in most, in many cases will be less than one and the impinging molecules will reach thermal equilibrium with the surface if, the time of collision is more. So, in rarefied systems or in systems at low pressure, in systems where the knudsen number is large, there is a possibility that such equilibrium will not be reached and therefore, the fraction of that fraction is called the accommodation coefficient.

So, it is the actual amount of energy transferred in the collision divided by the amount of energy which would have been transferred, had this been that, there been complete thermal equilibrium at the solid gas interface. When we talk about a similar to thermal accommodation coefficient, there is something called transverse momentum accommodation coefficient. So, it is a fraction of momentum normal to the wall which is transferred to the wall in terms of stress, the stress is no more commonly as pressure. So, by creating a pressure on the wall, some of the momentum is lost. So, this T M A C the transverse momentum accommodation coefficient takes into account the this loss of momentum of a molecule when it strikes another wall.

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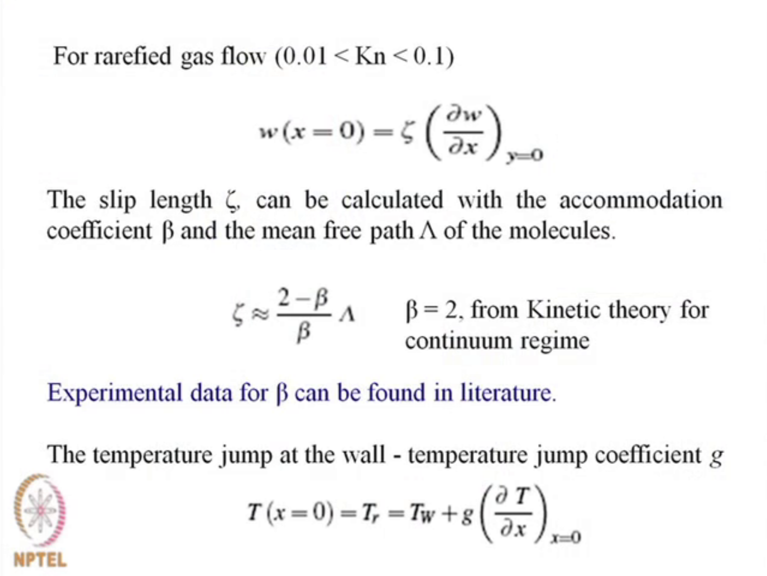

So, for rarefied gases, the velocity at the interface is not equal to 0, rather it is expressed it is modeled as a function of a slip length and a gradient of velocity at the wall. So, this is used as the boundary condition for rarefied gases and by rarefied gas we mean, when the knudsen number is greater than 0.01, but less than 0.1. For such cases, the value of the, value of velocity at the wall is not 0, it is rather expressed in terms of a property, which is called the slip length and the gradient of the velocity at that point.

Now, this slip length data are a large I mean, a large quantities of data for slip length are available for different fluids, under different conditions. So, this slip length obviously, can be correlated with the accommodation coefficient beta and the mean free path of the molecule. So, as you can see that, when beta is equal to 2, the from kinetic theory for continuum regime, we know that in continuum regime there is no that concept of slip does not arise.

So, from the definition of this slip length from the, as is shown as 2 minus beta by beta, the value of beta can be shown from kinetic theory, that for continuum. When the continuum approximation is valid, the value of beta is equal to 2 and the value of slip length would be 0 and the condition at the solid gas interface, would be the no slip condition. But the value of beta will not be equal to 2, for most of the other cases and obviously, as the mean free path increases, the value of the slip length will also increase, and the mean free path will increase for a system which is rarefied.

So, for a rarefied system will have finite slip and the slip coefficient, the slip length will increase with increase in mean free path. So, experimental data for beta can be found in the literature and if, we know, what is the pressure of the system? Then we should be able to find out, what is the mean free path? So, combining these two, the slip length for velocity at the interface, at the solid fluid interface can be evaluated, which will serve as the new boundary condition for solution of equation of motion and many such cases.

So, similar to the concept of slip length in fluid flow, there would be some sort of a temperature jump at the wall and that is the wall temperature jump coefficient, that is denoted by g and it is called temperature jump coefficient. So, the temperature at temperature at x equal to 0 that means temperature at the interface, if we call that as T r which is not going to be equal to T w. But another quantity is going to be added to it with g as the parameter and the temperature gradient as at the wall being the other factor. And the product of the two will have to be added to T wall, the temperature of the wall in order to obtain, what would be the temperature on the gas side of the interface.

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The temperature jump coefficient g via kinetic theory from the thermal accommodation coefficient γ, a material parameter f, and the mean free path Λ

$$g = \frac{2-\gamma}{\gamma} \frac{15}{8} f \Lambda$$

The material parameter f is calculated from the linearized Boltzmann transport equation and is given by

$$f = \frac{16}{15} \frac{\lambda}{\eta\, c_v} \frac{1}{\kappa+1} = \frac{16}{15} \frac{1}{\Pr} \frac{\kappa}{\kappa+1}$$

λ = thermal conductivity, η = viscosity, κ = isentropic exponent = c_p/c_v

So, conceptually, the accommodation coefficient gives us the slip length and if it does so if, accommodation coefficient gives us the slip length, then we should be able to correlate in a similar fashion, the temperature jump at the wall defined as T r minus T w with a parameter, which is called the temperature jump coefficient g and the gradient of temperature at x equal to 0. For situations, for rarefied situations where the collision between the gas molecule and the solid molecule would not take place for sufficient length of time, for equilibrium, thermal equilibrium for to reach thermal equilibrium at the interface. Now, this temperature coefficient g via the kinetic theory can be, can also be obtained via kinetic theory from the concept of thermal accommodation coefficient, a material parameter which is denoted by f and the mean free path.

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The temperature jump coefficient *g* can be regarded as an additional distance of the gap.

So, g is two minus gamma, where that is accommodation coefficient, the f which, whose formula is given and the mean free path. So, once we have the value of g, then the temperature jump at the interface can be evaluated. Now, this temperature jump coefficient g as we have mentioned can be regarded as an as additional distance of the gap.

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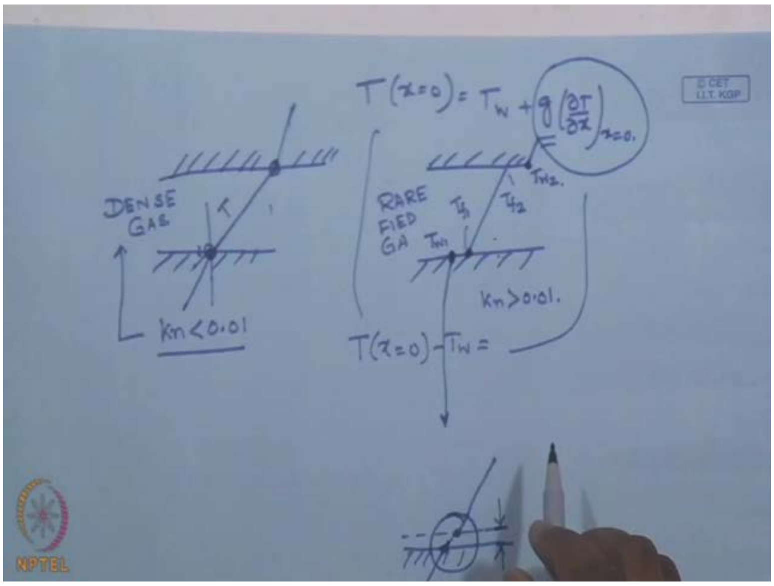

So, if we write the equation for the temperature at the wall, which is T at x equal to 0 is equal to T of the wall plus g times del T by del x at x equal to 0. So, of course, this temperature jump is T at x equal 0 minus T w and which would be equal to this term where g is the coefficient. So, what I have drawn here is, a situation in which the; this is the dense gas and we have the temperature gradient. So, this is the temperature gradient, temperature profile, temperature gradient. This is the wall temperature, which is the same for the solid as well for the liquid side, as well as the gas side interface and there is a definite drop and over at the interface also. The two temperatures at both sides of the interface is same and then we have the; another, we have the case.

So, this is a case for knudsen number less than 0.01 such that, we have this dense gas situation and there is no temperature jump across the interface. On the other hand if, it is rarefied gas, then what we find is that, this is the temperature on the solid sight of the interface and then, we have a temperature this is on the liquid sight of the interface, this is the liquid sight interface at the other end and here we have T w 2. So, this situation is for a rarefied gas, where the knudsen number is greater than 0.01.

So, this temperature jump at the interface can be thought as if, we have an additional distance of the gap. So, what we can think of this to be equivalent to? I have the solid, then as if the solid has been we have another gap. So, this is the solid temperature, this is imaginary gap and then, I have this temperature. So, in many it is conceptually easier to understand visualize the situation as if, the presence of a specific gap, additional gap in this, is providing the extra resistance to heat transfer for rarefied gases.

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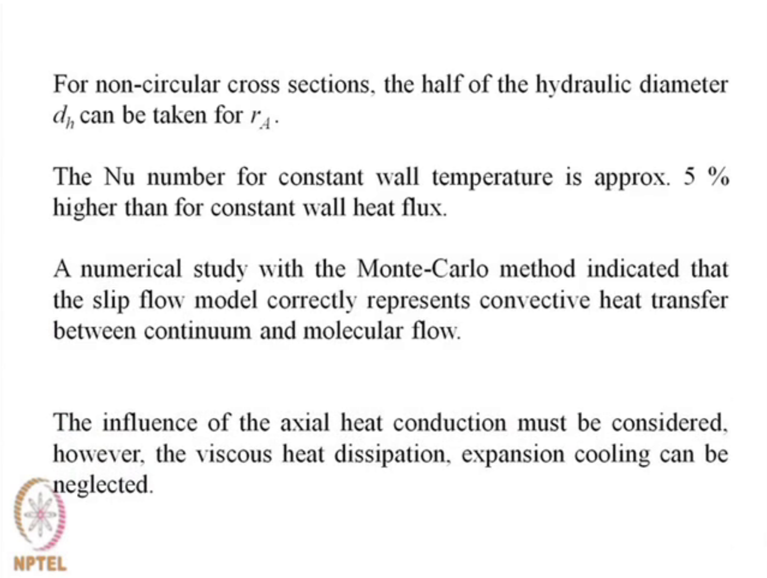

So, the other factors which we need to know is that, the nusselt number for constant wall heat temperature is going to be different from that of the constant wall heat flux. And a numerical study indicates that, the slip flow model correctly represents convective heat transfer between the continuum and the molecular flow. So, whenever continuum model breaks down, the use of slip flow model correctly pictures, the correctly represents heat transfer between heat transfer for such cases. And in microchannel flows, we also need to consider the effect of actual heat conduction, but generally the viscous heat dissipation and the expansion cooling can be neglected.

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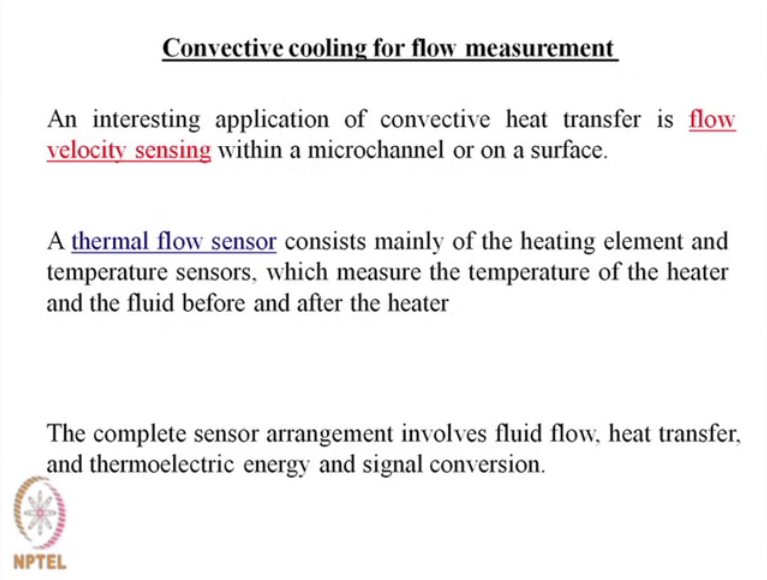

Now, we come into the, one of the applications of convective cooling for flow measurement, we all know that, as the flow over a hot surface is increased, we are increasing heat loss from the solid. So, is there a way to relate the cooling achieved, due to an enhanced flow on a solid substrate and more importantly, can that be used as a sensor to measure the velocity of the flowing fluid. So, in order to design such a sensor, certain considerations will have to be kept, will have to be kept in mind, and the governing equations for such cases need to be clearly stated.

So, what we are going to look at is the construction of a thermal flow sensor, which consists mainly of the heating element and temperature sensors that, measures the temperature of the heater and the fluid before and after the heater.

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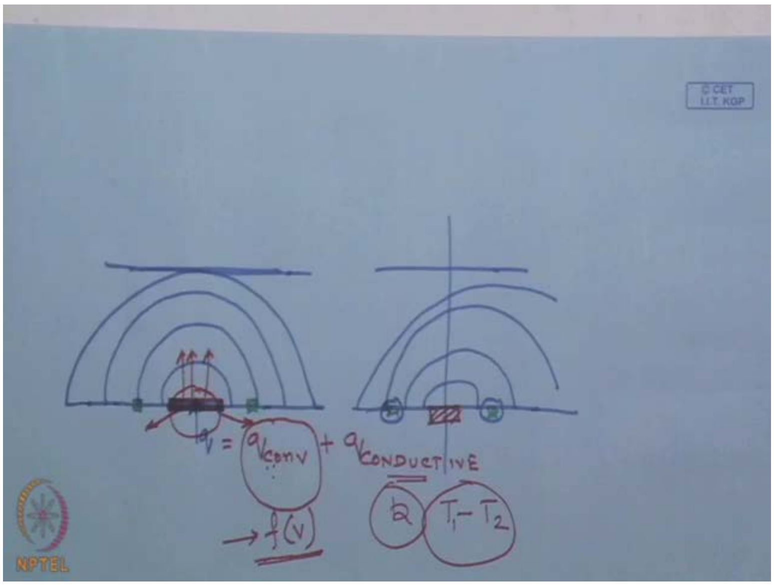

So, let us say, what we have in this case is, I have a heater embedded into a solid. So, this is my heater, this zone is the heater and the, it situated in a channel and I have a temperature sensor over before the heater and that equal distance after the heater. So, when we have no flow in the system, then what we have is a temperature profile, which. So, I have some heat, which is being added to the system and I can draw the temperature profiles, which are going to be symmetric in shape. But this symmetry is going to be disturbed the moment, I am going to have a flow present in the system.

So, the moment I have a flow in the system, then the profile will look something like this. So, as if everything will shift towards the right now, here also I am going to measure the temperature in the using these two sensors. Now, what happens then? We need to; we are going to find out the governing equations, we are going to write the governing equations for slow heat loss from this plate. Now, if you think carefully, the heat loss from this plate would be into the fluid, but some heat will also be conducted into the solid. So, that is going to be.

So, the total amount of heat which is being supplied to the heater will have a convective contribution, will have some convective losses plus there will be some conductive losses, from the heater to the solid. Now, this convective heat loss is going to be a function of the fluid velocity. So, we are going to see, what are the what is going to be the functional form of q convection in such a case? Whereas, in the case of q conductive, it is going to depend on the thermal conductivity, it is going to depend on the temperature difference and so on.

So, this is material property, this is a condition whereas, over here, this is going to be a function of the velocity and that would allow us, if you can calibrate properly that would allow us to find out, what is going to be the? What is going to be the velocity of this situation? So, if you look at the complete sensor arrangement, it involves fluid flow, heat transfer and thermoelectric energy. And we will also need to do some sort of signal conversion in order to achieve this.

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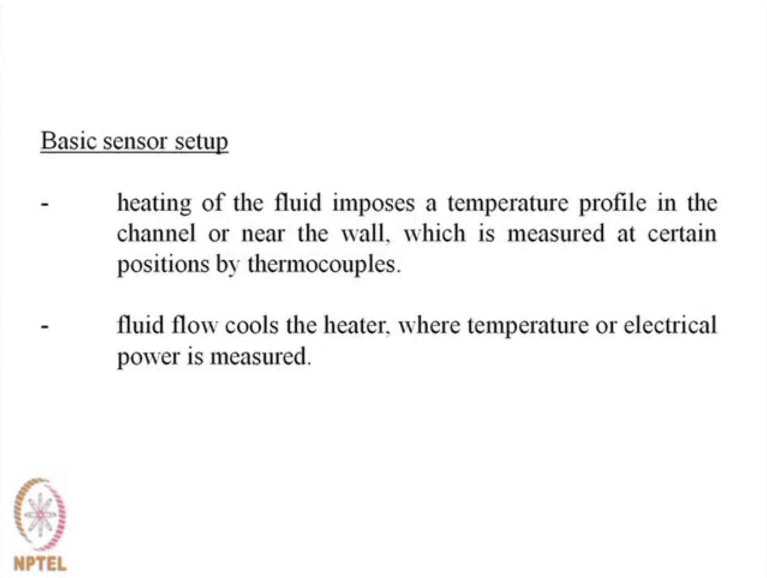

So, the basic sensor setups can be of two types. One is, we have to, the basic sensor setup is that, the heating of the fluid it is going to impose a temperature profile in the channel or near the wall, which is measured at certain position by thermo couples or some such sensors. The fluid flow cools the heater, whose? Where temperature electrical properties are measured?

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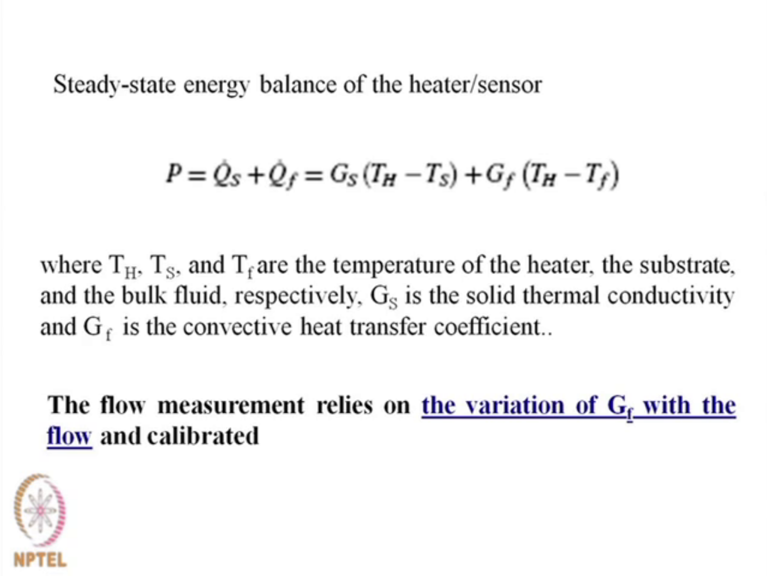

So, let us look at the energy balance for such cases. Now, if we; if you see the total power lost by the heater is a function of q convection and a function of q conduction. Which are the first and the second terms of the equation that, I have write now, where T H and T S and T f are the temperture of the heater, the substrate and the bulk fluid. So, T H minus T S provides the driving force for conductive heat transfer in the solid whereas, T H minus T f provides the gradient necessary for convective flow.

Now, G s and G f are two coefficients, one refers to conduction and the other refers to the convective heat transfer coefficient. The flow measurement essentially, relies on the variation of G f with the fluid flow and is calibrated. So, we know that from our calibration that, for the value of G f to be this, what would be the corresponding velocity of the fluid? So, from that one can find out, what is the velocity of the fluid?

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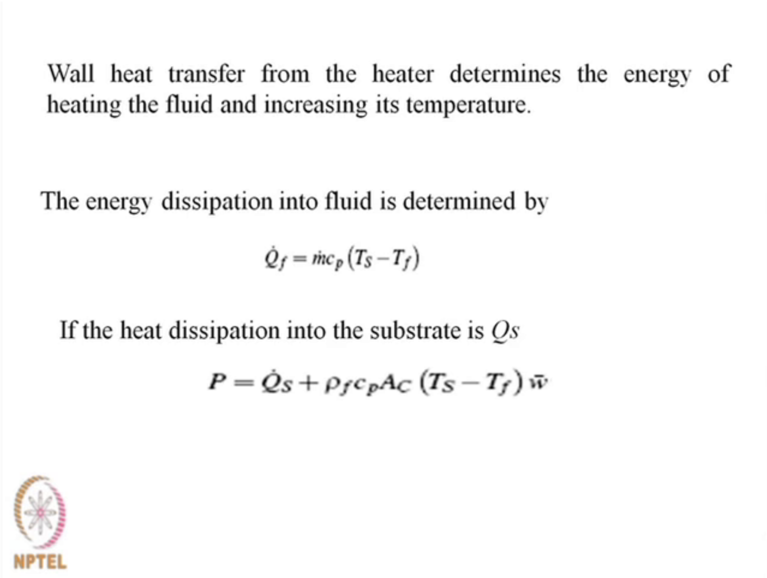

Now, wall heat transfer from the heater evaluates the energy of heating the fluid and increasing its temperature. Now, if you look at the last equation, the heat dissipation into the substrate if it is Q s, then the total power provided to the heater is a sum of Q s. Which is the heat dissipation of the, into the solid substrate plus rho of the fluid c p of the fluid, area of the heater, times T of the heat T of this; T of the heater minus T f multiplied by w, where w is the velocity.

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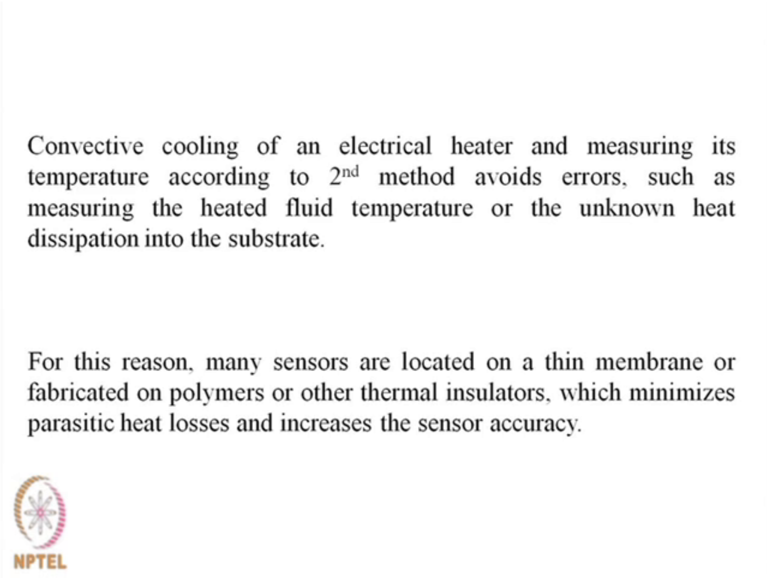

So, this gives us the complete picture, complete figure of complete equation for the heat loss from the sensor surface. Now, if you convective, if you use convective cooling for an electrical heater and you measure the temperature according to the second method, that we have just discussed, it involves lesser errors. So, what we do is, we do not have to measure in the second method, we do not have to measure the temperature of the fluid at different points, which is difficult, rather we measure, what is a temperature of the solid at different points?

So, as we understand that with increase in flow, we are going to provide more heat to this sensor and if you provide more; if you lose more heat through the sensor, then these solid temperature should also vary. And the variation of this solid temperature will provide us and the knowledge about, how much of heat is being lost? So, it is a much better method to measure the temperature of the solid compared to the temperature of the fluid over here.

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Basic equations - convective heat transfer into the fluid

$$\dot{q}_f = \frac{\dot{Q}_f}{A_H} = \alpha_W \left(T_H - T_f\right)$$

$$\mathrm{Nu} = \frac{\alpha\, z}{\lambda_f} = 0.332\ \mathrm{Pr}^{1/3}\mathrm{Re}^{1/2}$$

$$\mathrm{Nu}_L = \frac{\alpha\, l_H}{\lambda_f} = 0.664\ \mathrm{Pr}^{1/3}\mathrm{Re}^{1/2}$$

$$\frac{\dot{Q}_f}{A_H} = \frac{\mathrm{Nu}\,\lambda_f}{l_H}\left(T_H - T_f\right) = 0.664\,\lambda_f\,\mathrm{Pr}^{1/3}\left(\frac{\bar{w}}{l_H \nu}\right)^{1/2}\left(T_H - T_f\right)$$

$$P = G_S\left(T_H - T_S\right) + 0.664\,\lambda_f A_H \mathrm{Pr}^{1/3}\left(\frac{\bar{w}}{l_H \nu}\right)^{1/2}\left(T_H - T_f\right)$$

NPTEL

Reference: Transport Phenomena in Micro Process Engineering, N. Kockmann, SPRINGER 2008

So, as a result of which, most of the sensors that we see, are located on a membrane and this minimizes the parasitic heat losses and increases the sensor accuracy. So, these are some of the basic equations, which are given in any text book and here I am referring to the text book by cockman, transport phenomena in micro process engineering. So, they I have collected the equations from there. So, what you see first is the heat loss to the fluid. So, it is T h minus T f is a driving force and when you an alpha is the heat transfer coefficient.

So, if you think of the flow to be laminar over the heater, which in most cases of micro channels, it is going to be laminar flow. So, the nusselt number 0.12 of the nusselt number is simply equal to 0.332 prandtl to the point one third reynolds to the power half. And if we integrate this nusselt number expression for nusselt number over the entire length of the heater, what we get is, nusselt number based on the entire length to be equal to 0.664 prandtl to the power one third and reynolds to the power half.

So, in this case the reynolds number refers to, reynolds number is based on the entire length of the heater. So, putting the value of heat transfer coefficient in the first equation, one can obtain the heat lost over the heater area to be to expressed in terms of lambda, the property prandtl number, the similarity parameter and all the other things. So, the total power which is to be, which is dissipated by the heater is given by the last equation in terms of quantities, in terms of all the quantities, which can be measured. So, T the temperatures to be measured are temperature of the heater temperature of the solid and then the rest are dependent on the velocity and the thermo physical property of the fluid.

So, this way, once this is calibrated this way, detailed analysis of such correlation requires many parameters. If you look at the previous expression, then you would see that large number of measurements are to be carried out, the physical properties at different values of temperatures are to be known and then only one can use this expression to find out, what is a total power loss from such an heater?

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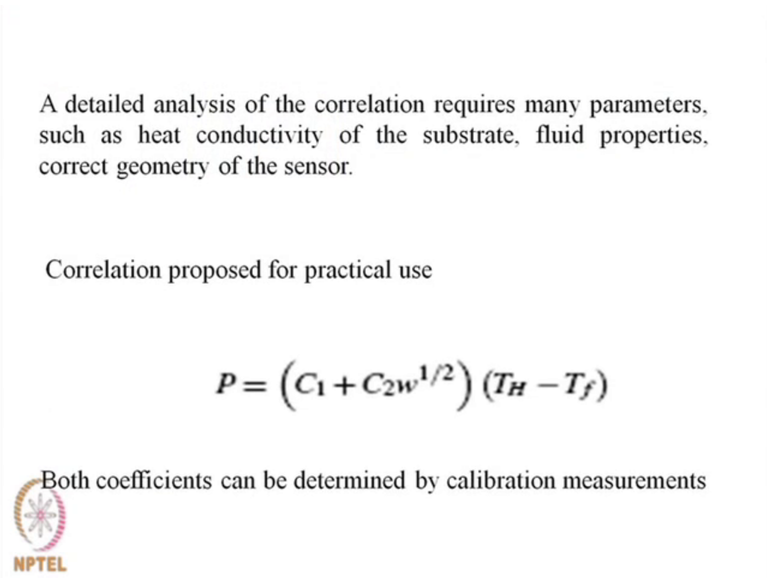

So, in order to extend the use of this, in order to ease the use of such a relation, a correlation of the following form is proposed and is used in most of the calculations. Which simply tells us, that the correlations are that is two coefficients C 1 plus C 2 into w, which is the velocity to the power half T of the heater minus T of the fluid. So, both these coefficients are calculated are evaluated from calibration measurements and therefore, we should be able to obtained from the known value of P and from the measured value of the temperatures. The unknown value of w, which is the flow velocity and that is the purpose of the flow sensor.

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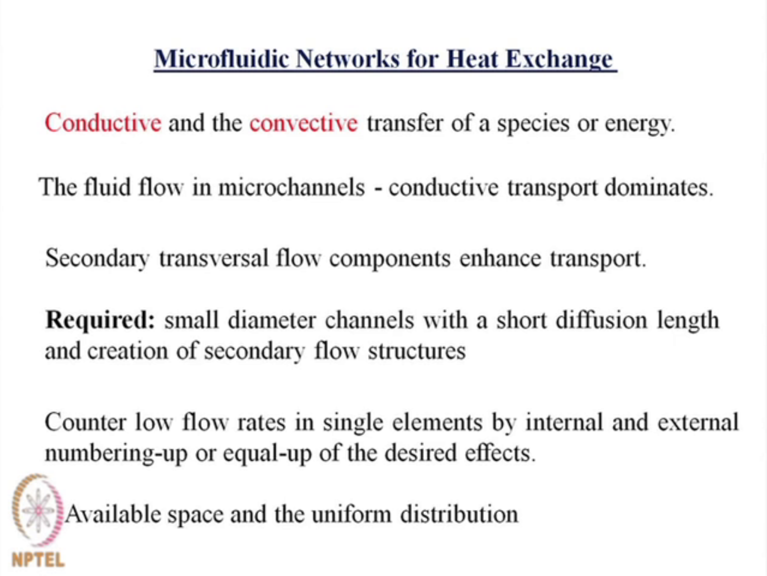

So, C1 and C2 are already known to us. So, measuring the two temperatures and the power input to the heater, we would be able to obtain, what is the unknown value of the fluid flowing; unknown value of the fluid flow velocity. Now, we come to an interesting part of it, which is the; which is what? I wanted to discuss in this class is the microfluidic networks for heat exchange. Now, conductive and convective transfer of a species or energy is the focus of creating microfluidic networks. So, it can be used for heat exchange, it can very well be used for mass transfer as well.

So, when we talk about fluid flow in microchannels, is a conductive transport which dominates over the, which is dominating? Now, you can have secondary transversal flow specially, when you have an obstruction in the path of the flow, or obstruction which is perpendicular to the direction of flow. Or you have a bend in the path of fluid thereby, causing secondary flows for example, dean flow which we? Which one can encounter? Whenever, there is a 90 degree or some such bend and which depends on the reynold's number and which also depends on the dimension of the channel and the radius of the curvature or the bend. So, that is how, dean flow number is define its reynold's numbers times, d pi the radius of the curvature to the power half.

So, these kind of secondary flows increase the value of heat or mass transfer now, the counter flow rate since the requirement here, is small diameter channels with a short diffusion length and creation of a secondary flow structures. So, if you can achieves that, we if, you can have a small diameter channel, whose diffusion length is obviously, small. And if, we can create a number of secondary flow structures, then we are going to get both conductive and convective flow and we have high, we can enhance a transport process. Now, the major drawback of some of these processes is the flow rates are going to be extremely small. So, if you think of a single element of a microchannel, the amount of liquid, it can handled is going to be very small.

So, even though the diffusion length is small and therefore, you can have very high transport rates in between two species or very high heat transfer rate, but the small size of the device means, that you can handle very small quantities of fluid at a time. So, you can increase that, by having a large number of such channels connected parallely, but you can. So, you can do internal and external numbering up or equal up two of the desired effects to enhance the rates, which are possible in such cases. But this internal or external numbering of or the equal of process needs space and uniform distribution of fluid into all these parallel channels is a major challenge. So, the space and the distribution, these two limit the use of numbering up or equal up strategies to encounter; to counter the small quantities of fluids, which can be handled at a time.

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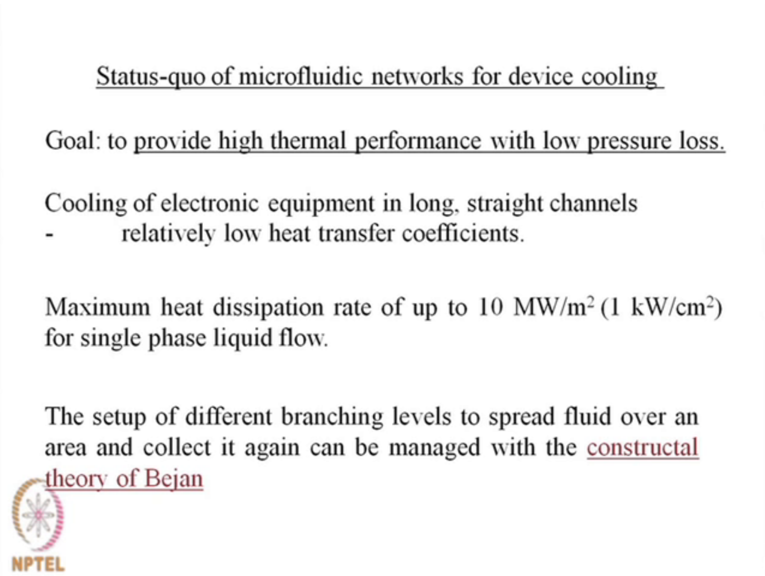

So, we are going to look at, what is the status of microfluidic networks for device cooling. Now obviously, what we would like to do is to provide high thermal performance with low pressure loss. So, one such requirement is the, cooling of electronic components in mostly long straight channels are used, to the long straight channels are relatively easy to fabricate. So, there is no question of distribution; no problem of distribution of liquid, but the problem is this, has relatively low heat transfer coefficients.

To give you an example, the maximum heat dissipation rate is of the order of 10 mega watt per meter square for single phase liquid flow. But so we, but in order to reach to all the places, one needs to to have a number of branches from the mean channel, from the principle channel. So, let us to spread fluid over an area and you not only have to spread the fluid over the, over an area, you have to collect it again. And how do you make these branches? It depends, it is governed by the constructal theory of Bejan.

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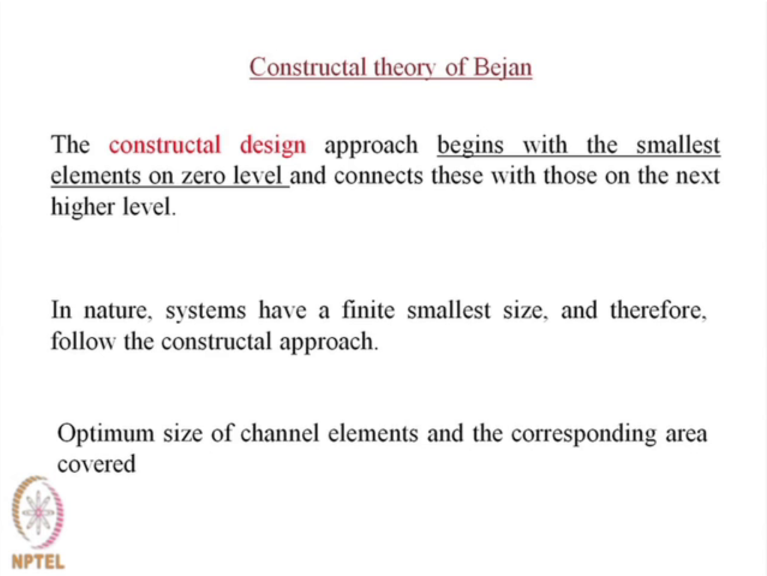

So, the constructal theory of Bejan is extremely interesting. So, the constructal design approach, it begins with the smallest element on zero level and connects these with those on the next higher level and this is very common in biological systems. Think about all the arterial network, that we have in our system. So, the arterial network, the smallest of these arteries, the capillaries reach each and every cell of our body. So, from a tiny capillary, then it is going to join with another once. So, that becomes the building block of the entire blood circulation system in our body.

So, from the tiniest capillary, they combine from the next level of blood vessels and those very small blood vessels keep on combining till we have the major arteries that supply blood from our heart to the remotest place of our body. So, the constructal theory of Bejan, mimics the natural process, mimics whatever, what is already available, already known, already established in biological system and tries to apply it for uniform distribution of fluid over a hot surface. Such that, it can be cold convectively and collects the hot liquid, once again at the end of the process and brings it back to the point from where, it starts its movement towards the hot surface once again.

So, in nature, the systems have a finite smallest size, and therefore, they follow the constructal approach. The question is, how do we finite, find out the optimum size of channel elements and the corresponding area covered?

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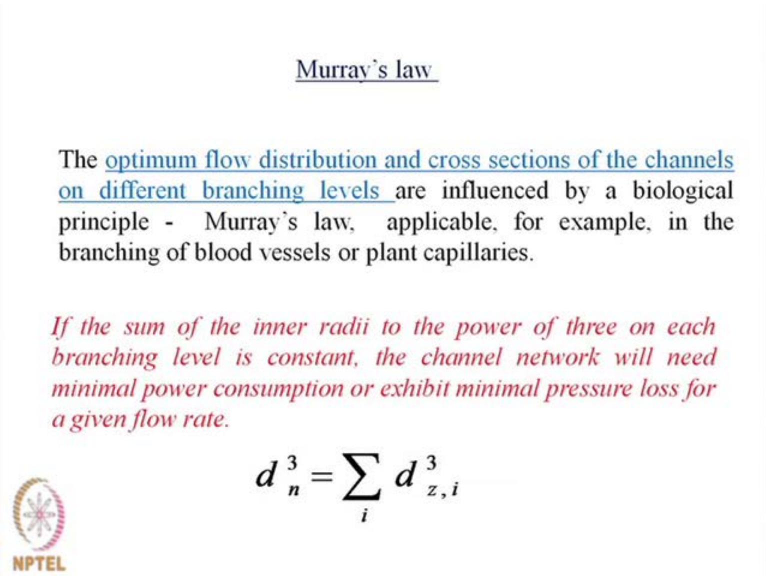

So, here we come across a very interesting law, which is known as the murray's law. It simply tells us that, the optimum flow distribution and cross sections of the channels on different branching levels are influenced by a biological principle, which is known as the murray's law. It is applicable for in the branching of blood vessels or plant capillaries. Now, when we see our body as a system? Many of the, finally, what is the size of the arteries, and size of the vessels? They are in the microscale.

So, what our heart does? It pumps blood from the heart to every section of our body. Now, when it does that, it the blood which flows from the heart encounters, a number of branching out at different places. So, from the main aorta it divides into two and two divides into another two and so on. At each of these branching, there would be some pressure loss, but the in order for a healthy heart, in order for the heart to do minimum amount of work, these branching should be such, that the pressure drop due to these branching is the smallest. And when the, when researchers compare the sizes of the branches with that of the mother artery, what they have found out is, that if, d 1 and d 2 are the diameters of the two branches and d 3 is that of the mother artery.

Then d 1 cube plus d 2 cube is equal to d 3 cube and it has been mathematically shown that, when the branching out follows this rule, that d cube is equal to d 1 cube plus d 2 cube. The pressure drop in such systems would be the least, and that is what, the; this was observed by murray and he proposed it as a law, which you known, which is known as the murray's law. And what is simply tells us is that, if the sum of the inner radii to the power of three on each branching level is constant, then the channel network will need minimal power consumption or exhibit minimum pressure loss for a given flow rate. And it is amazing to think that, all branches in our body fallow murray's law.

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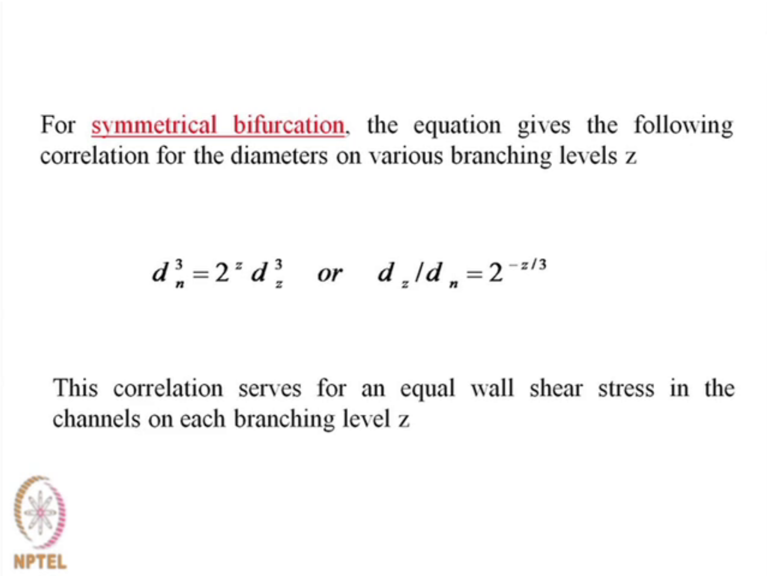

So, what has been proposed is that, the same law now can be used for branching out of microfluidic channels as well now. So, symmetrical bifurcation, the murray’s law gives the correlation for the diameters on various branching levels. So, if we have a d three. So, the first equation, if you see, the d three n, that is d three n d sorry d n cube is equal to 2 to the power z, when there are z number of bifurcations and d z cube.

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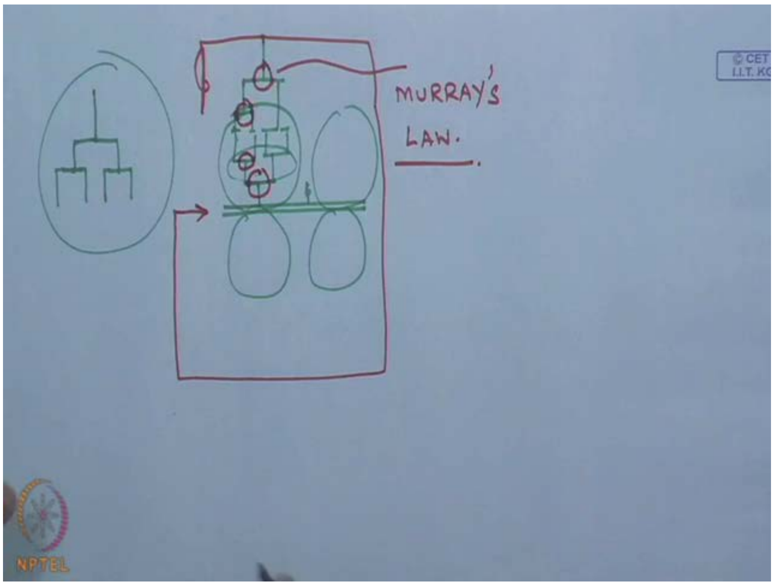

So, this whenever, there is any branching that is done on a system, it must follow murray's law. And some of the examples of murray's law are amazing and I am going to, I will try to draw this a two level heat exchanger network from branch channels is that, one is start with one channel. You create two channels, then these two channels will create another two and this is going to create one more channel. So, this is going to be an unit. So, this is the set up of a two level heat exchanger network from branched channels and the way it is done is let say, this is the main channel through which the liquid is being pumped. And then we are going to have a branching at this level, branching at this level and so on.

And there will be this unit. This unit will be repeated over here and over here. So, if you take the complete diagram, you would see that, these will later on be combined. So, this is the region, which you would like to cool and they are going to be combined into one. These are at the end, they are going to be combined into one, then these two are also will be combined into one. But at every stage, at every point of branching out and the point where they are going to be combined so that combine. So, that I can have flow again from here, at every point murray's law will have to be will have to be maintained, in order to obtain the minimum pressure drop.

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For symmetrical bifurcation, the equation gives the following correlation for the diameters on various branching levels z

$$d_n^3 = 2^z d_z^3 \quad or \quad d_z/d_n = 2^{-z/3}$$

This correlation serves for an equal wall shear stress in the channels on each branching level z

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So, this relation solves for an equal wall shears stress, in the channels on each branching level. So, this is branching level one, and this is branching level two. So, we have a two level branching and then two level combinations in order to irrigate a space on a solid to maintain to lower its temperature.

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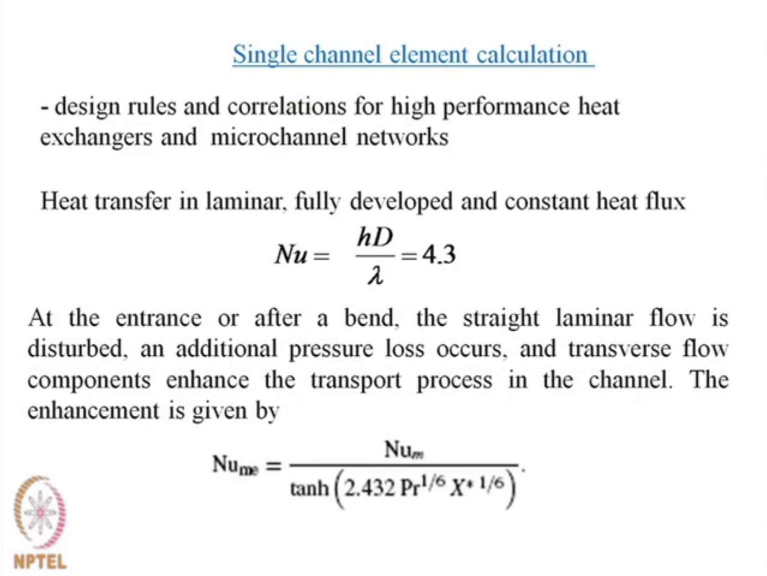

So, what are the single channel element calculations? We are already familiar with some of them. So, we need to look at the design rules and correlations for high performance heat exchangers and microchannel networks. So, if its laminar, fully developed and constant heat flux case, then the nusselt number is a constant equal to 4.3, but if you have the entrance or if you have a bend, then the straight laminar flow is disturbed. What we then have, just after a bend or near the entrance, developing flow taking place.

And this developing flow, in with the appearance of additional pressure loss is going to enhance the transport process in the channel. And this enhancement is given as the equations, suggests an nusselt number m e is equal to nusselt number m, where the nusselt number m, the right hand side denotes, where the flow is further developed. And the left hand side suggests, it is the nusselt number for the developing flow and these two are connected in terms of prandtl number, which is a thermo physical property of the liquid and X star, which is the dimensionless entry length.

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With the dimensionless entrance length X^* given by

$$X^* = \frac{L}{d_h} Pe = \frac{L}{d_h} Re \cdot Pr$$

The optimum channel length x,

$$\frac{x}{d_h} = Re \cdot \left(0.411 \cdot \operatorname{arctanh}\left(\frac{Nu_n}{Nu_{me}}\right)\right)^6$$

To yield a heat transfer enhancement of 30 % ($Nu_{me}/Nu_m = 1.3$), the entrance length divided by the hydraulic diameter should be less than 0.005Re.

For longer channels, the pressure loss increases without additional benefit to the heat transfer.

So, for with the dimensionless entry length X star given as a L, which is the length of the channel, d h which is the hydraulic diameter and peclet number, which is the product of reynolds into prandtl number. When we are talking about heat transfer, for mass transfer peclet number, it would be the product of reynold's and schmidt number, in one can obtain from this, what would be the optimum channel length for such cases? So, in order to, in just give you an example, in order to obtain a heat transfer enhancement of 30 percent.

Such that, nusselt number m e, the nusselt number for the developing flow of the nusselt number, when there are number of bends present in the flow path divided by nusselt number m, which is the full developed condition to be equal to be 1, to be equal 1.3. The entire length divided by hydraulic diameter, if you look at the left hand side of it, the entire length divided by the hydraulic diameter should be less than 0.005 reynold's number.

So, this gives a very good handled on the design, on the efficient design of a micro microchannel, which would provide certain quantity, certain amount of heat transfer or mass transfer enhancement. So, this tells us precisely, what for in order to obtain x percentage of heat transfer enhancement. The entire length divided by the hydraulic diameter x by d h should be less than 0.005 reynold's number. So, you know? If, you know your reynold's number, if you know the value of d h, then you know, what would be the entrance length, to obtain a certain percentage of heat transfer enhancement? But there is obviously, a limit to this, if you use a longer channel, then the pressure loss increases without any additional benefit to heat transfer.

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Simulations of the heat transfer in a microchannel with square cross section, constant heat flux and curved flow at bends demonstrate

- existence of vortices in curved flow, Dean flow.

- heat transfer enhancement directly after the bend

- phenomena dampened by viscous forces subsequently

- relaminarization in straight channel flow after curved flow

- Nu number asymptotically approaches the value of laminar flow in straight channels

So, simulation of heat transfer in a microchannel with a constant heat flux and with curved flow at bends, they demonstrate certain interesting trends, which we have already discussed, but I am summarizing it over here. So, if you have flow in a microchannel under constant heat flux condition and the curved, there is; there are bends present in the path and there is a curved flow. What you see first is the existence of vortices, which are all at the bends, which is called dean flow. So, the existence of the dean flow can be seen from the numerical simulations. The heat transfer enhancement, there would be heat transfer enhancement just after the bend, and once you have such enhancement and such existence of vortices the phenomena will be dampened by the viscous forces present in the path.

So, viscous forces present in the fluid. So, any vortices, which is a primary cause of heat transfer enhancement will be dampened and there will be laminar, in re introduction of laminar flow in the straight portion of the pipe, after the bend. Now, whenever you have straight portion of the pipe, the viscous forces will force all these vortices, to bring back laminarity in the system once again. So, any enhancement of heat transfer, any enhancement of transport process, that you have, may get, will be nullified, will be dampened once you move away from the bend and finally.

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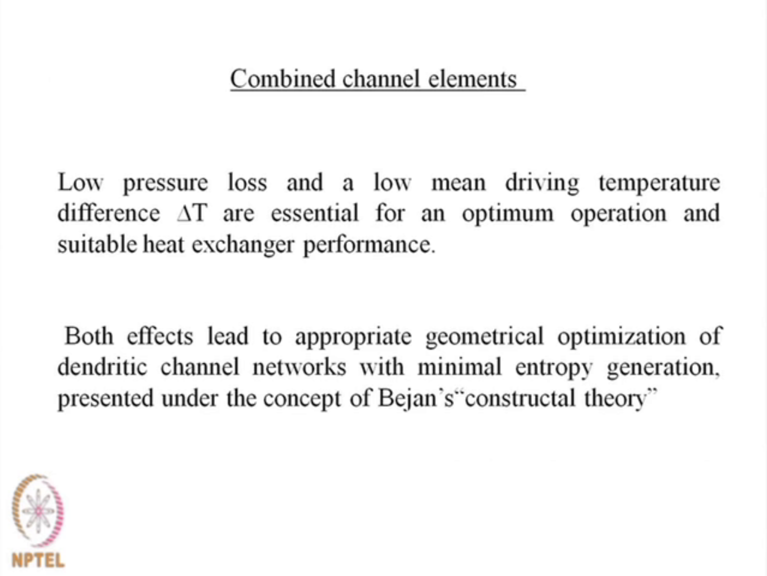

The nusselt number will asymptotically approach the value of the laminar flow in straight channels, once you flow passed the, such bends. So, if you a combine, if you have combine channel elements, then what you would want is, low pressure loss and low mean driving temperature difference. So, what you can have? You can enhance the performance of a heat exchanger. Now, both these effects lead to geometrical optimization of dendritic channel networks, which follows murray's law and which is under the broad concept of Bejan's constructal theory. And this ensures, minimum entropy generation and once you have minimum entropy generation, then that is the most optimum, that is the optimum channel network, that you can think of.

So, you work with single channel to get the maximum amount of heat transfer enhancement and then combine the channels as for Bejan's theory. As for murray's law to create more and more to create a network of channels, which will have the advantages of high transfer rate and at the same time, the least possible pressure drop.

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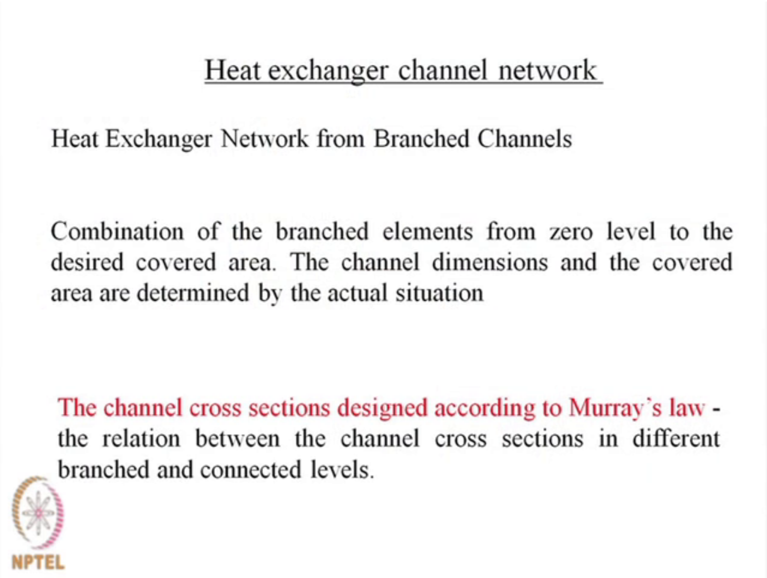

So, these are the points, which need to be kept in to mind, while designing a microfluidic network, and when we think about heat exchanger channel networks. So, from branched channels, we need to combine the branched elements from a zero level to the desired covered area. Whatever area, that you would like to cover, we would, we are going to combine the branch elements, the channel dimensions and the covered area determined by the actual situation, the channel cross sections are going to follow murray's law.

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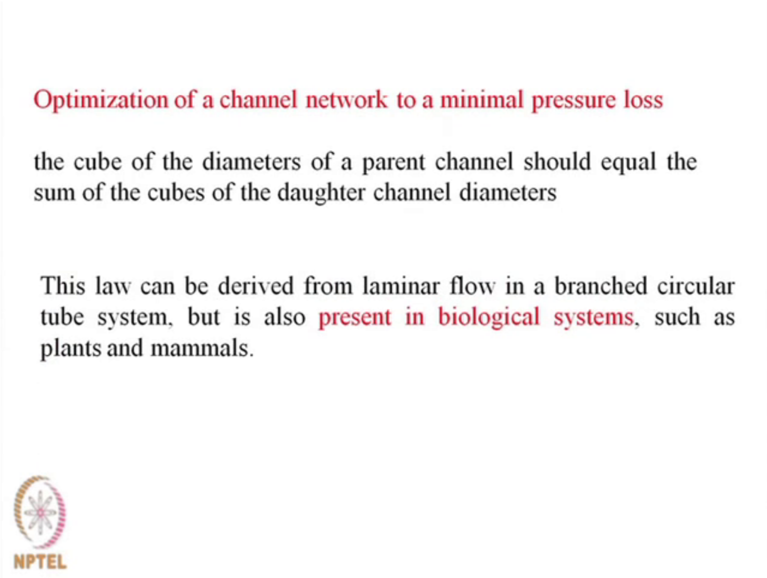

Now, when we would like to optimize the channel network, we are going to follow whatever is present in biological systems, such as in you and me and thereby obtain the channels dimensions.

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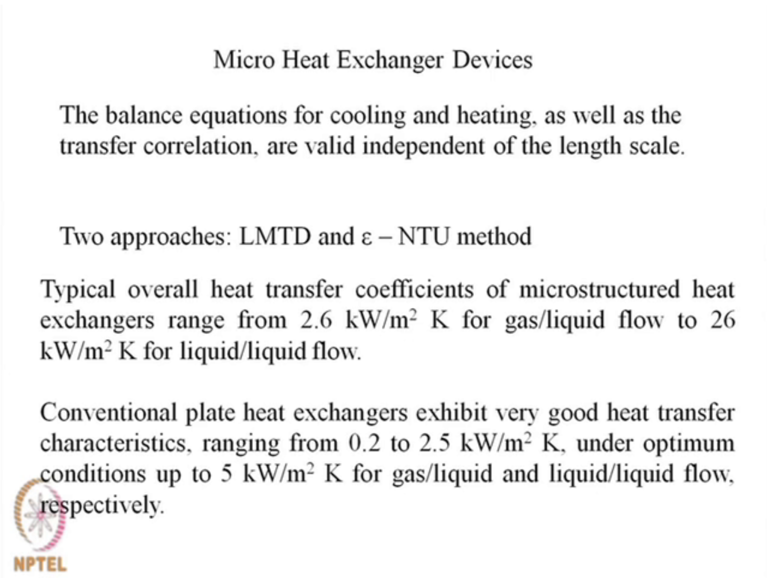

When we design a micro heat exchanger device, we have already covered, what are the balance equations for cooling, for heating, the transfer correlations mostly assuming laminar flow. And we know that, these expressions are valid independent of the length scale. So, these expressions can be used for a microchannel as well. Now, there are two obviously, two different approaches the L M T D method and the epsilon N T U method, while designing a heat exchanger and any of these methods can be followed depending on the situation at hand.

So, just to give you an idea that, what are the typical over all heat transfer coefficients of microstructured heat exchangers, the range from 2.6 kilo watt per meter square for gas liquid, 26 kilo kilo watt per meter square for liquid liquid flow. Now, if you compare that with the best conventional heat exchangers, that we plate heat exchangers, that we can have, they also exhibit very good heat transfer. But their range from 0.2 to 2.5 meters kilo watt per meter square per Kelvin under optimum conditions and may be up to 5 kilo watt per meter square per Kelvin for liquid liquid flow.

So, if you see the advantage of a micro heat exchanger, you would get about 5, it is five times more efficient. It can extract 5 times more heat from a region compare to a plate type heat exchanger. So, that is the unique feature, unique advantage of a micro heat exchanger in or any microscale device, which has initiated such intense research in recent years in the scale; in the, in the field of microscale transport processes. Not only newer and newer devices are being fabricated, being invented or there would be tremendous development, in the design of microstructured devices.

But to understand the basic physics of the process, there is, it is an area of intense, an intense research activity and which will continue for quite some time, because we still have not reach the point, where we cannot go any smaller. So, from micro device we are. So, we are probably going towards nano device, the problem that we have is, how to fabricate it? How to make the system leak proof? How to reduce the pressure drop? That is one of the major problems and how do we distribute the liquid? How do we distribute the fluid evenly in all the channels?

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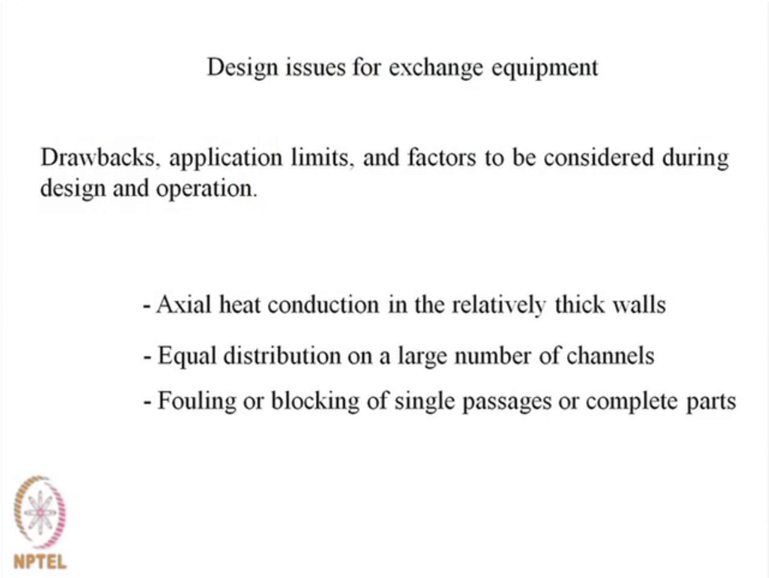

So, these are the major, some of the major drawbacks. So, and the design issue, if you think about the design issues for the exchange equipment, when we consider heat transfer, it is the actual heat conduction in the relatively thick walls, which reduces the delta T available. And if it reduces delta T, then our process efficiency gets hampered, how do we distribute it equally in all channels? and this, another interesting, another important thing that I have not spoken before is falling or blocking of single passage or complete parts. It is not possible in many most of the cases, to clean a microchannel once it gets fouled or it is blocked by a presence of suspended particles.

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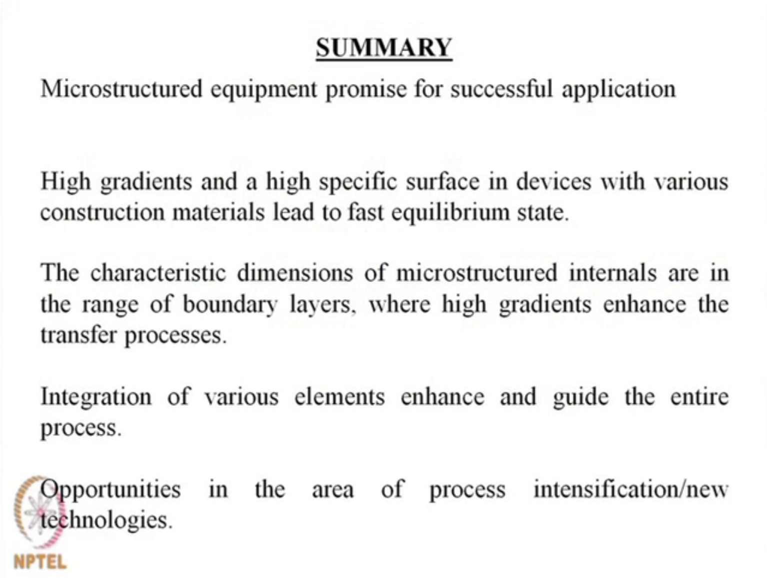

So, these are some of the issues that one keep, has to keep in mind during design and operation. So, as a summary what I am going to talk? What I can see is that, the microstructured equipment, they are promising and there are many successful applications, where microstructured, micro devices have been effectively utilized. The major reason why such devices are so successful is the, high gradients, high surface area, surface to wall, surface to area volume ratio and there are various construction materials depending on the application.

So, if you have. So, so many construction materials for each specific use and such high gradients allows the use of such devices in a variety of such situations. So, it is very flexible, it is range of applicability is quite large, and the characteristic dimensions of such devices are comparable to the thickness of the boundary layers. Now, that is one of the reason, why devices offer such high transport rates? So, there is no reason, which unutilized, where the concentration is the same. Where the concentration is uniformed or the temperature is uniform. Throughout everywhere in the device, the device is performing its transferring heat or transferring mass.

So, thereby the entire volume, entire cross section of the device is being utilized, an integration of various elements they enhance and they guide the entire process. So, there are defiantly, huge opportunity in the area of process intensification, in the area of new technologies. So far we have discussed, so many positive as well as some of the limitations of microscale processes, microscale devices and we may think that, we have invented so many things and we know so much about microscale processes. Our fabrication techniques of so wonderful, that we would be able to make almost anything we want with the property, that we desire and we can have fantastic applications out of them.

But just to bring you back to reality, I would like to show one picture and that would tell us, how far behind to nature? We are in terms of the design and fabrication of microfluidic devices.

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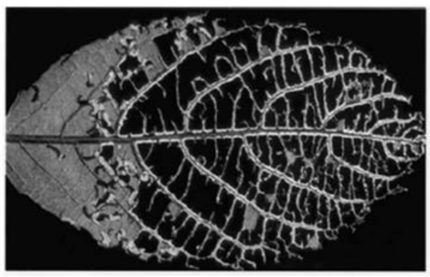

So, the next figure, if you see, I mean all of you can identify this. This is the picture of a dried leaf and if you look at some of the major channels, some of the major paths for the, to receive nutrients from the main tree. You would be amazed, we can never think of making a leaf with such intricate microfluidic channels carrying different fluids at all parts of the leaf, with the best technology that we have today. So, it is sometimes, it is good to know, where we stand in terms of nature. Thank you.